Limits of Arbitrage: Understanding How Hedge Funds Fail

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Abstract

Even if arbitrage opportunities are found in a statistical sense, they might not be exploitable due to unexpected widening of spreads. This paper models such a case in the framework of a hedge fund. Specifically, Long Term Capital Management is presented as a case study. In particular, we calculate the likelihood of hedge fund failure and survival given different statistical arbitrage opportunities and hedge fund risk management decisions. Dynamic relationships between a hedge fund, dealer, and market (investor) are modeled. The model explores phenomenon when a fund manager who engages in arbitrage and uses high leverage might lose all his money before realizing positions at a profit. As assets go down in value, the firm has to post more collateral or decrease position exposure. We observe if positions converge before all collateral has been exhausted, the most profitable strategy is to post more collateral and increase position exposure. However, if positions diverge beyond the point of remaining cash, a hedge fund will avoid collapse if it decreases position exposure. However, we find that a large and visible hedge fund like LTCM that affects asset prices as it attempts to unwind its positions is not going to escape the collapse by decreasing position exposure because the effect of its sales will drive the stock price further down. We propose that given positions are well diversified and not closely correlated, leverage by itself does not lead to the collapse of a fund. Correlated positions in the absence of leverage might lead to a loss, but are not subject to collateral collapse. However, the superimposition of both leverage and induced high correlation between assets can lead to a collapse. The paper explores these "flight to quality" and "collateral collapse" dynamics in depth.

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Table of Contents

1. Introduction

This paper presents framework for modeling limits of arbitrage using system dynamics methodology. Limits of arbitrage are well researched and categorized in finance literature. However, most of approaches include econometrics, linear extrapolations or dynamic programming. Although feedback is known to play a critical role in nonlinear systems such as open market trading, this analysis has not been introduced in previous works. This paper is one in a series that tries to explain limits of arbitrage using system dynamics method. In particular, the paper explores how hedge funds fail given arbitrage opportunities. The collapse of the Long Term Capital Management is used as a case study.

Limits of Arbitrage

One of the most fundamental notions in finance is arbitrage. Arbitrage is defined as "the simultaneous purchase and sale of the same, or essentially similar, securities in two different markets for advantageously different prices" (Sharpe and Alexander, 1990). This kind of arbitrage requires no capital and does not have any risk in the limit that the two securities are identical. Given that L and S are identical securities in different markets trading at different prices, the arbitrageur will be certain to make a profit, and his future cash flows will be zero. However, according to the efficient market hypothesis (Samuelson, 1965) and the Law of One Price, the profitable arbitrage cannot exist. The Law of One Price asserts that any two assets (or positions formed from traded assets) with the same payoff must have the same value or price. The premise of the efficient market hypothesis is that stock prices are always "right"; therefore, no one can predict the market's future direction, which, in turn, must be "random." For this to hold, prices have to be set by rational and well informed investors. The hypothesis was developed by Samuelson (1965) and Harry Roberts and expanded by Eugene Fama and Merton Miller.

As long as there is one investor in the market for whom more is better, such an investor would take advantage of any arbitrage and scale it up arbitrarily. Such behavior is inconsistent with economic equilibrium. This conclusion is formalized in the No Arbitrage Theorem.

A more common strategy implemented by hedge funds is the statistical arbitrage or risk arbitrage. In this case, an arbitrageur has a fundamental price in mind. He finds two securities that have similar payoffs and act essentially the same except that at some time, one security is overpriced and another is underpriced. The hedge fund manager will buy an underpriced security and short sell the overpriced one, making the profit, and hoping that the prices will converge in the future. To borrow a security, a hedge fund has to post a collateral, long security or cash, for example. However, if positions diverge before converging, the hedge fund is faced with a margin call. A hedge fund has to either post more collateral or close the positions. Even though a hedge fund is going to realize profits once positions converge, in the short run, the hedge fund must carry a loss. A hedge fund might fail if it does not have enough capital to cover the margin.

While a losing trade may well turn around eventually (assuming, of course, that it was properly conceived to begin with), the turn could arrive too late to do the trader any good – meaning, of course, that he might go broke in the interim. John Maynard Keynes in his famous quote said: “Markets can remain irrational longer than you can remain solvent.”

Shleifer and Vishny studied the limits of arbitrage and the impact of noise traders on the arbitrage opportunities (Shleifer and Vishny, 1997). They warned that an arbitrage firm of Long-Term’s type can collapse if the market is overwhelmed by noise traders who push prices away from the true value. It might lead to adverse price shock that can force LTCM to liquidate its positions at low prices.

