IDENTIFICATION OF DYNAMIC SYSTEMS

J.H BALBI - N. BALBI - G. GIROLAMI - P. ORENGA - G. SIMONNOT
Université de Corse - Laboratoire d'Hélioénergétique
Vignola - 20000 AJACCIO - France

Abstract. In this paper, the dominant approach to the modelling of physical systems is described : it uses local laws and powerful numerical tools. For linear problems, it leads to eigenvalues and eigenvectors, in a suitable functional space, from which it is possible to construct the response to any excitation using the Green's resolvant. This approach has led to important progress in engineering physics.

Nevertheless, a systemic approach is useful in physics and irreplaceable for living systems. This second way uses global laws of the phenomenon in addition with a dynamical identification of the system using some adequate experiments. We illustrate this method on the modelling of a solar plant, which is correctly represented by a simple ordinary differential equation.

INTRODUCTION

The 19^{th} century gave macroscopic physics its laws. The second half of the 20^{th} century gave it the tools necessary to apply these laws : power-ful computers, efficient algorithms and a rigorous mathematical basis.

Today's dominant approach to the modelling of physics systems, the local-numerical approach, uses these laws and tools, which has led to spectacular progress in all the fiels of engineering physics.

There is nevertheless a second and quite different approach, which could be called a systemic approach, it presents many advantages when applied to physics systems and is almost irreplaceable for the study of living systems (ecology, biology, economics, etc.).

This article presents these two approaches and proposes an example of the use of the systemic method applied to a solar energy system.

THE LOCAL-NUMERICAL MODELLING OF PHYSICS SYSTEMS

For all the systems of classical physics, a general method of problem solving, is used most of the time. It consists in writing the local equations describing the phenomenon, then in solving them numerically in the suitable space-time domain. This method is also applied to certain problems in biophysics such as circulation of the blood, Cherruault (1977) and marine ecosystems, Nihoul (1982). An outline of the method follows.

Definition of the system

First, it is usually necessary to describe the geometry of the structure, (domain and boundary), the phenomena which occur in it, the external actions, the state variables, and the internal parameters.

Local Equations

Physical phenomena are often described by partial differential equations with respect to state and space variables. For example, in structure mechanics, the problem can be represented by :

domain : $M[\ddot{y}(x, t)] + B[\dot{y}(x, t)] + K[y(x, t)] = F(x, t)$,
boundary : $l[y(s, t)] = f(s, t)$, (1)
where M, B, K, l are differential space operators, F and f express external actions and y expresses generalized displacements.

Rough Numerical Solution

As this point, most people using this method integrate these equations numerically by discretisation of space and time variables (finite differences, for example). This procedure has become a basic tool in physics and sometimes even replaces actual experimentations.

Eigen-model

For certain difficult and essentially linear problems, the approach can be quite different. An eigen-problem can be associated with the original one ; for example, in structure mechanics we can associate the free undamped system ($F = 0$, $f = 0$, $B = 0$) for wich harmonic vibrations are sought :

$$y(x, t) = u(x) \exp(j\omega t)$$

$$K[u(x)] - \omega^2 M[u(x)] = 0 \quad (2)$$

$$l[u(x)] = 0$$

The set of the solutions $[\omega_\nu, u_\nu(x)]$ constitutes the spectrum and the modal base of the system. From them, it is possible to obtain the response to any excitation (F, f), using Green's resolvent. These elements form a model of the system : the eigen-model.

Theoretical Resolution : Variational Formulation

Every time it is possible, it is very important to be able to give a theoretical solution. Which is a very good guide for a numerical solution. This can be done by replacing the local formulation (2) by a global one called variationnal :

$$a(u, v) - \omega^2 b(u, v) = 0 \qquad \forall\ v \in V \quad (3)$$

in wich V is a Hilbert's functionnal space and a and b are bilinear forms. In certain circumstances it is possible to establish the existence and the properties of the spectrum $[\omega_\nu]$ and of the modal base $[u_\nu]$, V being a space of infinite dimension, Doutroy (1984).

