Sensitivity Analysis Techniques for System Dynamics Models of Human Behavior

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Abstract

Human and social models are an important capability of system dynamics, and sensitivity analysis can be used to strengthen and better understand these models. To learn about which sensitivity analysis techniques are most suitable for models of human behavior, different promising methods were applied to an example system dynamics model, tested, and compared. The example model simulates cognitive, behavioral, and social processes and interactions, and involves substantial nonlinearity, uncertainty, and variability. Results showed that some sensitivity analysis methods create similar results, and can thus be considered redundant. However, other methods, such as global methods that consider interactions between inputs, can generate insight not gained from traditional methods.

INTRODUCTION

Human and social modeling has emerged as an important research due in part to its potential to improve decision-making in the presence of uncertainty. These models can be used to generate insight about human behavior, and can contribute to understanding of social systems, behavioral forecasting, and training, among other uses. These models can be built using various paradigms, including system dynamics, cognitive modeling, game theory, agent-based modeling, and others, or may use combinations of these techniques (NRC 2008).

The purpose of this work was to study which sensitivity analysis techniques are most applicable to models that simulate human behavior. Sensitivity analysis determines which model inputs have the largest impact on model response, and is a component of rigorous, data-based model validation. The results of sensitivity analysis can be used to strengthen a model and to understand its implications. Sensitivity analysis can be used to identify where valuable data collection resources should be directed to most effectively improve the model. It can be used to find leverage points where intervention into the system can have a substantial and robust effect on the results. Sensitivity analysis can also be used to understand model robustness and to find areas where a model can be simplified with minimal effect on outcomes.

System dynamics models of human behavior have inputs that are difficult to quantify and highly variable between people or groups. Furthermore, these models often simulate nonlinear, complex adaptive systems. This necessitates sensitivity analysis techniques that can deal with large variations in many model variables simultaneously, a challenge that has not yet been sufficiently explored (NRC 2008). The variability inherent in models of human behavior indicates that sensitivity analysis techniques designed to deal with the highly nonlinear nature of these models will be more effective than traditional techniques.

Various methods of sensitivity analysis are available. System dynamics modelers sometimes use one-at-a-time, exploratory methods or correlation coefficients over time. Engineering applications often use sampling based and metamodeling methods, among others. To learn about which sensitivity analysis techniques are most suitable for models of human behavior, different promising methods were applied to an example system dynamics model, tested, and compared. The example model simulates cognitive, behavioral, and social processes and interactions, and involves substantial nonlinearity, uncertainty, and variability. The results of this analysis are given below.

FOOD SUBSIDY MODEL

The example model used for this study is a system dynamics model that incorporates cognitive components. The model represents two cognitive, decision-making entities: the government, which makes policy decisions, and voters, whom the government aims to satisfy. This model, like many models of human behavior, involves substantial feedback and nonlinearity. Inputs to the model are highly uncertain (especially those involving cognitive processes) and highly variable (especially economic and social factors).

An overview of the food subsidy model structure is shown in figure 1. The population of the simulated society grows steadily over time. Food demand is based on population and the price elasticity of food. If the price of food grows too quickly voter satisfaction will decline, causing voters' support of the government to decline and protesting to increase. The government attempts to avoid this situation by implementing a food subsidy to artificially cap food prices. The government would like to use oil revenues to pay for the food subsidy. If these revenues are insufficient to cover the entire subsidy, the government must print money to pay for it. When the government prints money, inflation increases, which decreases voter satisfaction and thus further reduces voter support for the government and increases protests.

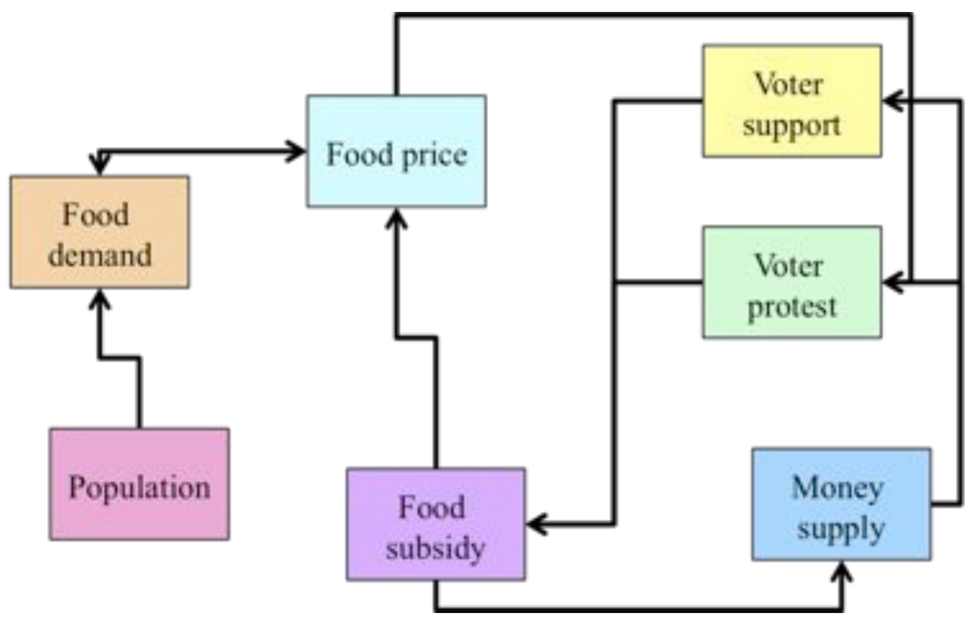

Figure 1. Overview of the model structure.

