

## Objective Analysis of Subjective Feedback Structures: The Problem of Consistency in Explaining Model Behavior

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### Abstract

Real-world concepts can be operationalized into variety of feedback structures which may be mathematically identical but diverse in the number of feedback loops. Factors including model purpose, the modelers' perspective and the intended audience all influence the final layout of a feedback rich model. One challenge in the analysis of model behavior is to account for the variations in the appearance of its structure and the feedback loops. This paper focuses on consistency in explaining model behavior and illustrates some of the issues related to the cancellation problem and figure-8 loops. Both conditions can potentially lead to poor and even contradictory explanations of model behavior based on its idiosyncratic feedback structure. The paper concludes by illustrating how the pathway participation approach addresses these two issues and calls for comparative studies to using alternative approaches to model analysis to better understand the general principles and subtleties in connecting the structure to the behavior and explaining observed dynamics. Different methods in formal analysis can learn from one another and expedite the development of user-friendly tools to aid model analysis that serve a wider audience.

### Introduction

At the heart of system dynamics are consistent, coherent and dynamically correct causal explanations about how the system's structure influences its behavior over time. It has been persuasively argued that intuitive understanding of dynamic systems is prone to error (Forrester, 1994; Peterson et al, 1994). Although simulation reveals the dynamics of complex systems and facilitates performing "what if" analysis, it is insufficient in providing consistent explanations about why the system does what it does. Significant progress has been made in the last several decade in developing tools and methods that can enhance modelers' intuition on dynamics consequences of feedback structures. Yet more challenges are in the way. (Richardson, 1996; Sterman, 2000)

One of the difficulties facing formal model analysis is that real-world concepts can be operationalized into a variety of feedback structures, which may be mathematically identical<sup>1</sup>, but diverse in the number of feedback loops. Such diversity can potentially lead to inconsistent and incorrect stories about how the structure contributes to the behavioral dynamics. "Non-dynamic" feedback loops (Lyneis and Lyneis, 2006), "phantom" loop (Kampmann and Oliva, 2006, 2008) and "figure-8" loops (Mojtahedzadeh, 1997;

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<sup>1</sup> Mathematically identical models are defined, here, as models that may differ the number of feedback loops they contain, but have the same reduced form. Reduced form of dynamic models can be obtained by substitution of auxiliary variables into the corresponding net rate equations (Sterman 2000, p. 203).

Güneralp, 2006) present a subset of a bigger challenge related to feedback representation of model structure that explains its dynamic behavior.

This paper aims to explore the issue of consistency in explanations for systems' behavior based on its feedback structure. It discusses how the pathway participation metric (PPM) approach maintains consistency in mathematically identical models despite variations in the number of feedback loops they contain. In doing so, the paper focuses on two outstanding issues, cancellation and figure-8 loops that defy intuitive descriptions of structure-behavior relationships. These two problems are merely a subset of the larger problem arising from different feedback-loop structures in mathematically identical models; they, nevertheless, help to understand the consistency issue in model analysis. The paper includes four case studies that present the cancellation and figure-8 loop problems. The first case study draws upon the recent work by Lyneis and Lyneis (2006) on simple epidemic models with mathematically identical equations but different feedback structure. The second and third case studies focus on figure-8 loops and illustrate how adding or omitting auxiliaries can hide important feedback loops in the visual diagrams which are vital in explaining the observed dynamic behavior. The fourth case study illustrates more subtle examples of cancellation problem. Both problems of cancellation and figure-8 loops come from including additional auxiliaries (or flows) into the structure and can potentially lead to incorrect and inconsistent explanation of model behavior. The case studies show how the PPM approach detects the dominant structure and avoids inconsistencies in explaining model behavior, regardless of the choice in auxiliaries, algebraic expressions and the layout of the model.

### **Consistency in Explaining Model Behavior**

Mathematical descriptions of dynamic systems require state variables and net-flows. It is only these two classes of variables, as well as their relationships, that determine the dynamics of the systems. However, for the purpose of better communication and clarity (Sterman 2000), it is often helpful to include auxiliaries, and to break down the net-flow into meaningful inflows and outflows in the model. Auxiliaries help to operationalize the model based on real-world concepts and variables. They, nevertheless, do not impact the dynamics of the closed-loop structure, although, they greatly influence the visualization of the structure and the number of feedback loops that the structure contains.

Auxiliaries are intermediate variables that "are algebraically substitutable into the subsequent rate equations and are structurally part of the rate equations" (Forrester, 1968). According to Road Maps<sup>2</sup>, an auxiliary is "a subdivision of rate equation that allows a model to be disaggregated into easier to understand equation statements." (Road Maps 9, D-4509-2). Sterman (2000, p. 203) encourages modelers to avoid "economizing on the number of equations" and to include auxiliaries that help to clearly express the main idea and relevant real world concepts. Similarly, Lyneis and Lyneis (2006, p. 4) state "using multiple algebraic expressions within variables violates a standard system dynamics modeling practice".

Despite their contribution in enhancing clarity and ease of communication, the inclusion of auxiliaries may increase the number of feedback loops within the structure<sup>3</sup>. Kampmann (1996) derived the number

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<sup>2</sup> Road Maps is a self-study guide to learning system dynamics developed at MIT under Jay Forrester's direction. For more information visit: <http://sysdyn.clexchange.org/road-maps/home.html>

<sup>3</sup> Additional flows (other than net flow) can also increase the number of feedback. Furthermore, any inference about the net rates of state variable may change the number of feedback loops visible in the structure. (For examples and further discussions see cases studies reported in next sections of this article).

of feedback loops in a maximally connected system—where each net rate is determined by all state variables-- given the number of state variables and auxiliary variables. The result of his study is summarized in Table 1. While most models are not maximally connected, Kampmann’s calculations show the potential impact of auxiliaries in the expansion of feedback loops in a dynamic model.

Table 1 makes the point that in large-scale models, adding one auxiliary to a model may have a larger impact on the number of feedback loops than adding one state variable (stock). For instance, adding the first auxiliary to a third order model may increase the number of feedback loops from 8 to 34 while an additional state variable will increase the number of feedback loops to 24. The difference is greater for higher order models. In a fourth-order system, while an additional auxiliary may increase the number of feedbacks by a factor of seven, with virtually no impact on the dynamics of the system; an additional state variable, which may dramatically change the dynamics, will only increase the number of feedbacks by a factor of three.

State variable	Auxiliary variables		
	0	1	5
1	1	2	32
2	3	8	1,088
3	8	34	68,704
4	24	192	10 <sup>6</sup>
5	89	1,458	10 <sup>8</sup>

Table 1: Number of Feedback Loops in a Fully Connected System  
 From: CE Kampmann, 1996, Feedback loop gains and system behavior

Notwithstanding their contribution in understanding the equations, enhancing clarity and ease of communication, auxiliaries present a challenge to model analysis, that is, *how to maintain consistency and correctness in the explanations for observed dynamics?* The generous use of auxiliaries exponentially increases the number of feedback loops in a model, greatly complicating the task of consistently explaining mathematically identical models. Since auxiliaries do not create any dynamics of their own, the 192 feedback loops in a fully connected fourth-order system with one auxiliary are essentially a “breakdown” of the 24 feedbacks found in the reduced form. Unless carefully analyzed, the additional feedbacks produced by the auxiliaries introduced in a model can mislead and cause errors and inconsistencies in detecting the dominant structures.

Although the issue of consistency in explaining model behavior is not adequately addressed in formal approaches to model analysis, a number of scholars have pointed out to the potential problems and challenges in working diagrammatic tools to understand the structure (Richardson 1986; Lane 2008) connecting the its feedbacks loops to the observed behavior. Lyneis and Lyneis (2006) show that due to alternative formulations some of the feedback loops in a model may be “non-dynamic” that can “obscure the focus on the essential dynamic loops” (page 19) and distort “an understanding of the direct relationship between feedback structure” (page 12). In reviewing eigenvalue elasticity approach to formal model analysis, Kampmann and Oliva (2008) recognize the problem of what they call “phantom” or “artificial” feedback loops and define as loops that “cancel each other by logical necessity and are essentially artifacts of equation formulation used in model”(page 513). The authors suggest that the phantom loops “could nonetheless have large elasticities and thus seriously distort the interpretation of the results”. Güneralp (2006) also notes that “elasticity of a feedback loop can be negated by elasticity of another if the gains of these loops contribute to exactly the same compact loop gains” (page 286) and calls for caution in “interpreting the weighted loop influence plots” when “opposing loops” are present. The problem of “artificial” feedback loops, according to Kampmann and Oliva (2008), “may not be intractable but their resolution will require careful mathematical analysis” (page 513).

### **Consistency in Pathway Participation Approach**

The story told about the observed behavior based on the underlying feedback structure using pathway participation approach remains consistent regardless of the number of auxiliaries and the number of feedback loops in the model. The main reasons that the PPM approach to model analysis avoids the problems of figure-8 loops and non-dynamic loops are two folds:

- Both identification of feedback loops and detection of dominant structure in this approach are based on pathway that are recognized with model equations.
- The search algorithm identifies the dominant structure in multiple stages.

