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CRYSTAL DEVELOPMENT AND GLACIATION OF A SUPERCOOLED CLOUD

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RÉSUMÉ

Un modèle numérique simple a été développé pour évaluer la croissance des cristaux de glace et le taux de glaciation des nuages surfondus en fonction de la vitesse du courant ascendant et de la concentration en noyaux glaçogènes. Les conditions choisies représentent au mieux les tempêtes de neige des Grands Lacs. Les données indiquent que, dans des circonstances typiques (courants ascendants $\leqslant 3$ *m/s), le givrage devient le mécanisme dominant de la croissance du cristal après seulement quelques minutes, et avec des concentrations en noyaux glaçogènes inférieures à quelques dizaines par litre, de la neige roulée devant se former par la suite. Avec des concentrations en noyaux dépassant 50 à 100 par litre, les nuages surfondus se congèlent rapidement (en moins de 15 minutes) avant qu'un givrage effectif puisse se produire. Alors qu'on pourrait ensuite s'attendre à des cristaux individuels qui grossiraient uniquement par diffusion de vapeur d'eau, les concentrations élevées en cristaux conduisent à une agrégation prononcée de flocons de neige. Le fait que des agrégats de flocons de neige s'observent dans la plupart des tempêtes d'hiver suggère l'existence de concentrations en cristaux bien supérieures à celles indiquées par les mesures de noyaux en chambre froide, et probablement l'existence de zones d'accumulation de cristaux dans les nuages. Il s'ensuit certaines implications concernant l'ensemencement des nuages.*

ABSTRACT

An unsophisticated numerical model was developed to estimate the growth of ice crystals and the glaciation rate of supercooled clouds as a function of updraft speed and ice nucleus concentration. The conditions chosen best represent Great Lakes snowstorms. The data indicate that under typical circumstances (updrafts $\leqslant 3$ *m/sec) riming becomes the dominant crystal growth mechanism after only a few minutes time, and with ice nucleus concentrations less than a few tens per liter, graupel will subsequently form. With nucleus concentrations in excess of 50—100* l^{-1}*, the supercooled clouds rapidly glaciate (< 15 minutes time) before substantial riming can take place. While one might then expect individual crystals that grow strictly by diffusion of water vapor, the high crystal concentrations lead to pronounced snowflake aggregation. The common occurrence of snowflake aggregates in most winter snowstorms suggest crystal concentrations well in excess of that indicated by cold-box nucleus measurements, and probably preferred cloud regions of high crystal congregation. Certain cloud seeding implications readily follow.*

I. — Introduction.

The mathematical modeling of a mixed-phase cold cloud has not received attention comparable to that of warm clouds, although some partial formulations have been reported (e.g., Houghton [1], Jiusto [2], Podzimek [3], Cotton [4]). Adding to the complexity of these clouds are such factors as varied crystal geometry, associated variations in vapor diffusion fields and crystal fall velocity, and the uncertainties of collision relationships in crystal-droplet interactions (riming) and crystal-crystal interactions (snowflake aggregation). Yet such mixed-phase clouds probably represent the most general type at mid-latitudes, and certainly are basic to a number of weather modification concepts.

Often there are insufficient ice nuclei present to glaciate supercooled clouds. This is especially true of highly convective systems where the supply rate of water vapor can exceed the extraction rate by deposition onto available ice crystals. There is a critical ice-to-water ratio beyond which the cloud will completely glaciate in a given period of time. This ratio, which varies with cloud updraft intensity, temperature, and crystal size and type, is important in studying the development of natural and seeded clouds.

Twomey [5] and Todd [6] have expressed the critical ice concentration for complete glaciation of a cloud in terms of fixed values of liquid-water content. A general expression for the critical concentration was obtained (Jiusto [2], [7]) that accounted for the variable generation rate of water vapor in a cloud and the rate of extraction of vapor onto crystals of given type and size. The results obtained were useful in evaluating the relative importance of the three mechanisms of ice crystal growth — diffusion, accretion of drops (riming), and snowflake aggregation — during the evolution of a mixed cloud.

The key expression in this derivation was the standard equation for supersaturation change with time, dS/dt, in a cloud :

$$dS/dt = \varphi_1 u - \varphi_2 \, dm/dt, \tag{1}$$

where u is updraft intensity, dm/dt is condensation rate, and φ_1 and φ_2 are thermodynamic functions defined in the list of symbols (Appendix). It was shown that the specification dS/dt = 0 implied that all water vapor generated in an updraft would be consumed by growing ice crystals (or droplets). Thus, the mass of water produced by a given updraft could be determined, and this quantity divided by the rate of vapor extraction per given crystal to yield the concentration of crystals N_c required for complete cloud glaciation in some desired time interval :

$$N_c = \frac{dm/dt}{dm/dt} \frac{(\text{Supply})}{(\text{Extraction})} = \frac{\varphi_1 u/\varphi_2}{4\pi C G^1 S_i \rho_i} \tag{2}$$

Table I illustrates the nucleation requirements as a function of temperature indicated by a solution of equation (2) for an updraft of 0.5 m/sec and planar crystals of 0.1 mm radius. Note that the critical number of nuclei N_c is directly proportional to the updraft and can be readily adjusted for other values.

