Logical vs Historical Time in A Price Adjustment Mechanism

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Abstract

Recent global financial crisis seems to be re-kindling the battle of economic thoughts which has been dominated by the neoclassical doctrine as free market fundamentalism. This paper first examines the neoclassical foundation of price adjustment mechanism built on logical time, using system dynamics modeling. Then it is argued that similar workings could be done in a real market economy running on historical time by the interplay of price, inventory and their interdependent feedback relations. This implies that off-equilibrium analysis built on historical time without neoclassical concept of auctioneer is a better way of representing market activities. This approach can be made possible by system dynamics modeling.

1 Adam Smith!

"There's a person who has influenced upon us more than Jesus Christ! Who's he?" An instructor of Economics 1, an introductory course for undergraduate students at the Univ. of California, Berkeley, challenged his students cheerfully. I was sitting in the classroom as a Teaching Assistant for the course. This was in early 80's when I was desperately struggling to unify three schools of economics in my dissertation; that is, neoclassical, Keynesian and Marxian schools of economics.

"He's the author of the Wealth of Nations written in 1776; his name is Adam Smith!", claimed the instructor. Adam Smith's idea of free market economy has

^{*}This paper is written during my short-term sabbatical leave at the Victoria Management School, Victoria University of Wellington, New Zealand in March 2009. I'm very grateful to Prof. Bob Cavana for his kindly hosting my visit and providing a wonderful research opportunity. It is submitted to the 27th International Conference of the System Dynamics Society, Albuquerque, New Mexico, USA, July 26-30, 2009. It is partly supported by the grant awarded by the Japan Society for the Promotion of Science.

been a core doctrine throughout the so-called Industrial Age which started in the middle of the eighteenth century. It has kept influencing our economic life even today with a simple diagram such as Figure 1.



Figure 1: Price Mechanism of Demand and Supply

Those who have studied economics are very familiar with this diagram of demand and supply, which intuitively illustrates a market mechanism of price adjustment processes. Price is taken on vertical axis and quantity is taken on horizontal axis. Demand is illustrated as a downward sloping curve, indicating the attitude of consumers that their demand decreases for higher prices and increases for lower prices. This relation is theoretically derived from a utility maximization principle of consumers. Supply is illustrated as an upward sloping curve which exhibits the behavior of producers that their supply increases for higher prices and decreases for lower prices. This relation results from a principle of profit maximizing behavior by producers. Market equilibrium, in which the amount of demand is equal to the amount of supply and market clears, is shown to exist at a point where demand and supply curves intersect in the diagram.

When price is higher than the equilibrium, there exists an excess supply or unsold and increased amount of inventory (which is also called a negative excess demand), and price is eventually forced to go down to attract more consumers to buy the product. On the other hand, if price is lower, there exists an excess demand or the shortage of product which eventually pushes up the price. In either case, price tends to converge to an equilibrium price. This adjusting market force is provided by an *invisible hand*, Adam Smith believed. It is called a price adjustment mechanism, or tâtonnement process, in modern microeconomics.

This price adjustment mechanism works not only in commodity markets but also in labor markets as well as financial capital markets. For instance, let us consider a labor market by taking a wage rate on the vertical axis and the quantity of labor on the horizontal axis. Then, demand curve is interpreted as the demand for labor by producers and supply curve represents the attitude of workers to work. Producers do not employ as many workers as before if wage rate increases, while more workers want to work or they want to work longer hours if their wage rate is higher, and vice versa. Market equilibrium in the labor market denotes full employment. If wage rate is higher than the equilibrium, unemployment comes off and eventually workers are forced to accept a wage cut. In the case of lower wage rate, labor shortage develops and eventually wage rate is pushed up. In this way, price adjustment mechanism works similarly in the labor market.

In a financial capital market, price on the vertical axis becomes an interest rate, and it become a foreign exchange rate in a case of a foreign exchange market. Price mechanism works in a similar fashion in those markets.

In this way, workings of a price adjustment mechanism could be explicated uniformly in all markets by the same framework. Our daily economic activities are mostly related with these market mechanisms governed by the *invisible hand*. This is why the instructor at the UC Berkeley amused his students, saying that Adam Smith has been more influential!

