Criterial Approach to Verification at Cognitive Mapping of Ill-Structured Situation Dynamics

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Increasingly popular cognitive mapping of complex and ill-structured situations dynamics carries risks for validity of end results both because of cognitive complexity of situations and features of modern cognitive map languages. To cope with typical human-induced formalization risks, verification of cognitive maps is proposed, based on the open system of more or less local expert criteria of absence of risks and direct errors in the map. Comparison of processes of model construction in system dynamics and cognitive mapping as well as analysis of processes of model understanding and mastering show that human-induced risk problems in system dynamics and cognitive mapping have much in common. Presented types of risks and criteria for their detection during verification of cognitive maps refer to early stages of modeling, when translating primary representations to a formal language has not passed into the phase of quantitative parameters definition, be it objective data or expert estimations. Similarity of early stages of modeling either with signed cognitive maps or causal loop diagrams as the intermediate language gives hope for integration of ideas improving end results validity and cross-fertilization between system dynamics and cognitive mapping, with the first steps seen today.

Keywords: cognitive map, causal-loop diagram, distortion effect, false transitivity, formal language, human-induced risk, validity criteria, verification

1. Introduction

For last decades there appear more and more publications, basic and applied, related to cognitive mapping and its applications to ill-structured socio-economic and other interdisciplinary objects, systems and problem situations. In the spectrum of covered problems essential place is taken with modeling and simulation of dynamics of ill-structured situations and decision-making on situation evolution control up to the regional and state level.

In the variety of approaches in cognitive mapping the special place belongs to cognitive maps (CMs), which (i) are aimed to represent the structure of causal influences in a mapped situation and (ii) are characterized with more or less formal behavioral semantics. It seems appropriate to identify such CMs as formal ones in order to distinguish them from informal maps usually applied in soft OR (Howick et al. (2008)). Namely formal CMs which are computable enable simulation of complex and ill-structured objects and situations with highly abstract qualitative (soft) variables, thus supporting the solution of problems of forecasting and control not amenable to classical econometric methods and models.

The scientific direction of cognitive mapping of complex and ill-structured situations and systems by means of formal CMs goes back to Maruyama (1963), Axelrod (1976), Roberts (1976a,b), Kosko (1988). In the field of formal CMs fairly extensive researches are carried out. (See, for example, reviews in Aguilar (2005), Peña et al. (2008), Avdeeva and Kovriga (2008), Carvalho (2010), Abramova et al. (2011)). In this direction the hallmark of “formal” is commonly not used in the naming. From now onward, just this kind of CMs will be discussed.

It is difficult not to notice the similarity of system dynamics and cognitive mapping in the applied problems and expressive means. Suffice it to say that causal loop diagrams (CLDs) and signed CMs, which are often used in the practice of cognitive mapping as the primary representation of a mental model of a ill-structured situation modeled for subsequent refinement to obtain a formal CM of some type(usually by assigning weights to influences), are not visually different from each other except for the explicit designation of cycles in CLDs. On the other hand, the large family of types of formal CMs which, according to the classification from Abramova et al. (2011) are identified as the functional CMs, in the formal aspect may be regarded as relatives with stock and flow diagrams (SFDs) in the family of dynamical systems, although the expressive power of functional CMs in the formal sense is more limited.

In the field of system dynamics, some studies refer to comparative analysis of approaches using cognitive mapping and causality (Sterman (2000), Giordano (2007), Schaffernicht (2007), Schaffernicht (2010)) and moreover the ideas of integration of system dynamics and cognitive mapping are developed (McLucas (2002), Giordano (2007)). As a rule, informal CMs in the style of the methodology by Iden and Ackerman are meant (Eden (1988), Eden and Ackermann (2001), Howick et al. (2008)). In contrast, in McLucas (2002) ideas and the expressive capabilities of formal CMs in modeling the behavior are not only used but also developed. However in general, judging by references to publications found in the sample reviews and analyses (Sterman (2000), Giordano et al. (2007), Schaffernicht (2010) and others), in the scientific field of system dynamics the knowledge about researches on CMs with formal semantics oriented to dynamics analysis and modeling is insufficient.

At the same time, there are numerous basic and applied researches on formal CMs. It is enough to give partial geography of these studies: Brazil, Chile, Portugal, Russia, South Korea, Turkey, USA etc. This geography is reflected in the review by Abramova et al. (2011) which is based on the representative set of 35 types of formal CMs. Since 2000, Institute of Control Sciences of Russian Academy of Sciences holds International Conference “Cognitive analysis and situations evolution control” (CASC). There are a number of cognitive mapping representatives who associate themselves with system dynamics (see, for example, Kim (2000), Carvalho and Tomé (2000, 2001), Ferrarini (2011)).

This work deals with the problem where, the authors believe, the integration efforts can be very fruitful. This is the problem of human-induced risks at modeling dynamics of ill-structured situations and its overcoming by means of verification. (Use of the term “verification” in this work and its relationship to other known interpretations will be refined.)

The human-induced risks for validity of end results in cognitive mapping are caused both by cognitive complexity of investigated problem situations and by features of modern CM languages.

Moreover, it is shown (Abramova (2006, 2007), Abramova at al. (2009)) that such risks may be induced not only by decision-makers, experts, analysts and composers of specific CMs and CM-based models of ill-structured situations (first kind risks) but also by developers of cognitive mapping tools such as theoretical models, languages, techniques and information technologies (second kind risks). From this division follow the corresponding objects of verification.

The main objective of verification of specific CMs and CM-based models (shortly verification of CMs) is defined as the early detection and blocking risks for validity of end results of modeling a situation and direct errors ranging from early conceptual stages of composing models. The vagueness of semantics of theoretical models and languages was found out for most types of CMs and CM-based models of situations, this leading to necessity of verification of theoretical models (Abramova (2011), Abramova et al. (2011)). (This aspect of researches in the paper is not presented.)

It is pertinent to point out that criticism of the modern languages adequacy for modeling complex situations in the context of their application for solving applied problems can be heard both in cognitive mapping (Carvalho (2010), Abramova et al. (2009), Abramova (2011)), and in system dynamics (Schaffernicht (2010)), although on a few different grounds. The attention to the question of their development is brought, taking into account semantic aspects quite often ignored.

This work focuses on the verification of CMs, based on predefined criteria of specific risks absence found out and tested by these authors and their colleagues, with some relevant criticism on semantic vagueness of CM-based theoretical models. More advanced techniques of expert-performed verification focused on actualization of expert's cognitive resources (cognitive dissonance, cognitive control, error detectors, known from cognitive science) are presented in Abramova and Kovriga (2011), Abramova (2012).