Leverage

L^R, leverage ratio, equals assets, A, divided by equity, E. $L^R = \dfrac{A}{E}$ (eq.1)

Return on equity, R^E or return on capital equals to $R^E = \dfrac{E_1 - E_0}{E_0}$ (eq 2)

However, return on assets, R^A equals to

$$R^A = \frac{A_1 - A_0}{A_0} = \frac{L_1 + E_1 - L_0 - E_0}{E_0 + L_0} = \frac{L_0(1+r) + E_1 - L_0 - E_0}{E_0 + L_0} = \frac{L_0 r + E_1 - E_0}{E_0 + L_0} \quad \text{(eq. 3)}$$

where L is leverage, and r is the interest rate that has to be paid back on leverage to the lender bank. For example, if $L^R = 2$ and r=0, then $R^E = L^R R^A$. A hedge fund will use leverage in order to increase return on equity.

The Federal Reserve Board, under a statutory provision known as "Regulations T," sets a limit on broker loans for stocks, or "margin." For the past twenty-five years, the Fed has set the maximum margin loan at 50 percent of the total investment which translates into the maximum $L^R = 2$.

Leverage is used to buy more securities than available cash allows you to. For example, say, stock XYZ is going to go up by 20% in a year. A customer puts $100 of his own money to XYZ. The early return is 20%, so in a year, the customer has $120. However, let's say that the customer borrowed another $100 (interest = 5 %). In a year, the stock XYZ is worth $240. After paying off the broker $105, the customer is left with $135. Therefore, with leverage, the customer's profit is 35%, which is significantly better than 20% with margin. However, if a position goes down, than a customer with margin can lose more money than if he did not use leverage. For example, the stock goes down by 20% in a year. Therefore, it would be worth $180 without a margin, or $160 with margin. After paying off $105, the customer is left with $55, which is a 45% loss, compared to 20% loss in the case a customer does not use leverage.

2. Hedge Fund Overview

The term 'Hedge Fund' originated when Alfred Winslow Jones founded a novel approach to investing in 1949. He discovered an innovative strategy for maximizing asset returns and minimizing market risk. The strategy was based on "hedging" long stock positions with short stock positions by using leverage to increase potential of returns. Jones bought seemingly cheap stocks and sold short overpriced stocks. In theory, the Jones's portfolio was "market neutral." Any market event will increase the value of one half of his portfolio and depress the second half. His net return would depend only on his ability to single out the relative best and worst. In 1966, Carol J. Loomis' article in Fortune magazine entitled *The Jones Nobody Can Keep Up With,* revealed that by using this double-parameter model, Jones outperformed the highest-ranking mutual funds of the 1950's and 1960's by over 44%. This breakthrough technique catalyzed the most lucrative and unregulated financial industry in the history of economics, a multi-billion-dollar industry consistently attracting smart and wealthy individuals.

As of October 2003, the size of the global single-manager hedge fund universe (not including funds of funds) is $650 - $700 billion (Tremont Company). There are about 5,000 global single-manager hedge funds in the hedge fund universe. There are about 1,200 – 1,400 funds of funds. There are about 3,000 distinct hedge fund managers that manage both offshore and domestic accounts. In 1990, there were 610 funds managing $39 billion. Despite spectacular growth and performance in double digits of various hedge funds, there have been many horrifying collapses and bankruptcies of hedge funds such as Granite Capital and LTCM.

Hedge funds differ from mutual funds and other investment vehicles by both internal structure and investment discipline. Hedge fund managers are not restricted to any particular type of investments. Hedge funds can buy (long) or sell (short) securities that they do not own. They are not restricted to common ``buy and hold'' strategies. Most U.S. hedge funds are limited partnerships, or limited liability companies, established to invest in public securities. However, there is no common definition of a hedge fund. U.S. hedge funds are defined by their freedom from regulatory controls stipulated by the Investment Company Act of 1940. Before 1996, a hedge fund had a 100 investor limit in order to qualify as a limited partnership. However, under the National Securities Markets Improvement Act of 1996, the 100 investor limit was lifted. The minimum new worth requirement for a qualified investor is $5 million and the minimum institution capital is $25 million. Companies can also become reporting companies voluntarily by filing with the SEC. Under the Exchange Act, a company must become a reporting company if it has at least 500 shareholders and $10 million in assets. The Exchange Act contains registration and reporting provisions that may apply to hedge funds.