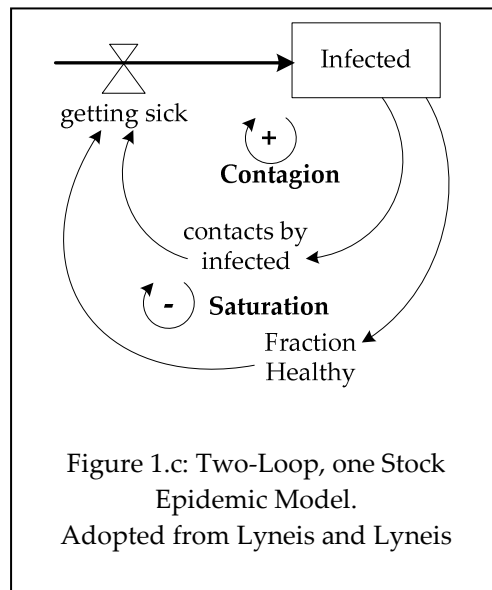
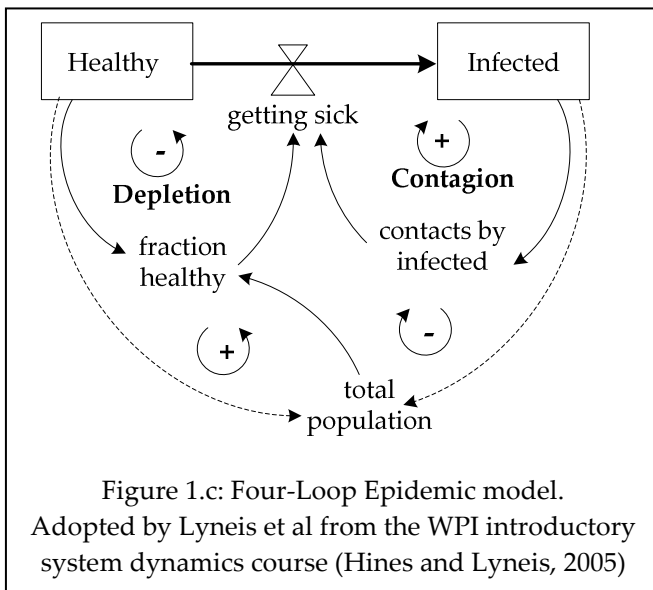
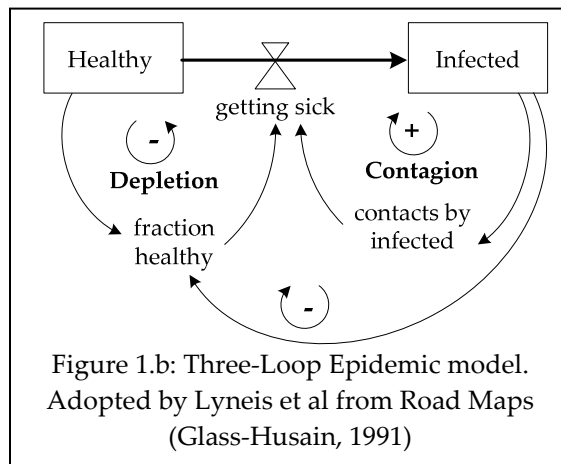
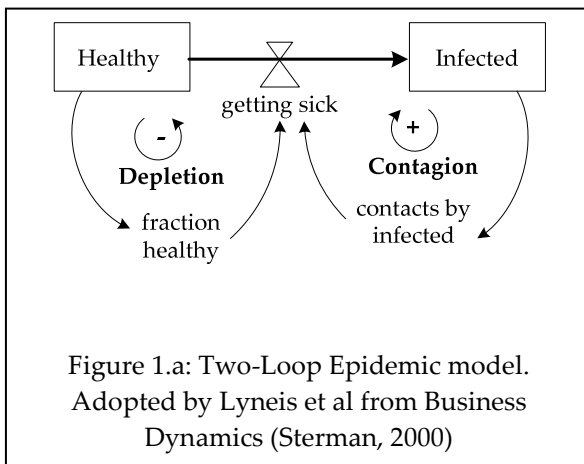
In pathway participation approach, pathways, links of causal structure between two system stocks, are envisioned as the primary building blocks of feedback loops. Pathways as construct of the feedback loops are identified by the model equations and not the schematic display of the model structure. As discussed, the second and third case studies, the method of indentifying feedback loop by visual inspection of the structure can be misleading. The second case study demonstrates how PPM reveals a hidden second-order feedback loop that is needed to explain an oscillatory behavior in the system.

Detecting the dominant structure in pathway participation approach is also based on pathways and not loops. According to pathway participation approach, the dominant structure is a set of most influential pathways that connects one state variable to another (and form a feedback loop). In most cases, in the absence of any auxiliaries there is only one pathway connecting two state variables, however, when auxiliaries are added to the model, the number pathways connecting two state variables increases, but one of the pathways will always be dominant. As a result, the order of dominant loop will remain the same when auxiliaries are inserted or eliminated. The third case study shows that a second order feedback loop, figure-8 loop, is dominant in the growth phase of urban dynamics regardless of the presence auxiliaries in the model. Indeed, to assure the consistency in explaining model behavior, it is essential that the order of the detected dominant feedback loop to remain identical regardless of the number of auxiliaries.

The pathway participation approach does not compare feedback loops around two different stocks; it only compares pathway (and first-order loop) reaching the state variable of interest. Any cancellation that occurs around a state variable is reflected in the total and partial derivatives in the participation metrics. Consequently, the “non-dynamic” or “phantom loop” will not be selected as dominant. The first case study demonstrates how non-dynamics feedback loops remain dormant even with alternative formulation and operationalization of the model. Furthermore, to avoid choosing a feedback loop or a pathway with large participation metrics that may be canceled out by another, the search algorithm for selecting dominant structures groups (aggregates) pathways according to the state variables at the head and the tail of the pathways and select the most influential aggregate pathways. The pathway with largest the participation metric at the aggregate and individual level will be considered as dominant. The fourth case study provides two examples containing opposing feedback loops and shows how pathway participation approach can avoid the potential errors and inconsistencies related to the cancelation problem.

**Case 1: Alternative Feedback Structure for Epidemic Model:**

In their article Lyneis and Lyneis (2006) present different versions of the epidemic model which are “exactly the same equation structure, all producing identical behavior” but they differ in the number of feedback loop they contain. Figure 1 shows the feedback structures for the four versions of the epidemic model. All these four structures in Figure 1 are all correct and pass alternative tests developed for examining the system dynamics models (Sterman 2000), including dimensional consistency, integration error and extreme condition tests. These models are mathematically identical and produce identical behavior. Through careful comparison of the four epidemic model versions, Lyneis and Lyneis (2006) raise an important challenging question related to the analysis of model behavior: “How can the same behavior be explained with such different feedback structures?” While only the reinforcing Contagion and balancing Depletion (or Saturation) feedback loops are sufficient to explain the S-shaped growth in the behavior of the model, the challenge is to make sure that the additional feedback loops do not become part of the dominant structure in the three and four-loop models.



The concepts of “relevance”<sup>4</sup> and “elegance” help to compare and contrast the four alternative structures of the epidemic model. Lyneis and Lyneis (2006) argue that the two-loop epidemic models are more elegant because they contain just the two loops needed to explain the S-shaped pattern in the behavior of Healthy and Infected population. The four-loop model, Figure 1.c, is more relevant to the real world situation as total population is operationalized in the model; however, they are not as elegant as the two-loop model because the two extra loops can be misleading in the analysis of simulation outcomes. The relevance criterion seems to influence some tests outlined for model validation process such as the one developed for structural assessment (Sterman, 2000). Practitioners working on developing real-world business models are often concerned about relevance to build on their clients’ confidence that the model adequately and accurately represent the system. The elegance principle, however, merely help modelers to assure correct and consistent explanations of the system’s behavior based on its feedback structure. The balance between elegance and relevance that Lyneis and Lyneis (2006) arguably call for is a nice solution that may be hard to achieve, if not impossible, in complex large scale models. In fact, the need for elegance principle mainly comes from lack of tools and techniques for model analysis regardless of how the model is operationalized and formulated.

The three-loop model possibility leans toward the relevance criterion because the total population is implicitly operationalized. In the other hand, it only comes with one additional loop which diminishes the chance of incorrectly picking the dominant structure. However, it is at odd with another standard which suggests avoiding multiple algebraic expressions within a variable. Compliance with this standard can lead to even more additional feedback loops. Adding total population as an auxiliary to model equations helps to avoid multiple equations in fraction healthy, but as Lyneis and Lyneis (2006) have pointed out, it increases the number of feedback loops as shown in Figure 1.b and 1.c.

The biggest challenge is in commercial models. It is likely that in larger models, a number of feedback loops will often remain dormant and do not contribute much or even at all in creating the observed dynamics of the system. Much of these feedback loops are needed to include different perspectives on how the system work in order to build confidence. However, identifying and highlighting the part of feedback structure that remain dormant require extensive experience in working with large scale models. As a result, formal model analysis is perhaps the only practical solution to this outstanding issue of connecting system’s behavior to its structure.

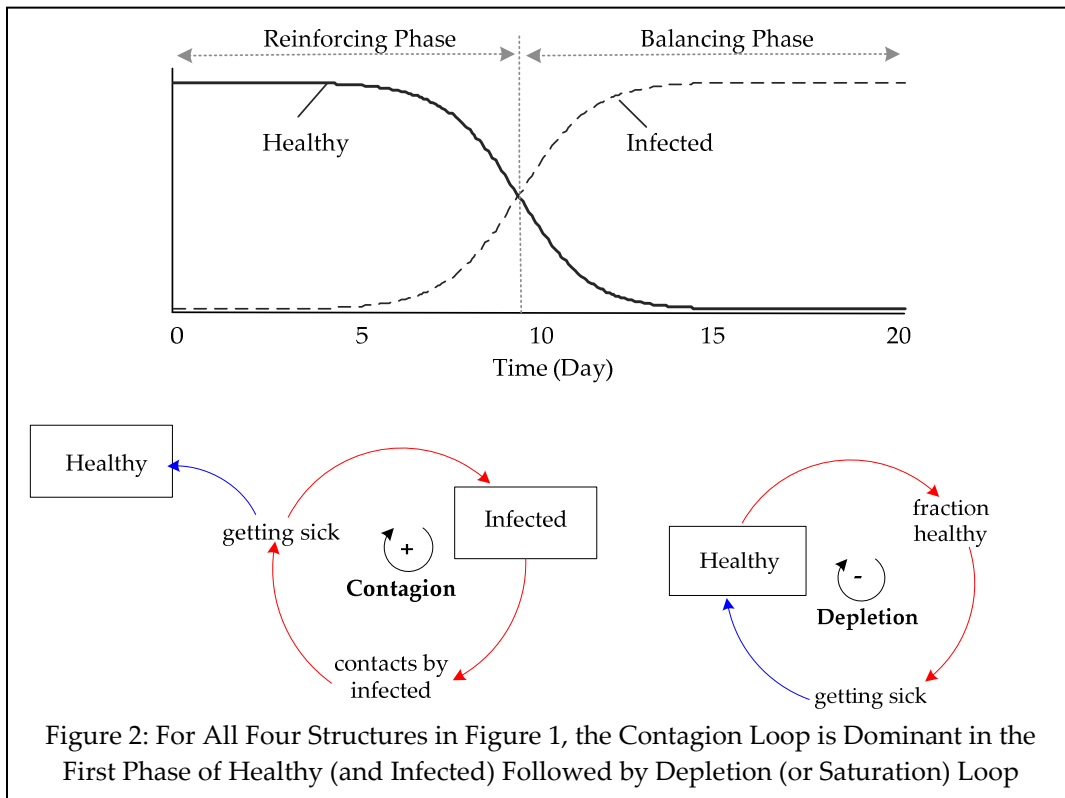
***Using PPM Approach to detect the Dominant Structure in Epidemic model:***