Numerical determination of the eigen-model

Basic functions associated to a descretisation of the space variables are chosen to facilitate numerical computations (triangle functions in the case of finite elements, Ciarlet (1978)).
These functions define a finite dimension sub-space for V ; the problem (3) is then reduced to finding the eigen-values of a matricial problem.

$$A_n u_n - \omega_n^2 B_n u_n = 0 \quad (4)$$

wich leads to a spectrum $[\omega_{n,\nu}]$ and a modal base $[u_{n,\nu}]$, $\nu = 1 \ldots N$, which constitutes an approximation of the base of the system described in equation (3).

Controlling the Model

Although these methods are very efficient it is nevertheless necessary to control the validity of the model. By changing certain parameters, it is possible to adjust the theoretical model to the reality of the experiments. This is in general

essential because either consciously or not many simplifications have been made to find a more or less simple model. In addition to this, the numerical computations can also bring about errors.
In conclusion, we can say that this general method, justified by rigorous theory and poweful algorithms, is a very good tool and has given many examples of success in technology, such as the design of planes using the finite elements method.

A SYSTEMIC APPROACH : A GLOBAL-EXPERIMENTAL MODELLING

Considering all this, it might seem that there is nothing to add. However, this approach has weak points and, thus limits. First of all, it cannot generally be applied to ecosystems, since most of these systems (economics, biology, ecology, etc) do not have universal local laws, but only global and empirical laws. And even in physics systems, the local-numerical method is not necessarily the only or the best way to tackle a problem. As soon as a complex structure is dealt with, the method becomes awkward and costly, and can only be used if it is profitable.

Therefore we propose a very different modelling which can be applied to a large class of systems in various fields : physics, chemistry, biology, economics, etc.

Principle of the Method

The method concerns systems which are essentially linear within their normal limits.

It determines a reduced model experimentally and at the same time, takes into account the global laws of the phenomenon.

Description of the Method

- Definition of the System

By observing reality, the experimenter specifies the limits and the variable necessary to define the system unambiguously.

The choice of the variables is one of the major difficulties encountered : how many are necessary ? How many sub-systems must be chosen ? Generally the answers to these questions can only be obtained by the analysis of the dynamic behavior.

For physics systems with feedback, it will be necessary to modelize the passive system obtained by doing away with the feedback, whenever possible. (This can also do away with major non-linearity).

- Structuring the System

Systems can be structured by a fundamental experiment. The parameters must be kept constant to make the system invariant. It is then excited, appropriately : for oscillating structures a harmonic excitation is widely used whereas for dissipative systems, a step is often chosen. Invariant linear sub-systems, their eigenvalues and therefore the number of necessary state variables, as well as the relations among the sub-systems, can be determined by the spectral analysis of the response, combined withs the knowledge of the phenomena.

- Identification of the System

This same experiment is performed again with a number of captors equal to

the number of state variables determined previously in order to measure the eigenvectors of the sub-systems. Then the experiment is repeated again a certain number of times in order to determine the relations between the coefficients used in the model and the physical parameters of the system.

Ordinary differential equations for each sub-system and functional relations among these sub-systems can then be derived.

$$\dot{x}_i = A_i(q_i)\, x_i + B_i\, \phi_i \quad (t)$$
$$f_i\,(x_1, x_2, \ldots x_i \ldots) = 0 \qquad (5)$$

where x_i are the state variables

q_i are the parameters of the system

f_i are the functional relations and

ϕ_i are the external excitations.

For every sub-system, the eigen problem is then

$$\dot{x}_i = A_i(q_i)\, x_i$$

Conclusion

The proposed method, like the local numerical method, leads to a reduced model, but much more directly ; it is also inexpensive and easy to use. In fact, its simplicity is an important point in its favor for its widespread use in various fields.

APPLICATIONS

Structure Mechanics

This method is widely used in structure mechanics where it is in competition with the finite element method, Fillod (1985). It is easily applied to this field, because of advanced phenomenological knowledge, and also because it is possible, using a sweep in the frequency of the excitation, to the precise eigenvalues of the reduced model find even in presence of some non-linearity in the phenomenon.