To study and systematize detected risks and criteria for their early detection language-oriented approach is developing proposed earlier (Abramova (2011), (Abramova and Kovriga (2011)). According to this approach, composing maps is considered as the translation of human's substantive knowledge about a problem situation into the mathematical language and reading maps is the backward translation (interpretation) with the inevitable distorting effect in both cases (Abramova (2007)). Thus structuring types of risks goes in accordance with the logic of the composition of CMs from elementary semantic constructs and associated risks to more complex ones (Abramova and Kovriga (2011)).

The idea to form the validity criteria (so-called Goldratt's Categories of Legitimate Reservation) for cause-and-effect logic is found also in system dynamics (Burns and Musa (2001)). The comparison shows that both regular approaches to forming the criteria of validity presented in Abramova and Kovriga (2011) and Burns and Musa (2001), have some common features (focus on locality of the criteria, some close criteria).

According to these authors' estimate, the language-oriented approach has proved to be richer in the repertoire of partial criteria and its capability to detect risks and direct errors of formalization, with integration obviously preferable.

The further development of the approach is connected with allocation of two levels of formalization at composing CMs. Considering the risks of formalization which arise in the translation of initial representations of experts about the problem situation and its dynamics into a formal language, one can with some degree of conventionality distinguish two levels of risks according to typical stages of such translation.

The first level relates to the stage of qualitative modeling, when translation of initial representations (that is a human's cognitive model) of a situation to a formal language has not yet passed into the second stage of quantitative parameters definition, whether they be objective data or expert estimations.

Our analysis has helped to notice similarity in early stages of modeling dynamics of ill-structured situations either in terms of signed CMs or CLDs as an intermediate language for subsequent refinement of the intermediate model correspondingly in terms of some type of formal CMs or SFDs. Based on similarity of the languages of signed CMs and CLDs, it seems reasonable to suggest commonality in mechanisms of risk in their application. In agreement with Schaffernicht (2007) who believes that CLDs allow to model "fast-and-dirty" we are inclined to believe that not only CLDs but other first-level languages of qualitative modeling such as signed CMs are risky when modelers rely upon their intuitive semantics.

The principal role of early verification in the first (qualitative) stage of modeling is to identify risks of inconsistency with the semantics of selected language of the second level of formalization (some type of CMs or SFD) in view of correction or even rejection of the selected language, if the relevant substantive features of the situation are not expressible by its expressive means. Types of risks of the early stage of qualitative formalization presented in the article and the relevant criteria for their detection during verification are based on the authors' experience of the expert verification of applied and research maps.

This paper is organized as follows. In Section 2 the initial concepts are refined, including the notion of formal CMs, as well as the concept of verification in the context of this study and relations to system dynamics. In Section 3 firstly the rationale and general features of the open system of partial criteria of absence of risks for validity of formal CMs are given. Then the most typical and effective qualitative criteria are described in short. Only the criteria associated with the detection of fragments of false transitivity in formal CMs are presented and illustrated in the separate section 4 within the description of the false transitivity phenomenon itself and its origins in the conceptual modeling of ill-structured situations. Some types of risks for validity of the end results with the criteria for their detection will be shown in the examples (section 3.2 and section 4). In conclusion some practically significant research problems and opportunities for future research are denoted.

2. The concepts of formal CMs and verification in the context of this study and relations to system dynamics

2.1. Formal CMs and their relation to languages of system dynamics

Diversity of research approaches to ill-structured socio-economic and other inter-disciplinary objects, systems and problem situations in terms of cognitive maps has with inevitability led to ambiguity of terms "cognitive map" and "cognitive mapping" themselves. The diversity is largely caused by inventing and using many types of cognitive maps, and in particular, by various degrees of formalization of knowledge and beliefs of experts about a problem situation, beginning from maps with informal semantics (such as CMs in the methodology of Eden and Ackermann (2001), Howick et al. (2008)) and up to formal maps.

This work deals with CMs that may be assigned with attributes of “cognitive”, “causal” and “formal” on various grounds. They are causal since they represent the structure of causal influences of a mapped situation. They are cognitive since they are the product of cognitive transformations of the primary (internal) perceptions and beliefs of people into a formal language. Such cognitive nature of producing the formalized maps is essential for complex and ill-structured situations due to inevitable distorting effect discussed below. Finally, they are (more or less) formal since they are computable and afford application of formal methods (such as simulation, inference and others) to forecast and to search and make decisions on the control of complex and ill-structured situations. Today in publications and communications of the research domain of cognitive mapping the uniting attribute “cognitive” is dominant though in adjacent domains they often speak about causal maps or diagrams. On the contrary, the demarcative attribute “formal” which means that the maps are computable is not commonly used. The distinctive feature of the family of formal CMs is that the semantics of a given type of maps in the family is determined with the corresponding theoretical model which defines behavior of situations modeled as the maps. In other words, one can say that a theoretical model defines the behavioral semantics of the language, aimed to describe maps of a give type (either a graphical language or matrix one, or a language of structural equations).

Consideration of the basic mathematical properties of the family of formal CMs helps to understand the sources of distorting effects in the translation of signed CMs of the first level formalization in the formal language of the second level.

The obligatory base of formal CM definition is the directed graph which nodes are associated with factors (or concepts) and arcs are interpreted as *direct causal influences* (or causal relations, connections, links) between factors.

Functional CMs

This work mainly deals with the large subfamily of formal CMs which can be referred to as *functional* CMs, on classification from Abramova et al. (2011). In functional CMs factors are mathematically represented as variables. Arcs in the graph of functional CMs are usually added with influence weights (intensities), resulting in a *weighted* CM. From the behavioral point of view, the basic element of the theoretical model of a given type of functional CMs is the type of function representing behavior of any *bundle* in a map i.e. of a factor with all incoming direct influences (Fig.1). This function is sometimes referred to as the *influence aggregation function* although in a number of publications it is called simply the rule for aggregating influences onto a factor or like that.

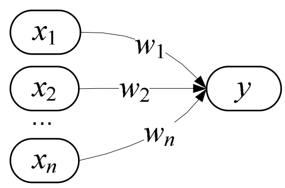

Fig.1. A bundle in a weighted CM

If instead of weights in a CM there are given signs (polarities of influences), such a *signed CM* may be considered as a weighted CM with uncertain magnitudes of weights.

Typical examples of functional CMs are maps belonging to the family "in the spirit of Roberts"; they are different modifications of the dynamic maps proposed in Roberts (1976). For all types of maps in this family the aggregation function of the bundle is pseudo linear, i.e. some modification of the classical linear function with the one-clock delay. One more family of formal cognitive maps is formed with maps "in the spirit of Kosko" usually referred to as fuzzy maps. It seems more pertinent to name many maps of this sort pseudo-fuzzy because they use classical mathematics, rather than fuzzy, as it might be expected from the title. They differ from modern maps "in the spirit of Roberts" mainly by a type of the aggregation function.

