DOMINANT STRUCTURE ANALYSIS: ANOTHER APPROACH[1]

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ABSTRACT

In this paper, we put forth another approach to the dominant structure analysis, which we call parameter elasticity method. This method is based on the law of differentiability of solution with respect to parameter. It concerns itself with the dominant structure that contributes most to a particular behavior or behavior change. By applying this law, we develop the new method and new quantitative indexes to determine the dominant structure of a system.

This new method has some advantages. One is that it can be applied directly to nonlinear systems without linearization. Another is that it can be accomplished within DYNAMO. In some condition, it may be a good guide to model simplification.

INTRODUCTION

The dominant structure of a system dynamics model is usually understood as the key structure, that is responsible for a particular behavior of the model.

There are three existing approaches available to determine the dominant structure of dynamic systems. One is known as the experimental approach, which depends on repetitive simulations, and thus is quite time-consuming for complex systems. Another is the frequency response approach, which is applicable to linear systems. Up to now, there is no paper being seen that tries to apply this mehtod to nonlinear systems in the system dynamics community. The third one is the eigenvalue approach, which is also applicable to linear systems. Though it may also be applied indirectly to nonlinear systems by linearizing different operation points of the systems, however, because the operation point of nonlinear systems may change with time,

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this approach, therefore, requires a lot of effort and may seem not easy to handle for some system dynamics practitioners. Here in this paper, we put forth another method to analyze dominant structure. By applying the law of differentiability of solution with respect to parameter, the new method involves parameter sensitivity analysis of model behavior. The method can be accomplished with DYNAMO. Therefore it is easy to handle. And it also has some other useful application.

DIFFERENTIABILITY OF SOLUTION WITH RESPECT TO PARAMETER

Our method is based on the law of differentiability of solution with respect to parameter. Let us first introduce the law concisely in the following.
For an initial value problem

$$x'=f(t,x;\mu) \quad (1)$$
$$x(t_0,\mu)=x_0 \quad (2)$$

where $t\in R$, $x\in R^n$, $\mu=(\mu_1,\ldots,\mu_m)^T\in R^m$, and x' is the first derivative of x with respect to t, if function $f(t,x;\mu)$ is continuous in the closed region G_μ, and $\partial f_i/\partial x_j$ and $\partial f_i/\partial \mu_k$ are also continuous in G_μ, then there is continuous partial derivative of solution $x=\emptyset(t,\mu)$ with respect to μ_k $(k=1,\ldots,m)$ and the partial derivative $\partial\emptyset(t;\mu)/\partial\mu_k$ $(k=1,\ldots,m)$ satisfies the linear differential equation

$$z'=f_x(t,\emptyset(t;\mu);\mu)z+f_{\mu k}(t,\emptyset(t;\mu);\mu) \quad (3)$$

and its initial condition

$$z(t_0;\mu)=0 \quad (4)$$

where

$$f_x(t,x;\mu)=[\partial f_i(t,x;\mu)/\partial x_j]_{n\times n} \quad (5)$$
$$f_{\mu k}(t,x;\mu)=(\partial f_1(t,x;\mu)/\partial\mu_k,\ldots,\partial f_n(t,x;\mu)/\partial\mu_k)^T \quad (6)$$
$$G_\mu=\{(t,x,\mu)\mid |t-t_0|=<a,\ ||x-x_0||=<b,\ ||\mu-\mu_0||=<c\} \quad (7)$$

and z is also a vector, z' is the first derivative of z with respect to t, f_x is a square matrix.
A system dynamics model can usually be reduced to the form represented by equations (1) and (2). x' is the rate variable vector, x is the level variable vector, and μ is the parameter vector. These parameters determine the structure of the system. When a parameter changes, its effect on level variables can be calculated with equations (3) and (4).
Because

$$x=\emptyset(t;\mu) \tag{8}$$

therefore,

$$\partial x/\partial\mu_k=\partial\emptyset(t;\mu)/\partial\mu_k=z(t;\mu) \tag{9}$$

By integration, we can get z value from equations (3) and (4).