The complete structure of the food subsidy model is shown in figure 2. The model works as described above, but detail is included to specify how decisions are made and how non-cognitive variables are calculated. The model simulates behaviors based on utility functions and qualitative choice theory (Ben-Akiva and Lerman 1985; Train 1986). The voters in this model have three decisions to make. Their demand for food is based on the price of food. Voter protest is determined by the price of food and a general price index of goods in the society. Voter support is also based on food and general price indices, but also takes protesting activity into account. The government has just one decision to make in this model: where they would like to set food price, using the food subsidy. This decision is based on voter support and voter protest, and is aimed at keeping voter satisfaction with the government high.

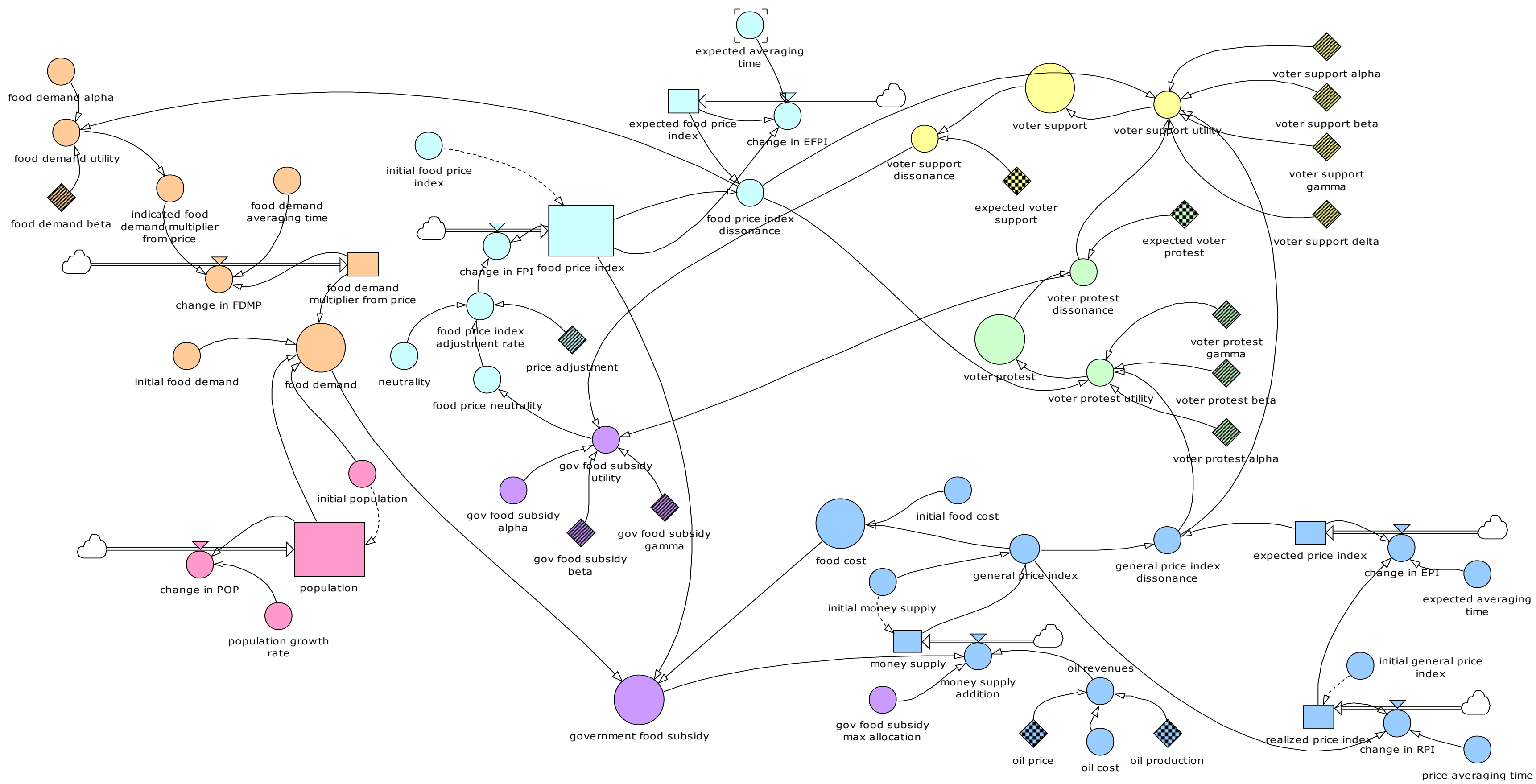

Figure 2. Complete structure of the food subsidy model.