For most of the presently known types of functional CMs (see Abramova et al. (2011)) the general form of the influence aggregation functions may be represented as the compositional formula:

$$V_{y/\Delta y}^{t} = f\left(\sum_{i=1}^{n} w_i \cdot V_{x_i/\Delta x_i}^{t-1}\right) \qquad (1)$$

where $V_{y/\Delta y}^{t}$ is the value (or increment) of factor y at time t, $V_{x_i/\Delta x_i}^{t-1}$ is the value (or increment) of factor x_i at time t-1, n is the number of cause factors in the bundle, w_i is the weight of influence of factor x_i on factor y, and f is some type of function. The values of variables in different types of CMs can be set at different scales, often finite and / or verbal ones, typically normalized.

The role of the outer function f, say, sigmoid, is that in some approximation composite function (1) models the intuitively natural properties of *monotonicity* of individual influences and *additivity* in the integrity of influences onto a factor for limited scales of variables and\or their increments, however without leaving the permissible range of values. Note that the limitation of the range, typical for various types of functional CMs, creates systemic risk of distortions in the formalization of intuitively monotonic influences. Note also that the above formula does not take into account the additional influences on the dependent factors that are considered to be independent within a specific model from other factors, including the influences of the environment and control actions. (Such "conditionally independent" influences are explicitly distinguished only in some theoretical models of CMs.)

One more important behavioral characteristic of dynamic models in the formalism of CMs is the mode of dynamics of conditionally independent factors (i.e. factors not influenced by other factors within a specific model). Both qualitative features (i.e. events, or continuous behavior, or mixture) and quantitative ones should be specified. By our estimation, the role of conditionally independent factors in correct understanding of behavior of models is comparable with cycles.

Finally, it is necessary to name one more mathematical property which is fundamental not only for all types of formal CMs but for other languages and models as well. We speak about *transitivity of cause-effect influences*. This property is usually considered as the universal principle both by mathematicians, and the problem area experts. The need for adequate application of this property, as well as the properties of monotonicity and additivity of influences is taken into account in the framework of the proposed approach to verification of CMs.

In the present brief description of functional CMs such advanced types of CMs as rule-based CMs, those with variable delays, with externally controlled weights and some others, are omitted. We suppose that our approach to verification with the validity criteria found out can be extended to such CMs; however this requires more careful study on the practice.

A rigorous comparison of the language of SFDs and different types of functional CMs is still waiting its researcher. However, even today it is clear that (i) they have approximately the same level of formality; (ii) the language of SFDs is more expressive than those of functional CMs due to a larger number of types of variables; (iii) the languages of functional CMs appear to have more opportunities for expression of ill-defined, vague causal relations.

Comparison of the role of signed CMs and CLDs in the process of composing dynamic models of ill-structured situations

Analysis of the practice and some of the methodologies for composing dynamic models of ill-structured situations in the formalism of functional CMs shows that typical is a primary representation of the situation as a *signed CM*, which differs from a weighted functional CM only in that no weights of direct influences are defined in it, but only their signs (polarities) are assigned to arcs of the graph. Further a signed CM is concretized by specifying the signed weights instead of the signs, and by further definition of the initial data and the mode of dynamics of conditionally independent factors.

An example of such a CM is shown in Fig. 2. The example is chosen intentionally to emphasize the kinship of languages of signed CMs and CLDs: it was originally developed by Taber (1991) as a cognitive map. (By the classification of Abramova et al. (2011), it refers to pseudo fuzzy CMs, and then to functional CMs.) Later it was renamed by McLucas as CLD (McLucas (2002)), this differing only in adding explicit designation of cycles except for details of visualization.

Analysis of a number of published examples like this, both CMs and CLDs (Kwahk and Kim (1999), Sterman (2000), McLucas (2002), Binder et al. (2004), Crescitelli and Figueiredo (2009), Schaffernicht (2012)), performed by our colleague A. Fedotov has helped to comprehend deep similarity in early stages of modeling dynamics of ill-structured situations in terms of signed CMs or CLDs. Moreover, the experiential knowledge of typical risks in the early stages of constructing CMs allowed accepting as reasonable the suggestion of commonality in mechanisms of risk in the application of signed CMs and CLDs as the intermediate language for subsequent refinement in terms of some type of formal CMs or SFDs.

From the standpoint of verification, a number of types of risks for validity and errors may be involved in the stage of composing a signed CM, and they should be identified and blocked with early verification. Similar role could be performed if composing SFDs proceeds through intermediate CLDs. It both cases verification means identification of risks of inconsistency with the semantics of a selected target language of the second level of formalization or direct errors whether they would result in correction or even rejection of the selected target language (if its expressive means are limited relative to the relevant substantive features of the situation modeled).