TABLE I

Critical concentration of ice nuclei (N_c) for complete cloud glaciation within ~ 5 minutes (Updraft u = 0.5 m/sec)

T (°C)	φ_1/φ_2 (g cm^{-4})	Vapor supply dm/dt (g sec^{-1} cm^{-3})	Vapor extraction per crystal (0.1 mm r)	Critical ice nuclei N_c
— 5	1.285×10^{-11}	6.42×10^{-10}	1.9×10^{-9} g sec^{-1}	340 l^{-1}
—10	1.045	5.23	2.9×10^{-9}	180
—20	0.623	3.12	3.0×10^{-9}	105
—30	0.323	1.62	1.9×10^{-9}	85
—40	0.147	0.735	0.92×10^{-9}	80

The following work represents an extension of the above calculations with a numerical model that allows considerably greater variation of parameters. Notably, supersaturation changes in a mixed-phase cloud are determined as a function of ice nucleus concentration and the continuous interplay among the updraft, the growth of crystals, and the growth or evaporation of droplets.

II. — FORMULATION.

At this stage, the cloud model is single layered with constant temperature and uniform hydrometeor distribution. The goal was to realistically represent snow crystal morphology and cloud microphysics on a local scale before adding the complexities of spatial variations. Owing to the limited vertical development ($\lesssim$ 3.5 km) of most Great Lakes snowstorms, this approach appears permissible. Also verification of seeding effectiveness in a NOAA field program (Weickmann et al. [8]) relied heavily on the size, type, and concentration of falling snow crystals, giving added impetus for stressing these variables in the numerical model.

Crystals are allowed to grow by diffusion of water vapor and by accretion of supercooled droplets, with the dominant growth mechanism being indicated by the model. A distinction is made between nonrimed crystals ; partially rimed crystals, where the initial crystal form (hexagonal plates and plane dendrites) is preserved ; and heavily rimed three-dimensional pellets (graupel). Two key input quantities are updraft intensity and concentration of ice nuclei, both being specified for a given run of, typically, 55-60 minutes of cloud development time. Ice nucleus concentrations of 1 l^{-1} to 1000 l^{-1} have been considered. Output variables as a function of time include : cloud saturation with respect to ice ; crystal type ; crystal mass, size, and fall velocity ; droplet size and cloud liquid-water content ; ice water content ; and total fall distance of crystals.

1°) *Equation set.*

The main equations used in the program are discussed below, with symbols defined there and/or in the Appendix. The c.g.s. system of units is used throughout unless otherwise indicated.

a) Cloud supersaturation.

Amplifying upon equation (1) to include both droplets (d) and crystals (c) in a mixed-phase cloud yields :

$$(dS_i/dt) = \varphi_1 u - \varphi_2 N_c (dm/dt)_c - \varphi_3 N_d (dm/dt)_d \quad (3)$$

where S_i is supersaturation with respect to ice and N_c and N_d are the concentrations of ice crystals and cloud droplets respectively. Note that the third term on the right can be either positive or negative, depending on whether droplets are evaporating (vapor source) or growing (vapor sink). The latter occurs when the updraft is vigorous and the ice crystals are too few or growing too slowly to prevent supersaturation with respect to water.

b) Crystal growth by diffusion.

The mass growth of an ice crystal by diffusion of water vapor is given by the so-called classic equation :

$$(dm/dt)_c = 4 \pi CG' S_i \rho_i \quad (4)$$

where C is the crystal shape factor, G′ a thermodynamic function accounting for latent heat and vapor diffusivity, and ρ_i the density of ice.

To convert from crystal mass to size (radius r_c) at any given point, the empirical relationships of Nakaya and Terada [9] were used. These are listed in Table II, along with values of the shape factor C assumed for each crystal type.

TABLE II

Crystal mass-size relationships and C values

Crystal habit	C	Mass vs. size
Planar unrimed	$2 r_c/\pi$	$r_c = (m_c/0.00152)^{1/2}$
Partially rimed	$0.8 r_c$	$r_c = (m_c/0.0108)^{1/2}$
Heavily rimed	r_c	$r_c = (m_c/0.52)^{1/3}$

c) Droplet evaporation (or growth).

The equation used for droplet growth by diffusion is that given by Fletcher [10] :

$$(dm/dt)_d = 4 \pi \rho_l G r_d \left(S - \frac{a}{r_d} + \frac{b}{r_d^3}\right), \quad (5)$$

where ρ_l is the density of water, S the supersaturation with respect to water, a the Kelvin curvature term, and b the nucleus solubility term. It was assumed that droplets form on 0.1 μ radius sodium-chloride condensation nuclei.

d) Crystal growth by riming.

The rate of increase of mass m of a falling crystal by accretion of supercooled cloud droplets is commonly given by

$$(dm/dt)_R = \pi r^2_c E \omega (v_c - v_d), \quad (6)$$

where r_c is the crystal radius, E the collection efficiency, ω the liquid water content, and (v_c — v_d) the difference in fall velocity of crystal and collected droplets, respectively. (Droplet fall velocity $v_d \ll v_c$, and can be neglected).

Collection efficiencies of ice crystals for cloud droplets are not well known. A collection efficiency of 0.5 was assumed for disc type crystals (hexagonal plates, solid stellars) and an E of 1 for dendrites, where the capture cross-section area reduces to approximately $\pi r_c^2/2$. Based on the collection theory of Ranz and Wong [11] for discs and representative cloud drops of 10-20 μ, and dendrite collection estimates of Weickmann [12], these E values appear reasonable.

The conversion to partially rimed crystals is specified when the droplet accretion growth rate (eqn. 6) exceeds the vapor diffusion growth rate (eqn. 4) of the crystals. When the crystal mass equals an equivalent sphere radius of 1 mm (occurring approximately when the crystal c axis exceeds the a axis), graupel growth is assumed.

e) Fall velocity of crystals.

The fall velocity of unrimed planar crystals is taken as a constant 30 cm/sec, as given by Nakaya [13]. Based on the cited crystal-mass relationships and some measured velocity data of Langleben [14], the terminal velocity of riming crystals can be expressed [2] as

$$v_{cR} = 210\ r_c^{0.3}\ \text{cm/sec.}$$

This expression appears quite valid for partially rimed crystals, but underestimates the fall velocity of graupel.