Eco-Systems

Applications to biology are beginning to appear under the name of "compartment models" Atkins (1969). They are less frequent in economics in spite of some recent uses, Aracil (1984), Pupion (1980).

Thermal System

This procedure has been adapted to a thermal system in the field of solar energy, Balbi (1986). This application is intersesting because of the contrast between the complexity of the system, and the simplicity of the model. The system is composed of an array of solar collectors, using 1200 m^2 of mirrors which focalize the sun on eight twenty meter long pipes. (fig. 1)

Let us now apply the above method to this particular system.

Fig. 1 - General view of the solar plant.

- Definition of the System

Like most artificial systems, this system is clearly defined. The extenal action is produced by the solar flux, the out put temperature can be used as a state variable. Other important parameters are :
the rate of flow of the fluid in the pipes (q)
the input temperature Te of the same fluid,
the ambient temperature, Ta.

The output temperature can be maintained constant by the action of a feedback loop on the rate of flow q, no matter what the value of the solar flux is.

Due to the propagation delays of the fluid, this kind of command is not very well suited to the system and must be replaced by a previsional command based on the system model.

Following what was said above the feedback loop was disconnected to maintain the rate of flow constant during the experiment. T_a and T_e were also constant.

- Structuring the Model

A solar flux step can be obtained by simultaneously defocalising the eight lines of collectors. The response of the array to this excitation is given in Fig. 2. The analysis of this curve shows that it is necessary and sufficient to consider two identical invariant linear subsystems. The output temperature of the array can then be defined by computing the half sum of the output temperatures of the two subsystems, each temperature being delayed differently, (τ_1, τ_2). Each subsystem is caracterised by two eigenvalues α and β Here, it is not necessary to use the notion of eigenvectors, and because the sub-systems are identical, it is possible to describe the system by just one differential equation of second order and not two. It is then necessary to include the delays τ_1, τ_2 in an equivalent solar flux ϕ^*.

The final modal can be written

$$\ddot{T} + (\alpha + \beta)\,\dot{T} + \alpha\beta\,(T - T_d) = a\,\phi^*(t)$$

$$\phi^*(t) = \frac{1}{2}\,(\phi(t - \tau_1) + \phi(t - \tau_2))$$

Where T is the output temperature,
T_d is this temperature when the array is not focalised,
a is an equivalent optical coefficient.

- Identification

The dependence of the coefficients (α, β, T_d, a, τ_1, τ_2) with respect to the physical parameters (q, T_e, T_a) is determined by performing several similar experiments with different sets of parameters.

The model can then provide a correct response even if the parameters are time-dependent.

The efficiency of the model is shown in Fig. 3, where the correspondance between the experimental and theoretical responses to a variable flux can be seen.

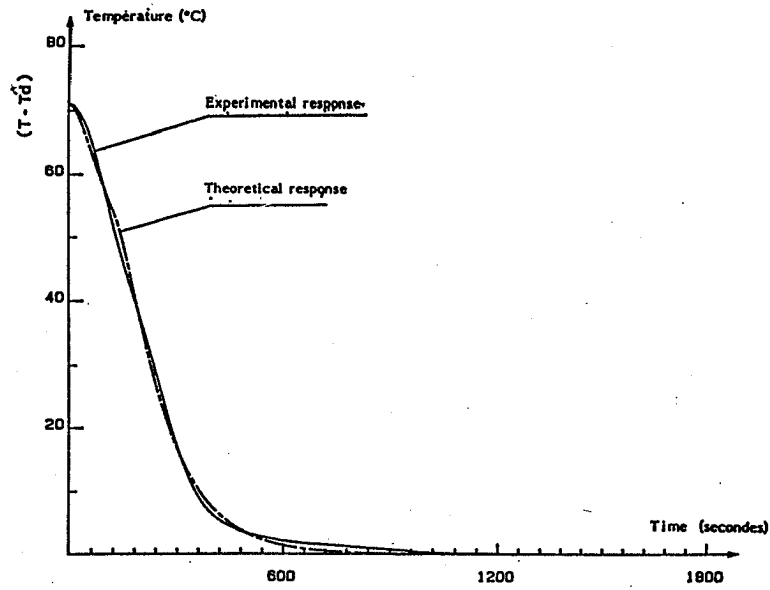

Fig 2 - Experimental and theoretical responses to a solar flux step.