There are 12 inputs to the food subsidy model that were considered uncertain for this sensitivity analysis. These inputs, as well as the distributions used in the analysis, are shown in table 1. The expected voter protest and support indicate levels that the government considers desirable. Oil price is defined by a log-normal distribution. The price adjustment describes the fraction of the indicated change in price that will actually occur. The remaining uncertain inputs are coefficients on utility functions. These inputs indicate the magnitude of the effect that a particular societal event or trend will have on a decision.

Table 1. Uncertain inputs to the food subsidy model.

Variable	Distribution	Details
Expected Voter Protest (EVP)	Uniform	[0.05,0.15]
Expected Voter Support (EVS)	Uniform	[0.6,0.8]
Oil Price (OP)	Log-normal	μ=4, σ=0.55
Price Adjustment (PA) - Fraction of indicated change in price	Uniform	[0.05,0.5]
Food Demand β (FDβ) - How much food price affects demand	Uniform	[0,10]
Government Food Subsidy β (GFSβ) - How much voter support affects GFS	Uniform	[0,5]
Government Food Subsidy γ (GFSγ) - How much voter protest affects GFS	Uniform	[0,10]
Voter protest β (VPβ) - How much food price affects protests	Uniform	[-10,0]
Voter protest γ (VPγ) - How much general prices affect protests	Uniform	[-10,-1]
Voter support β (VSβ) - How much food price affects support	Uniform	[0,10]
Voter support δ (VSδ) - How much protests affect support	Uniform	[0,10]
Voter support γ (VSγ) - How much general prices affect support	Uniform	[0,10]

Figure 3 shows the results of voter support for a 50-run Monte Carlo simulation of the food subsidy model. Each line in figure 3 represents a full time series for the output voter support for a simulation with random values for each of the 12 uncertain inputs in table 1. While each simulation exhibits a different pattern over time, there is a somewhat robust pattern that is shared between the simulations. At the beginning of each simulation, the government tries to gain support from voters by subsidizing food. This causes voter support to increase. However, the government has to print money to pay for the subsidy, which, after a time lag, causes inflation to increase. Thus, after an initial rise, voter support declines below its initial level.

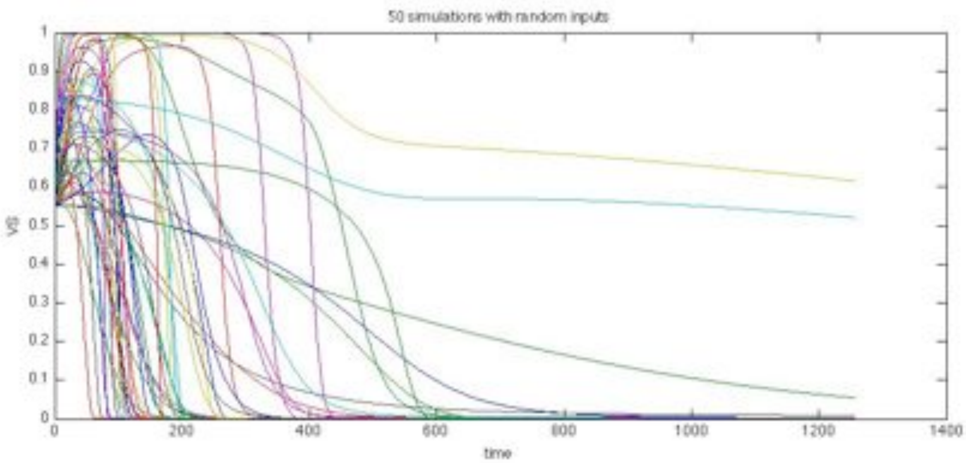

Figure 3. Voter support for Monte Carlo simulation (N=50).

3. COMPARISON OF METHODS

The goal of this project was to compare different sensitivity analysis techniques to gain insight into which methods are most appropriate for models of human behavior. Different promising methods were applied to the food subsidy model, tested, and compared. While different outputs are certainly of interest in this model, results presented here focus on one output: voter support. Each sensitivity analysis given here used a sample size of 1,000, except sensitivity indices for which N=10,000. Static and dynamic sensitivity were both considered. For the static analyses, the highest value of voter support over the time horizon was used as a metric. The dynamic analyses were based on voter support at each point throughout the time horizon.