A number of other standard equations are used in the program. These include expressions for the cloud liquid water content, ice content, net vertical velocity and vertical displacement of crystals, and water vapor pressure.

2°) *Computer program.*

A fourth-order Runge-Kutta method was used to numerically solve the foregoing set of equations with a Univac 1108 computer. While the model is capable of generating a cloud from an initial updraft, the computations reported here are begun with an established supercooled cloud at water saturation. Then the effects of introducing given concentrations of ice nuclei are examined. The starting conditions for all the cases discussed in the next section are as follows :

$T = -20\ ^oC$	$N_d = 300\ cm^{-3}$
$p = 800$ mb	$r_d = 7\ \mu$
$S_i = 21.5\ \%$	$\omega = 0.43\ g/m^3$

III. — Results.

Five ice nucleus concentrations ($N_c = 1, 10, 50, 100$, and $1000\ l^{-1}$) were successively run for each specified updraft case. The low concentrations of 1 and 10 l^{-1} (and sometimes even 50 l^{-1}) are fairly representative of natural precipitating clouds while concentrations of 100 and 1000 l^{-1} simulate heavily seeded clouds. Nonprecipitating supercooled clouds can, of course, have ice nucleus concentrations substantially lower than 1 l^{-1}.

Four fixed updraft cases were treated ; 5 cm/sec, 20 cm/sec, 1 m/sec, and 3 m/sec. One might think of these values as representing average updraft conditions in winter cloud types varying from weak stratus to vigorous cumulus clouds. In a given intense Great Lakes snowstorm, all these values can and do occur.

Based on calculations of precipitation rate for given updrafts, it is felt that an overall average value of 0.5 m/sec represents quite well the typical Great Lakes snowband. For example, with such an updraft and an average cloud temperature of —15 °C, Table I predicts a cloud-water generation rate of 4.18×10^{-10} g/sec cm^3. Assuming a cloud depth of 3 km and that 50 % of the condensate falls out as precipitation, this yields a precipitation rate of 0.226 g/cm^2 hr, which translates to a snowfall rate of approximately 1 to 3 inches per hour — commonly measured values in these intense snowstorms. Thus the updraft values chosen for the model seemingly bracket quite well average lake storm conditions. The 3 m/sec updraft case would apply best to buoyant turrets in the narrow core of an intense storm cell.

The time history of cloud supersaturation and liquid water content are shown in Figures 1 through 3 as a function of updraft and ice nucleus concentration. Also shown on the supersaturation traces are the times when crystals (which initially grow strictly by diffusion of water vapor) commence to partially rime (P) and then heavily rime (R). If sufficient crystals are present, a return to strict vapor diffusion growth (D) may occur as the cloud glaciates.

Examining the figures, it is evident that one ice nucleus per liter of air is not adequate to glaciate the model cloud even in a weak updraft of 5 cm/sec (extending the time to 60 minutes produced little change). A crystal concentration of 10 l^{-1} will consume all the cloud liquid water in approximately 20 to 55 minutes, depending on updraft strength. Complete cloud glaciation occurs in approximately 8 to 12 minutes for $N_c = 100\ l^{-1}$, and in 3-5 minutes for $N_c = 1000\ l^{-1}$.

It should be noted that the model does not presently account for droplet population decreases by riming. While it can be argued that new drops are constantly being generated in the updraft, these neophyte droplets would not be as large as those lost through riming. Hence, this model deficiency tends to exaggerate cloud liquid water (and riming) in some cases. Clearly it causes no error when $N_c = 1000\ l^{-1}$ and riming never occurs, and only slight water exaggeration in the low updraft cases of 5 and 20 cm/sec. For the 1 and 3 m/sec updraft cases and intermediate N_c values of 10-100 l^{-1}, the glaciation times noted should be considered conservative.

Figures 1 and 2 indicate that light riming in the model cloud commences in about 2 minutes almost independent of updraft strength and crystal concentration ($\leqslant 100\ l^{-1}$). For example, the onset of riming occurs at 1.7 minute when $u = 3$ m/sec and about 2.6 minutes when $u = 5$ cm/sec. Thus, some light riming of crystals in clouds of such liquid water content is to be expected and theoretically, if perhaps not practically, can be avoided only by seeding with massive concentrations of ice nuclei.

Heavy riming to the graupel stage, however, can be avoided with ice nucleus concentrations of about 50 l^{-1} for updrafts up to 1 m/sec and 100 l^{-1} for 3 m/sec updrafts. The onset time of heavy riming, if it is to occur at all, and its cessation time vary considerably with updraft strength and crystal concentration.

For comparison purposes, certain model output parameters are listed in Table III after 30 minutes of cloud development time, a reasonable lifetime for a given cloud cell. The data trends are probably more useful than the absolute numerical values, owing to the specification of realistic but singular initial cloud conditions and to model limitations. Some of the relevant items that might be pointed out are as follows :

a) Crystal sizes predicted by the model for updrafts $u \leqslant 100$ cm/sec and natural cloud crystal concentrations of 1-10 l^{-1} are approximately 2 to 4 mm in size. This is consistent with our field measurements of single crystals in natural storms.

b) N_c must be increased by about three orders of magnitude to decrease crystal

TABLE III

Cloud model variables after 30 minutes

(Model cloud : T = —20 °C, p = 800 mb ; initial conditions, S_i = 21.5 %, ω = 0.43 g/m³)