The application of pathway participation metrics in the four different feedback structures shown in Figure 1 suggests that the reinforcing Contagion loop is dominant in the early phase of the behavior of the Healthy and Infected population which later shifts to Depletion loop. Figure 2 depicts the two phases of the behavior of Healthy and the dominant structure in each phase, according to pathway participation metrics. In the first phase that lasts until day 9.5, Healthy population follows a reinforcing pattern. During this period, Healthy population is highly influenced by the pathway that connects Infected to Healthy population through contacts by infected and getting sick. At the same time, the behavior of Infected is driven by the reinforcing Contagion loop. As a result, the Infected-Healthy pathway (that goes through contacts by infected, getting sick and riches Infected) together with the Contagion loop explain the reinforcing decline in the Healthy population. Around day 9.5, the Health population experiences a shift in its behavior pattern from reinforcing to balancing and the Depletion loop dominates for the rest of

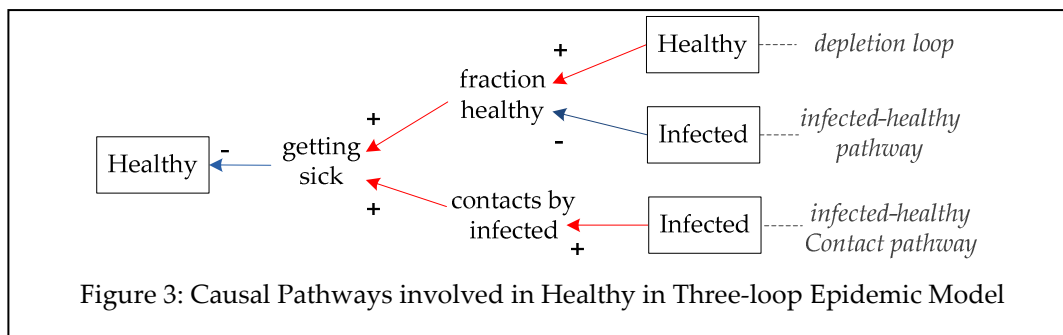
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<sup>4</sup> Lyneis and Lyneis (2006) use the word “operational” to describe the concept of “relevance” to the real world situation.

the simulation. Different algebraic expressions that led to different feedback structures shown in Figure 1 does not seem to change the dominant structure detected by pathway participation metrics.



The difference between the three and four-loop model is the use of multiple algebraic expressions in the variable total population. This leads to additional feedback loop. Figure 3 and Figure 4 depict the pathways that are involved in the Healthy population in three-loop (Figure 1.b) and four-loop (Figure 1.c) models, respectively. Healthy population in the three-loop model contains one first-order feedback loop and two pathways the starts with Infected, whereas in the four-loop model, it involves two first-order loops and two pathways starting with Infected. The difference in the number of loops and pathways in the two models comes from additional auxiliary, total population, explicitly formulated in the four-loop model.



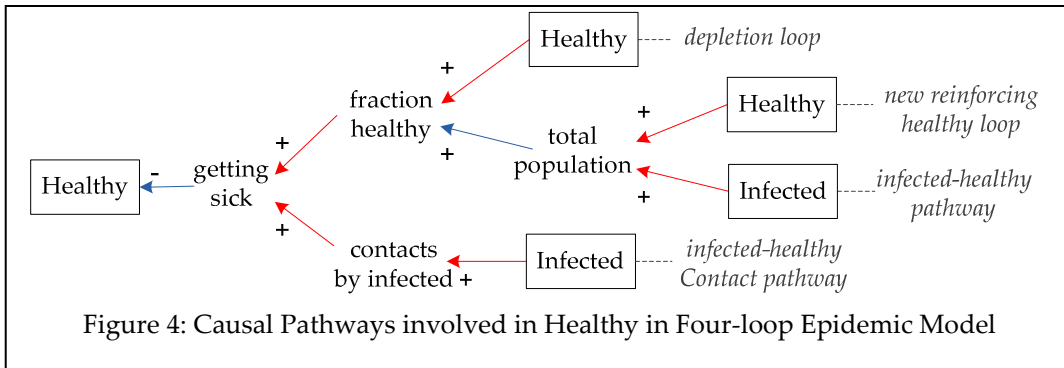


Table 1.a, 1.b, 2.a and 2.b show the participation metrics for pathways involved in Healthy and Infected population for the three-loop (Figure 1.b) and four-loop model (figure 1.c), respectively. According to the tables, for both models, the total participation metrics for Healthy and Infected population and the aggregate pathways (and first-order loops) are equal. In both models Infected-Healthy Contact pathway is dominant in early phase of the Healthy population. During this phase, in both models Contagion loop is mainly responsible for the reinforcing growth in Infected population. In both models, Depletion loop remains dominant as long as Healthy population follows a balancing pattern -- total pathway participation metrics is negative. Therefore, avoiding multiple algebraic expressions and including additional auxiliary, total population, the new feedback loop that it creates does not change the dominant structure identified by pathway participation approach.

Time (Days)	0	2	4	6	8	9.5	10	12	14	16	18	20
<b>Total PPM for Healthy</b>	<b>1</b>	<b>1</b>	<b>0.99</b>	<b>0.94</b>	<b>0.61</b>	<b>-0.02</b>	<b>-0.27</b>	<b>-0.86</b>	<b>-0.98</b>	<b>-1</b>	<b>-1</b>	<b>-1</b>
<b>Aggregate first-order loops</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>-0</b>	<b>-0.04</b>	<b>-0.26</b>	<b>-0.4</b>	<b>-0.87</b>	<b>-0.98</b>	<b>-1</b>	<b>-1</b>	<b>-1</b>
Depletion loop	0	0	0	-0	-0.04	-0.26	-0.4	-0.87	-0.98	-1	-1	-1
<b>Aggregate Infected-Healthy pathways</b>	<b>1</b>	<b>1</b>	<b>0.99</b>	<b>0.94</b>	<b>0.65</b>	<b>0.24</b>	<b>0.13</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
Infected-Healthy pathway	0	-0	0	-0.03	-0.16	-0.25	-0.23	-0.06	-0.01	-0	0	0
Infected-Healthy Contact pathway	1	1	0.99	0.97	0.81	0.49	0.37	0.07	0.01	0	0	0

Table 2.a: Pathway Participation Metrics for Healthy in Three-loop Epidemic Model (Figure 1.b)

Time (Days)	0	2	4	6	8	9.5	10	12	14	16	18	20
<b>Total PPM for Infected</b>	<b>1</b>	<b>1</b>	<b>0.99</b>	<b>0.94</b>	<b>0.61</b>	<b>-0.02</b>	<b>-0.27</b>	<b>-0.86</b>	<b>-0.98</b>	<b>-1</b>	<b>-1</b>	<b>-1</b>
<b>Aggregate first-order loops</b>	<b>1</b>	<b>1</b>	<b>0.99</b>	<b>0.94</b>	<b>0.65</b>	<b>0.24</b>	<b>0.13</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
Contagion	1	1	0.99	0.97	0.81	0.49	0.37	0.07	0.01	0	0	0
New balancing infected loop	0	-0	-0	-0.03	-0.16	-0.25	-0.23	-0.06	-0.01	-0	0	0
<b>Aggregate Infected-Healthy pathways</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>-0</b>	<b>-0.04</b>	<b>-0.26</b>	<b>-0.4</b>	<b>-0.87</b>	<b>-0.98</b>	<b>-1</b>	<b>-1</b>	<b>-1</b>
Healthy-Infected Contact pathway	0	0	0	-0	-0.04	-0.26	-0.4	-0.87	-0.98	-1	-1	-1

Table 2.b: Pathway Participation Metrics for Infected in Three-loop Epidemic Model (Figure 1.b)



Time (Days)	0	2	4	6	8	9.5	10	12	14	16	18	20
<b>Total PPM for Healthy</b>	<b>1</b>	<b>1</b>	<b>0.99</b>	<b>0.94</b>	<b>0.61</b>	<b>-0.02</b>	<b>-0.27</b>	<b>-0.86</b>	<b>-0.98</b>	<b>-1</b>	<b>-1</b>	<b>-1</b>
<b>Aggregate first-order loops</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>-0.04</b>	<b>-0.26</b>	<b>-0.40</b>	<b>-0.87</b>	<b>-0.98</b>	<b>-1</b>	<b>-1</b>	<b>-1</b>
Depletion loop	0	0	0	-0.03	-0.19	-0.51	-0.63	-0.93	-0.99	-1	-1	-1
New reinforcing healthy loop	0	0	0	0.03	0.16	0.25	0.23	0.06	0.01	0	0	0
<b>Aggregate Infected-Healthy pathways</b>	<b>1</b>	<b>1</b>	<b>0.99</b>	<b>0.94</b>	<b>0.65</b>	<b>0.24</b>	<b>0.13</b>	<b>0.01</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
Infected-Healthy pathway	0	0	0	-0.03	-0.16	-0.25	-0.23	-0.06	-0.01	0	0	0
Infected-Healthy Contact pathway	1	1	0.99	0.97	0.81	0.49	0.37	0.07	0.01	0	0	0

Table 3.a: Pathway Participation Metrics for Healthy in Four-loop Epidemic Model (Figure 1.c)

Time (Days)	0	2	4	6	8	9.5	10	12	14	16	18	20
<b>Total PPM for Infected</b>	<b>1</b>	<b>1</b>	<b>0.99</b>	<b>0.94</b>	<b>0.61</b>	<b>-0.02</b>	<b>-0.27</b>	<b>-0.86</b>	<b>-0.98</b>	<b>-1</b>	<b>-1</b>	<b>-1</b>
<b>Aggregate first-order loops</b>	<b>1</b>	<b>1</b>	<b>0.99</b>	<b>0.94</b>	<b>0.65</b>	<b>0.24</b>	<b>0.13</b>	<b>0.01</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
Contagion Loop	1	1	0.99	0.97	0.81	0.49	0.37	0.07	0.01	0	0	0
New balancing infected loop	0	0	0	-0.03	-0.16	-0.25	-0.23	-0.06	-0.01	0	0	0
<b>Aggregate Infected-Healthy pathways</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>-0.04</b>	<b>-0.26</b>	<b>-0.40</b>	<b>-0.87</b>	<b>-0.98</b>	<b>-1</b>	<b>-1</b>	<b>-1</b>
Healthy-Infected pathway	0	0	0	0.03	0.16	0.25	0.23	0.06	0.01	0	0	0
Healthy-Infected Contact pathway	0	0	0	-0.03	-0.19	-0.51	-0.63	-0.93	-0.99	-1	-1	-1

Table 3.b: Pathway Participation Metrics for Infected in Four-loop Epidemic Model (Figure 1.c)

In both models the new feedback loops, new reinforcing healthy loop and the new balancing infected loop that emerge because of explicitly formulating total population are dormant. Consequently, the new feedback loops do not become part of the explanation of the observed behavior of Infected and Healthy population, and the story about the connection between behavior and structure remains consistent

Inspecting Table 3.a and 3.b indicates the new balancing loop and reinforcing loop have the same participation metrics in magnitude and different in sign. Because of the similarities between the two feedback loops, one might conclude that the two first-order loops cancel each other out. Pathway participation approach does not compare the feedback loops around different state variables. It is true the two new feedback loops in Figure 1.c disappear by replacing the equation for total population with its constant value. In practice, it is hard to make such generalization particularly when additional structure can change total population while maintaining the similarities between the two first-order loops. Using algebraic equation for total population, in three and four-loop models, introduces additional nonlinearity that can potentially have adverse impacts on the role of different feedback in the model. In the current model, such nonlinearity has no dynamic impact since total population is constant. Further, applying the concept of “cancelation” in the three-loop model does not help much to explain the role of the third loop around Infected in Figure 1.b.