Depending upon their activities, in addition to complying with the federal securities laws, hedge funds and their advisers may have to comply with other laws including the Commodity Exchange Act ("CEA"), rules promulgated by the National Association of Securities Dealers ("NASD") and/or provisions of the Employment

Retirement Income Security Act ("ERISA"). In addition, hedge funds may be subject to certain regulations promulgated by the Department of the Treasury, including rules relating to the prevention of money laundering. Moreover, hedge fund advisers are subject to certain state laws.

Offshore hedge funds are typically corporations registered in a tax haven such as the British Virgin Islands, the Bahamas, Bermuda, the Cayman Islands, Dublin, or Luxembourg, where tax liabilities to non-U.S. citizens are minimal. In general, the hedge fund industry is not transparent to regulators unlike the mutual funds industry. Like mutual funds, hedge funds are actively managed investment portfolios holding positions in publicly traded securities. However, unlike mutual funds, hedge funds have greater flexibility in the kind of securities they can invest in. Hedge funds can invest in domestic and international debt and derivative securities. They can take undiversified positions, sell short, and lever up their portfolios. These alternative investments mainly attract institutions and wealthy individuals with minimum investments typically in the range of $250,000 - $1 million. Hedge funds are also characterized by a substantial managerial investment and strong managerial incentives. On average, hedge fund managers receive a 1% annual management fee and 20% of the annual profits. Most of funds employ a bonus incentive fee: managers are paid a percentage of the excess of a fund's return over some level, commonly called a "high-water mark." If a hedge fund incurred losses in the past, its managers can be paid in present period only if return in this period exceeds the "high-water mark" plus past losses.

Hedge funds seek to generate above-average returns to their investors. For many investors, hedge funds act as risk managers since their returns are often not correlated with equities or fixed-income securities. Most hedge funds use the following strategies:

- Short selling. The strategy involves the sales of borrowed securities hoping the price of these securities will go down. A hedge fund manager should have sufficient skills and expertise to identify overvalued securities and being able to cost-efficiently borrow the overpriced stocks.
- Hedging. The strategy involves decreasing risk inherent in hedge fund's portfolio. The risks might be the following: political, economic, company, interest rate and market. Hedging can use the combination of derivatives and short sales. Hedge fund

managers should be able to use efficient hedging techniques. For example, it is very costly and not efficient to hedge by shorting a share of a stock for every share held long in the portfolio. It might be more economical to short contracts or shares of different assets which are highly correlated with the underlying asset.

- Arbitrage. The strategy involves finding any price inefficiencies or discrepancies between securities or markets. The strategy is risk-free; however, in current efficient markets it is very hard to find any price inefficiencies. Even if such inefficiencies are found, they do not last. Therefore, fund managers tend to use leverage in order to enhance returns due to such minuscule short-term opportunities.
- Leveraging. The strategy involves either borrowing money, to increase the size of the portfolio; or assigning cash or securities as down payment, collateral, or margin for a percentage of the position one seeks to establish.
- Synthetic positions or derivatives. The strategy involves using derivative contracts to establish certain positions or strategies in the hedge fund.

There are many hedge fund types. The list of hedge fund types is the following:

- Macro funds
- Special-situation funds
- Pure equity funds
- Convertible arbitrage funds
- Funds of funds
- Market-neutral funds
- Commodity trading advisor funds
- Private equity funds
- Risk arbitrage funds
- Long or short funds
- Emerging market funds
- Event risk funds
- Restructured or defaulted security funds

The recent hedge fund collapses and developments in hedge fund industry make SEC anxious. At the end of July 2002, 55% of hedge funds in the TASS database were down in asset values for the year. Because of high-water marks, the need to recoup losses before taking incentive fees on gains, it will be difficult for many hedge funds to obtain a profit soon. That increases the probability of default for many hedge funds. The average hedge fund advisor is 35 years old, very young. The average age for a hedge fund (not included funds of funds) is 46 months with a median of 35 months (Getmansky, 2003). Also, the recent "retailization" of the industry – the introduction of products that make hedge funds available to investors with as little as $25,000 to invest, makes SEC worried.

3. Long Term Capital Management Hedge Fund

Long Term Capital Management (LTCM) was started in February, 1994 by the infamous Salomon Brother's arbitrage trader John Meriwether. The beginning of LTCM was very rocky, having trouble gathering enough investors to trust John Meriwether. After hard work from its prime broker, Merrill Lynch and its many talented partners, who included Nobel prize winners Myron Scholes and Robert Merton, LTCM eventually raised 1.25 billion dollars of assets to launch the hedge fund.