DEFINITION AND INDEX OF THE DOMINANT LOOP

The dominant loops are those that contribute most to a specific system behavior and have the strongest influence on system behavior when something in them changes. We may also say that the dominant loops are those that contribute most to a specific behavior change when something in the system changes. In the following, we use the parameter elasticity and design an index which is consistent with this definition and both can be used in dominant loop analysis.
In order to analyze the dominant loops with the above law, we define the elasticity of solution with respect to parameter as the following

$$E_{\mu k}=(\partial x_1/\partial\mu_k\cdot\mu_k/x_1,\ldots,\partial x_n/\partial\mu_k\cdot\mu_k/x_n) \tag{10}$$

or $E_{\mu k}=(EX1_{\mu k},\ldots,EXn_{\mu k})$

This elasticity vector measures the percentage change in solution as a result of percentage change in a parameter. The solution is a function of time, and so does the elasticity. Therefore, at a specific point of time, the value of the elasticity measures the percentage change in solution value at that point of time caused by the percentage change of a parameter.
It is our basic idea to analyze the dominant structure with the assistance of the elasticities of parameters. If the elasticity of a parameter is equal to zero, then the parameter must has no contribution both to the existing behavior and to any behavior change in system. On the other hand, if the elasticity of a parameter is large in absolute value, it means that the parameter change has strong influence on the existing system behavior and because of this characteristics we infer that the parameter also takes an important position in the solution that determines the existing behavior. The loops are the candidate dominant loops in which some of their parameters elasticities are the greatest in all system parameters. Identifying those parameters that have largest elasticities in absolute value then becomes the first step to analyze the dominant structure.

Because the elasticities change with time, if we want to compare different parameter elasticities, we have to take into consideration their behaviors in the entire period of time we are interested. One way to do so is to to divide the whole time interval into n equal short intervals and observe their values at the n points of time. If we add the absolute values of a parameter elasticity at the n points of time and divide the sum by n, we get the average absolute elasticity. We will use this value as an index to identify the important parameters. We use $AE_{\mu k}$ to denote the average absolute elasticity of parameter μ_k.

$$AE_{\mu k}=(AEX1_{\mu k},\ldots,AEXn_{\mu k}) \qquad (11)$$

$$AEX1=(\sum_{i}^{n}|\partial \emptyset_1(i;\mu)/\partial\mu_k \cdot \mu_k/\emptyset_1(i;\mu)|)/n$$

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$$AEXn=(\sum_{i}^{n}|\partial \emptyset_n(i;\mu)/\partial\mu_k \cdot \mu_k/\emptyset_n(i;\mu)|)/n \qquad (12)$$

Another more accurate way to compare different parameter elasticities is to depict the elasticities on a figure and compare their absolute values.

PARAMETER ELASTICITY AND LOOP POLARITY

Through examining and analyzing some nonlinear systems with the parameter elasticities, we get the following important and useful rules.

Rule 1.

If a parameter elasticity is the largest of all the parameters in a system, then the dominant loop must go through that parameter.

Rule 2.

If the sum of all parameters' elasticities of a certain level variable is larger than zero, then the level must increase. If the sum is less than zero, then the level must decrease.

Rule 3.

If the above sum is equal to zero, then the level must reach its extreme point.

Rule 4.

If a parameter is a multiplier and its elasticity of a level variable is positive, then its effect on the level variable is positive and the loop that connects the parameter and the level is positive. If the elasticity is negative, then its effect is negative and the loop is also negative.

Rule 5.

If a parameter is divider and its elasticity of a level variable is positive, then its effect on the level is negative and the loop that connects the parameter and the level is negative. If its elasticity is negative, then its effect is positive and the loop is also positive.

Rule 6.

If in a positive loop, a multiplying parameter elasticity is negative, then there must be another negative loop which shares the same level with that positive loop. And this negative loop must produce a stronger negative effect on the level, when and because the multiplying parameter exerts a positive force on the same level, therefore, as a result of the two, causes decrease in the level variable.

Rule 7.

If in a negative loop, a multiplying parameter elasticity is positive, then there must be another negative loop which shares the same level with the former negative loop. And this negative loop must produce a stronger reversing effect on the level, when and because the multiplying parameter exerts a negative force on the same level, therefore, as a result of the two, causes increase in the level variable.

Rule 8.