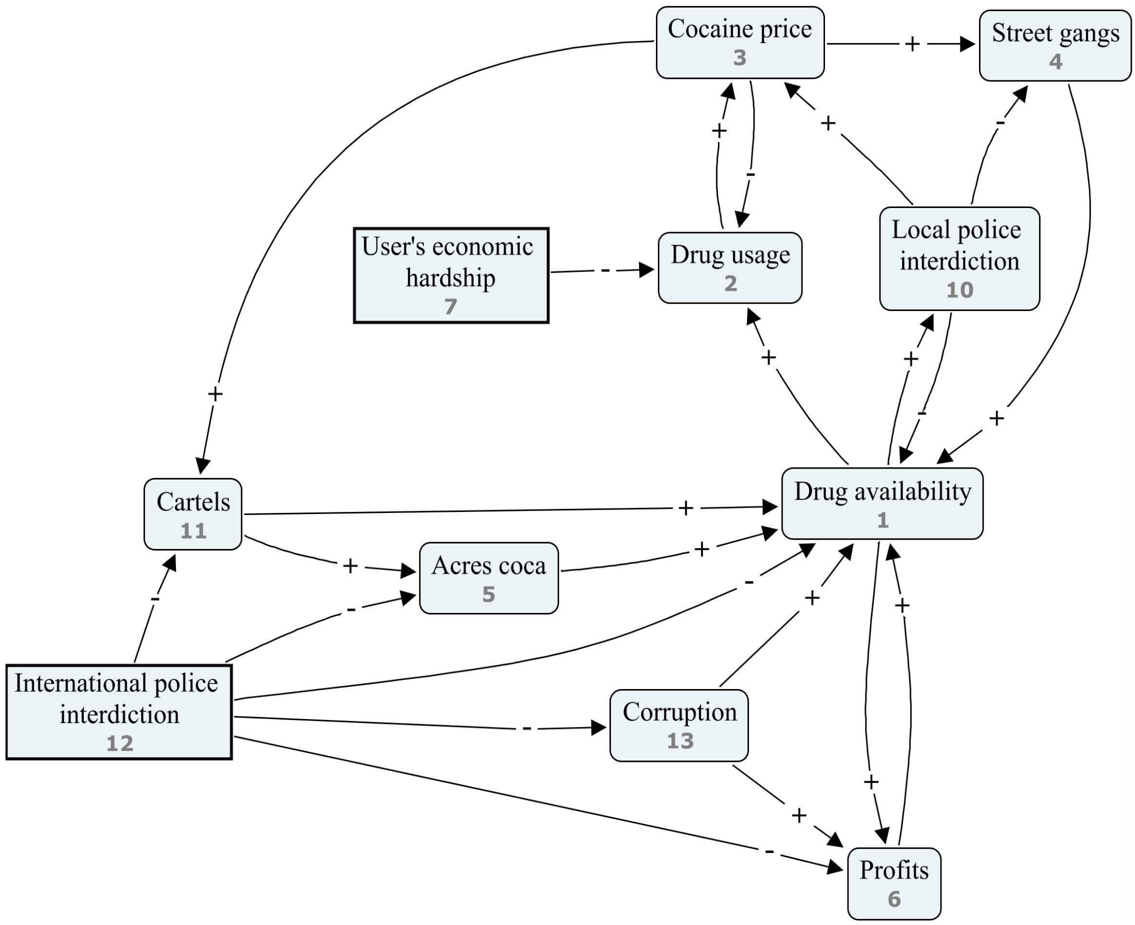

Fig. 2. The map of cocaine use (Taber (1991), McLucas (2002))

2.2. The verification and validation concepts in the context of cognitive mapping and system dynamics

Application of verification ideas to new types of objects demands refinement of the concept of verification which covers the most different objects, from objects of designing and data to theories.

Study of different meanings of this term and its interrelation with the term of validation at the obvious polysemy is far beyond the given paper.

It seems enough to say that, on the one hand, the term “verification”, as it is known, has differing interpretations today as well as “validation” with a host of accompanying concepts, e.g. “confidence”, “credibility”, “dependability”, “soundness” and so on. (See, for example, the critical analysis by Pala et al (1999) which covers the history of system dynamics in comparison with hard OR and soft OR as to validation ideas.)

On the other hand, different approaches draw the demarcation line between the two terms on different grounds.

In the field of system dynamics the point of view according to which the term “validation of models” covers narrower term “verification” is more common.

Thus, according to (Forrester and Senge, 1980) validation means “…the process of establishing confidence in the soundness and usefulness of a model with respect to its purpose”. Herewith structural verification is considered as a specific mode of tests within validation process.

On the other hand, Sargent (Sargent, 2003) distinguishes stage of checking whether the computer model is programmed correctly and calls it “computerized model verification”.

The opposite point of view that the verification and validation are different stages of a single process, represented in (Coyle, 2000), where in part verification of the simulation model and validation with the simulation model are differed.

A similar position with the opposition of verification and validation is characteristic in the field of safety-related software.

From these authors’ viewpoint, most relevant interpretation for cognitive mapping defines verification “in computer modeling and simulation” as “the process of determining that a model or simulation implementation accurately represents the developer's conceptual description and specifications” (Dictionary (2005)). However, in accordance with the known tradition to separate verification and validation stages in the process of the executable model creation, it seems preferable to refer testing of the model in real or similar conditions to the stage of validation. On the contrary, verification is usually referred to the earlier stages, so that its main objective is defined as the early detection and blocking risks to validity of end results and direct errors.

Moreover, early verification of descriptive models of situations in some language may not relate to underlying software implementation at all.

Typical for the development of verification techniques in many areas is that the emphasis is placed on formal methods and on the predefined criteria of conformity. The approach to verification in cognitive mapping which is developed by these authors (Abramova (2010a), Abramova and Kovriga (2011)) differs from the tradition in that the verification is regarded primarily as the human activity with the inevitable making decisions by experts-verifiers. It is caused by specificity of human-induced risks and direct errors in the case of complex and ill-structured situations with the high level of abstraction and qualitative variables. These factors bear risks and errors not only from composers of CMs and CM-based models of problem situations (first kind risks), but also from developers of cognitive mapping techniques (second kind risks) which should be identified by verifiers as well (Abramova (2006, 2007), Abramova at al. (2009)).

In this work we restrict ourselves namely to the criterial approach to verification with criteria that are predetermined beforehand, putting aside the more advanced verification techniques mentioned in the introduction. But the criteria themselves to be described below are largely the product of namely experience of expert verification without of predefined criteria (Abramova (2012)).

3. Verification of formal CMs with predefined qualitative criteria

3.1. Rationale for the system of validity criteria for CMs and their general features

The basic idea of the proposed criterial approach to verification of formal CMs is language-oriented. Composing CMs is considered as the translation of human’s substantive knowledge about a problem situation into the mathematical language, and reading CMs as the backward translation (interpretation) with the inevitable distorting effect in both cases (Abramova (2007), Abramova and Kovriga (2011) and earlier publications). The matter is that in fact there are two languages for knowledge representation and understanding with close, but not coincident semantics: intuitive and mathematical (Fig.3).

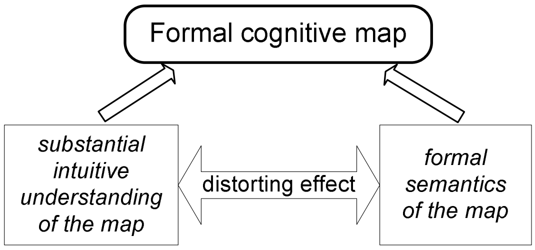

Fig.3. Distorting effect between two understandings of a formal CM

Naturally, this idea applies not only to formal CMS but to other schemes for formalized knowledge representation including CLDs or SFDs as well.

If not to concern psychological, linguistic or philosophical rationale for such distortions, in short they are caused with at least two reasons. On the one hand, a subject area specialist “sees” more than what is presented in the mathematical model. For example, he sees a significant factor in the situation, not just a variable. On the other hand, a person tends to reduce the cognitive load in the intuitive understanding. For example, the understanding of a causal effect in accordance with a truncated definition of the semantics of the relationship between factors, for him (or her) is simpler than complete one. (We refer to opposition of the definitions of the semantics of the polarity of the influences widely discussed in system dynamics (for example, Richardson (1997), Sterman (2000), Schaffernicht (2010)).

Vagueness and distortions of the substantive sense of model constructs with regard to their mathematical interpretation may cause the risk of unreliable estimation of factor influence intensities, especially for the lack of data for such estimation. One of the risk factors is incomplete understanding of mathematical sense of constructs by a problem area specialist. Such incomplete understanding, in turn, creates the risk of inadequate application of the chosen general formal model to a particular problem situation.