Unfortunately, however, this doctrine of *invisible hand*, or neoclassical school of economic thought has failed to obtain unanimous acceptance among economists, and two opposing schools of economics eventually have been struggling to fight against the workings of market price mechanism depicted by Figure 1 They are Keynesian and Marxian schools. Mutually-antagonistic dissents of these school created the East-West conflicts, Cold War since the World War II, and domestic right-left wing battle till late 80's when these battles of ideas finally seemed to have ended with a victory of neoclassical school. Since then, the age of the so-called privatization (of public sectors), and globalization with the help of IT technologies have started as if the doctrine of the *invisible hand* has been the robust foundation of free market fundamentalism similar to religious fundamentalisms.

Accordingly most of us believed there would be no longer conflicts in economic thoughts as well as in our real economic life until recently when we were suddenly hit by severe financial crises in 2008; the worst recession ever since the Great Depression in 1929. The battle of ideas seems to be re-kindled against the doctrine of the *invisible hand*. Today Adam Smith seems to be getting more influential globally, not because his doctrine is comprehensive enough to accomplish a consensus on the workings of a market economy, but because it caused many serious socio-economic conflicts and wars instead.

2 Three Schools in Economics

As a graduate student in economics in late 70's and early 80's, I was struggling to answer the question: Why did three schools disagree? As a proponent of Adam Smith's doctrine, neoclassical school believes in a price adjusting mechanism in the market. As shown above, however, this price mechanism only works so long as prices and wages move up and down flexibly in order to attain an equilibrium. Therefore, if disequilibria such as recession, economic crisis and unemployment happen to occur, they believe, it's because economic agents such as monopoly, government and trade unions refuse to accept price and wage flexibility and distort the workings of market mechanism.

Keynesian school considers that market has no self-restoring forces to establish an equilibrium once economic recessions and unemployment occur, because prices and wages are no longer flexible in a modern capitalist market economy. To attain an equilibrium, therefore, government has to stimulate the economy through fiscal and monetary policies. In Figure 1 these policies imply to shift the demand curve to the right so that excess supply (and negative excess demand) will be eliminated.

Marxian school believed that market disequilibria such as economic crisis and unemployment are inevitable in a capitalist market economy, and proposed a planned economy as an alternative system. After the collapse of the Soviet Union in 1989, Marxian school ceased to exercise its influence because the experiments of a planned economy in the former socialist countries turned out to be a failure. Even so, they manage to survive under the names of post-Keynesian, environmental economics and institutional economics, etc.

Accordingly, only neoclassical and Keynesian schools remain to continue influencing today's economic policies. In the United States, Republican policies are deeply affected by the doctrine of neoclassical school such as free market economy and small government through deregulation. Meanwhile, Democrats favor for Keynesian viewpoint of public policies such as regulations by wise (not small) government. Current financial crises may reinforce the trend of regulation against hand-free financial and off-balance transactions.

Why do we need three different glasses to look at the same economic reality? Why do we need three opposing tools to analyze the same economic phenomena? These were naive questions I posed when I started studying economics as my profession. In those days I strongly believed that a synthesis of three schools in economics is the only way to overcome Cold War, East-West conflicts and domestic right-left wing battles. By synthesis it was meant to build a unified general equilibrium framework from which neoclassical, Keynesian and Marxian theories can be derived respectively as a special case. My intention was to show that different world views were nothing but a special case of a unified economic paradigm.

While continuing my research toward the synthesis, I was suddenly encountered by a futuristic viewpoint of *The Third Wave* by Alvin Toffler [1]. It was on December 23, 1982, when I happened to pick up the book which was piled up in a sociology section at the Berkeley campus bookstore. The most unimaginable idea to me in the book was the one that both capitalism and socialism were the two sides of the same coin in the industrial age against the leftist doctrine that socialism is an advanced stage of economic development following capitalism. What's an economic system of the Third Wave, then? Can a new economic system in the information age comply with either neoclassical or Keynesian school of economics developed in the industrial age? I kept asking these questions many times in vain, because Tollfer failed to present his economic system of the information age in a formal and theoretical fashion.