### Case 2: Figure-8 Loop: When Epidemic Model Oscillate

Interestingly, adding one reinforcing loops around Healthy population and one balancing loops around Infected population in the epidemic model shown in Figure 1.a causes the model to oscillate, while the addition do not have such impact on the three-loop and four-loop models<sup>5</sup>. The new feedback loops

<sup>5</sup> The reason that the three and four-loop models does not oscillate in the presence of the two new loops is that the total population does not remain constant. In fact, the structures shown in the Figure 1 are mathematically identical given their existing structure but they may not be, if new structures are added.

brings the number of first-order loops in epidemic model (in Figure 1.a) up to four: Two first-order loops around Healthy and two first-order loops around Infected. The structure with the additional feedback loops may not necessarily be meaningful but it helps to analyze the system under different conditions. Figure 2 shows the feedback structure and the resulting behavior of the new “epidemic model”.

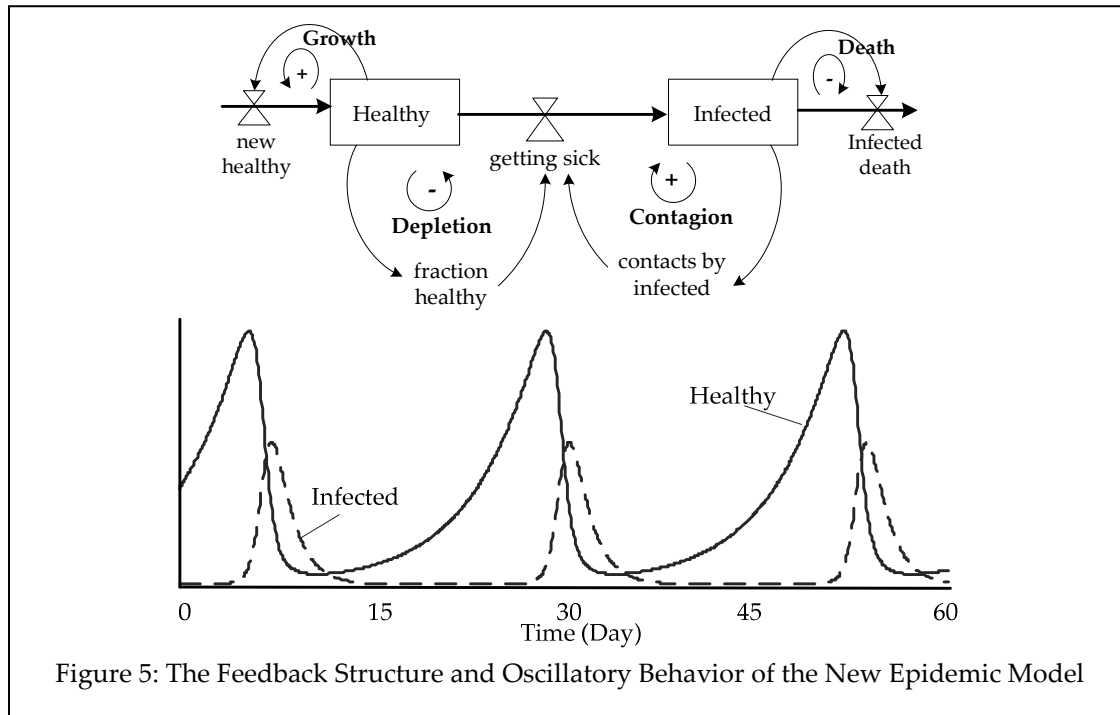
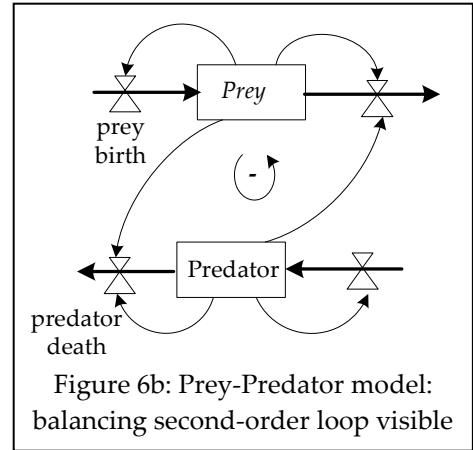
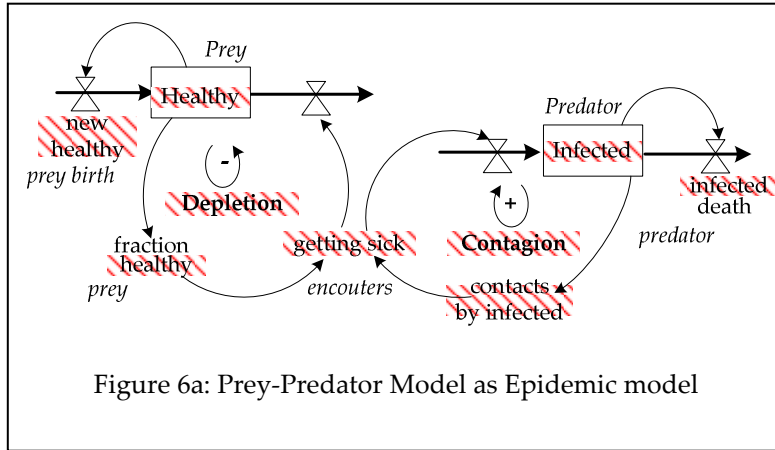


Figure 5: The Feedback Structure and Oscillatory Behavior of the New Epidemic Model

The challenge in the analysis of the ‘epidemic model’ shown in Figure 5 is to explain its oscillatory behavior based on the feedback structure involving four first-order loops around Healthy and Infected. However, at least one second-order loop (or higher) is necessary to make sense of cycles in oscillatory systems (Graham, 1974, Sterman, 2000). Another classical example is the Prey-Predator model. In fact, the modified ‘epidemic model’ is a special case of Prey-Predator model, when the outflow of Prey, prey death, happens to be exactly equal to the inflow of Predator, predator birth. Figure 6.a shows the similarity between the new epidemic and Prey-Predator models. The figure clearly depicts the four feedback loops around Prey and Predator, still no second-order loop visible in the system to explain the cycles in the behavior of variables of interest. Figure 6.b presents another layout of the Prey-Predator model that is exactly the same as the structure in Figure 6.a, and produces exactly the same behavior. The latter resulted from the omission of auxiliary variables from the former structure and including the corresponding equations in the inflow of Predator and the outflow of Prey. Consequently, the two structures in Figure 6.a and 6.b are mathematically identical. The second-order feedback loop around Prey and Predator is now visible as a result of removing the auxiliaries. With the evident second-order loops, it is possible to explain the cycles in the oscillatory behavior in Prey and Predator in terms of their feedback structure.

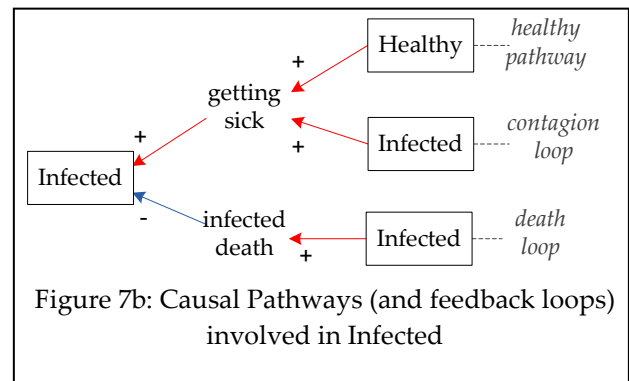
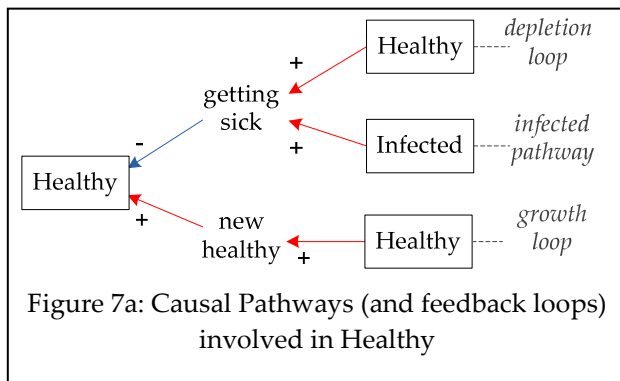


The emergence of the fifth feedback in Figure 6.b comes with the possibility of, at least, two diverse explanations for the behavior of two algebraically identical structures. The feedback structure shown in Figure 6.a is known as figure-8 loops and present a special structure where, at least, two loops passing through two stocks have at least one auxiliary variable in common. In the presence of the auxiliary, the graphical representation of the structure tends to hide the second-order loop that may play an important role in explaining the observed behavior. The issue of figure-8 loops is discussed in Mojtahedzadeh (1997) while analyzing the dynamics of URBAN1 model (Alfeld et al, 1976) using pathway participation metrics. Güneralp (2006) applies the concept of pathways to identify the hidden loop in Prey-Predator model and Kampmann and Oliva (2008) describe figure-8 loops as “specific puzzles relating to pathological cases” (page 518). Figure-8 loops are indeed puzzling as they can lead to inconsistent and incorrect explanations of observed dynamics.

**Using PPM Approach to detect the Dominant Structure in Observed Cycles:**

**Characterizing the Structure:**

Identifying pathways and feedback loops based on the model equations eliminate the error that can occur in visual inspections of the diagrams. The pathway participation approach identifies feedback loop in the model based on the pathways that connect on state variable recognized by the equation of the model, not the visual diagrams. As a result, the presence of auxiliaries and alternative algebraic expression does not prevent identifying dominant loops that remain hidden in schematic diagrams.