	Ice crystal concentration N_c (l^{-1})				
	1	10	50	100	1000
a) u = 5 cm/sec					
Crystal type	Graupel	Rimed C.	Rimed	Ligth rime	Unrimed
Crystal radius (mm)	1.7	1.6	1.0	0.5	0.20
Drop radius (μ)	6.9	0	0	0	0
Liquid water (g/m³)	0.41	0	0	0	0
Supersaturation-ice (%)	21.5	0.4	0.3	0.2	0.1
Crystal fall velocity (cm/sec)	210*	133	94	84	30
Riming to diffusion (min)	—	22	11	8	D only
b) u = 20 cm/sec					
Crystal	Graupel	Graupel	Rimed	Light rime	Unrimed
r_c (mm)	1.8	1.6	1.1	0.6	0.23
r_d (μ)	7.7	0	0	0	0
ω (g/m³)	0.58	0	0	0	0
S_i (%)	21.5	5.0	0.9	0.8	0.3
v_T (cm/sec)	220*	210*	97	84	30
R→D (min)	—	27	11	8	D only
c) u = 100 cm/sec					
Crystal	Graupel	Graupel	Rimed	Rimed	Unrimed
r_c (mm)	2.6	1.9	1.3	1.0	0.32
r_d (μ)	10.4	8.0	0	0	0
ω (g/m³)	1.44	0.63	0	0	0
S_i (%)	21.6	21.5	2.1	2.3	1.2
v_T (cm/sec)	260*	230*	113	94	30
R→D (min)	—	51	15	10	D only
d) u = 300 cm/sec					
Crystal	Graupel	Graupel	Graupel	Rimed	Unrimed
r_c (mm)	5.7	4.4	1.5	2.0	0.48
r_d (μ)	14.2	12.0	0	0	0
ω (g/m³)	3.61	2.19	0	0	0
S_i (%)	21.7	21.5	2.2	2.1	2.3
v_T (cm/sec)	>400*	>300*	200*	126	30
R→D (min)	—	56	22	12	D only

* Velocities from Nakaya [13] graupel data.

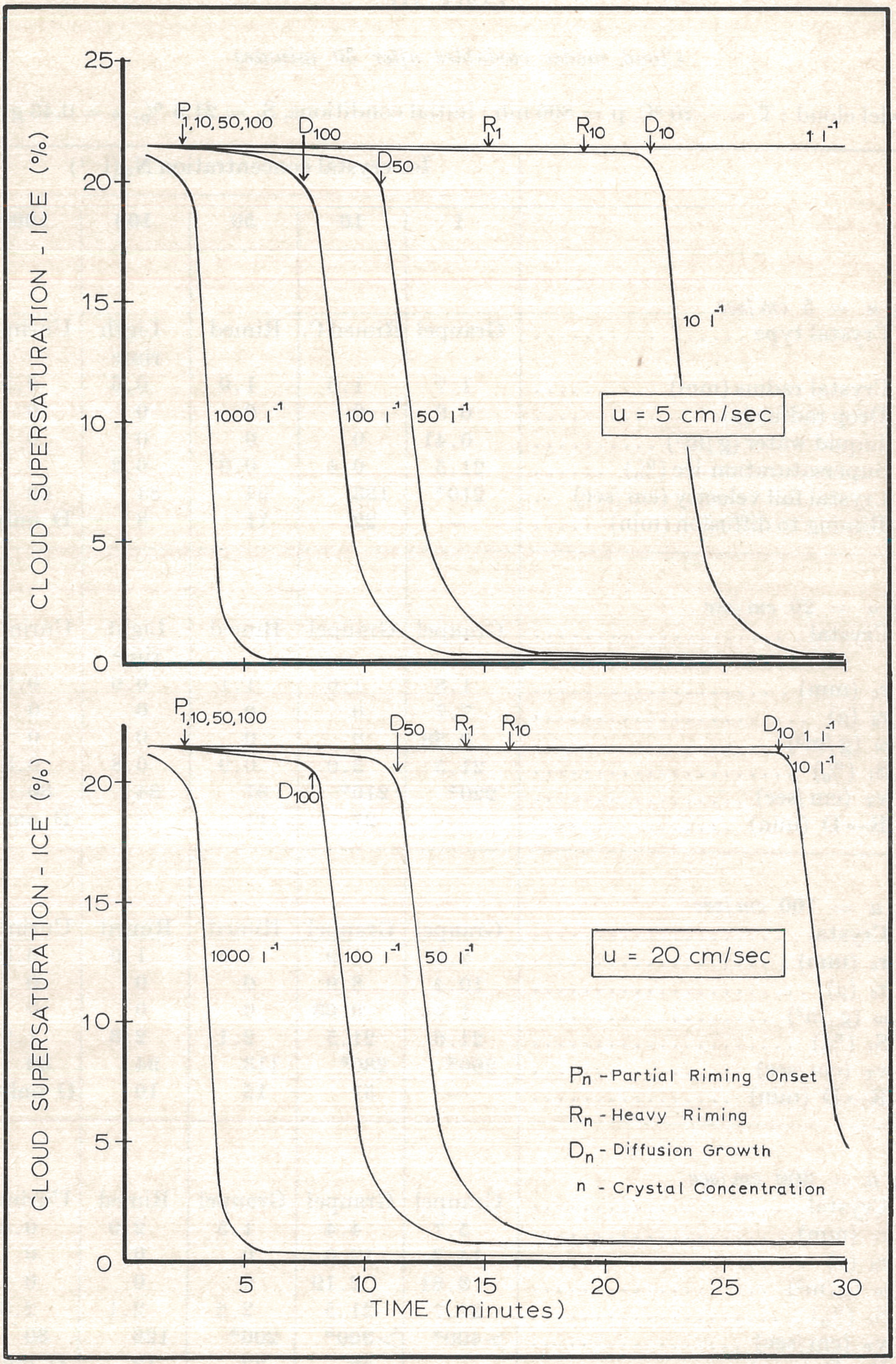

FIG. 1. — Cloud supersaturation (ice) vs. crystal concentration N_c and time (model cloud — 5 and 20 cm/sec updraft cases).