Being convinced by Toffler's basic idea, however, I immediately decided to develop a simple economic model which could be a foundation of a new economic framework for the information age. In this way, the Third Wave became a turning point of my academic research in economics, and since then my work has been focused on a new economic system of the information age. My effort of synthesizing three schools in economics and creating a future vision of a new economic system fortunately resulted in a publication of the book [3]. Its main message was that three schools in economics are effete in a coming information age, and a new economic paradigm suitable for the new age has to be established.

My idea of economic synthesis is to distinguish logical time on which neoclassical school's way of thinking is based, from historical time on which Keynesian and Marxian schools of economic thought are based. My working tools in those days are paper and pencil. I was fortunate to encounter by chance system dynamics in middle 90's through the activities of futures studies. System dynamics modeling re-kindled my interest in economics recently. This paper examines a true mechanism of the working of market economy, which is made possible by the application of system dynamics modeling.

3 Tâtonnement Adjustment by Auctioneer

Let us now construct a simple SD model to examine how a market economy of demand and supply works. In this simple economy buyers and sellers have demand and supply schedules of shirts per week as shown in Table 1. These figures are taken from a paper in [2] under the supervision of Professor Jay W. Forrester¹. The reader can easily replace them with his or her own demand and supply schedules.

In microeconomics these schedules are called demand and supply functions of market prices and derived rigorously from the axiomatic assumptions of consumers and producers. Demand and supply schedules (or functions D = D(p)and S = S(p)) are illustrated in Figure 2 in which price is taken on horizontal axis while demand and supply are plotted on vertical axis. This is a standard presentation of functions in mathematics. On the other hand, in standard textbooks of economics price has been traditionally taken on vertical axis as illustrated in Figure 1.

Now buyers and sellers meet in the market to buy and sell their products according to their schedules of demand and supply. In order to make this market economy work, we need the third player called *auctioneer* who quotes a price. His role is to raise a price if demand is greater than supply, and lower it if demand is less than supply. His bids continue until the equilibrium is attained

 $^{^1\}mathrm{MIT}$ System Dynamics in Education Project (http://sysdyn.clexchange.org/sdep.htm) offers a collection of SD models and papers called Road Maps for self-taught learning of system dynamics. The reader is encouraged to explore these profound resources of SD modeling.

Price	Quantity Demanded	Quantity Supplied			
	D = D(p)	S = S(p)			
\$ 5	100	0			
\$ 10	73	40			
\$ 15	57	57			
\$ 20	45	68			
\$ 25	35	77			
\$ 30	28	84			
35	22	89			
\$ 40	18	94			
\$ 45	14	97			
\$ 50	10	100			

Table 1: Demand and Supply Schedules in [2]



Figure 2: Demand and Supply Functions

where demand is simply equal to supply. This process is called Walrasian or neoclassical price adjustment mechanism or tâtonnement.

The important rule of this market game is that no deal is made until market equilibrium is attained and buyers and sellers can make contracts of transactions. In this sense, time for adjustment is not a real time in which economic activities such as production and transactions take place, but the one needed for calculation. The time of having this nature is called *logical time* in [3]. In reality, there are very few markets that could be represented by this market except such as stock and auction markets. Even so, neoclassical school seems to cling to this framework as if it represents many real market transactions.

Equilibrium

Does this market economy work? This question includes two different inquiries: an existence of equilibrium and its stability. If equilibrium does not exist, the auctioneer cannot finish his work. If the equilibrium is not stable, it's impossible to attain it. Let us consider the existence problem first.

The auctioneer's job is to find an equilibrium price at which demand is equal to supply through a process of the above-mentioned tâtonnement or groping process. Mathematically this is to find the price p^* such that

$$D(p^*) = S(p^*) \tag{1}$$

In our simple demand and supply schedules in Table 1, the equilibrium price is easily found at \$ 15. The existence proof of general equilibrium in a market economy has annoyed economists over a century since Walras. It was finally proved by the so-called Arrow-Debreu model in 1950's. For detailed references, see [3]. Arrow received Nobel prize in economics in 1972 for his contribution to "general economic equilibrium and welfare theory". He was a regular participant from Stanford University to the Debreu's seminar on mathematical economics in Berkley. Debreu received Nobel prize in economics in 1983 for his contribution to "new analytical methods into economic theory and for his rigorous reformulation of the theory of general equilibrium". I used to attend his seminar on mathematical economics in early 80's, and still vividly remember the day of his winning the prize, followed by a wine party spontaneously organized by faculty members and graduate students.

