Sustainable Growth Rate for Emerging Firms

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Introduction

Financial planners define the sustainable growth rate (SGR) as the maximum rate of growth that an organization can maintain without issuing additional equity. SGR ensures that growth is funded from retained earnings and additional debt, without modifying the firm's debt ratio. While the SGR is easy to understand, develop, and implement in well-established firms in mature industries, the financial SGR rarely represents the "speed limit" for emerging firms or firms competing in dynamic industries. Two reasons push organizations beyond their SGR. First, there are strategic imperatives that suggest aggressive growth rates beyond what is financially sustainable in the short-term—first mover advantage, leveraging on economies of scale, "winner-takes-all" markets, "get-big-fast" strategies, etc. Second, dynamic capital markets, investors' optimism, and corporate spin-offs with secured resources, distort or eliminate the funding limitation that traditionally limit growth rates. Aggressive growth rates, however, overstretch the firms' resources, leading to a poor work environment, inadequate customer service, and / or disappointing returns to shareholders. Overstretched firm's resources frequently result in reinforcing feedback processes (vicious cycles) that take the firm out of business.

This research's premise is that –even assuming unconstrained access to financial resources (as apparently was possible during the recent dot.com bubble)– there are other limits to how fast a firm can grow. The study develops an integrated model of the firm, modeling explicitly the acquisition and disposal of resources (labor, capital and technology) and customers, and has the firm working under GAAP principles and embedded in a financial market. For each sector, I identify the steady state conditions for sustainable growth, and closed-form expressions for the growth rates that maximize productivity, sustainable output, and income growth. These different rates delimit four growth regions for the sector, each region having direct consequences to the SGR of the firm.

In the attached presentation, as a way of illustration, I have outlined the analysis done in the labor sector of the firm. Once the conditions and limits to the SGR for each sector are identified, I use simulation to assess by how much, and how long, can a firm exceed each sector's SGR, and to explore the implications of exceeding the firm's SGR and changes of the limiting factors to the SGR.

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Motivation

- **Sustainable Growth Rate (SGR) – finance**
  - **Max. growth rate without issuing additional equity**
  - **Growth is funded through retained earnings and additional debt, without affecting the debt ratio**
  - **Definition based on the accounting valuation of equity**
- **SGR is easy to understand and follow for well established firms**
- **But is rarely the "speed limit" for emerging firms**
  - **Strategic pressures to growth**
    - **First mover advantage**
    - **Economies of Scale**
    - **"Get-big-fast" -- "Winner-takes-all" markets**
  - **Investor's "exuberance" eliminate the funding limitation**

Research Questions

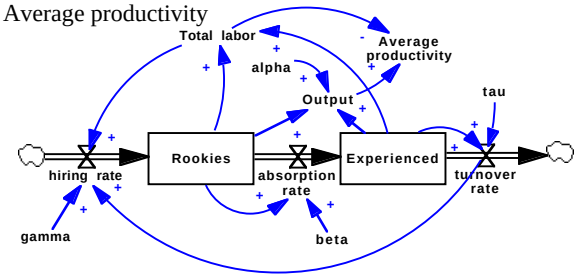
- What is the growth "speed limit" for an emerging firm?
- What is the firm's Sustainable Growth Rate?
- What are the determinants of the SGR?
- By how much, and for how long, can a firm exceed the SGR?
- What are the implications of exceeding the SGR?
- What are the implications of shifting limiting factors to the SGR?

Research Approach

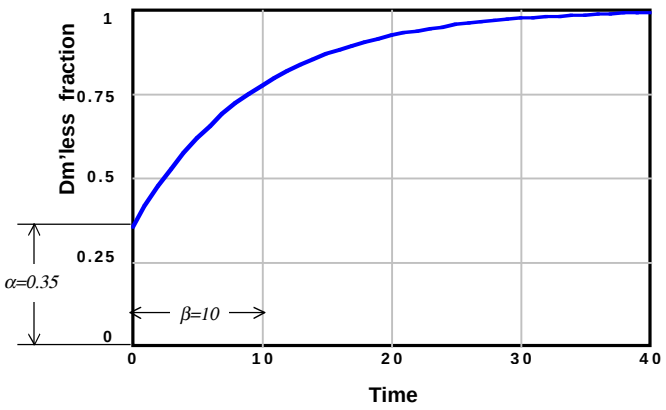
- Develop integrated model of firm
  - Production is a function of labor, capital and technology
    - Acquisition and disposal of resource explicitly modeled
  - Market for the firm's service needs to be develop
  - GAAP accounting and embedded in financial market
- "No-Fat" modeling
  - Very clear premises
  - Closed form solutions
- Identify the SGR for each sector
  - Conditions of steady state growth
- Identify conditions for overall SGR
- Use SD model to explore deviations from and shifts in SGR