It may be assumed that the distortion between formal and intuitive understanding of a dynamic model depends on the quality of formal language and its intuitive clarity. Besides, the distortion should increase with increase in the level of abstraction of a conceptual model of the situation.

The proposed language-oriented approach to verification of formal CMs (Abramova (2011), Abramova and Kovriga (2011)) is based on the idea of protection against the distorting effect of formalization due to the difference between intuitive and mathematical language semantics.

For considerations given the general translation adequacy criterion was proposed for CMs verification (Abramova (2007)). Today it is implemented in the series of partial criteria, reflecting the logic of composing CMs with the elementary language constructs (Abramova and Kovriga (2008a), Abramova (2010b), Abramova et al. (2010), Abramova and Kovriga (2011)).

The open system of partial validity criteria for formal CMs

The represented partial criteria of absence of risks to validity of CMs and direct errors, or shortly *partial validity criteria* belong to the open system in which all the criteria known to date support the general criterion for adequate translation of substantive knowledge into some mathematical language and vice versa. The system is characterized with the following general features.

(i) With regard to capabilities of estimation of conformity, the criteria are not formal but *expert* ones. This means that decision on conformity is made by the expert composing a formal CM or verifier carrying out control of formalization correctness with account of knowledge not captured with formalizations. It may involve not only the content of a specific construct under verification, but also its context in the CM and even the problem domain context.

(ii) With regard to the representation form, the criteria are *weakly formalized*, when it is possible. It means that criterion $K_i(Cj)$ is represented as a verbal template (scheme) which, in the logic sense, is a predicate with free variable *Cj* denoting a construct.

(iii) With regard to the scope, the criteria are more or less local, i.e. they mostly refer to separate constructs of a CM starting with elementary ones.

As *elementary constructs* factors-variables, direct causal links and bundles are considered. It is important that when talking about construct of “factor-variable”, we actually consider factors, not only as variables, as is typical, but as substantive entities of the problem domain, denoted by concepts with some linguistic risks. We also emphasize that in the traditional descriptions of formal CMs only factors and links are usually considered as elementary constructs. A similar picture is typical for describing the semantics of the language system dynamics (Schaffernicht (2012)). With our viewpoint, just nodes (together with independent factor dynamics mode) determine the behavioral semantic of dynamic models (whether discrete, continuous, or mixed), and the effect of risks in their description and understanding on the validity of a CM as a whole is essential (Abramova et al. (2011), Abramova (2011)). (In our classification these types of description and understanding of the language semantics of language are denoted as the edge-semantics and vertex-semantics respectively.)

Finally, when considering the links and bundles, we limit ourselves to the qualitative aspects, without touching risks of the quantitative estimation inherent in the functional CMs.

3.2. The basic partial validity criteria for functional CMs

The paragraph contains some partial qualitative validity criteria for verification of functional CMs with rationale and examples. The criteria are described in the order of increasing complexity of constructs to which they are applicable including those related to a factor, to a link, to a bundle, to some compound constructs, up to a map as a whole.

The criteria related to elementary constructs

- *The criteria related to a factor*

The criterion of normality of factor concept name, $K_c(p)$ (Abramova and Kovriga (2008a), Abramova et al. (2010), Abramova (2010b) is applicable to any factor p or, more exactly, to the concept denoting a factor significant for the situation in the cognitive map of the situation. (In short, it is referred to as a factor concept.)

It is assumed that factor concept p is named in the normal form if it may naturally be interpreted (understood) both as a factor in substantive sense and as a variable taking on values at a definite measuring or estimating scale, in the mathematical sense. If $K_c(p)$ is met for factor *p* its concept occurs to be "two-faced", with one face turned to subject matter experts and the other one turned to mathematicians.

In the above example of map of cocaine use (Fig. 2) positive examples with the criterion satisfied are factors named as "drug usage", "user's economic hardship" and others, which names are natural in such linguistic contexts as "increase in drug usage", "the more is user economic hardship, the less…". Negative examples are the factors named as "cartels" and "street gangs" which are not quite clear as variables in similar contexts (It is not clear whether it is about the quantity, or perhaps something else.)Even more incomprehensible as variables are, for example, the factors of "environment condition", "environment", "federal regulators" (the latter in the context of evaluating the strength of influence on the environment and population incomes) found out in other published CMs.

Mismatch of concept *p* to criterion $K_c(p)$ may be interpreted as vagueness (insufficient clearness) of substantive sense of the factor with respect to the required mathematical sense. On the logical arguments, such vagueness is considered as a risk factor for the final model validity which operates in the process of composing a model involving vague concepts of factors. Its action actually begins on the qualitative level with adding causal influences into the map and determining their polarity (sign). And it becomes more obvious when determining the quantitative parameters of the model. Recall that in essence this process can be treated as translation of intuitive understanding of a situation into the mathematical language even if the composer of a map does not realize this fact.

If the mismatch of concept *p* to criterion $K_c(p)$ is found out, it is desirable to clarify the meaning of *p* in the context of its relations in the map.

The criterion of context-freedom of factor concept name, $K_o(p)$ (Abramova and Kovriga (2008a), Abramova et al. (2010)) is a particular case of criteria of context-free clarity of map constructs. Context-free clarity of a map (in particular, of each factor and direct link) means that in order to read (understand) a given construct the subject matter experts do not need any additional context from this map.

A typical source of implicit contexts is consecutive character of a map development process, when separate constructs of a map, in particular, concepts of factors, are formed (i.e. identified and named) in the context of the preceding ones. When reading a map this order and its context are lost.

Criterion $K_o(p)$ is naturally to be checked with respect to the normally named factors, i.e. to those satisfying criterion $K_c(p)$.

In the map of fig. 2 characteristic examples of mismatches to criterion $K_c(p)$ are concepts of factors named as "corruption" (13) and "profits" (6).

They turn out disproportionately general, if considered free of the context of such factors as "drug availability" (1).

Such extra generality sometimes referred to as indefinitizing in linguistics, at least, it creates a risk of washing out the actual meaning of factors and influences (for example, in case of influences 12→6, 12→13 “international police intervention”).

In more difficult cases erroneous conclusions about the situation and its dynamics may be the result. (See criterion for the absence of false transitivity below).

- *The criteria related to a link*

Unclear semantics of causal links in CLDs is well known and, to some extent, it is discussed as a source of risk in cognitive mapping. Taking into account extensive discussion of this problem in the field of system dynamics, we restrict ourselves to only the summary of results to ensure the validity of causal links. Our approach starts from the concept of *cognitive clarity* (Abramova (2010b)). The concept of cognitive clarity of some information, messages, descriptions, etc., characterizes easiness of intuitive understanding. The lack of cognitive clarity appears when a person has much ado, hesitates trying to understand what has been said or written. This can appear in observed deceleration of understanding process.