Stability

The second question is how to find or attain the equilibrium. From the demand and supply schedules given above, there seems to be no difficulty of finding the equilibrium. In realty, however, th auchtioneer has no way of obtaining these schedules. Accordingly, he has to grope them by quoting different prices. To describing this groping process, a simple SD model is built as in Figure 3.

Mathematically, the model is formulated as follows:

$$\frac{dp(t)}{dt} = f(D(p) - S(p), \lambda) \tag{2}$$

where f is excess demand function and λ is a price adjustment coefficient. In the model f is further specified as

$$f = \lambda p \frac{D(p) - S(p)}{D(p) + S(p)} \tag{3}$$

From the simulations in our simple model the idea of tâtonnement seems to be working well as illustrated in Figure 4. The left-hand diagram shows that the initial price of \$10 tends to converge to an equilibrium price of \$15. Whatever values of initial price are taken, the convergence can be similarly shown to be attained. In this sense, the market economy can be said to be globally stable.

Figure 3: Auctioneer's Tâtonnement Model

With this global stability, the auctioneer can start with any quotation of initial price to arrive at the equilibrium successfully.

In the right-hand diagram, demand schedule is suddenly increased by capricious buyers by 20 units at the week of 15, followed by the reactive increase of the sellers in the same amount of supply at the week of 30, restoring the original equilibrium. In this way, the auctioneer can easily respond to any changes or outside shocks and attain new equilibrium states. These shifts of demand and supply curves are well known in microeconomics as comparative static analysis.

Figure 4: Stability of Equilibrium

Chaos

So far, neoclassical price mechanism seems to be working well. To attain the equilibrium in our model, a price adjustment coefficient λ is set to be 0.4. What will happen if the auctioneer happens to increase the adjustment coefficient from 0.4 to 3 in order to speed up his tâtonnement process? Surprisingly this has caused a period 2 cycle of price movement with alternating prices between 10.14 and 18.77, as illustrated in the left-hand diagram of Figure 5. When

the coefficient is increased a little bit further to 3.16, price behavior suddenly becomes very chaotic as the right-hand diagram illustrates. I encountered this chaotic price behavior unexpectedly when I was constructing a pure exchange economic model using S language under UNIX environment in [4].

Figure 5: Chaotic Price Behavior

Under such a chaotic price behavior, it is obvious that the auctioneer fails to attain an equilibrium price. Accordingly, under the failure of finding the equilibrium, market transactions can never take place according to the neoclassical rule of the market game. This indicates a fundamental defect in neoclassical framework of market economy based on the idea of *logical* time.

Short-side Transactions

Tired with an endless struggle by the auctioneer to attain an equilibrium in a chaotic price behavior, buyers and sellers may force their actual transactions to resume at a short-side of demand and supply. In other words, if demand is greater than supply, the amount supplied at that price is traded, while the amount demanded is purchased if supply is greater than demand.

To allow this off-equilibrium transactions, the auctioneer has to have enough amount of inventory at hand before the market starts. To calculate the enough amount of inventory, a slightly revised model is built as shown in the left-hand diagram of Figure 6.

When the auctioneer quotes an initial price below equilibrium at \$10, allowing the short-side trade, unrealized excess demand keeps piling up as backlog due to an inventory shortage ant the amount accumulates up to 325.30 shirts. When market price is initially quoted above equilibrium at \$25, excess supply causes inventory of unsold shirts to piles up to 137.86 shirts, as illustrated in the right-hand diagram of Figure 6. If the auctioneer is allowed to have these amount of inventories from the beginning, he could find an equilibrium price even by allowing these inter-auction transactions. Since no shirts are made available until the equilibrium contract is made and production activities start under the neoclassical rule of market game, this short-side off-equilibrium deal is logically impossible. In other words, no feedback loop is made available without

Figure 6: Short-side Transaction Model and Inventory

inventory from the viewpoint of system dynamics. In conclusion, the existence of chaotic price behavior and neoclassical assumption of market economy are inconsistent.