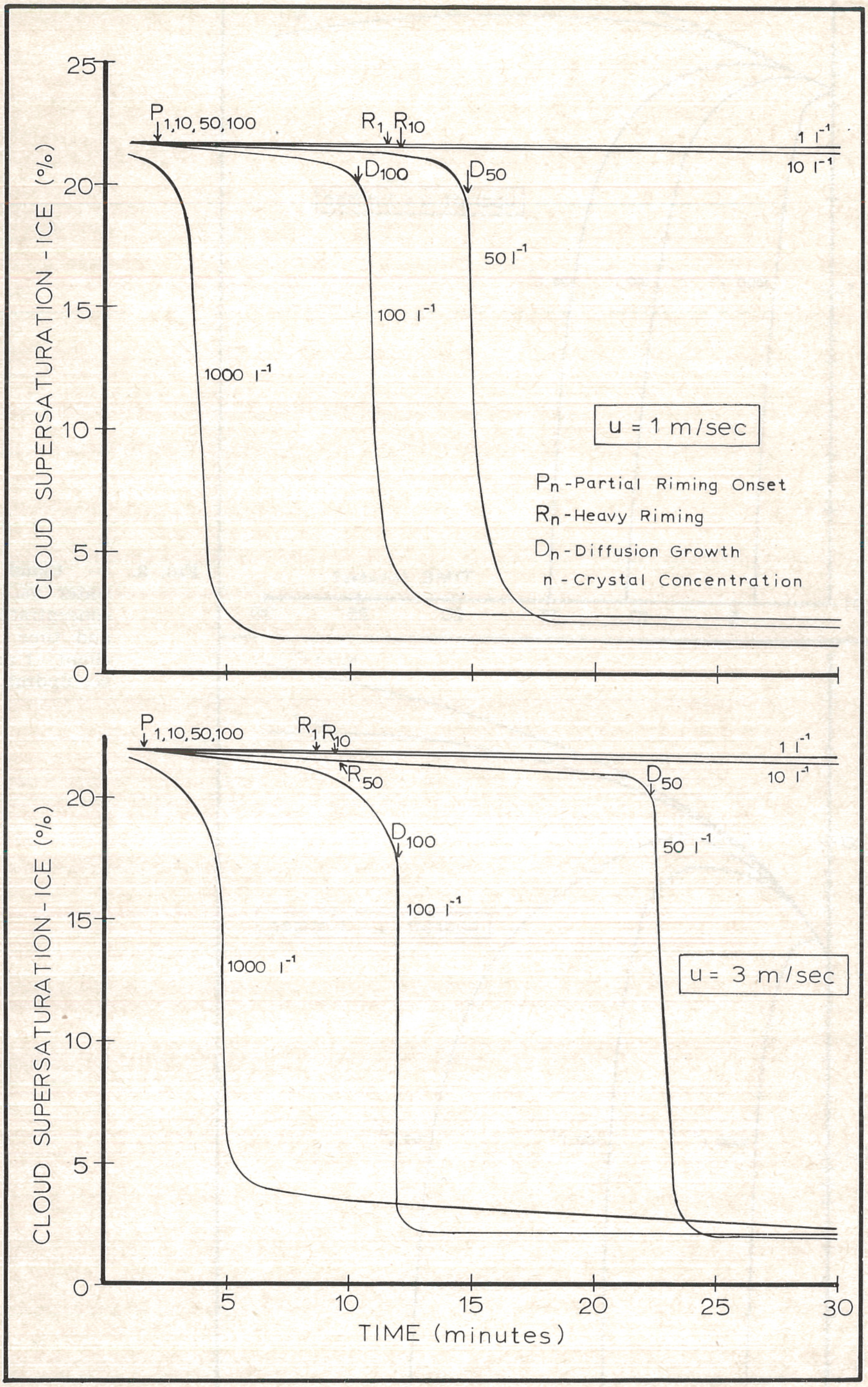

FIG. 2. — Cloud supersaturation (ice) vs. crystal concentration N_c and time (model cloud — 1 and 3 m/sec updraft cases).