The structure of LTCM was drastically different from other hedge funds. For example, investment fee paid to the partners was 25% instead of the usual 20%, and yearly management fee was 2% instead of the usual annual 1%. LTCM also required investors to invest at least $3 million. Investors were forced to sign a contract of holding their investments for at least three years. LTCM was also extremely secretive. LTCM had about 100 investors and 200 employees.

LTCM's financial strategy concentrated on "relative value" trades in bond markets. Long-Term would buy underpriced bonds and sell overpriced ones. It would bet on spreads between pairs of bonds to either converge or diverge. For example, they bought underpriced off-the-run US treasury bonds (because they are less liquid) and shorted on-the-run (more liquid) treasuries, betting on the convergence of the two assets. The government has the same likelihood of paying off off-the-run and on-the-run bonds. The net risk was minimal because long and short positions were highly correlated. Bonds usually rise and fall in sync; therefore, spreads don't move as much as the bonds themselves.

Another trade example is the following: If interest rates in Italy were significantly higher than in Germany, making Italian bonds cheaper than German ones, the hedge fund would invest in Italy and short Germany. The fund would profit if this differential narrowed. Since most of the spreads discovered by LTCM were very small, LTCM had to have huge leverage in order to make significant profits. The leverage rate was about 20 to 30 times the investment. The Federal Reserve Board, under a statutory provision known as "Regulation T," sets a limit on broker loans for stocks, or "margin." For the past twenty-five years, The Fed has set the maximum margin loan at 50 percent of the total investment. When LTCM purchased stocks, it was subject to Reg T. However, the fund rarely purchased stock outright; instead, it entered into derivative contracts such as swaps, that mimicked the behavior of stocks. LTCM also used highly complicated mathematic models to achieve elevated returns and control risk. They utilized swaps options and other derivatives to control their trades.

The firm earned 20% net of fees in 1994. In 1995 it earned 43%, in 1996 - 41%, and in 1997 – 25% net of fees return on equity. Including the money from new investors, the company's equity capital had, in less than two years, tripled, to a total of $3.6 billion. The assets also grew to $102 billion. Thus, at the end of 1995, it was leveraged 28 to 1. Leverage did not include derivatives. The return on total capital was approximately 2.45%. By the spring of 1996, the Long-Term grew to $140 billion in assets. By 1997, it had more than $5 billion in equity. By 1998, the worst month was the loss of 2.9%. According to their models, the maximum that they could lose on any single day was $45 million.

By borrowing or selling bonds that were in high demand with a smaller interest rate and by purchasing bonds that were slightly less in demand and that therefore yielded a little bit higher interest rate, LTCM was in effect a liquidity provider to capital markets. As a bank which earns money on a spread by charging borrowers a slightly higher interest rate than it paid to depositors, the hedge fund was earning profit on the spread between the two assets. LTCM in effect was buying assets that everybody wanted to sell. Therefore, those assets were not totally independent. In case of a mass selling panic, the fund could default if everybody wanted to sell and nobody wanted to buy.

LTCM also had several brokers lending money to the fund. Brokers involved were Bear Stearns, Goldman Sacks, Morgan Stanley, JP Morgan, Lehman Brothers, Chase Manhattan, Banker's Trust, Union Bank of Switzerland, UBS Warburg and Salomon Smith Barney. Long-Term would place orders of each leg of a trade with a different broker, so nobody could see the whole trade. LTCM could get rid of the haircut fee required to be paid to brokers for borrowing money. All of its brokers complied with the LTCM's strict requirements, allowing the fund to be the most unregulated hedge fund during that time.

LTCM disclosed its total assets and liabilities to its banks each quarter and to investors each month. It also reported those numbers to the Commodity Futures Trading Commission. It reported its derivative totals only annually. People were aware of high leverage and exposure; however, nobody thought that it might lead to LTCM failure. However, LTCM did not disclose details of assets. Banks only knew their own exposure to Long-Term, but not exposures of others. About 55 banks were doing financing for LTCM.