The first sensitivity analysis technique considered for the food subsidy model was scatterplots. Scatterplots necessitate a static output metric, so the highest voter support metric was used as the output. Each uncertain input was plotted against this metric to look for patterns in the relationships between inputs and the output metric (figure 4). Scatterplots are especially useful in finding unusual or unanticipated patterns, such as thresholds and nonlinearities, in the data (Ford and Flynn 2005; Helton et al. 2006). Relationships to the output metric are apparent for some of the uncertain inputs to the food subsidy model, particularly EVS and GFSGamma (positive correlations), and VPBeta (negative correlation). However, none of the inputs are obviously dominant in determining the highest voter support.

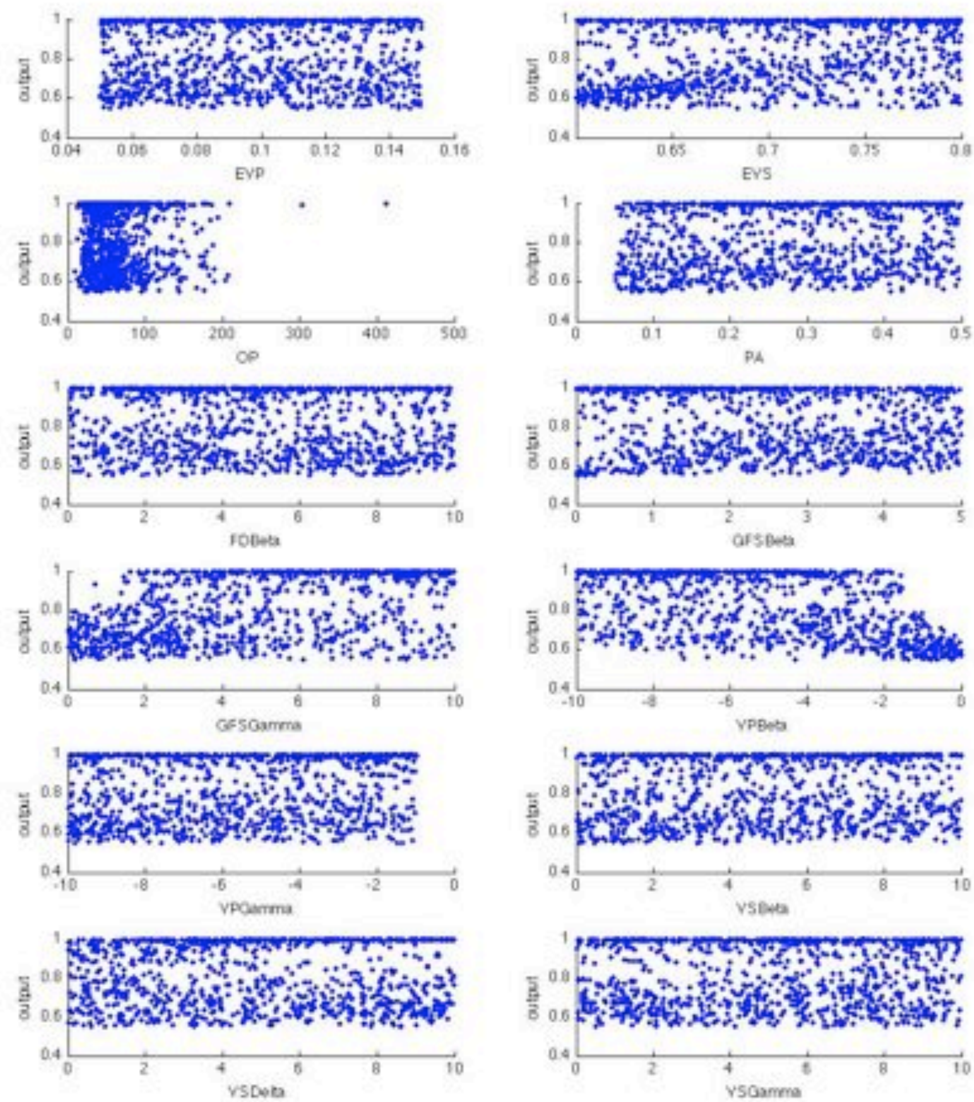

Figure 4. Scatterplots of different inputs compared to the highest voter support.

The next method considered was correlation coefficients (Helton et al. 2006). These are used to measure the strength of the linear relationship between each uncertain input and the output of interest. Correlation coefficients can be used to rank inputs by importance. Variations of correlation coefficients are also available. Partial correlation coefficients correct for the linear effects of other inputs. Rank correlation coefficients consider monotonic, rather than linear relationships between inputs and outputs. Partial rank correlation coefficients combine these qualities. Correlation coefficients vary from -1 to 1, with a stronger correlation indicated when the coefficient is farther from 0. The p-value associated with a correlation coefficient describes the significance of the correlation, with a small p-value (for instance, $p<0.05$) indicating high significance.

Two different ways of using correlation coefficients were considered for this analysis. The first was a static analysis, which looked at how each uncertain input to the food subsidy model correlates with the output metric highest voter support (table 2). According to the p-values for each of the different types of correlation coefficients, most of the uncertain inputs have a significant non-zero correlation with the output metric. Furthermore, the different methods agree on most of the top (most highly correlated) inputs.