In order to adapt the mathematical language of a given model of cognitive maps for problem domain specialists and increase its cognitive clarity, theorists and developers of information technologies often create verbal definitions and\or templates to translate separate causal links in a map into the natural language, or in short, *link translation templates.* The link translation template is the verbal formulation of the semantics of an arbitrary link in a map that is specified by substitution of particular names of factors linked with direct influence for free variables of the template. Such kind templates are more or less explicitly represented, for example, in Maruyama (1963), Roberts (1976), Abramova et al. (2010). In system dynamics, different polarity definitions are formulated in a form which is naturally viewed as a template or easily converted into it. Here is just one case of the template commonly used in cognitive mapping for a couple of factors linked with the direct positive influence, $p1 \xrightarrow{+} p2$:

increase in <name of factor 1>, *other things being equal, causes increase in* <name of factor 2>;
decrease in <name of factor 1>, other *things being equal causes decrease in* <name of factor 2>.

In the context of translation adequacy we can speak of the cognitive clarity from two points of view. On the one hand, stereotyped manner of link understanding via a template really assists intuitive cognitive clarity of links. But on the other hand, with regard to published templates and definitions for map link understanding, nowadays one can see the tendency to oversimplification of the templates themselves for the sake of their clarity to problem domain specialists. However *cognitive clarity of a mathematical model* is often decreased for problem domain specialists who need to understand the mathematical sense via verbal templates in order to escape negative distorting effects. (In particular, we are talking about well-known opposition of the “truncated” and “full” definitions of polarity in system dynamics.)

Our main conclusions on the semantic templates used today are as follows.

- We agree with the conclusion of Schaffernicht (2010) that each definition each definition (and hence each template) has its shortcomings.

• Moreover, in terms of cognitive clarity of mathematical sense, none of the templates cannot be good enough if the truth of an assertion about the semantics of a particular influence composed with the template is evaluated the originator or the map or a verifier, without taking into account the context of a bundle, which includes this link. For example, the expression "other things being equal" perceived simplified out of context (and this is confirmed by experience).

• Based on practice, it is difficult to agree with that experienced system dynamicists are protected against errors in identifying the semantics of the causal influences modeled. In this respect example from Maruyama (1963) i.e. M. Maryuama's map describing city pollution with garbage well known in scientific literature is didactic. The author introduces the definition of the influence which by today's norms refers to complete (not truncated) definitions. However, in his example there is a positive influence (Fig. 4), which does not meet this definition.

Rather, we can assume that inconsistent casual influences found out by Richardson (1997) refer to cognitive biases (as introduced by Kahneman and Tversky) when some type of inconsistence is systematically not noticed by a person.

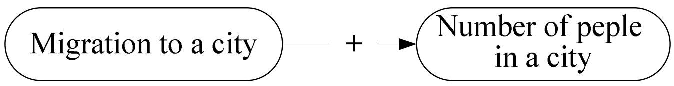

Fig. 4. A fragment of Maruyama's map with the inconsistent casual influence

In Abramova et al. (2009) the hypothetical example is constructed with erroneous recommendations to decision-makers inferred from the Maruyama's map due to assigning formally correct sense to the given influence ("Decrease in migration to a city results in decrease of the number of people in a city")

- *The criteria related to a bundle*

To date, *the criterion of (proportional) completeness of influences on the factor*, $K_B(p, B(p))$ may be regarded as the most important for a bundle that is for a dependent factor *p* with the set of all direct influences on it, *B*(*p*) (Abramova and Kovriga (2008a), Abramova et al. (2010)). The criterion is met if, according to the expert's estimation, there are no other factors of direct influence on factor *p*, besides those from *B*(*p*), which have proportionally significant influence with regard to other factors from *B*(*p*). It is assumed that there could be other factors influencing *p*, including unknown ones, but their influence is comparatively negligible for the analysis carried out.

For CM of Fig.2 doubts about the (proportional) completeness of the influences onto a factor are in a greater or lesser extent justified for all the factors except for the factors 7 ("user's economic hardship") and 12 ("international police interdiction") which appear as conditionally independent in this CM.

However, the most obvious example is bundle (13, {12}), that is factor 13 "Corruption" with the only factor 12 "international police interdiction" on which it depends. Incompleteness is obvious even if we consider "corruption due to drug availability" as the name of factor 13 instead of "corruption" which is context free. It is doubtful whether the absence of other significant factors of influence on corruption. In particular, factors which generate it. Accounting for only the factor "international police interdiction" which is deterrent could lead to incorrect administrative decisions in the development of policies to fight against corruption.

The risk of missed influences in the case of non-compliance with criterion $K_B(p, B(p))$ is quite obvious: in accordance with (1) it distorts the proportions of the aggregate effect of all the commensurate effects on the factor, especially if the force of influences is non-linear. Nevertheless, widespread is the point of view articulated by Carvalho (2010): “since the effect of each concept antecedent is independent from the effect of other concept antecedents, it is possible to remove or add concepts and links without having to change the rest of the system”.

It is worth noting that the criterion of cause insufficiency proposed by Burns and Musa (2001), is close to our criterion $K_B(p, B(p))$, however we additionally consider the balance of influences with regard to their importance. Note also that, as shown in the psychological research by Dörner (1997), the errors of incompleteness are both typical and significant.

One more criterion practically important for the bundle is *the criterion of additivity of influences*, $K_A(p)$. Practice and publications show that the multiplicative function, rather than (1) often makes sense as a function of influence aggregation, without saying about other monotonic functions. Nevertheless, usually an additive function is taken by default.

The criteria related to more complex constructs

The set *of criteria related to more complex constructs* the most actively applied in our practice includes:

− *criterion of absence of false transitivity of causal influences,* $K_{FT}(S)$ (where S is a chain of two or more direct influences);

− *criterion of absence of duplicating influences* $K_D(Q)$ (where Q is a risky configuration in which duplication of a direct influence with indirect ones is possible).

The phenomenon of false transitivity in CMs and some the criteria associated with its detection are presented and illustrated in the following section 4.

The criterion $K_D(Q)$ is connected with *duplication of influences* which quite often takes place in practice of composing CMs (Abramova (2010a). It means that the same, as a matter of fact, influence is specified both directly and through indirect influence by transitivity. Such duplication is risky in types of CMs where separate influences on the factor are summarized due to exaggeration of the force of an influence. (The same influence is accounted twice). Moreover, duplication of influences greatly complicates human’s understanding of CMs and interpretation of simulation results.