Suppose there are two negative loops, loop 1 and loop 2, do not share a same level variable, and loop 1 has its own level 1 and parameter 1 (abbreviated as L1 and P1), loop 2 has its own level 2 and parameter 2 (L2 and P2), if P2 elasticity of L1 is positive, then P2 has an indirect positive effect on L1 and these two loops are in a larger loop and are connected by positive links.

Rule 9.

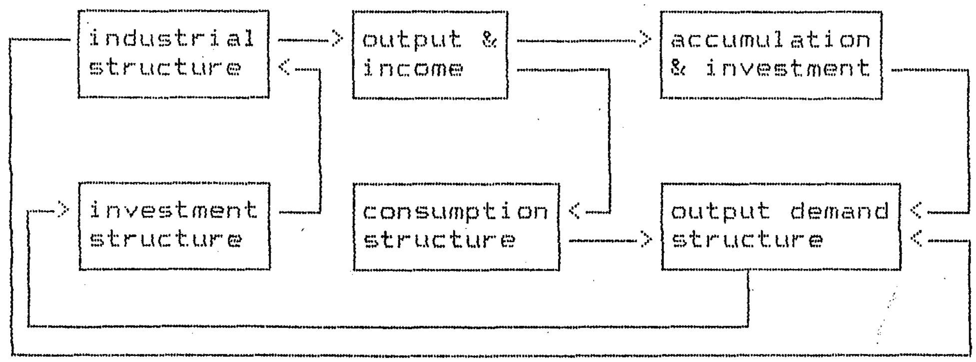
Suppose there are two loops which do not share a same level variable, one loop is positive and has L1 and P1, another is negative and has L2 and P2, if P2 elasticity of L1 is positive, then P2 has an indirect negative effect on L1 and these two loops are in a larger loop and are connected by positive links.

These rules are very useful in analyzing system behavior and loop importance.

APPLICATION IN LOOP ANALYSIS

We apply this method to analyze the national industrial structure model of China. We first simplified the industrial model into two sector model, the capital and consumption goods production sectors. The model has four level, i.e., capital stocks in capital and goods production sectors, capital stock under construction in capital and goods

production sectors, and has eleven parameters and more than thirty loops.
The general structure of the model is shown in the following figure.



Because the dominant loops have these parameters which have the largest elasticities in absolute value, therefore,we first identify a few parameters which have larger average absolute elasticities of the four levels and then depict these parameters elasticities of each level. Then list these parameters in the following.

LEVEL NAME	DOMINANT PARAMETER NAME
capital in capital goods industry	COIH: capital output/input ratio in capital goods industry
capital in consumption goods industry	COIH
capital under construction in capital goods industry	COIH
capital under construction in consumption goods industry	COIH ALK : average life of capital

By analyzing all these parameters' elasticities, we find that they have positive effect on the four levels and keep they increasing continuously. The parameter COIH is the most important. It exerts a dominant positive force both on the capital in capital goods industry and that in the consumption goods industry through increasing the production in capital goods, thus decreasing the delivery delay of capital goods and finally increasing the two capital

stocks. As a divider, it exerts a dominant positive force on capital under construction in capital goods industry through increasing desired capital in capital goods industry, then its desired correction of capital from production capacity, its desired total capital order, then increasing the accumulation rate, the total capital order, then increasing the .capital under construction in capital goods industry, its capital stock, production capacity, total output, and then increasing further demand and desired demand of its production and then its desired capital. This parameter exerts a dominant positive force on the capital under construction in consumption goods industry in a similar way, however, in about the tenth year of simulation, parameter ALK replaces its dominance. As a divider, it exerts positive force on the capital under construction through increasing the capital stocks in both sectors, the total output, the demand and the desired production in both sectors, then desired corrections of capitals, total desired capital order, accumulation, then capital order in consumption goods industry, and its capital stock again.
Through examining the simulation result, we find that the accumulation rate and accumulation keep increasing. The desired correction of capital in capital goods sector from its production capacity also keeps increasing and makes an about eighty percent contribution to the increase of accumulation. These results are consistent with our dominant loop analysis result.