Labor Sector

$L = Ex + R$	Total labor		
$dR/dt = hr - ar$	Rookies	$\tau \geq \beta > 0$	Absorption period is less than 50% of total tenure time
$dEx/dt = ar - tr$	Experienced	$\alpha \leq 1$	Rookie productivity is lower than Experienced productivity
$ar = R/\beta$	absorption rate	$\gamma \geq 0$	Growth rate is positive
$tr = Ex/\tau$	turnover rate		
$hr = L\gamma + tr$	hiring rate		
$O = Ex + \alpha R$	Output		
$A = O/L$	Average productivity		



Implied Learning Curve



Steady State Condition

- **Steady State – invariant relationships among the variables**
- **The proportion of R to Ex remains constant**

$$d(R/Ex)/dt = \frac{Ex(dR/dt) - R(dEx/dt)}{Ex^2} = 0$$

$$R/Ex = \beta(\gamma + 1/\tau)$$

  - **Increasing in γ**
    - **Aggressive hiring keeps more Rookies in the mix**
  - **Increasing in β**
    - **Longer training time keeps more Rookies in the mix**
  - **Decreasing in τ**
    - **Longer residence time keeps more Experienced in the mix**

$$\lim_{\gamma \to 0} R/Ex = \beta/\tau$$

- **Assuming constant R/Ex, Average productivity is**

$$A = \frac{Ex + \alpha R}{Ex + R} = \frac{\alpha\beta + \tau + \alpha\beta\gamma\tau}{\beta + \tau + \beta\gamma\tau}$$

  - **Increasing in α**
  - **Decreasing in R/Ex**
    - **Inverse effect of β, γ, and τ**

Learning Constraints
Mentoring Limitation

- **If the absorption rate is limited by the Ex available to train R …**

$$ar = \frac{R}{\beta} * Min\left(1, \kappa\frac{Ex}{R}\right) = \begin{cases} \kappa Ex/\beta & \text{if } \kappa Ex \le R \\ R/\beta & \text{if } \kappa Ex > R \end{cases}$$

  - **This formulation is not robust to an initial condition of Ex=0**
  - **Where κ is the number of R that each Ex can train**
- **… this limits γ**

$$\beta\left(\gamma + \frac{1}{\tau}\right) \le \kappa \quad \Rightarrow \quad \gamma \le \frac{1}{\beta}\kappa - \frac{1}{\tau}$$

  - **Function of the absorption rate minus the turnover rate**
    - **How fast you can develop talent minus how fast you loose it**
  - **Upper limit for γ is the absorption rate ($1/\beta$) times the training capacity (κ)**
- **A at γ_m**

$$A_m = \frac{1 + \alpha\kappa}{1 + \kappa}$$

Learning Constraints
Maximum Growth Rate

- **Maximum Growth Rate (γ_M) is at the point where the fractional output growth rate is maximized**
  - **It only makes sense if α<0 … otherwise there is no limit to the growth rate**

$$\gamma_M \Rightarrow \underset{\gamma}{Max}\left(\frac{dL/dt}{L}A\right)$$
$$\gamma_M \Rightarrow \gamma\, dA/d\gamma = 0$$
$$\gamma_M = \frac{1}{\beta}\left(\sqrt{\frac{\alpha-1}{\alpha}}\sqrt{\frac{\beta+\tau}{\tau}}-1\right)-\frac{1}{\tau}$$

- **Absorption rate increased by the Sqrt of the ratio of overall residence to residence as Ex**
- **First radical defined only for α<0. It is neutral on the worse case scenario, and the closer α is to 0 the further it increases the growth rate**

$$\underset{\alpha\to-\infty}{Lim}\sqrt{(\alpha-1)/\alpha} = 1$$
$$\underset{\alpha\to 0}{Lim}\sqrt{(\alpha-1)/\alpha} = \infty$$

Financially Constrained Growth
Sustainable Growth Rate

- **Sustainable Growth Rate (γ_S) is at the point where the fractional revenue growth rate is equal to the fractional cost growth rate ($r \geq c$)**

$$rA\frac{dL/dt}{L} - c\frac{dL/dt}{L} = 0$$
$$rA\gamma_s - c\gamma_s = 0$$
$$\gamma_s = \frac{1}{\beta}\left(\frac{r-c}{c-r\alpha}\right)-\frac{1}{\tau}$$

- **Function of the absorption rate minus the turnover rate**
- **Absorption rate adjusted by the ratio of margin (r-c) to training cost -- the cost of having an R minus the revenue that can be extracted form her work (c-$r\alpha$)**
- **If training cost is negative (α>c/r), it means that from the first day is possible to extract value from R and there is no limitation to γ_S**