It is, in fact, the presence of the auxiliary variable, encounters, in Figure 6.a that makes the second-order loop in the Prey-Predator model invisible. To identify the hidden second-order loop in the new epidemic model in Figure 5, the pathways involved in Healthy and Infected population are identified based on the model equations. Figure 7.a and 7.b depicts pathways involved in Healthy and Infected population of the new epidemic model, respectively. According to Figure 7.a, there are three pathways involved in Healthy population: The two pathways that begin and end with Healthy are in fact first-order loops. The Infected pathway starts with Infected, goes through getting sick and ends with Healthy population. On the other hand, Infection population, as shown in Figure 7.b, contains two first-order loops and one pathway that start with Healthy population, passes through getting sick and riches Infected.

The pathway the starts with Infected, goes through getting sick and riches Healthy population in Figure 7.a along with the pathway involve in Infected in Figure 7.b that starts with Healthy form a second-order loop. This second-order loop, Interaction, as well as the four first-order loops in the new epidemic model is depicted in Figure 8.

### Dominant Structure in Observed Cycles

The application of pathway participation approach for identifying the dominant structure for observed cycles in Healthy and Infected population suggests the second-order loop, Interaction, is responsible for the periodicity of the cycles. Both Healthy and Infected experience longer and shorter half-cycles. For Healthy population, the longer half-cycles are unstable and their instability is mainly influenced by the Growth loop. The shorter half-cycles in Healthy population are stable and their stability is driven by the Depletion loop. In Infected population, the longer half-cycles are unstable and are mainly influenced by the balancing Death loop while the shorter half-cycles are unstable and are influenced by Contagion loop.

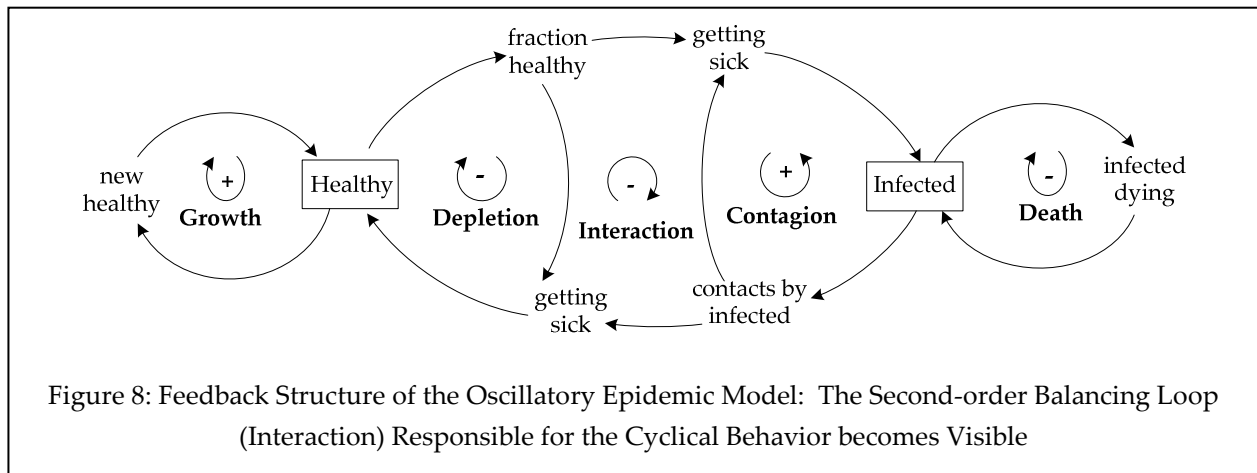


Table 1 and 2 summarize the pathway stability and pathway frequency factors<sup>6</sup> for half-cycles<sup>7</sup> in Healthy and Infected. According to Table 1, the first half-cycle in Healthy population begin at time 6.46 and last

<sup>6</sup> Pathway frequency ( $pf_{ij}$ ) and stability factors ( $psf_{ij}$ ) are derived from the participation metrics ( $ppm_{ij}$ ) in the beginning of the middle of a half-cycle.

$$pf_{ij} = \frac{\pi}{\omega} \cdot \frac{ppm_{ij}^*}{tppm_i^*} \Big|_{t=\omega} \quad \text{and} \quad psf_{ij} = ppm_{ij}^{\circ} \Big|_{t=\omega/2} \quad \text{or} \quad psf_{ij} = -pf_{ij} / \tan(\pi \omega_{ij}^{\circ} / \omega)$$

where  $ppm^*$  is the pathway participation metrics in the beginning of a half is-cycle,  $\omega$  is the duration of a half-cycle and  $ppm^{\circ}$  is the pathway participation metrics in the middle of a half cycle. Notice that these

about 5.33 days followed by another longer half-cycle that is 17.9 days long. These two half cycles form a complete 23.23 days cycle. Table 2 shows that Infected population also experiences 23.23 days cycles, however, the first half-cycle is 14.88 long and the second half-cycle is shorter and it is only 8.23 days.

Half-cycles	Time	duration	factors	Infected pathway	Healthy first-order loops			Total	Dominant
					Total	Depletion	Growth		
1	6.46	5.33	Freq.	<b>0.59</b>	0	0	0	<b>0.59</b>	Infected pathway
			Stab.	-0.62	-0.93	<b>-1.13</b>	0.2	<b>-1.55</b>	Depletion loop
2	11.79	17.9	Freq.	<b>0.18</b>	0	0	0	<b>0.18</b>	Infected pathway
			Stab.	0	0.2	0	<b>0.2</b>	<b>0.20</b>	Growth loop
3	29.69	5.33	Freq.	<b>0.59</b>	0	0	0	<b>0.59</b>	Infected pathway
			Stab.	-0.62	-0.93	<b>-1.13</b>	0.2	<b>-1.55</b>	Depletion loop
4	35.02	17.9	Freq.	<b>0.18</b>	0	0	0	<b>0.18</b>	Infected pathway
			Stab.	0	0.2	0	<b>0.2</b>	<b>0.20</b>	Growth loop

Table 4.a: Pathway Frequency and Stability Factors for the Half-Cycles in Healthy

Half-cycles	Time	duration	factors	healthy pathway	Infected first-order loops			Total	dominant
					Total	Contagion	Death		
1	8.23	14.88	Freq.	<b>0.21</b>	0	0	0	<b>0.21</b>	Healthy pathway
			Stab.	-0.06	-0.61	0.19	<b>-0.8</b>	<b>-0.67</b>	Death loop
2	23.11	8.35	Freq.	<b>0.38</b>	0	0	0	<b>0.38</b>	Healthy pathway
			Stab.	0.34	1.03	<b>1.83</b>	-0.8	<b>1.37</b>	Contagion loop
3	31.46	14.88	Freq.	<b>0.21</b>	0	0	0	<b>0.21</b>	Healthy pathway
			Stab.	-0.06	-0.61	0.19	<b>-0.8</b>	<b>-0.67</b>	Death loop
4	46.34	8.35	Freq.	<b>0.38</b>	0	0	0	<b>0.38</b>	Healthy pathway
			Stab.	0.34	1.03	<b>1.83</b>	-0.8	<b>1.37</b>	Contagion loop

Table 4.b: Pathway Frequency and Stability Factors for the Half-Cycles in Infected

As indicated in Table 4.a, the dominant pathway in creating the frequency of Healthy population is the Infected pathway that starts with Healthy, passes through getting sick and ends with Infected. Table 2 indicates that the dominant pathway responsible for the frequencies in Infected population is the Healthy pathway that starts with Healthy and ends with Infected. Therefore, based on pathway frequency factors the balancing second-order feedback loop (named Interaction in Figure 8) containing both Healthy and Infected is dominant for the cyclical behavior of the systems.

Table 4.b shows that the short half-cycle in Healthy has a negative total pathway stability factor (-1.55) which indicates stability. The Depletion loop is mainly responsible for the stability of the short half-cycles. The longer half-cycles in Healthy population is unstable because of its positive total pathway stability factor (0.2). The dominant structure for the instability of this half-cycle is the Growth reinforcing loop around Healthy population. The longer half-cycles in Infected population, based on the information

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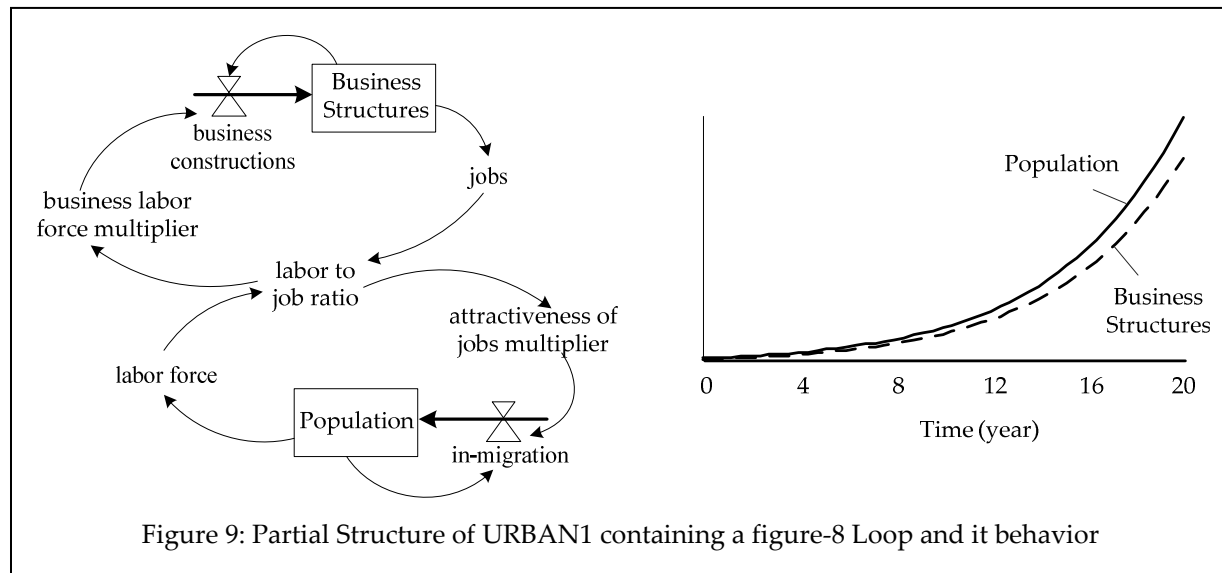
properties holds for linear systems in steady-states, therefore applying them to nonlinear systems is just rough approximations. (Mojtahadzadeh, 2009)

<sup>7</sup> A half-cycle defined as a period in which the total pathway participation metric changes its sign from positive to negative.

in Table 4.b, is stable (total stability factor is -0.67) and balancing death loop appears be dominant. For the shorter half-cycle in Infected population is unstable, the total stability factor is positive and the reinforcing Contagion loop in mainly responsible for the instability of the half-cycle.