Table 2. Correlation coefficients for highest voter support output metric.

	Correlation Coefficient	CC p-value	Partial Correlation Coefficient	PCC p-value	Rank Correlation Coefficient	RCC p-value	Partial Rank Correlation Coefficient	PRCC p-value
EVP	-0.012	0.716	-0.041	0.196	-0.019	0.541	-0.056	0.080
EVS	0.211	0.000	0.363	0	0.213	0.000	0.375	0
OP	-0.086	0.007	-0.046	0.152	-0.078	0.013	-0.073	0.022
PA	0.288	0	0.408	0	0.324	0	0.468	0
FDBeta	-0.022	0.496	-0.035	0.275	-0.014	0.649	-0.033	0.296
GFSBeta	-0.140	0.000	-0.161	0.000	-0.138	0.000	-0.165	0.000
GFSGamma	0.383	0	0.548	0	0.365	0	0.556	0
VPBeta	-0.519	0	-0.646	0	-0.528	0	-0.676	0
VPGamma	0.160	0.000	0.220	0.000	0.149	0.000	0.219	0.000
VSBeta	0.103	0.001	0.158	0.000	0.158	0.000	0.248	0.000
VSDelta	0.021	0.509	0.071	0.027	0.101	0.001	0.201	0.000
VSGamma	-0.009	0.789	-0.021	0.519	-0.021	0.500	-0.050	0.120

The second correlation coefficient analysis calculated correlation over time (Ford and Flynn 2005) for each uncertain input in relation to voter support. By plotting each type of correlation coefficient over time, the relative strength of correlations for different inputs during different times can be seen (figures 5-8). For most inputs, he different types of correlation coefficients were similar. In the beginning of the simulation, when voter support is increasing, a selection of inputs is apparently highly correlated with voter support. The collection of highly correlated inputs seems to change, however, when the behavior of voter support shifts to declining. One input, VSDelta, is highly negatively correlated toward the end of the simulation only in the rank and partial rank analyses, indicating a monotonic but not linear relationship between VSDelta and voter support.

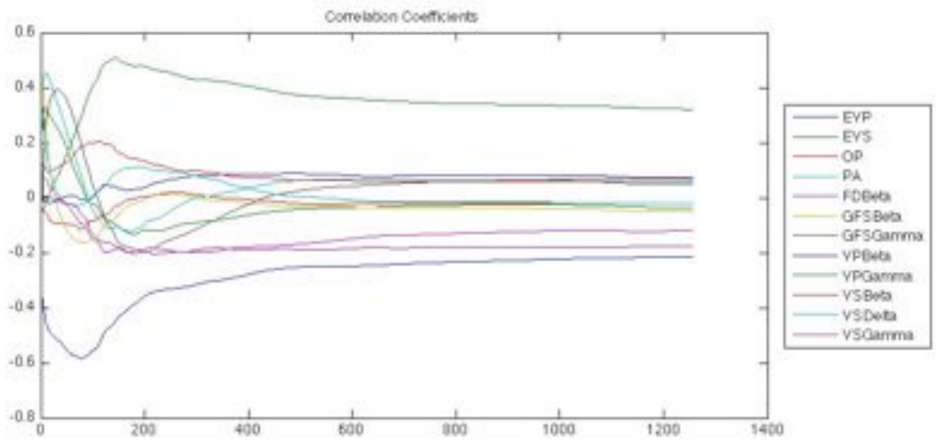

Figure 5. Correlation coefficients for voter support over time.

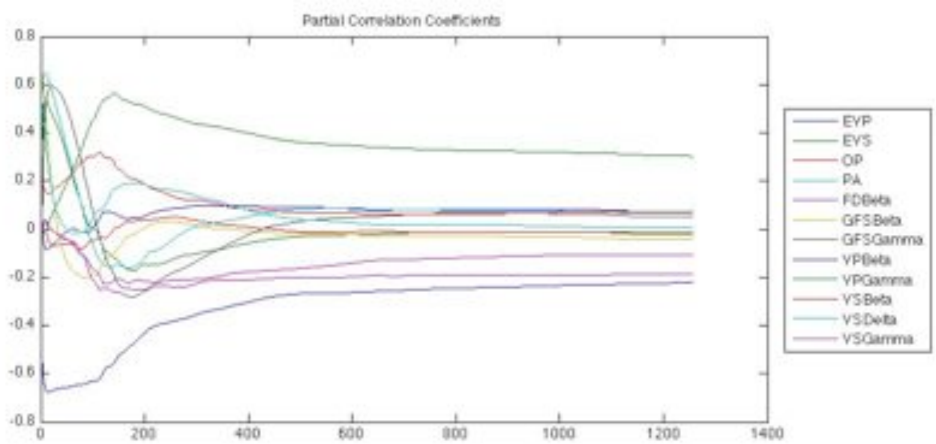

Figure 6. Partial correlation coefficients for voter support over time.