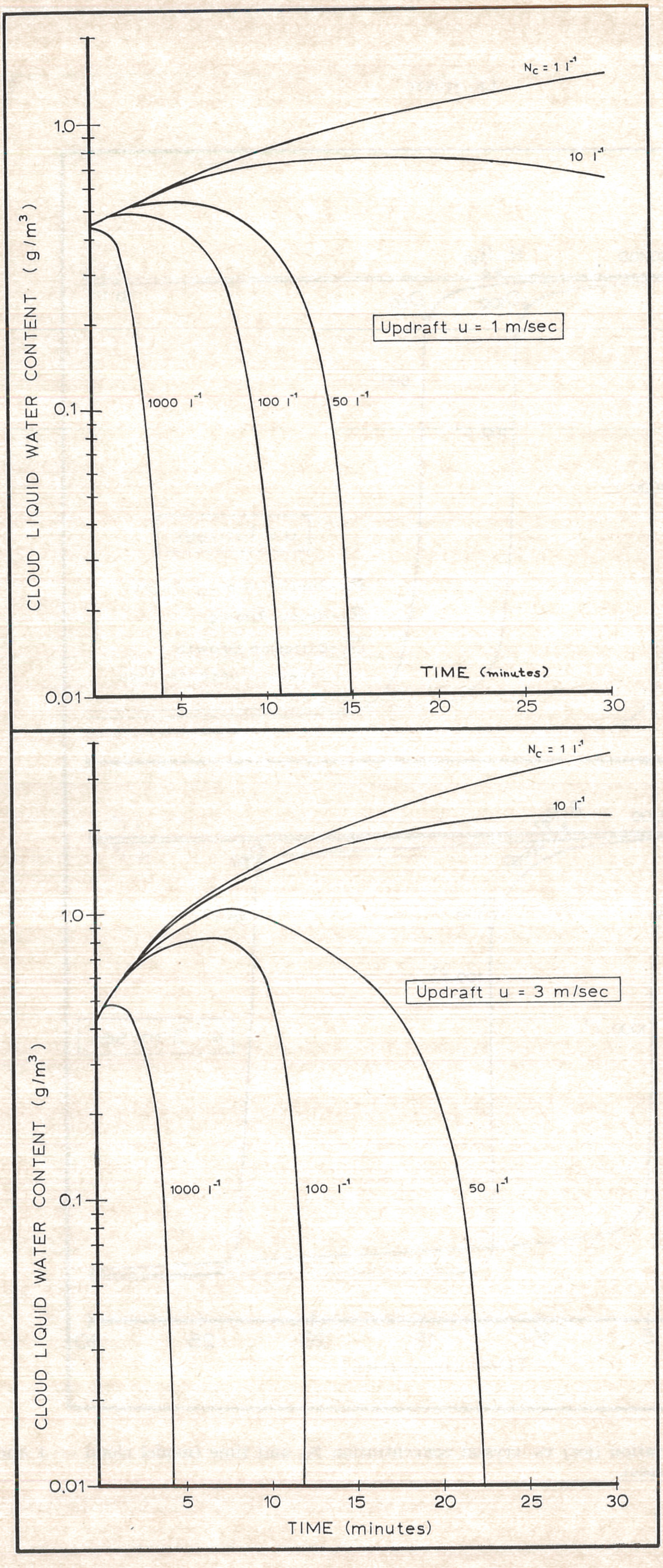

FIG. 3. — Cloud liquid water vs. crystal concentration N_c and time (model cloud - 1 and 3 m/sec updraft cases).

size by one order of magnitude. The largest nucleus concentrations (1000 l^{-1}) limit individual planar crystals to approximately 0.4 to 1 mm sizes.

c) The data suggest that a seeding concentration of the order of 100 l^{-1}, while not capable of suppressing light riming, will reduce the fall velocity of crystals by a factor of 2 to 3 over graupel pellets. The restriction of fall velocities to 30-50 cm/sec, characteristic of unrimed planar crystals, suggests massive seeding concentrations of the order of several 100 to 1000 l^{-1}.

Note that the critical concentrations given simply by Equation (2) and Table I are roughly comparable to these more detailed model estimates.

Table IV lists the approximate time and cloud layer thickness required for the formation of graupel of 1 mm radius. Again, the initial cloud model conditions of Section II 2°) were assumed. Note that the stronger the updraft, the less is the time required for graupel to form. This is due to the corresponding increase in cloud liquid water and, hence, increase in crystal riming rate as the updraft becomes more intense.

Of particular interest is the fact that the required cloud layer thickness can be as shallow as about 0.18 km, which occurs with updrafts of 1 m/sec. Such updraft magnitudes correspond rather closely with the terminal fall velocity of partially rimed crystals so that a growing crystal remains relatively « stationary » with respect to the ground. It is also evident from the table that ice nucleus concentrations of 50-100 l^{-1} can prevent the formation of graupel.

TABLE IV

Required cloud layer depth (km) and time (min) for graupel ($r = 1$ mm) formation

(Model cloud conditions of section II 2°)

Updraft	Ice nucleus concentration N_c (l^{-1})			
	1	10	50	100
u (cm/sec)	km (min)	km (min)	km (min)	km (min)
5	0.70 (15.2)	0.96 (19.2)	*	*
20	0.51 (14.2)	0.62 (16.0)	*	*
100	0.18 (11.4)	0.18 (11.9)	*	*
300	1.17 (8.7)	1.20 (8.9)	1.30 (9.6)	*

* Graupel does not form.

Thus far we have not considered the implications of crystal aggregation on the cloud development process. As will be shown in the following section, high concentrations of ice nuclei lead to enhanced crystal aggregation and snowflake fall velocities of 1-1.5 m/sec. In terms of merely glaciating the cloud, the model predictions indicate that the nucleus concentrations of 100 l^{-1} will do almost as well as 1000 l^{-1} except that a bit more time is needed (4 to 8 minutes). Conversely, the extreme concentration conceivably could produce snowflakes with a higher fall velocity than individual lightly rimed crystals (see Table III velocity values.) In a practical sense, then, there appears to be little virtue in attempting to seed Great Lakes snowstorms with nucleus concentrations greater than 100 l^{-1} or so.

If the goal merely is to stimulate snowfall in nonprecipitating supercooled clouds, the model suggests that nucleus concentrations of 1-10 l^{-1} will suffice.

IV. — Snow crystal aggregation.

Snow crystal aggregation — even in the absence of seeding — is very common in lake storms, increasing in importance as ambient temperature decreases and as inland distance increases. In general, it appears that in the north-eastern United States, snowflake aggregates constitute the most common type of snowfall. Dendritic crystals, formed at temperatures near —15 °C and humidities close to water saturation, favor the aggregation process as probably does some light riming of these crystals. Field observations of snowflakes invariably revealed dendritic forms with sometimes hundreds of crystals per cm-size snowflake.

Snow crystal concentrations measured at ground level often ran high, i.e., tens per liter of air. Freezing nucleus concentrations at —18 to —20 °C were at least an order of magnitude lower. This not uncommon paradox in clouds has been reviewed by Mossop [15].