### Case 3: Figure-8 Loop in Urban Dynamics

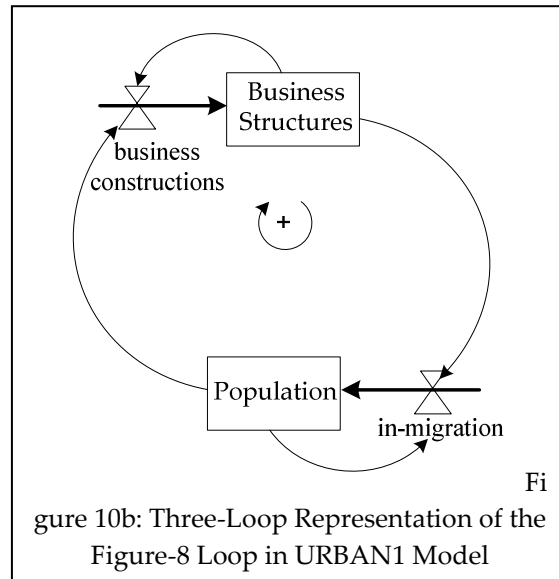
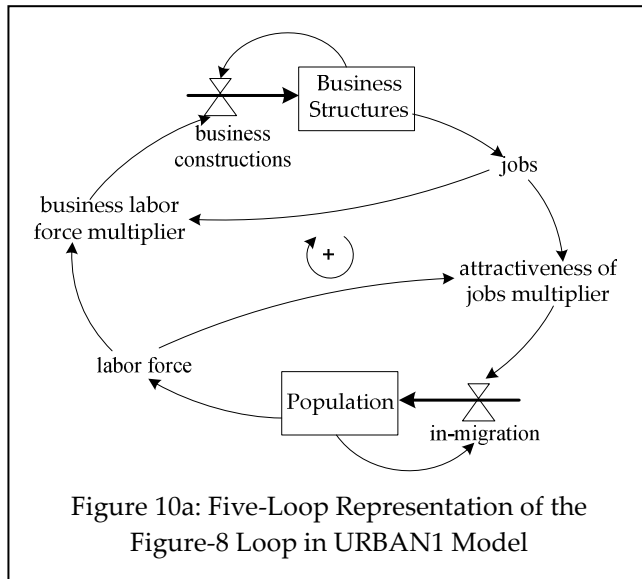
Another structure involving Figure-8 loop is the URBAN1 model (Alfeld et al, 1976). Overshoot behavior of the URBAN1 model is created by the in interactions among Business Structures, Population and Housing. Figure 9 shows the part of the model structure the entails the Figure-8 loop and its dynamic behavior. According to this structure, business constructions is determined by the size Business Structures, a fraction, business construction normal, and business labor force multiplier. In the other hand, in-migration is propositional to the Population size and is influenced by attractiveness of job multiplier. Both Attractiveness of job multiplier and business labor force multiplier are functions of labor to job ratio.



For the acceptable range of the model parameters, the structure shown in the above produces a reinforcing growth observed in the early phase of overshoot patterns. Visual inspection of the diagram indicates four first-order feedback loops; Business Structures and Population each contain one reinforcing and one balancing first-order loops. Given the four feedback loops identified from the structure, there are number of possible ways to explain the reinforcing growth in Business Structures and Population. One may explain the reinforcing growth in terms of one or both reinforcing first order loops around the state variables. Conversely, the balancing loops around one state variable may play the role of “catching up” if the reinforcing loop around another state variable is perceived as dominant. While this explanation for the reinforcing pattern may sounds plausible, the question is whether it holds if the very same model is presented with different auxiliaries and algebra?

Depending on the operationalization and the use of auxiliaries, the model can be represented in different feedback structures. Figure 10.a and 10.b depict two alternative feedback structures for the figure-8 loop in URBAN1 model. The two new structures are mathematically identical to the original model in Figure 9. The structure in Figure 10.a appears as a result of substituting for labor to job ratio in attractiveness of job multiplier and business labor force multiplier. It contains five feedback loops including the balancing

loops. The structure also reveals a second-order loop involving both Business Structures and Population with reinforcing polarity. Figure 10.b depicts the feedback structure of the reduced form where the equations are written in terms of net rates and state variable. The structure contains three reinforcing feedback loops; two first-order loops around Population and Business Structures and one second-order loop containing both stocks.



The “catching up” story told for the four-loop model (Figure 9) is supported by the five-loop model depicted in Figure 10.a as the structure preserves both balancing feedback loops. However, these balancing loops disappear in the three-loop model and thus the “catching up” story is no longer valid. On the other hand, the appearance of the reinforcing second-order feedback in Figure 10.a and 10.b can easily become part of the explanation of the exactly the same dynamics observed by the figure-8 loop, but it is essentially inconsistent with the original story. The challenge in model analysis is to provide consistent explanation for the observed behavior of mathematically identical models despite their differences in the number of feedback loop they contain. What happens when some feedback loops appear and disappear as a result of different representation of the same equations? Which one is correct? Which one is more elegant? Which feedback loops are dominant under what conditions?

**Using PPM Approach to Detect the Dominant Structure in URBAN1:**

Pathway participation approach identifies the same dominant feedback loop, and therefore tells consistent stories, regardless of variations in the number of feedback loop resulting from the choice of auxiliaries and algebra. Since pathway participation approach detects feedback loops based on pathways that connect one state variable to another, it recognizes the reinforcing second-order loop in the structure in Figure 9. In fact, according to pathway participation metrics, the second-order loop is dominant in creating the reinforcing growth in the model.

Figure 11.a and 11.b depicts pathways for Business Structures and Population in original figure-8 structure. The pathways involved Businesses Structures and Population form a second-order feedback loop which is difficult to detect by visual inspection of the structure in Figure 9. When both pathways are identified as the most influential in creating the reinforcing growth in Business Structures and Population, the second-order loop will be considered as dominant.

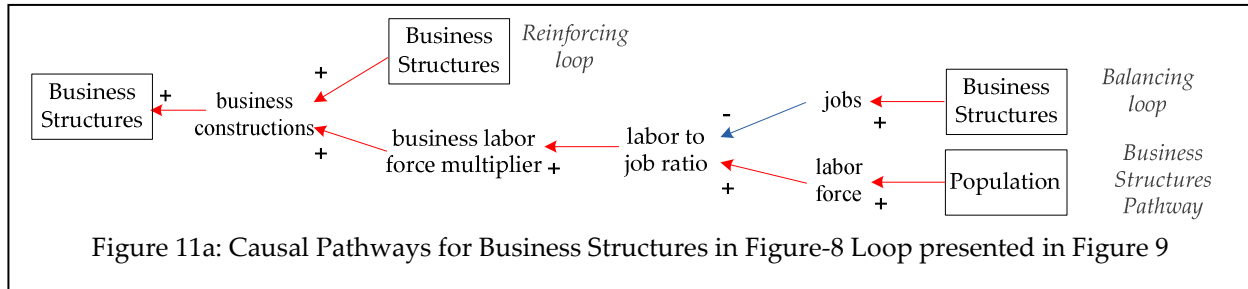


Figure 11a: Causal Pathways for Business Structures in Figure-8 Loop presented in Figure 9

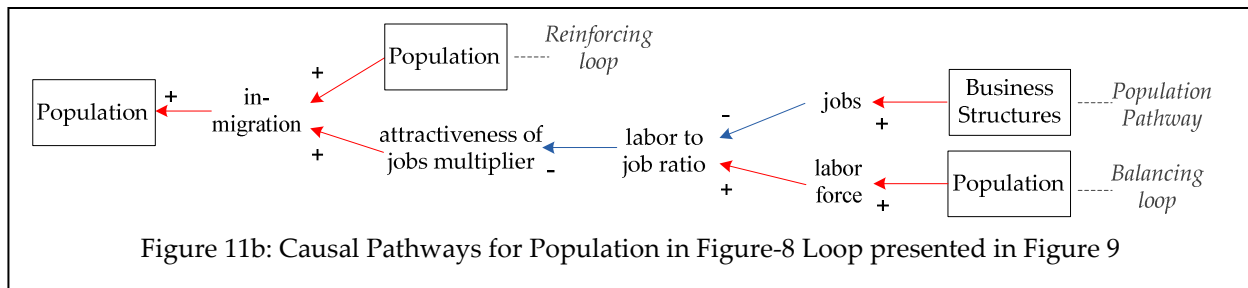


Figure 11b: Causal Pathways for Population in Figure-8 Loop presented in Figure 9

Table 5.a and 5.b display the participation metrics for pathway involved in Business Structures and Population shown in Figure 11.a and 11.b. The total participation metrics for Business Structures and Population is positive indicating a reinforcing growth in the behavior of both state variables. According to Table 6.a, the dominant pathway for Business Structures is the pathway that begins with Population and ends with Business Structures. The dominant pathway mainly responsible for the behavior of Population based on Table 6.b is the Business Structures-Population pathway. The two dominant pathways are in fact the second-order feedback loop that is invisible in Figure 9 but easily detectable in Figure 10.a and 10.b. Note that the participation metrics for pathways involved in Population and Business Structures in Figure 10.a and 10.b are the same those of Figure 9 indicating the appearance of the structure and the number feedback does not change the outcome of the analysis.

Time (Years)	0	2	4	6	8	10	12	14	16	18	20
<b>Total PPM for Business Structures</b>	0.12	0.11	0.09	0.09	0.08	0.08	0.08	0.08	0.08	0.08	0.08
<b>Agg. Business Structures first-order loops</b>	-0.04	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
Business Structures reinforcing loop	0.07	0.07	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Business Structures Balancing loop	-0.10	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13
<b>Agg. Business Structures-Population path</b>	0.16	0.16	0.14	0.14	0.13	0.13	0.13	0.13	0.13	0.13	0.13
Business Structures-Population path	0.16	0.16	0.14	0.14	0.13	0.13	0.13	0.13	0.13	0.13	0.13

Table 5.a: Pathway Participation Metrics for Business Structure in Figure 9

Time (Years)	0	2	4	6	8	10	12	14	16	18	20
<b>Total PPM for Business Structures</b>	0.04	0.04	0.06	0.07	0.08	0.08	0.08	0.08	0.08	0.08	0.08
<b>Agg. Population first-order loops</b>	-0.07	-0.16	-0.18	-0.19	-0.19	-0.19	-0.19	-0.19	-0.19	-0.19	-0.19
Population reinforcing loop	0.10	0.09	0.09	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Population Balancing loop	-0.17	-0.26	-0.26	-0.27	-0.27	-0.27	-0.27	-0.27	-0.27	-0.27	-0.27
<b>Agg. Population-Business Structures path</b>	0.11	0.20	0.24	0.25	0.26	0.27	0.27	0.27	0.27	0.27	0.27
Population-Business Structures path	0.11	0.20	0.24	0.25	0.26	0.27	0.27	0.27	0.27	0.27	0.27

Table 5.b: Pathway Participation Metrics for Population in Figure 9



In pathway participation approach the balancing loops around Population and Business Structures will not be identified as dominant as long as both stocks experience a reinforcing growth. As a result, the “catching up” story that is based on a significant role for the balancing loops is not supported by the pathway participation approach.