APPLICATION IN BEHAVIOR AND POLICY EFFECT ANALYSIS

By applying the above rules, we can possibly do the following analysis.
The first. By applying rules 1, 2 and 3, we can evaluate each parameter importance and its related loops importance, determine their polarities, analyze individual contribution of each parameter and its related loops to a specific behavior, and explain the behavior.
The second. When candidate dominant loops are identified according to their related parameters' elasticities, we can possibly identify the dominant loops and shift in dominant loops by depicting and analyzing these parameters' elasticities.
The third. By depicting and analyzing parameters' elasticities, and with the assistance of rules 6, 7, 8, and 9, we can analyze the relation and interaction among different loops and get some insights into the system behavior.
The fourth. Because a system behavior at a specific point of time is determined by its parameters, we can use parameter

elasticities to estimate the effect of small changes in policy parameters and its contribution of each policy parameter by applying the following formula.

$$dx=\sum_{i} \partial x/\partial \mu_i \cdot d\mu_i \tag{13}$$

Where x is the level variable, and μ_i $(i=1,\dots,n)$ is the policy parameter.

APPLICATION IN POLICY SELECTION

We can select such two policy parameters and their percentage change relation that their effect on one level variable is minimized and on the other one is maximized by applying the above differential formula. Suppose we have two level variable x_1 and x_2, and two policy parameters. If the percentage change of two parameters are proportional, i.e.,

$$d\mu_2/\mu_2=\sigma\cdot(d\mu_1/\mu_1)=\sigma\cdot(d\mu/\mu) \tag{14}$$

and the other parameters percentage changes are zero, then the two parameters compound elasticities of the two levels can be expressed as

$$(dx_1/x_1)/(d\mu/\mu)=(\partial x_1/\partial \mu_1)(\mu_1/x_1)+\sigma\cdot(\partial x_1/\partial \mu_2)(\mu_2/x_1) \tag{15}$$
$$(dx_2/x_2)/(d\mu/\mu)=(\partial x_2/\partial \mu_1)(\mu_1/x_2)+\sigma\cdot(\partial x_2/\partial \mu_2)(\mu_2/x_2) \tag{16}$$

let $E_{11}=(\partial x_1/\partial \mu_1)(\mu_1/x_1)$ (17)

$E_{12}=(\partial x_1/\partial \mu_2)(\mu_2/x_1)$ (18)

$E_{21}=(\partial x_2/\partial \mu_1)(\mu_1/x_2)$ (19)

$E_{22}=(\partial x_2/\partial \mu_2)(\mu_2/x_2)$ (20)

If we want to evaluate the effect of the parameters change, we can evenly divide the simulation time into n short intervals and sum up the squared or absolute values of the compound elasticities at the n points of time.

$$\text{Let } J=\sum_{0}^{n}(E_{11}+\sigma E_{12})^2-\sum_{0}^{n}(E_{21}+\sigma E_{22})^2 \tag{21}$$

$$a=\sum_{0}^{n}(E_{12})^2-\sum_{0}^{n}(E_{22})^2 \tag{22}$$

$$b=\sum_{0}^{n}E_{11}E_{12}-\sum_{0}^{n}E_{21}E_{22} \tag{23}$$

$$c=\sum_{0}^{n}(E_{11})^2-\sum_{0}^{n}(E_{21})^2 \quad (24)$$

then we have

$$J=a\sigma^2+2b\sigma+c \quad (25)$$

Function J reaches extreme point when $\sigma=-b/a$. When $a>0$, it is a minimum point. Such parameter change produces minimum effect on x_1 and maximum effect on x_2. When $a<0$, it is a maximum point, and the effects are just opposite.

APPLICATION IN MODEL SIMPLIFICATION

If a parameter elasticity of a level variable is equal to zero, then the parameter has no influence on the level. This does not mean that the parameter and the link it represents can be eliminated from the model. Because change in parameter value may also change its elasticity. However, if a parameter elasticity is always equal to zero or approximately equal to zero and much less than the other parameters' elasticities in average absolute value when its value changes, then the link represented by it can be eliminated.

CONCLUSION

By applying the law of differentiability of solution with respect to parameter, we can get the parameter elasticity through integration. Thus we can calculate and depict parameter elasticities within DYNAMO. In order to guarantee the accuracy of calculation, Professional DYNAMO is preferred.

We have identified the relations among parameter elasticity value, its influence on levels, and its loop polarity. The 9 rules about these relations are useful in analyzing system behavior and identifying dominant loops.

Application and some possible application are discussed. We hold that more effort to apply the method will help to test it and possibly make further improvement to it.

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