In the map of cocaine use in Fig.2 there are a number of configurations which are risky by formal indication. Some of them can be surely attributed to the redundant influences by the expert estimation. For example, the direct influence 13→6 duplicates the indirect influence by transitivity 13→1→6 (the factors “corruption” (13), “drug availability” (1) and “profits” (6)). (Formal identification of risky configurations in the CM of cocaine use (Fig.2) is performed by our colleague R. Portsev.)

The criteria of completeness related to the whole map

At last, there are following more or less obvious expert *criteria of completeness related to the whole map*: criterion of presence of essential factors, criterion of presence (completeness) of essential links, criterion of completeness of target factors, criterion of completeness of control factors, criterion of completeness of the environmental influences (Abramova (2010a). In all criteria of the last group, as well as in case of criterion $K_B(p, B(p))$, the proportional completeness is meant (when ignoring leads to essential decrease of the model adequacy and validity).

4. False transitivity of causal influences and some criteria of its detection

Under the false transitivity of causal influences the authors mean the situation when, according to the expert, $A \to B$ ("*A* is a reason of *B*") and $B \to C$, but $\neg(A \to C)$ instead of $A \to C$, expected according to the transitivity principle. In other words, essentially *C* does not depend (or partially depend) on A so that A is not an indirect reason of *C*.

Situations with false transitivity, contradicting the "common sense" and formal semantics of CMs, have been discovered by the authors at first in the chains of two influences (Abramova and Kovriga (2008a, 2008b)). In this paper the phenomenon of false transitivity is shown at the more complex cases

– false transitivity through long chains of influences (Abramova et al. (2010));

– false transitivity cycles.

The complex cases of false transitivity through long chains of influences

The fragment of the applied CM with discovered more complex case of false transitivity through long chains of influences between factors is presented on Fig. 5. The map has been created to analyze the problems connected with narcobusiness and drugs use in country N which has transit narcotraffic on its territory.

Presented fragment of the signed CM is added with two indirect influences $2 \xrightarrow{+} 3$ and $1 \xrightarrow{+} 4$ which "are logically deduced" from the chains of direct influences $2 \xrightarrow{+} 4 \xrightarrow{+} 5 \xrightarrow{+} 6 \xrightarrow{+} 3$ and $1 \xrightarrow{+} 3 \xrightarrow{+} 5 \xrightarrow{+} 6 \xrightarrow{+} 4$ accordingly the transitivity axiom. (Indirect influences are shown by a dotted line).

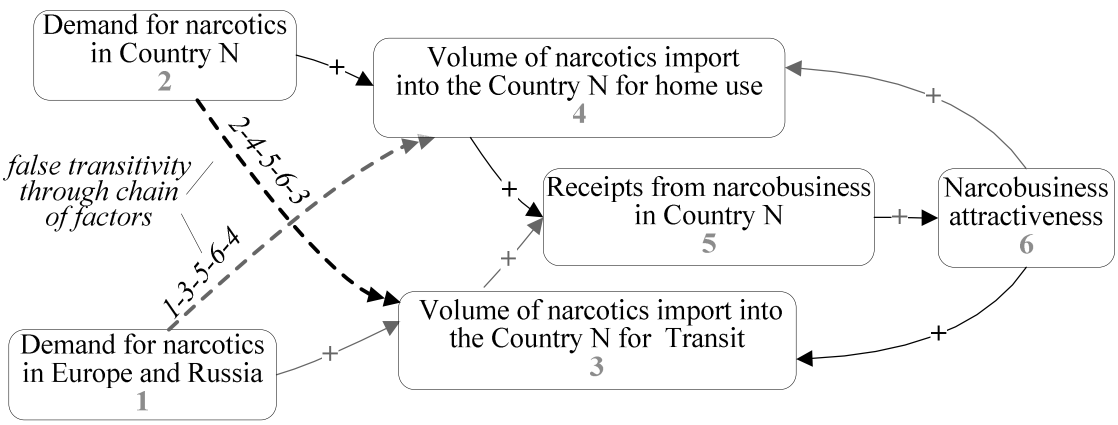

Fig. 5. Fragment of a real-life cognitive map of narcosituation with false transitivity

The mentioned indirect influences $2 \xrightarrow{+} 3$ and $1 \xrightarrow{+} 4$ mean the following:

an increase (decrease) in demand for narcotics in Country N, *other things being equal, causes an increase (decrease) in* volume of narcotics import into the Country N for transit;

an increase (decrease) in demand for narcotics in Europe and Russia, *other things being equal, causes an increase (decrease) in* volume of narcotics import into the Country N for home use.

However the direct estimation of presence of influences in pairs 2→3 and 1→4 for substantive considerations says that actually in each pair the factors are independent. Thereby in both cases false transitivity takes place.

Substantially it is possible to explain the false transitivity by the presence of risky (in logic sense) combinations of causal influences in the chain that generates it (transitivity). In this case, in the chain 2→4→5→6→3 not any change in receipts from narcobusiness in Country N (5), and therefore in narcobusiness attractiveness (6) is caused by change in volume of narcotics import into the Country N for home use (4), and in the following influence 6→3 a change in narcobusiness attractiveness (6) not necessarily changes volume of narcotics import into the Country N for transit (3). Therefore the given chain of factors does not mean obligatory indirect influence on all chain, and more detailed analysis of influences is required. In the second chain 1→3→5→6→4 the situation is similar. (For brevity, we omit the signs of influences. The sign of the total indirect effect is estimated by the signs of direct influences in the usual way.)

For revealing of risky fragments of CMs *criterion of absence of false transitivity of causal influences* $K_{FT}(S)$ is proposed. It is realized through a group of private criteria, applicable to links.

The analysis of fragments of CMs with false transitivity shows that their general feature is presence of concepts of factors which appear to be disproportionately general in extension of concept comparatively to other factors in the chain of direct influences. However the presence of such concepts not always leads to false inferences through transitivity, so we can talk only about risks that require further analysis.

Earlier, authors found the *criteria of factor concept extensions' proportionality in separate links* (Abramova and Kovriga (2008a,b)). In essence these criteria meant a search for new factor concepts which are more correct. Today, easier (for a practical use) and more pragmatic criteria are found out, which are based on formal logic. A check with the criteria precedes a search for more correct concepts.

One criterion from the group, $K_{g2}(A,B)$ which seems to be the easiest and most useful to reveal pairs of factor concepts, which create the risk of false transitivity of influences, is considered. Its weakly formalized expression looks as follows

$K_{g2}(A,B)$: *Situations when the change in A does not cause the change in B are unknown* (*or insignificant for the situation considered*),

where *A* and *B* are the names of factors p_1, p_2, such that $p_1 \rightarrow p_2$.