The failure of LTCM came on as a thundering shock to the financial world. When the Russian government defaulted on its debts in August 17, 1998, liquidity suddenly evaporated from international financial markets. Instead of converging, LTCM's positions began to diverge. The partnership knew perfectly well that over the short term, prices might diverge. But they always calculated the risks and the consequences of divergence with special statistical "value-at-risk" models. In August 1998, asset prices plummeted. LTCM lost lots of money because it could not liquidate its assets before the value of its portfolio dropped. LTCM was a victim of "flight to liquidity." People wanted to buy less risky Treasuries and get rid of risky bonds. People were afraid of going short on Treasuries. Only LTCM held short positions on Treasuries and long positions in riskier bonds. And as Treasuries rallied, spreads between them and other bonds widened. Mortgage-backed securities jumped from 96 basis points over Treasurys to 113 points. Corporate bonds rose from 99 to 105, and junk bonds rose from

224 to 266. Even seemingly safe off-the-run Treasurys climbed from 6 points over to 8 points over. In every market, the spreads widened leading to LTCM losing money.

In June, the fund lost 10%. On a single day, August 21, the LTCM portfolio lost $553 million – 15% of its capital. It had started the year with $4.67 billion. Suddenly, it was down to $2.9 billion. On September 2, 1998 Meriwether sent a letter to his investors saying that the fund had lost $2.5 billion or 52% of its value that year, $2.1 billion in August alone. LTCM capital base had shrunk to $2.3 billion. The fund had $125 billion in assets – 98 % of its prior total and the leverage increased to 55:1 due to the now-shrunken equity – in addition to the massive leverage in its derivative bets, such as equity volatility and swap spreads. At that point, leverage was very high, and the fund's partners were looking forward to sell some positions and raise more money before the end of the month. LTCM had a difficulty of reducing its positions with the markets under the stress. There was no liquidity in the market. Everybody wanted to be out at the same time – something that models missed. When losses mount, leveraged investors such as Long-Term are forced to sell, lest their losses overwhelm them. When a firm has to sell without buyers, prices are very high. In addition, Wall Street players learned more about the fund's positions, and went against them. They wanted to "squeeze" as much as possible from the fund, knowing that if the fund gets help from the government, it would be able to buy back its shorts. Therefore, anybody who held those securities would make money.

In September 1998, many banks were exposed to the same positions as LTCM. Therefore, to cut their losses, they unwound those positions, thus, hurting LTCM. Therefore, both cutting the losses and predatory trading led to the collapse of the fund. Also, Long-Term trades were in highly specialized instruments, such as equity volatility. Only a handful of banks traded them. LTCM was short on the equity volatility, and sooner or later they would have to buy. The dealers refused to sell, only at very high prices.

On Thursday, September 10, the firm had lost $145 million; on Friday, $120 million. The next week on Monday it lost $55 million: on Tuesday, $87 million, and on Wednesday $122 million. LTCM was down to $1.5 billion. Due to the excess leverage of LTCM, the potential failure of the hedge fund triggered the attention of the Fed. On

September 20th, 1998, the fed representatives visited the office of LTCM in Greenwich, CN. They were amazed to find that LTCM's on balance sheet assets totaled around $125 billion, on a capital base of $4 billion, a leverage of about 30 times. But that leverage was increased tenfold by LTCM's off balance sheet business whose notional principal ran to around $1 trillion. On September 21, 1998, LTCM had its second biggest loss of $500 million. At that point, the assets were worth $100 billion. Thus, even omitting derivatives, its leverage was greater than 100 to 1. Now, if LTCM lost 1%, it would be wiped out. LTCM exposed its books to Peter Fisher of New York Fed. He saw that in all markets LTCM was badly hurt. All its positions became perfectly correlated in the crisis period. Fisher was not worried that the markets would go down; he was afraid that they would not trade at all. Bankruptcy was out of the question because bankruptcy filing would make all counterparties go after the collateral further depressing the value of the collateral. Also, nobody wanted to buy the firm and obtain assets such as equity volatility or sophisticated derivatives. If one bank bought the firm, then it would be in the same position as LTCM, and given that by now positions of LTCM were exposed, other banks would try to trade against it. Therefore, the only solution was for all banks to work together.

The Fed convinced all the LTCM's major brokers to bail out the fund's losses, believing that if LTCM was allowed to fail, the world financial market would be at risk. If Long-Term defaulted, all of the banks that lent to LTCM would be left holding one side of a contract for which the other side no longer existed. Undoubtedly, there would be a frenzy as every bank rushed to escape its now one-sided obligations and tried to sell its collateral from Long-Term. LTCM had lots of derivatives which were relatively new. Officials were afraid that the financial system could crash. The consortium of 14 banks got $3.65 billion in exchange of 90% of the equity in the fund. The LTCM's existing investors would retain the rest 10%. On July 6, 1999, LTCM repaid $300 million to its original investors. It also paid out $3.65 billion to the 14 consortium members. LTCM met all margin calls. All of its debts to creditors were repaid in full. Through April 1998, the value of a dollar invested in Long-Term quadrupled to $4.11. By the time of the bailout, only five months later, 33 cents were remained. After fees, each invested dollar has grown to $2.85 and then shrank to 23 cents. In net terms, LTCM lost 77%.