4 Price Adjustment with Inventory

The above analysis indicates it's time to abandon the neoclassical framework based on logical time. In reality, production and transaction activities take place week by week, and month by month at short-side of product availability, accompanied by piled-up inventory or backlog. Time flow on which these activities keep going is called *historical time* in [3]. In system dynamics, demand and supply are regarded as the amount of flow per week, and flow eventually requires its stock as inventory to store products. Thanks to the inventory stock, transactions now need not be waited until the auctioneer finishes his endless search for an equilibrium. This is a common sense, and even kids understand this logic. In other words, a price adjustment process turns out to require inventory from the beginning of its analysis, which in turn makes off-equilibrium transactions possible on a flow of historical time.

This disequilibrium approach is the only realistic method of analyzing market economy, and system dynamics modeling make it possible. The model running on historical time for simulations, which is based on [2]. is drawn in Figure 7.

Price no longer need to respond to the excess demand, instead it tries to adjust to the gap between inventory and desired inventory. To avoid a shortage under off-equilibrium transactions, producers usually try to keep several weeks of the demanded amount in inventory. This amount is called desired inventory. An inventory ratio is thus calculated as the inventory divided by the desired inventory. And market prices are assumed to respond to this ratio. Table 2

Figure 7: Price Adjustment Model with Inventory

specifies the effect of the ratio on price in this model. For instance, if the actual inventory is 20% larger than the desired inventory, price is assumed to be lowered by 25%. Vice versa, if it's 20% smaller, then price is assumed to be raised by 35%.

Inventory Ratio	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
Effect on Price	1.8	1.55	1.35	1.15	1	0.875	0.75	0.65	0.55

Table 2: Effect of Inventory Ratio on Price

Mathematically, the model is formulated as follows:

$$\frac{dp(t)}{dt} = \frac{p^* - p(t)}{PCD} \tag{4}$$

where PCD is a parameter of price change delay, and p^* is a desired price such that

$$p^* = p(t)g\left(\frac{x(t)}{x^*}\right) \tag{5}$$

Function g is a formal presentation of the numerical relation in Table 2, and x(t) and x^* denote inventory and desired inventory, respectively, such that

$$\frac{dx(t)}{dt} = S(p) - D(p) \tag{6}$$

$$x^* = \alpha D(p) \tag{7}$$

where α is a parameter of desired inventory coverage.

Under such circumstances, the initial price is here set at \$10 as in the case of the auctioneer's tâtonnement. Price (line 5) now fluctuates around the equilibrium price of \$15 by overshooting and undershooting alternatively, then tends to

converge to the equilibrium as illustrated in Figure 8. Inventory gap (= desired inventory - inventory) is the gap between line 4 and 3, and price responds to this gap rather than the excess demand (the gap between line 1 and 2). The reader can easily confirm that price tends to rise as long as the inventory gap is positive, or inventory ratio is lower than one, and vice versa.

Figure 9: Effect of the Change in Demand, Supply and Inventory Coverage on Price

In the left-hand diagram of Figure 9, demand is increased by 20 units at the week of 15, followed by the increase in the same amount of supply at the week of 30, restoring the original equilibrium as in the case of the auctioneer's tâtonnement, though overshooting this time. These shifts of demand and supply curves, however, may no longer be appropriate to be called comparative static analysis method in microeconomics, because we are no longer comparing two different states of equilibrium points. Right-hand diagram illustrates how price cycle is triggered by reducing the original inventory coverage of 4 weeks to 2.3 weeks. In conclusion, system dynamics modeling makes it possible to describe the actual off-equilibrium transactions and price behaviors along the historical time.

5 Logical vs Historical Time

A combined model is created in Figure 10 to compare how the above two price adjustment processes behave differently; one is running on logical time and the other on historical time.