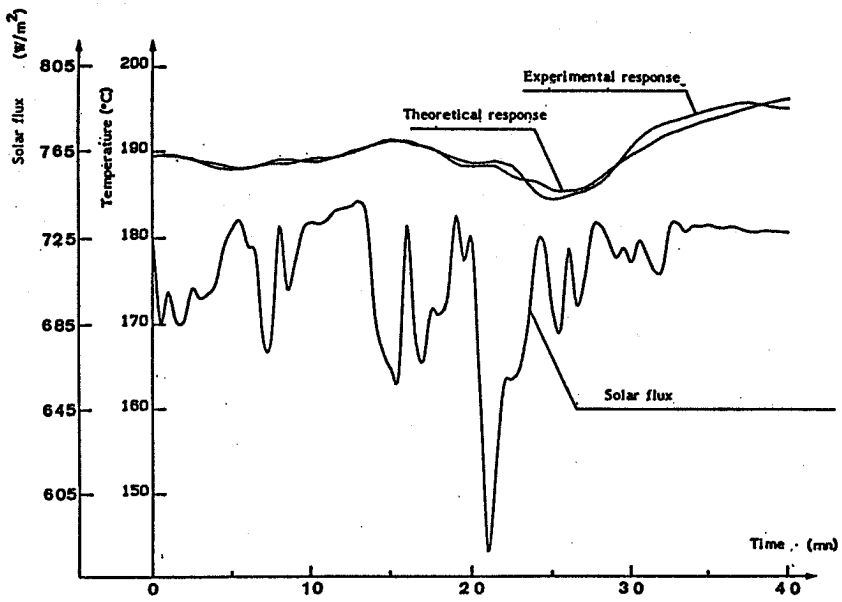

Fig 3 - Theoretical and experimental responses to a variable solar flux.

- Command

Due to the simplicity of the model, a real time previsional command can be considered. If it is necessary to maintain the output temperature cosntant : $T = T_o$. The equation $\alpha\ \beta(T_o - T_d) = a\ \phi^*(t)$ must hold, that is, if T_e and T_o are cosntant, the parameter q must be of the form :

$q = \psi(t)$, which is obtained by solving the implicit equation above ; this can be done in real time with the help of a microcomputer, thanks to the algebraic form of this equation.

CONCLUSION

The local-numerical method has many advantages, but the systemic approach can nevertheless be very useful for complex and inexpensive systems, particularly if automatic control is needed. All scientists should be familiar with it. Future research should attempt to extend it to essentially non-linear systems.

REFERENCES

ARACIL , J. (1984), Dynamique des Systèmes - Presses Universitaires de Lyon

ATKINS, (1969). Modèles à compartiments mutiples pour les systèmes biologiques Gauthiers - Villars - Paris.

BALBI, J.H., BALBI N., ORENGA, P., SIMONNOT, G.. Modélisation du champ de capteurs de la Centrale Solaire de Vignola - Revue Physique Appliquée, 21, 16.9.80 - Paris.

CHERRUAULT, Y., LORIDON, P., (1977). Modélisation et méthodes mathématiques en biomédecine - Masson, Paris.

CIARLET, P.G., (1978). The finite element method for elliptic problems. North-Holland, Amsterdam.

DAUTRAY, R., LIONS, J.L., (1984). Analyse mathématique et calcul numérique pour les sciences et les techniques, Tomes 1, 2 et 3. Masson, Paris.

FILLOD, R., LALLEMENT, G., PIRANDA, J. et RAYNAUD, J.L., (1985). Global method of modal identification. Third IMAC, 28-31, Janv. 85, Orlanda - Floride - U.S.A.

NIHOUL, J., RONDAY,F., (1982). Three-dimensional marine models for impact studies. Procedings of the ASCE. Cape Town, South.Africa, pp. 745-764.

PUPION, G., POULALION, G., (1980). Macro-Economie Vuibert - Paris.