There are many speculations of major reasons why the hedge fund failed. Many believed that it wasn't going to fail at all. In fact, the position taken by LTCM was simply going to take time to recover and eventually make a profit for the firm. There are other reasons for the collapse besides LTCM's strategy. First, the "value-at-risk" model used by LTCM did not anticipate the "flight to liquidity" taken place in August and September of 1998. Second, there were other hedge funds and major investment banks that mimicked strategy used by LTCM in convergence arbitrage. Third, LTCM partners lost faith in the strategy and started closing positions using the firm's assets. Fearing the failure, they made it inevitable by draining the firm of its remaining capital. Fourth, LTCM had about 8% of its book exposure to Russia, which could come to about $10 billion exposure. Fifth, LTCM took speculative positions in takeover stocks, such as Tellabs whose share price fell over 40% when it failed to take over Ciena. Sixth, LTCM was exposed to mortgage-backed securities, which experienced a downturn in 1998.

4. Brokerage Accounts

Brokerage houses offer clients numerous types of accounts. The most common ones are cash and margin accounts. These accounts represent different levels of credit and trustworthiness of the account holder as evaluated by brokerage houses.

A cash account is generally called "Type 1" account. A customer who has a cash account can make trades in that account, but he has to pay in full for all purchases by the settlement date. Again, different brokerage houses have different rules depending on the relationships with an entity. Several brokerage houses would allow a customer to execute buy or sell orders before cash is deposited in the account. The requirements to open a cash account are very minimal. However, several brokerage houses may require a significant deposit, of as much as $10,000, before customers can open the account.

A margin account is a type of brokerage accounts that allows customers to borrow money against securities they own. This account is sometimes called a "Type 2" account. In order to obtain this account, an entity must pass a security and background check. Short sales also occur in a margin account. Having a margin account makes it possible to take a margin load. A customer can buy securities or short sell securities on margin, or he can extract cash from an equity position without having to sell it (thus avoiding the taxes on selling positions or the chance of missing a run-up). In order to sell-short a stock, a broker needs to lend the stock to sell. The broker goes to another client's account or to his own account and borrows those shares to lend it to the customer for the short-sale. Interestingly, when a customer borrows money from the brokerage firm, the customer has to pay a fee to the dealer. However, when a broker lends a security for short-sell, he is not going to pay any interest on the proceeds from the short. There are exceptions to this rule: really big funds can negotiate a full or partial payment of interest on short sales funds provided there is sufficient collateral and the dealer does not want to lose the client.

5. Regulations – Margin Requirements

The basic rules for margin requirements are set by the Federal Reserve Board, the New York Stock Exchange, and the National Association of Securities Dealers. Every broker must apply the minimum rules to customers, but a broker is free to apply more stringent requirements. The amount an entity can borrow from a broker is closely regulated. The Federal Reserve Board's Regulation T states how much money an entity can borrow to establish a new position. The NYSE's Rule 431 and the NASD's Rule 2520 both state how much money an entity can continue to borrow to hold an open position. Federal requirement is 50% for long positions and 150% for short positions. Note, that the first 100% of the short sale can be satisfied by the proceeds from the short sale, leaving just 50% for the customer to maintain in margin. In the end, it looks similar to maintaining a long position. For example, $100,000 of cash can be used to buy $200,000 worth of stock. Maintenance margins can change based on brokerage houses and client relationships.

Here are few examples how to account for margin requirements. For simplicity, we assume a conservative estimate that the house margin requirement equals the Regulation T margin requirement of 50%.

Accounting for Cash, Long and Short Positions.

Case 1.

A hedge funds has $50 in cash and $100 in long positions. Therefore, Equity = $50 + $100=$150.

Assets	**Liability**
$50 – Cash	$0
$100 – Long position	
	Equity
	$150

Table 1_1: Balance Sheet for Case 1

Case 2.