Financially Constrained Growth
Optimal Growth Rate

- **Optimal Growth Rate (γ_O) is at the point where the difference between the revenue growth rate and the cost growth rate is maximized**

$$\gamma_O \Rightarrow \underset{\gamma}{Max}\left(rA\frac{dL/dt}{L} - c\frac{dL/dt}{L} \right)$$

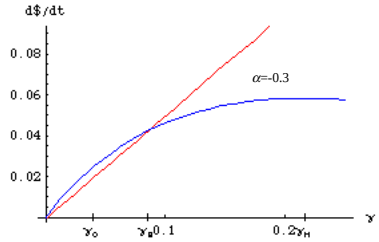
$$\gamma_O \Rightarrow d\left(rA\gamma - c\gamma\right)/d\gamma = 0$$

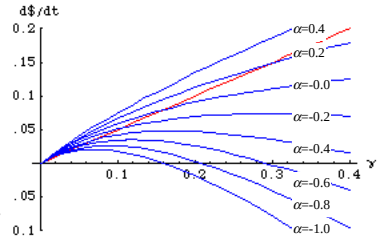
$$\gamma_O = \frac{1}{\beta}\left(\sqrt{\frac{r-r\alpha}{c-r\alpha}}\sqrt{\frac{\beta+\tau}{\tau}} - 1 \right) - \frac{1}{\tau}$$

  - **Function of the absorption rate minus the turnover rate**
  - **Absorption rate increased by the Sqrt of the ratio of overall residence to residence as Ex**
  - **Absorption rate adjusted by the ratio of value of experience (r-$r\alpha$) to training cost (c-$r\alpha$)**
  - **If training cost is negative (α>c/r), it means that from the first day is possible to extract value from R and there is no limitation to γ_O**

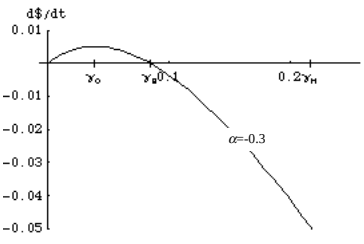
Intuition

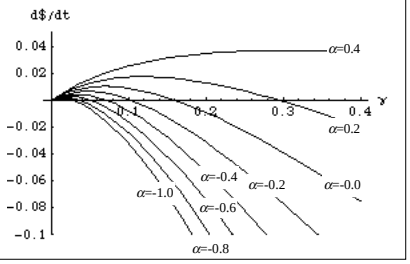
Effect of γ and α on revenue and cost growth rates





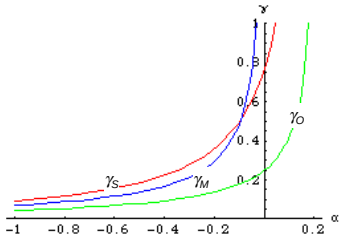
Effect of γ and α on gross profit growth rate

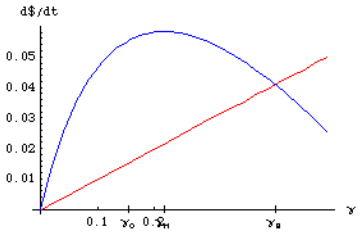




Intuition

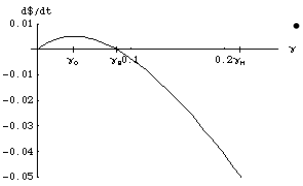
- **For all parameter values $\gamma_O < \gamma_M$ and γ_S**





γ_M and γ_S cross depending on the cost structure (shifts the slope of the cost line) or rookie productivity (shifts the curvature of the revenue line)

Regions for the growth rate



- **Learning constraints**
  - **Firm is limited either by the amount of mentoring that can be done (κ) or if alpha is negative**
    - **If α>0: no limitation to the output growth rate (no γ_M) – any additional R will effectively increase the total output**
  - **If $\gamma > \gamma^*_L = \text{Min}(\gamma_m, \gamma_M)$**
    - **The firm is increasing its cost base and decreasing its revenue base – not sustainable**
- **Financial constraints**
  - **If $\alpha \geq c/r$**
    - **No financial limitation to growth – the firm extracts more value out of each R that what it costs you to train him, only learning constraints**
  - **If $\alpha < c/r$**
    - **If $\gamma < \gamma_O$**
      - **The firm is not growing as fast as its labor sector could – sustainable**
    - **If $\gamma_O < \gamma < \gamma_S$**
      - **The firm is increasing its revenue base faster than its cost base – sustainable**
    - **If $\gamma_S < \gamma$**
      - **The firm is increasing its cost base faster than its revenue base – not sustainable**

Next Steps

- **Similar analysis for**
  - **Capital/Technology sector**
  - **Market sector**
  - **Financial sector**
- **Determine overall conditions for SGR**
  - **Very small set of parameters**
    - **Labor was captured with three unique parameters**
    - **Growth rate was determined with two additional parameters from financial sector**
- **Simulate to see effects of shifting constraints in SGR**

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