Using the presented criterion an expert can easily understand in the example considered (Fig. 5), that, for example, the concept (5), "Receipts from narcobusiness in Country N", designating the influence receiver, is excessive in its extension comparatively to the source of influence (4), “Volume of narcotics import into the Country N” for home use as far as the influence actually concerns only receipts from narcobusiness in Country N coming from home use. Similarly excessive in their extensions are the concept of the factor (5) as the influence receiver comparatively to (3) and concept of the factor (6) as an influence source comparatively to (3) and (4).

From the applied point of view, significant aspects of a problem of false transitivity of causal influences when solving concrete practical problems, are the early recognition of situations modeled by means of CM, that can become the reason of false inferences, and their diagnostics for decision-making on correction or even, if the one is impossible, to refusal from the technique chosen.

False transitivity cycles

The phenomenon of the false transitivity cycle in a CM consists in that a single influence (impulse) on one of factors, according to formal semantics of a CM, generates the cyclic sequence of impulses formally deduced by transitivity from a structural cycle of direct influences of factors. However, a problem domain expert does not recognize such behavior “by transitivity” in the real situation modeled with a given CM.

False cycles of transitivity with different mechanisms of their occurrence are found out by the authors of the given research and their colleagues in a number of research and applied CMs that enables to assume the regular nature of this phenomenon. It is interesting, that such cycles of false transitivity are found out already in the elementary structural cycles when two factors are directly linked with each other by a feedback cycle. It seems reasonable to consider such elementary cycles as the risk factors demanding expert verification by criterion of conformity of cyclic formal and modeled actual behavior.

A relatively simple example of false cycles of transitivity it is presented in Fig. 6 with the fragment of the applied CM from a research of the problem of complex safety of a region. The proposed conception of safety is presented in the form of structure of causal influences between the complex factor 6 “social and economic safety of region” and its components. (In the given fragment only two component factors are shown: 13 “the level of development of shadow economy” and 7 “ecological conditions”.)

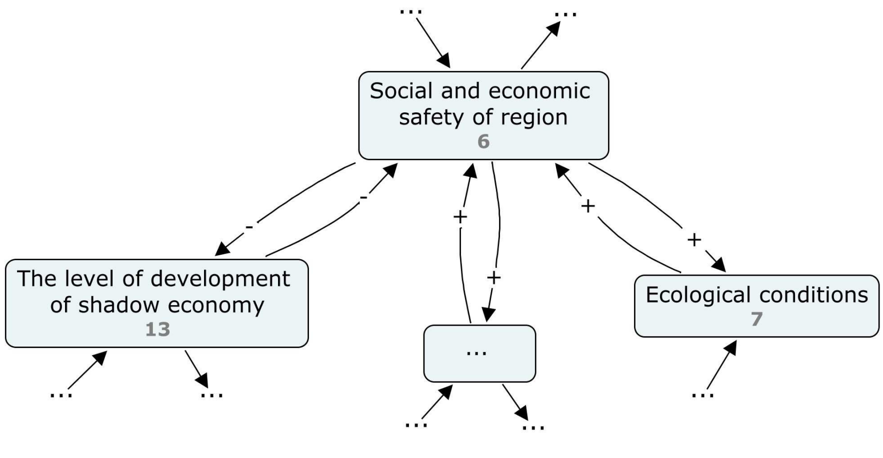

Fig.6. The fragment of the CM of complex safety of a region.

According to the formal behavioral semantics of such kind CMs (with any assignment of weights of individual influences in the CM), a single external impact on any of the component factors should cause, by transitivity of the influences (1) single change in value of not only the central factor 6, “social and economic safety of region”, but also of all its components, (2) cyclic processes of change in all factor values.

For example, an unsuccessful economic decision attracting increase of factor 13 in respect of leaving from taxes would formally generate degradation of ecological conditions (factor 7) with its subsequent cyclic change (13→6↔7). It is no wonder if such formal behavior is not recognized by experts what means detection of false transitivity cycles (6↔13, 6↔7, 6↔…).

In this rather simple example the root cause of false transitivity is the mechanism discussed earlier, i.e. excessive generality of the intermediate concept of factor of social and economic safety of region relative to concepts of the component factors in chains of influences 13→6→13, 7→6→7. It is easily found out with criteria of factor concept extensions’ proportionality in separate links.

5. Conclusion

A number of studies show that human-induced risks for validity of end results in cognitive mapping of ill structured situation dynamics are practically significant. Such risks are caused both by cognitive complexity of investigated problem situations and by features of modern CM languages. Comparison with processes of models construction in system dynamics, with account of criticism of modern system dynamics languages, beginning from CLDs, as well as analysis of processes of their understanding and mastering show that human-induced risk problems in system dynamics and cognitive mapping have related character.

In cognitive mapping two main problems are identified for which it is proposed to use different types of verification, and on the ways of solving the problems it seems reasonable to integrate the efforts on both fronts.

The main, more obvious direction is the early verification of specific models of complex situations by means of predefined criteria, beginning from the stage of qualitative modeling.

The aim is to reduce the first kind risks and direct errors, involved by decision-makers, experts, analysts and composers of specific models.

According to our research the principal role of such early verification whether in system dynamics or cognitive mapping is to identify risks of inconsistency with the semantics of languages in the second level of formalization (stage of quantitative parameters definition) in view of correction or even rejection of the selected language (either some CMs language or SFDs) if the relevant substantive features of the situation are not expressible by its expressive means.

Further cross-fertilization in this direction is possible through the formation of the compatible family of practical validity criteria and practical evaluation of their performance in the early stages of formalization, both in cognitive mapping and in system dynamics. As the ground for such integrated family of validity criteria could serve validity tests and criteria proposed in the known works on validation and verification in system dynamics (Forrester and Senge (1980), Burns and Musa (2001), Barlas (1996) and others), along with the results of this work.

The second, less obvious direction of applying verification to improve validity of modeling stems from criticism of modern languages adequacy for modeling complex situations for solving applied problems. It can be heard both in cognitive mapping (Carvalho (2010), Abramova et al. (2009), Abramova (2011)) and in system dynamics (Schaffernicht (2010)). More or less explicitly the object of criticism turns to be the quality of defining the semantics of modeling languages by theorists whether it be languages of cognitive mapping or of system dynamics. Thus the problems of modern modeling languages development (Schaffernicht (2010) and verification (Abramova et al (2011)) are putted forward, with taking account of semantic aspects. However, the question of what should be "good" languages under the requirements of safe use and conflicting requirements of cognitive clarity discussed in the paper is open, except for separate proposals. Moreover, our first appeals to various branches of cognitive science have shown that the question of how to systematically develop such languages is also open. Interdisciplinary research is needed.

Even today it is clear that such studies should focus not only on the available theoretical knowledge and trends but also on the experience of training and work of professionals in the applied domains, including model verification and validation.

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