A hedge fund has $50 in cash and $100 in long positions. Given 50% margin requirements, a hedge fund can at most carry ($50+$100)*2=$300 positions. It already has $100 long positions; therefore, can carry $200 short positions. In order to carry $200 short and $100 long positions, the equity in the account should be at least ($200+$100)/2=$150. The hedge fund has exactly $150 in equity.

Assets	**Liability**
$50 – Cash	$200 – Short position
$100 – Long position	
$200 – Proceeds from Short position	**Equity**
	$150

Table 1_2: Balance Sheet for Case 2

For a more detailed picture, in the margin account, the value of the long position is $100. Debit = $50. Therefore, equity = market value of the long position – debit = $50. Therefore, the maintenance margin = equity/value of the long position = $50/$100 = 0.5 is within the allowed limit and no margin calls are issued. In the short account, the value of the short position = $200. Credit = proceeds from the short position + credit from the margin account + cash = $200+$50+$50=$300. Equity = credit - market value of the short position = $300-$200 = $100. The maintenance margin = equity/value of the short position = $100/$200 = 0.5, which is within the allowed limit and no margin calls are issued.

Case 3.

Given case 2, the short position went up to $220. The new balance sheet looks as following:

Assets	**Liability**
$50 – Cash	$220 – Short position
$100 – Long position	
$200 – Proceeds from Short position	**Equity**
	$130

Table 1_3: Balance Sheet for Case 3

The equity in the account is $100+$50-$20=$130. However, equity needed is ($220+$100)/2=$160. Therefore, extra margin needed is $30. The hedge fund has a choice: either to come up with more cash- at least $30, or sell $60 worth of long positions.

Analyzing more in detail, the equity in the margin account = value of the long position – debit = $100 - $50 = $50. Therefore, maintenance margin = $50/$100 = 0.5. The maintenance requirement is met. In the short account, the value of the short position = $220. Credit = proceeds from the short position + debit from the margin account + cash = $200 + $50 + $50 = $300. Therefore, equity in the short account = credit – value of the short account = $300-$220 = $80. Note, that the sum of equity in margin and short accounts equals to the total equity of $130 that is obtained earlier. Maintenance margin for the short account = equity in the short account/market value of the short stock = $80/$220 = 0.36, which is less than 0.5, the required margin. The margin call equals to Market value of the short * (1+ Maintenance margin) – credit = $220*1.5-$300 = $30. As obtained above, the extra margin needed is $30. The hedge fund has a choice: either to come up with more cash- at least $30, or sell $60 worth of long positions.

Generally, given changes in margin and short accounts, margin call equals to:

Margin Call = $S_t^{H,S} P_t^S * (1+m) - S_t^{H,L} P_t^L (1-m) + Debit_t - Credit_t$ (eq. 4)

where m is the required maintenance margin.

Case 4.

Given case 3, if a hedge fund decides to come up with $30 of cash to cover margin, then new equity will be $160. New equity = $50+$30+$100-$20=$160. This is exactly what is needed. The new balance sheet looks as following:

Assets	**Liability**
$80 – Cash	$220 – Short position
$100 – Long position	
$200 – Proceeds from Short position	**Equity**
	$160

Table 1_4: Balance Sheet for Case 4

Case 5.

Given case 3, if a hedge fund decides to sell $60 worth of long security to cover margin, then new equity will be $130. New equity = $50+$60-$20+$40=$130. New equity needed = ($40+$220)/2=$130.   Therefore, extra margin is 0. The balance sheet looks as following:

Assets	**Liability**
$110 – Cash	$220 – Short position
$40 – Long position	
$200 – Proceeds from Short position	**Equity**
	$130

Table 1_5: Balance Sheet for Case 5

Case 6.

Given case 2, now the short position becomes worth $180 instead of $200. The balance sheet looks as follows:

Assets	**Liability**
$50 – Cash	$180 – Short position
$100 – Long position	
$200 – Proceeds from Short position	**Equity**
	$170

Table 1_6: Balance Sheet for Case 6

Equity is worth $170=$50+$100+$20. However, equity needed = ($180+$100)/2=$140. Therefore, $170-$140=$30 is an excess equity.

Case 7.

Given case 6, there is $30 excess equity. A hedge fund can decide to free up $30 worth of cash and use this cash to borrow $30*2=$60 worth more short security. The balance sheet will look as follows:

Assets	**Liability**
$50 – Cash	$240 – Short position
$100 – Long position	
$260 – Proceeds from Short position	**Equity**
	$170

Table 1_7: Balance Sheet for Case 7

Equity needed is ($100+$240)/2=$170. Current equity = $50+$100+$20=$170.