Explanations for the discrepancy usually invoke some type of droplet splintering mechanism (Koenig [16], Hobbs [17]), cirrus seeding of lower cloud layers (Braham [18]), or doubts as to the relevance of cold box measurements of ice nuclei. Another mechanism, that of an accumulation of crystals within the cloud where they are approximately balanced by the updraft, is rather appealing, at least for lake storm bands. Also appealing is some type of crystal fragmentation process. The common occurrence of snowflake aggregates requires a mechanism whereby high concentrations of crystals are produced within a cloud.

1°) *Snow crystal congregation.*

The idea of storage or accumulation of water in excess of the moist adiabatic production in certain restricted regions of a warm cloud is by no means new. Such observations have been reported by Weickmann and aufm Kampe [19], Battan and Reitan [20], and others. Warner and Squires [21] concluded that these somewhat rare observations would have to be restricted to layers almost negligible in size by comparison with the cloud as a whole. Squires [22] also inferred that such water accumulation could only be explained by selective droplet sedimentation that enriched some parts of the cloud. Gokhale and Rao [23], with an idealized physical model of a thunderstorm, argue that water accumulation is likely in the upper portion of the cloud where settling drops are balanced as they descend into layers of stronger updrafts.

Recognizing that the foregoing observations and interpretations are not universally accepted, it is suggested that *if* some updraft balance-type mechanism possibly can operate in convective warm clouds to explain large liquid water concentrations, it is more than likely to occur in cold clouds to produce high crystal concentrations. The conditions for ice crystal congregation appear far more favorable as reasoned by the following :

a) The terminal fall velocities of ice crystals comprise a much narrower spectrum than water droplets. Most ice crystals possess fall velocities less than 1.5 m/sec, with the average likely between 0.5 and 1.0 m/sec.

b) Average updrafts in these lake storm bands (and probably many winter snow clouds), as inferred from precipitation rate computations, closely coincide with typical crystal fall velocities. In intense cloud cores, more vigorous updrafts carry the crystals to the upper cloud levels.

c) One implication of *a*) and *b*) seems rather obvious — a substantial percentage of the ice crystal population could become concentrated in certain (probably upper) cloud layers to facilitate snowflake aggregation. (Lateral divergence would obviously tend to dilute crystal concentrations).

d) The occurrence of large snowflakes mathematically precludes (section IV 2°) a homogeneous distribution of ice crystals of say 0.01 to 1 l^{-1}, as suggested by ice nucleus measurements.

e) While observations of large water accumulations in warm clouds are rare, high crystal concentrations in cold clouds and at the ground appear common.

Crystal fragmentation by collisions with droplets and other crystals must account for some of the observed high crystal concentrations, as might the sublimational break-up of crystals suggested by Schaefer and Cheng [24]. Large numbers of crystal fragments are observed in some lake storms ; in one case, 80 % of the replicated crystals consisted of fragments (Jiusto and Holroyd [25]). Supercooled drop splintering also must occur to some extent in clouds, but the difficulty of duplicating this phenomenon in the laboratory tends at present to make one skeptical of its common occurrence in the atmosphere. Further the cold temperature of typical lake storm clouds results in small droplets less likely to shatter.

2°) *Snowflake aggregation rates.*

The collision rate (dC_o/dt) of a snowflake falling through a population of individual crystals can be expressed as,

$$(dC_o/dt) = \pi\, r_f^2\, E\, N_c\, (v_f - v_c), \tag{8}$$

where r_f is the radius of the flake, N_c the concentration of individual ice crystals in the cloud, and v_f and v_c the fall velocity of flake and crystals, respectively.

Derived equations (Jiusto and Holroyd [25]) for the terminal fall velocity v_f of snowflakes, and of flake radius r_f as a function of crystal concentration n_c per flake and individual crystal radius r are as follows :

$$v_f = 150\, r_f^{0.2}, \tag{9}$$

$$r_f = 0.25\, n_c^{1/2}\, r. \tag{10}$$

These equations permit, with some simplifying starting assumptions, an estimation of crystal aggregation rates and the size and velocity changes of falling snowflakes. It was assumed that a three-crystal snowflake initially existed, that $v_c = 30$ cm/sec, and $E = 1$. The equation set was then solved numerically for the following cloud crystal concentrations (N_c) and fixed crystal sizes (r) :

a) $N_c = 1{,}10\ l^{-1}$	$r = 1$ mm
b) $N_c = 10, 50, 100\ l^{-1}$	$r = 0.5$ mm
c) $N_c = 100\ l^{-1}$	$r = 1.0$ mm
d) $N_c = 1000\ l^{-1}$	$r = 0.2$ mm

Cases a) and b) represent conditions predicted by the microphysics model after about 10 minutes of cloud development time in updrafts varying from 5 cm/sec to 1 m/sec. Case c) represents crystal growth in an extreme 3 m/sec updraft while case d) is intended