#### Case 4: Back to Loop Cancellation:

Adding an inflow and outflow to the partial URBAN1 structure in Figures 9, 10a or 10b would not change the behavior of the model, if both flows are sufficiently close to one another. Figure 12 displays the partial URBAN1 structure shown in Figure 10b. In the new structure,  $h_1$  and  $h_2$  are assumed to be large, but the difference between the two is very small, so the impact of the new reinforcing loop on the behavior is almost cancelled out by that of the balancing loop. The question is whether the formal methods would mistakenly pick the new first-order loops for large values of  $h_1$ , instead of the second-order feedback loop containing Business Structures and Population. Of course, this is a very simple example and it is easy to recognize the two new first-order feedback loops cancel each other out. In more complex models, identifying what Güneralp (2006) calls “opposing” loops, especially higher order ones, may be more difficult.

As discussed in the previous section, according to pathway participation approach, under certain conditions, the second-order reinforcing feedback loop is dominant. The presence of the two new loops, regardless of the value of  $h_1$  and  $h_2$ , does not influence the outcome. The reason lies in the search algorithm for choosing the dominant structure based on pathway participation metrics. In choosing the dominant structure, the PPM approach first groups all pathways with the same state variables in the head and tail of pathways and then it picks the most influential aggregate pathway. In the second round, the most influential pathways is selected within the grouped pathways leaving and reaching the same state variables. In the partial URBAN1 structure in Figure 12, the two new feedback loops are almost cancelled out, therefore the aggregate pathway participation metrics for pathway that starts and end with Business Structures remains unchanged.

Table 6 provides the pathway participation metrics for Business Structures in Figure 12. The aggregate participation metrics for the first-order loops in Business Structures do not change in the present of the new loops as they cancel each other out although not exactly. As a result the second-order loop involving Business Structures and Population remains dominant.

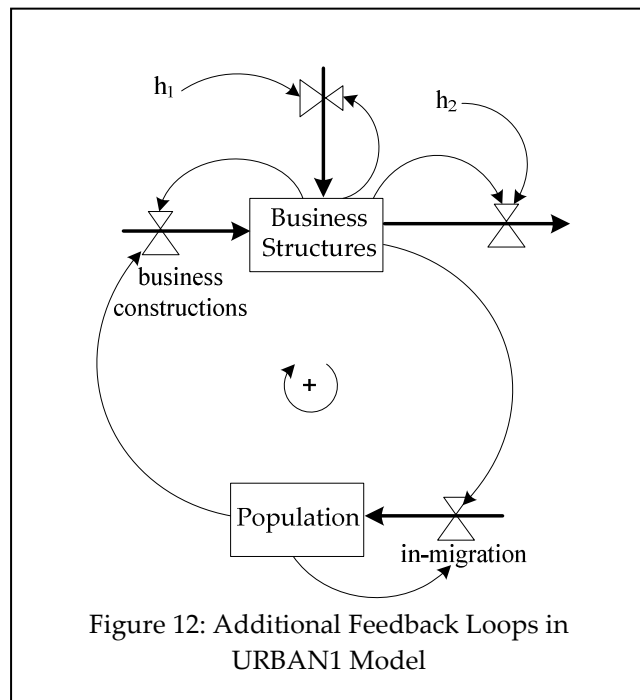
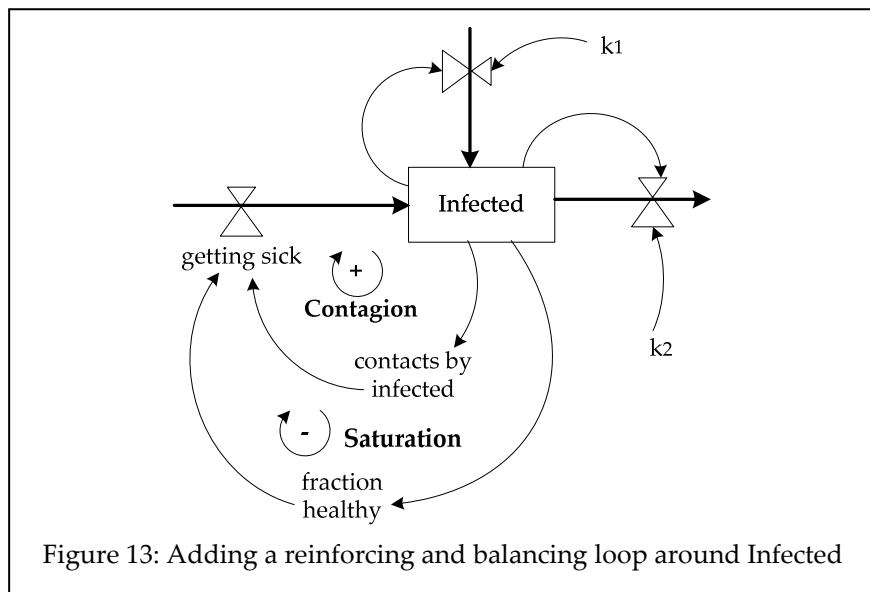


Figure 12: Additional Feedback Loops in URBAN1 Model

Time (Years)	0	2	4	6	8	10	12	14	16	18	20
<b>Total PPM for Business Structures</b>	0.13	0.12	0.11	0.11	0.11	0.10	0.10	0.10	0.10	0.10	0.10
<b>Agg. Business Structures first-order loops</b>	-0.03	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04
Business Structure reinforcing loop	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Business Structure balancing loop	-0.10	-0.12	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13
New Business Structures balancing loop	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100
New Business Structures reinforcing loop	100.01	100.01	100.01	100.01	100.01	100.01	100.01	100.01	100.01	100.01	100.01
<b>Agg. Business Structures-Population path</b>	0.16	0.17	0.16	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
Business Structures-Population path	0.16	0.16	0.14	0.14	0.13	0.13	0.13	0.13	0.13	0.13	0.13

Table 6: Pathway Participation Metrics for Business Structures in Figure 12

Another and even more subtle problem of cancellation can be observed in the epidemic model. To illustrate the problem, two additional inflow and outflow are included in the one-stock-two-loop model shown in Figure 1.d. The new structure is shown in Figure 13. Again,  $k_1$  and  $k_2$ , are assumed to be large numbers but the difference between the two is very small. As a result the impact of the new reinforcing loop on the behavior is almost cancelled out by that of the balancing loop. The challenge is to select the dominant feedback for the balancing phase of the behavior of Infected for large values of  $k_2$ .



For large values of  $k_1$  and therefore  $k_2$ , the two new feedback loops may sound reasonable candidates to replace Contagion and Saturation feedback. Drawing upon the elasticities of the feedback loops around Infected may also lead to the similar conclusion as the eigenvalue elasticities of the new loops appear to be larger than those of the old ones. However, the new balancing first-order loop is obviously a wrong choice for explaining the balancing growth in Infected as it lacks the nonlinearity to facilitate the shift in from reinforcing to balancing growth.

According to the pathway participation approach, the new reinforcing feedback loop is dominant in the reinforcing growth of Infected population, however, the dominant loop shifts to the Saturation balancing loop and drives a balancing growth in Infected. Table 7 shows the pathway participation metrics for Infected broken down by linear and nonlinear feedback loops. According to the table, the total pathway participation metrics for Infected remains positive in the first 9.5 day indicating a reinforcing growth in Infected. During this period, aggregate first-order linear loops, including Contagion, new reinforcing and balancing loops, and therefore, the new reinforcing loop is dominant. After day 9.5, the total participation metrics are negative and the dominance shifts to the Saturation loop.

Time (Days)	0	2	4	6	8	9.50	10	12	14	16	18	20
<b>Total PPM for Infected</b>	1	1	0.99	0.93	0.62	-0.02	-0.26	-0.86	-0.98	-1	-1	-1
<b>Agg. linear first-order loops</b>	<b>0.99</b>	<b>0.99</b>	<b>0.99</b>	<b>0.99</b>	<b>0.99</b>	<b>0.99</b>	<b>0.99</b>	<b>0.99</b>	<b>0.99</b>	<b>0.99</b>	<b>0.99</b>	<b>0.99</b>
Contagion loop	1	1	1	1	1	1	1	1	1	1	1	1
New reinforcing first-order loop	100	100	100	100	100	100	100	100	100	100	100	100
New balancing first-order loop	-100.01	-100.01	-100.01	-100.01	-100.01	-100.01	-100.01	-100.01	-100.01	-100.01	-100.01	-100.01
<b>Agg. nonlinear first-order loop</b>	<b>0.00</b>	<b>0.00</b>	<b>-0.01</b>	<b>-0.06</b>	<b>-0.38</b>	<b>-1.02</b>	<b>-1.26</b>	<b>-1.86</b>	<b>-1.98</b>	<b>-2.00</b>	<b>-2.00</b>	<b>-2.00</b>
<b>Saturation Loop</b>	<b>0.00</b>	<b>0.00</b>	<b>-0.01</b>	<b>-0.06</b>	<b>-0.38</b>	<b>-1.02</b>	<b>-1.26</b>	<b>-1.86</b>	<b>-1.98</b>	<b>-2.00</b>	<b>-2.00</b>	<b>-2.00</b>

Table 7: Pathway Participation Metrics for Infected Population in Figure 13

As a heuristic, the search algorithm in pathway participation approach also groups pathway, according to their degree of nonlinearity and then selects the dominant structure based on the magnitude of the participation metrics. In the above example, the Contagion and two new loops are aggregated in one group because they have the same degree of nonlinearity. In the second phase of the behavior of Infected, the Saturation alone outweighs the significance of the other three loops together and dominant.

### Conclusions and Discussions:

Real-world concepts can be operationalized into variety of feedback structures which may be mathematically identical but diverse in the number of feedback loops. Auxiliaries that help to better operationalize system dynamics models, achieve clarity and avoid confusion in algebraic equations can increase the number of the feedback loops without contributing to the dynamics of the system under study. As a result, consistency in explaining model behavior in terms of its feedback structure can present a challenge for formal approaches to model analysis.