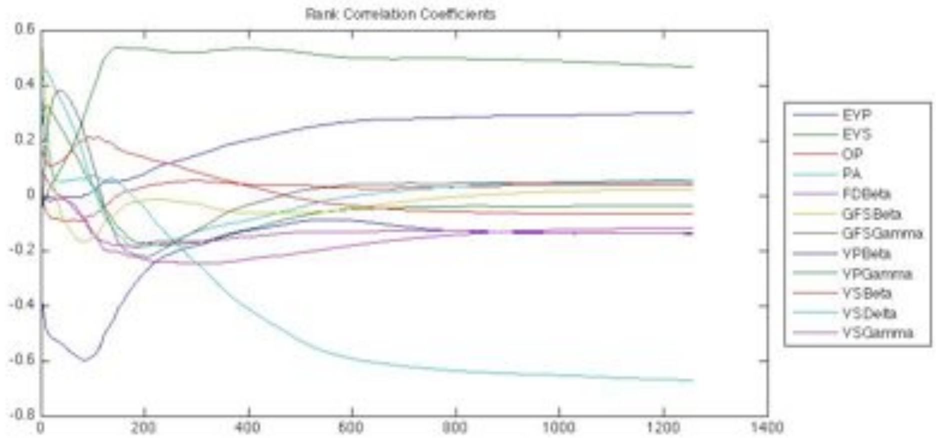

Figure 7. Rank correlation coefficients for voter support over time.

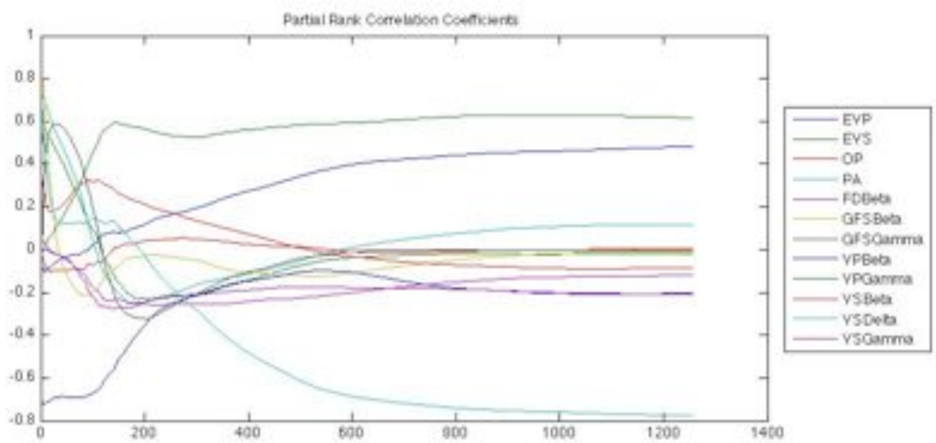

Figure 8. Partial rank correlation coefficients for voter support over time.

The next sensitivity analysis technique implemented was a stepwise regression (Helton et al. 2006) based on the highest voter support output metric. Stepwise regression creates a linear regression model by repeatedly adding the most important variable to the model. The process begins by determining for which input, by itself, would lead to the highest R^2 value. R^2 measures the fraction of output variance that is explained by the model. After the first, most important, input is found, the next most significant input is searched for and added to the model. This process goes on until the regression model would not be significantly improved by adding any of the remaining inputs.

Results for the stepwise regression of the food subsidy model and the output metric highest voter support are shown in table 3. Nine of the uncertain inputs were useful in the regression model, with only three considered insignificant. Even with all nine of the useful inputs included, the R2 value is still only 0.6134.

Table 3. Stepwise regression for highest voter support output metric.

Step	Variable	Regression Coefficient	R squared
1	GFSGamma	0.0254	0.1998
2	EVS	0.6017	0.2605
3	VPBeta	-0.0289	0.5314
4	PA	0.2663	0.5743
5	VPGamma	0.0074	0.5907
6	GFSBeta	-0.0103	0.6001
7	VSBeta	0.0050	0.6080
8	VSDelta	0.0032	0.6113
9	EVP	-0.2453	0.6134

The next sensitivity analysis method implemented for the food subsidy model was the elementary effects method (Saltelli et al. 2008). This method looks at the average difference in output when one input is perturbed. This is similar to a derivative. The process involves perturbing one input repeatedly in different locations within its domain, each time measuring the associated change in the output. Elementary effects results for the highest voter support output metric are shown in table 4. μ is the average of the changes in output after perturbing the input. μ^* is the average of the absolute value of these changes. σ^2 is the variance. Since derivatives will necessarily be larger during times when the output is exhibiting larger changes, elementary effects over time are not comparable for this model, and therefore not included here.

Table 4. Elementary effects results for highest voter support output metric.