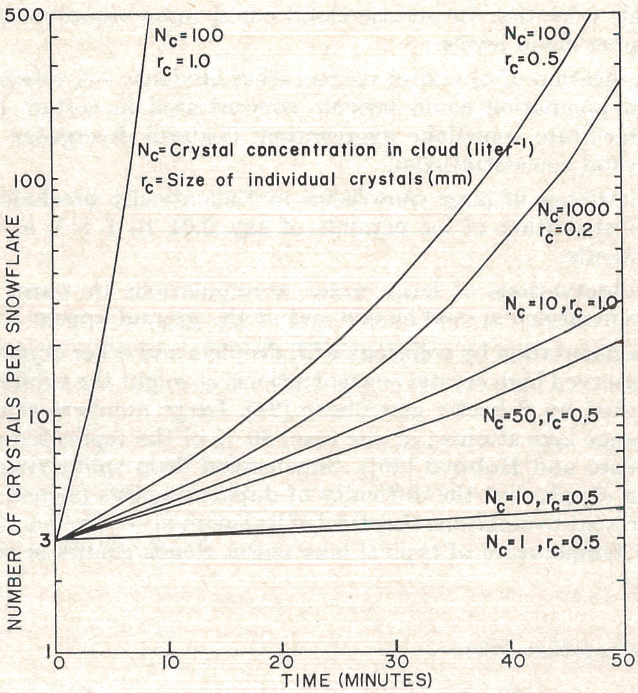

FIG. 4. — Snowflake aggregation rates (preliminary model).

to simulate a NOAA overseeding experiment of 7 December 1968, in which cloud properties and crystal type and sizes were reported (Holroyd and Jiusto [26]) to change in the manner predicted by the microphysics model.

Figure 4 shows the degree of aggregation predicted in each case, expressed as the number of crystals per snowflake with time. A fixed population of cloud crystals was assumed in this first-approximation calculation. Without a depletion term, the results tend to exaggerate aggregation. Nevertheless, certain informative points emerge, as follows :

a) with cloud crystal concentrations of $1\ l^{-1}$, aggregation is slight to nonexistent ;

b) the same is true for concentrations of $10\ l^{-1}$ until individual crystal sizes reach radii of 1 mm or so ;

c) with crystal concentrations of about $50\ l^{-1}$ or more, aggregation is apparently the dominant growth mechanism overshadowing riming and diffusion growth ;

d) the simulated overseeding case ($N_c = 1000\ l^{-1}$) shows crystal aggregation values after 30 minutes that are quite compatible with those observed (several tens to several hundreds of crystals per snowflake).

V. — SUMMARY.

The relatively simple microphysics model presented enables one to evaluate the respective importance of crystal growth by diffusion, riming, and snowflake aggregation. With clouds of the type involved in Great Lakes snowstorms (i.e., $\omega \simeq 0.5 — 1$ g/m^3 ; $u \leqslant 3$ m/sec), certain generalizations appear valid :

a) Crystal concentrations of only 1 l^{-1} are not sufficient to glaciate these clouds. Even with 10 l^{-1}, riming is the dominant growth mechanism leading to the formation of graupel in times as short as 8 minutes (and cloud layer depths as shallow as 180 m).

b) Ice nucleus concentrations in the neighborhood of 100 l^{-1} will prevent graupel and quickly glaciate a cloud but some crystal riming still is to be expected. Massive seeding of several 100 to 1000 nuclei per liter is needed to completely eliminate riming.

c) Snowflake aggregation becomes the dominant growth mechanism when crystal concentrations exceed about 50 l^{-1}. Hence, massive cloud seeding which formerly was felt might stabilize clouds, will only succeed in accentuating aggregation (and in producing snowflakes that precipitate at about 1 m/sec).

d) The common observation of snowflake aggregates in the north-eastern United States suggests regions within clouds where high concentrations of individual crystals congregate. An updraft balance mechanism appears reasonable. Crystal fragmentation within a mixed phase cloud involving dendritic forms could considerably enhance concentrations, as might also in rare cases the often speculated but tenuous droplet shattering.

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APPENDIX — LIST OF SYMBOLS

a	droplet curvature term ($a = 3.3 \times 10^{-5}/T$)
b	droplet solubility term ($b = 4.3\ i\ m_s/M_s$)
c_p	specific heat of air at constant pressure
C	crystal shape (capacitance) factor; degrees centrigrade
C_o	number of collisions of falling snowflake with ice crystals
D	diffusivity of water vapor
e	vapor pressure of the environment
E	collection efficiency of a falling hydrometeor
f	subscript denoting snowflake
g	gravitational acceleration; or subscript denoting graupel
G	thermodynamic function in drop growth equation

$$G = \frac{D\rho_v}{\rho_l}\left[1 + \frac{DL^2\,\rho_v\,M_o}{RT^2\,\varkappa}\right]^{-1}$$

G'	thermodynamic function in ice crystal growth equation (like G above except ρ_v, ρ_l, and L are replaced by ρ_{vi}, ρ_i, and L_s, respectively)
i	van't Hoff factor; or size interval
L, L_s	latent heat of condensation, and of sublimation
m	mass
M_a, M_o, M_s	molecular weight of air, water vapor, and soluble condensation nucleus, respectively
n_c	number of crystals per snowflake aggregate
N_c, N_d	concentration of ice crystals, and of cloud droplets, per unit volume of air
p	atmospheric pressure
r_c, r_d, r_f	radius of crystal, droplet, and snowflake, respectively
R	universal gas constant; or subscript denoting riming
S, S_i	supersaturation with respect to water, and ice
t	time
T	temperature
u	updraft velocity
v, v_T	fall velocity (often with a subscript denoting object)
$\varkappa$	thermal conductivity of air
ρ_{air}, ρ_f	density of air, and of a snowflake
ρ_i, ρ_l	density of ice, and of water
ρ_v, ρ_{vi}	saturation vapor density over water, and over ice
φ_1, φ_2, φ_3	thermodynamic functions in supersaturation equation (3)

$$\varphi_1 = \frac{g}{RT}\left[\frac{L\,M_o}{T\,c_p} - M_a\right]$$

$$\varphi_2 = \frac{L_s^2\,M_o}{T\,p\,M_a\,c_p} + \frac{RT}{M_o\,e}$$

φ_3: like φ_2 except L_s is replaced by L

ω	liquid water content of cloud