Figure 10: Auctioneer vs Inventory Price Mechanism Compared

Left-hand diagram of Figure 11 is produced to show similar patterns by setting the auctioneer's adjustment coefficient to be 2.7. In both cases it takes about 100 weeks to attain the equilibrium. The difference is that under logical time production and transactions never take place until the equilibrium is attained around the logical time of 100 weeks, while a real economy running on the historical time is suffering from the fluctuation of inventory business cycles for 100 weeks until a real equilibrium price is attained.

What will happen if the demand suddenly increases by 20 at week 50. Righthand diagram illustrates the real economy can no longer attain the equilibrium in 100 weeks. In this way the market economy is forced to be fluctuating around

Figure 11: Auctioneer vs Inventory Price Behavior

off-equilibrium points forever in face of continued outside shocks, compared with a quick realization of the equilibrium by the auctioneer around the logical time of week 70.

The meaning of logical and historical times is now clear. Microeconomic textbooks are full of logical time analyses when dynamics of price movements are discussed. The reader now has the right to ask if the time in textbooks is logical or historical. If historical, price has to be always accompanied by the inventory on historical time.

6 Stability on A Historical Time

Which path, then, should we follow to analyze free market economic activities? Neoclassical analysis of logical time is mathematically rigorous, yet free price behavior is no longer stable, as preached by market fundamentalists, due to the appearance of chaos as shown above. In other words, market economy could be chaotic even on the basis of neoclassical doctrine.

On the other hand, analysis running on historical time is off-equilibrium and looks unstable full of business cycles; that is, chaotic as well. Yet, there's a way to make the historical time analysis stable and free from business cycles. To do so, let us now change the seller's supply schedule so that it can reflect the inventory gap as follows:

Mathematically, equation (6) is replaced with the following:

$$\frac{dx(t)}{dt} = S^*(p) - D(p) \tag{9}$$

$$S^{*}(p) = S(p) + \frac{x^{*} - x(t)}{IAT}$$
(10)

where IAT is inventory adjustment Time.

Figure 12: Historical Price Stability wiht Adjusted Supply Schedule (1)

Left-hand diagram of Figure 12 illustrates how price behaviors are different between logical time (line 1) and historical time (line 2) when demand is increased by 20 units at the week of 15, followed by the increase in the same amount of supply at the week of 30. In both cases prices try to restore the original equilibria, though their speed and meaning are different. In the right-hand diagram, newly adjusted supply schedule is now applied with the inventory adjustment time of 3 weeks. To our surprise, almost the same price behavior (line 2) is obtained as the one on the logical time (line 1).

Figure 13: Historical Price Stability with Adjusted Supply Schedule (2)

In the left-hand diagram of Figure 13, price behavior on the logical time is illustrated as line 1 for the initial price at \$10, while the same price behavior on the historical time is illustrated as line 2 for the inventory coverage of 2.3 weeks, similar to the right-hand diagram of Figure 9. Now the new supply schedule is applied to the same situation, which results in line 3. Again, the line 3 becomes very similar to the price behavior (line 1) on the logical time.

Finally let us apply the new supply schedule to the right-hand diagram of Figure 11, that is previously explained as the case in which "the real economy can no longer attain the equilibrium in 100 weeks." Right-hand diagram of Figure 13 is the result obtained by newly adjusted supply with the inventory adjustment time of 3 weeks. Again almost similar price behavior is restored as the one on the logical time.

These simulation results may indicate that our market economy could behave as close as the one predicted by neoclassical equilibrium analysis on logical time so long as economic agents behave appropriately on the historical off-equilibrium time. In other words, we no longer need a help from auctioneer running on logical time to attain an equilibrium in a market economy. Price, inventory and their interdependent feedback relations can do the same job in a real market economy.

Conclusions

Recent global financial crisis may be an indication that the dominance of neoclassical doctrine of free market fundamentalism since 1980's may be close to its end. By constructing a simple system dynamics model, two price adjustment mechanisms are compared; one running on logical time, the other on historical time. It is shown that the attainment of equilibrium is possible even under off-equilibrium processes of transaction on historical time, if interplay of price, inventory and their interdependent feedback relations are well modeled using system dynamics. We no longer need neoclassical concept of auctioneer and logical time for an economic analysis of real market economy.

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