	μ	μ^*	σ^2
EVP	0.0091	0.0915	0.0002
EVS	-0.0077	0.0855	0.0006
OP	-0.0038	0.0809	0.0001
PA	-0.0003	0.0987	0.0014
FDBeta	0.0005	0.0916	0.0001
GFSBeta	-0.0031	0.0865	0.0002
GFSGamma	-0.0003	0.1127	0.0006
VPBeta	0.0071	0.1028	0.0006
VPGamma	0.0073	0.0915	0.0012
VSBeta	-0.0030	0.0870	0.0009
VSDelta	0.0080	0.1008	0.0003
VSGamma	-0.0054	0.0927	0.0006

The final method considered in this study was sensitivity indices (Saltelli et al. 2008; Weirs et al. 2010). Two measures result from this type of analysis. The first is the main effect, S_i, which describes the proportion of the variance in the output of interest that can be attributed to variation in a particular input. The second measure is the total effects index, S_{Ti}. This describes the proportion of the variance of the output of interest that can be attributed not only to one particular input, but also to all of the interactions that input has with other inputs. These measures can be used to indicate where reduced uncertainty in inputs would allow the output variance to be reduced.

The sensitivity indices for the static metric, highest voter support, are shown in table 5. Sensitivity indices over time in relation to voter support are shown in figures 9 and 10. The main effects plot shows that a few inputs emerge as very important in determining output variance. The total effects results show that interactions between inputs are very important in this model.

Table 5. Sensitivity indices for output metric highest voter support.

	SI	STI
EVP	-0.0001	0.0005
EVS	0.0621	0.1210
OP	0.0000	0.0000
PA	0.0633	0.1247
FDBeta	0.0013	0.0013
GFSBeta	0.0112	0.1110
GFSGamma	0.2255	0.3628
VPBeta	0.3773	0.5154
VPGamma	0.0097	0.0381
VSBeta	0.0146	0.0577
VSDelta	0.0154	0.1142
VSGamma	0.0008	0.0107

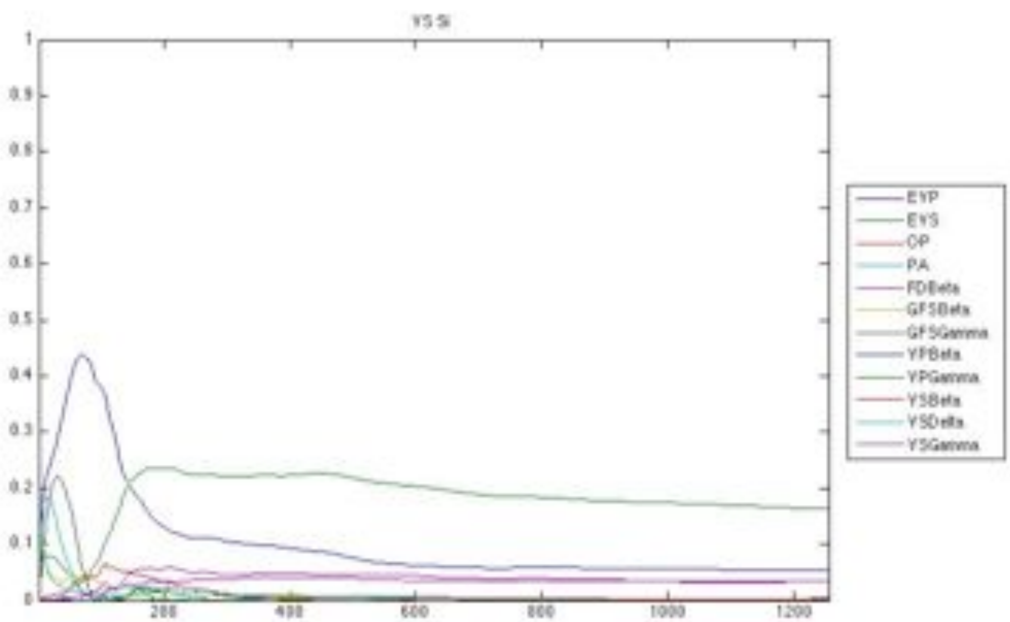

Figure 9. Main effects over time for voter support output.