The case studies reported in this paper focus on two important issues of loop cancellation and figure-8 loops. These two problems are merely a subset of the larger problem arising from the role of auxiliaries and the variations in the feedback-loop structures of the same model; they, nevertheless, help to understand the consistency issue in model analysis. Although the issue of consistency in explaining model behavior is not adequately addressed in formal approaches to model analysis, a number of scholars have pointed out to the potential problems and challenges in connecting the structure and behavior. Kampmann and Oliva (2008) describe figure-8 loops as “specific puzzles relating to pathological cases” that can “seriously distort the interpretation of the results” of eigenvalue analysis. Güneralp (2006) calls for caution in “interpreting the weighted loop influence plots” when “opposing loops” are present. Lyneis and Lyneis (2006) recognize the problem of “non-dynamic” loops that can “obscure the focus on the essential dynamic loops”.

Both cancellation and figure-8 problem arise from adding auxiliaries --as well as additional flows and any inference about the flows-- to the model to achieve various purposes such as clarity, ease of communications and relevance to real-world. The first case study shows that it is, in fact, focusing on relevance to real-world concepts and introducing new auxiliaries leads to additional feedback loops, some of which may be non-dynamics or dormant. The second case study makes the point that merging the outflow in Healthy population that happens to be equal to the inflow of Infected population— to make a different point not related to the concept of feedback loops-- hide a second-order loop (Figure 5). This feedback loop would have been visible if the two flows were not merged (Figure 6.b).

The third case study focuses on the partial structure in URBAN1 model and demonstrates that operationalizing labor to job ratio and formulating it as a separate auxiliary masks the second-order feedback loop. The second-order loop would be visible in the diagram when labor force to job ratio is formulated in the multipliers (Figure 10.a) or the rates (Figure 10.b). The point of examples in the fourth case studies is to reiterate the additional flows (or auxiliaries) can lead to new pathways and feedback loops that may cancel each other out.

Modelers developing real-world business models are often required to focus on “relevance” to assure that the model adequately and accurately represent the actual system. As a result, the final model may contain more feedback loops needed to explain observed behavior. User friendly tools and easy to interpret techniques for model analysis may be a viable solution to achieve consistency in explaining observed dynamics regardless of how the model is operationalized and formulated.

The story told about the observed dynamics based on the underlying feedback structure using pathway participation approach remains consistent regardless of the number of auxiliaries and the number of feedback loops in the model. The application of pathway participation metrics in case studies reported in this paper demonstrate that identifying feedback loops in the model through pathways and model's equation, and not the schematic display of the structure, help to avoid the problem of cancellation and figure-8 loops.

The paper calls for comparative studies using alternative formal methods in model analysis. Comparing the outcome of different formal as well as intuitive approaches to model analysis can help to better understand the general principles and subtleties in explaining observed behavior in terms of its feedback structure. Different methods in formal model analysis can learn from one another and expedite the development of user-friendly tools to aid model analysis and serve a wider audience.

**Appendix A:**

This Appendix present the equations for the models used in the case studies.

**Case Study 1: Epidemic Models****Two-Loop Epidemic Model**

$$\text{Healthy} = \text{Healthy}_0 + \int -\text{getting sick} * dt$$

$$\text{Infected} = \text{Infected}_0 + \int \text{getting sick} * dt$$

$$\text{getting sick} = \text{fraction healthy} * \text{contact by infected} * \text{infectivity}$$

$$\text{fraction healthy} = \text{Healthy} / \text{total population}$$

$$\text{contacts by infected} = \text{Infected} * \text{contact rate}$$

$$\text{contact rate} = 2 ; \text{infectivity} = 0.5 ; \text{total population}_0 = 10000 ; \text{Healthy}_0 = \text{total population}_0 - \text{Infected}_0 ;$$

$$\text{Infected}_0 = 1$$

**Three-Loop Epidemic Model**

$$\text{Healthy} = \text{Healthy}_0 + \int -\text{getting sick} * dt$$

$$\text{Infected} = \text{Infected}_0 + \int \text{getting sick} * dt$$

$$\text{getting sick} = \text{fraction healthy} * \text{contact by infected} * \text{infectivity}$$

$$\text{fraction healthy} = \text{Healthy} / (\text{Infected} + \text{Healthy})$$

$$\text{contacts by infected} = \text{Infected} * \text{contact rate}$$

$$\text{contact rate} = 2 ; \text{infectivity} = 0.5 ; \text{total population}_0 = 10000 ; \text{Healthy}_0 = \text{total population}_0 - \text{Infected}_0 ;$$

$$\text{Infected}_0 = 1$$

**Four-Loop Epidemic Model**

$$\text{Healthy} = \text{Healthy}_0 + \int -\text{getting sick} * dt$$

$$\text{Infected} = \text{Infected}_0 + \int \text{getting sick} * dt$$

$$\text{getting sick} = \text{fraction healthy} * \text{contact by infected} * \text{infectivity}$$

$$\text{fraction healthy} = \text{Healthy} / \text{total population}$$

$$\text{contacts by infected} = \text{Infected} * \text{contact rate}$$

$$\text{total population} = \text{Infected} + \text{Healthy}$$

$$\text{contact rate} = 2 ; \text{infectivity} = 0.5 ; \text{total population}_0 = 10000 ; \text{Healthy}_0 = \text{total population}_0 - \text{Infected}_0 ;$$

$$\text{Infected}_0 = 1$$

**Two-Loop, One Stock Model**

$$\text{Infected} = \text{Infected}_0 + \int \text{getting sick} * dt$$

$$\text{getting sick} = \text{fraction healthy} * \text{contact by infected} * \text{infectivity}$$

$$\text{fraction healthy} = (\text{total population}_0 - \text{Infected}) / \text{total population}_0$$

$$\text{contact rate} = 2 ; \text{infectivity} = 0.5 ; \text{total population}_0 = 10000 ; \text{Infected}_0 = 1$$

**Case Study 2: Epidemic Model that Oscillates**

$$\text{Healthy} = \text{Healthy}_0 + \int (\text{new healthy} - \text{getting sick}) * dt$$

$$\text{Infected} = \text{Infected}_0 + \int (\text{getting sick} - \text{Infected death}) * dt$$

$$\text{new healthy} = \text{Healthy} * \text{healthy fraction}$$

$$\text{Infected death} = \text{Infected} * \text{infected fraction}$$

$$\text{getting sick} = \text{fraction healthy} * \text{contact by infected} * \text{infectivity}$$

$$\text{fraction healthy} = \text{Healthy} / \text{total population}_0$$

$$\text{contacts by infected} = \text{Infected} * \text{contact rate}$$

$$\text{healthy fraction} = 0.2 ; \text{infected fraction} = 0.8 ; \text{contact rate} = 2 ; \text{infectivity} = 0.5 ;$$

$$\text{total population}_0 = 10000 ; \text{Healthy}_0 = \text{total population}_0 - \text{Infected}_0 ; \text{Infected}_0 = 1$$

**Case Study 3: URBAN1 Partial Structure, figure-8 Loop**

$$\text{Business Structures} = \text{Business Structures}_0 + \int \text{business constructions} * dt$$

$$\text{Population} = \text{Population}_0 + \int \text{inmigration} * dt$$

$$\text{business constructions} = \text{Business Structures} * \text{business construction normal} * \text{business labor force multiplier}$$

$$\text{inmigration} = \text{Population} * \text{inmigration normal} * \text{attractiveness from job multiplier}$$

$$\text{business labor force multiplier} = \text{business labor force multiplier function} (\text{labor force to job ratio})$$

$$\text{business labor force multiplier function} = (0,0.2),(0.2,0.25),(0.4,0.35),(0.6,0.5),(0.8,0.7),(1,1),(1.2,1.35),(1.4,1.6), \\ (1.6,1.8),(1.8,1.95),(2,2)$$

$$\text{attractiveness from job multiplier} = \text{attractiveness from job multiplier function} (\text{labor force to job ratio})$$

$$\text{attractiveness from job multiplier function} = (0,2),(0.2,1.95),(0.4,1.8),(0.6,1.6),(0.8,1.35),(1,1),(1.2,0.5),(1.4,0.3), \\ (1.6,0.2),(1.8,0.15),(2,0.1)$$

$$\text{jobs} = \text{Business Structures} * \text{jobs per business structures}$$

$$\text{labor force} = \text{Population} * \text{labor participation fraction}$$

$$\text{Business Structures}_0 = 1000 ; \text{Population}_0 = 50000 ; \text{business construction normal} = 0.07 ;$$

$$\text{inmigration normal} = 0.1 ; \text{labor participation fraction} = 0.35 ; \text{jobs per business structures} = 0.18$$

**Case Study 4: Back to the Cancellation Problem**

**Example 1:**

$$\text{Infected} = \text{Infected}_0 + \int (\text{getting sick} + k_1 * \text{Infected} - k_2 * \text{Infected}) * dt$$

$$\text{getting sick} = \text{fraction healthy} * \text{contact by infected} * \text{infectivity}$$

$$\text{fraction healthy} = (\text{total population}_0 - \text{Infected}) / \text{total population}_0$$

$$k_1 = 100 ; k_2 = 100.01 ; \text{contact rate} = 2 ; \text{infectivity} = 0.5 ; \text{total population}_0 = 10000 ; \text{Infected}_0 = 1$$

**Example 2:**

$$\text{Business Structures} = \text{Business Structures}_0 + \int \left( \begin{array}{l} \text{business constructions} + h_1 * \text{Business Structures} \\ - h_2 * \text{Business Structures} \end{array} \right) * dt$$

$$\text{Population} = \text{Population}_0 + \int \text{inmigration} * dt$$

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business constructions = Business Structures \* business construction normal \* business labor force multiplier

immigration = Population \* immigration normal \* attractiveness from job multiplier

business labor force multiplier = business labor force multiplier function (labor force to job ratio)

business labor force multiplier function = (0,0.2),(0.2,0.25),(0.4,0.35),(0.6,0.5),(0.8,0.7),(1,1),(1.2,1.35),(1.4,1.6),  
(1.6,1.8),(1.8,1.95),(2,2)

attractiveness from job multiplier = attractiveness from job multiplier function (labor force to job ratio)

attractiveness from job multiplier function = (0,2),(0.2,1.95),(0.4,1.8),(0.6,1.6),(0.8,1.35),(1,1),(1.2,0.5),(1.4,0.3),  
(1.6,0.2),(1.8,0.15),(2,0.1)

jobs = Business Structures \* jobs per business structures

labor force = Population \* labor participation fraction

Business Structures<sub>0</sub> = 1000 ; Population<sub>0</sub> = 50000 ; business construction normal = 0.07 ;

immigration normal = 0.1 ; labor participation fraction = 0.35 ; jobs per business structures = 0.18 ;

h<sub>1</sub> = 100.01 ; h<sub>1</sub> = 100

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