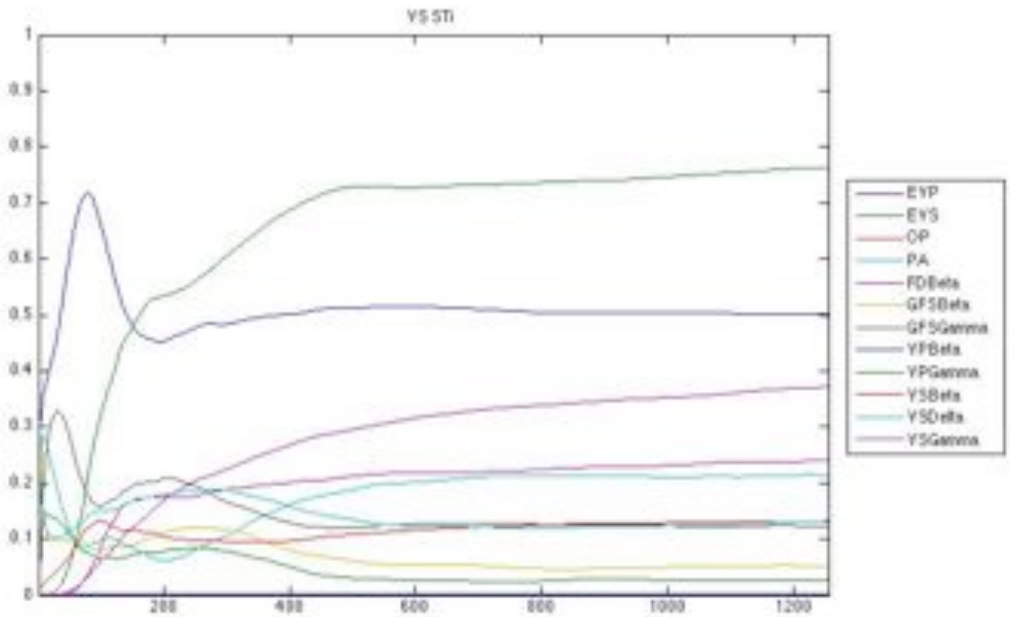

Figure 10. Total effects over time for voter support output.

CONCLUSIONS

Table 6 shows how each of the above methods of sensitivity analysis ranks the uncertain variables in the food subsidy system dynamics model in order of importance. The different types of static correlation coefficients give similar results, at least for the most important inputs. However, the dynamic analysis of different types of correlation coefficients (figures 5-8) indicates that one of the inputs, VSDelta, has a strong monotonic but nonlinear correlation with voter support in the second half of the time horizon. Stepwise regression gives similar results to the static correlation coefficient methods. The elementary effects and sensitivity index rankings differ from the others somewhat.

Table 6. Comparison of importance rankings of uncertain variables.

	Correlation Coefficient	Partial Correlation Coefficient	Rank Correlation Coefficient	Partial Rank Correlation Coefficient	Stepwise Regression	Elementary Effects Mustar	Sensitivity Indices Si	Sensitivity Indices STi
EVS	1-4	1-7	1-4	1-8	2	11	4	4
PA	1-4	1-7	1-4	1-8	4	4	3	3
GFSGamma	1-4	1-7	1-4	1-8	1	1	2	2
VPBeta	1-4	1-7	1-4	1-8	3	2	1	1
VPGamma	5	1-7	7	1-8	5	8	8	8
VSBeta	6	1-7	5	1-8	7	9	6	7
GFSBeta	7	1-7	8	1-8	6	10	7	6
EVP	8	11	9	12	9	7	12	11
FDBeta	9	8	11	9	X	6	9	10
OP	10	10	12	11	X	12	11	12
VSGamma	11	12	10	10	X	5	10	9
VSDelta	12	9	6	1-8	8	3	5	5

Table 7 gives an overview of the different methods of sensitivity analysis described here, as well as some of the main differences and conclusions found in the sensitivity analysis of the food subsidy model. Scatterplots showed apparent correlations for only a few of the uncertain inputs, and no unusual relationships were obvious. Correlation coefficients showed that many of the uncertain inputs were significantly correlated with the static output metric. The different methods of calculation did not result in very different coefficients for the static analysis, but understanding of one uncertain input benefited from calculation of rank correlation coefficients in the dynamic analysis. The stepwise regression analysis gave similar results to the correlation coefficient analysis, and thus was probably an unnecessary calculation for the food subsidy model. The elementary effects analysis showed that many of the inputs were similar in importance. The sensitivity indices showed that interactions between inputs were significant, especially during the beginning of the simulation.

Table 7. Comparison and implications for different methods of sensitivity analysis.

Method	What is measured	Comparison and implications
Scatterplots	Subjective relationship between inputs and outputs	•Good first method for identifying patterns •No very obvious patterns
Correlation Coefficients	Strength of linear (or monotonic) relationship	•Useful in ranking inputs •Static results similar for different types of CC, dynamic results differed for one input
Stepwise Regression	Coefficients for linear model that best predicts output	•Most inputs were significant •Results similar to correlation coefficients
Elementary Effects	Average derivative when one input is perturbed over different points in its domain	•Little variation in μ^* between inputs
Sensitivity Indices	Proportion of output variance attributed to input variance	•Interactions were significant •Especially important at the beginning of the simulation

No one, or few, inputs dominated the results of the food subsidy model. Interactions between inputs, however, did play a large role, particularly at the beginning of the simulation. It is important to note that the results above apply to only the food subsidy model. More investigation, into different models, different output metrics, and different techniques of sensitivity analysis, is needed to determine if these results apply more generally to models of human behavior.

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