Case 8.

Given case 6, there is $30 excess equity. A hedge fund can decide to free up $60 worth of long positions, and use it to borrow more money = $60. The balance sheet will look as follows:

Assets	**Liability**
$50 – Cash	$240 – Short position
$100 – Long position	
$260 – Proceeds from Short position	**Equity**
	$170

Table 1_8: Balance Sheet for Case 8

Equity needed is ($100+$240)/2=$170. Current equity = $50+$100+$20=$170.

Case 9.

Given case 1, say both long and short positions change. The value of the long position went down to $80 from $100, and the value of the short position went up to $220.

Assets	**Liability**
$50 – Cash	$220 – Short position
$80 – Long position	
$200 – Proceeds from Short position	**Equity**
	$110

Table 1_9: Balance Sheet for Case 9

Current equity = $50+$80+$200-$220 = $110. Equity needed is ($80+$220)/2 = $150. Therefore, margin call is $40.

To look in detail in margin and short accounts, the value of the long position in the margin account equals to $80. Debit = $50. Therefore, equity = $30. Margin = $30/$80 = 0.38, which is less than the required 0.5. Therefore, margin needed = value of the long position * margin – equity = $80*0.5-$30 = $10. In the short account, the value of the short position is $220. Credit = proceeds from the short position + debit from the margin account + cash = $200+ $50 + $50 = $300. Therefore, the equity in the short account = credit – value of the short account = $300-$220 = $80. Note, that the sum of equity in margin and short accounts equals to the total equity of $110 that is obtained earlier. Maintenance margin for the short account = equity in the short account/market value of the short stock = $80/$220 = 0.36, which is less than 0.5, the required margin. Margin call equals to Market value of the short * (1+ Maintenance margin) – credit = $220*1.5-$300 = $30. As obtained above, extra margin needed is $40 = $10 (from margin account) + $30 (from short account).   The hedge fund has a choice: either to come up with more cash- at least $40, or sell $80 worth of long positions.

6. Broker-Dealer

Broker-dealers (dealers) are intermediaries between a client and a bank. They purchase, sell, and short-sell securities for a client, borrow securities from one client or form their own inventory and lend them to another client for short-selling. Dealers collect fees on all transactions and the amount borrowed. Dealers are responsible to monitor trading and credit risks. They are responsible to do background checks on their counterparties and establish credit limits. They are responsible for monitoring collateral which they pledge with a bank (lender).   The dealer's balance sheet looks the following with respect to short-sells of a hedge fund. Note, we are omitting assets and liabilities from other accounts.

Assets	**Liability**
Cash	Proceeds from Short position
$100	$100

Table 2: Balance Sheet for a Dealer

Proceeds from Short position is the credit balance payable. For example, a hedge fund short sells $100 worth of securities. The $100 proceeds from short position are recorded on the Assets side of the hedge-fund balance sheet, to be given to a hedge fund once it decides to buy back the short positions. Therefore, the Proceeds from Short position are recorded on the Liability side of the Dealer's balance sheet. This $100 cash is deposited in a bank under the dealer's name.

7. Model Conceptualization

Model Purpose

Even if arbitrage opportunities are found in a statistical sense, they might not be exploitable. Moreover, a fund manager who engages in such arbitrage might lose all his money before realizing the positions at a profit. For example, lots of hedge funds find arbitrage opportunities that are usually very miniscule considering almost efficient markets, and leverage up the positions in order to make high profit margins. As assets go down in value, the firm has to post more collateral or unwind positions. If it is unavailable, this often leads to a hedge fund collapse.

However, given that positions are well diversified and not closely correlated, leverage by itself, does not lead to the collapse of a fund. Correlated positions in the absence of leverage might lead to a loss, but are not subject to collateral collapse. Given diversified positions in a fund, a price drop in one asset does not necessarily correspond to a price drop in another asset, even less likely there is a possibility of a cascade in drop in prices of all assets. However, the superimposition of both leverage and induced high correlation between assets can lead to a collapse. This is something that sophisticated hedge funds like LTCM did not take into equation in determining risk exposure. Their decisions were bounded rational. The managers separately managed leverage and diversification of positions, not thinking that two can feed on each other during a period of a crisis.

Unlike other financial institutions such as mutual funds and banks, hedge fund can get exposed to various kinds of assets and borrow on margin. Therefore, the dynamics of "flight to liquidity" and "collateral collapse" can be best studied in the framework of a hedge fund. Even if a hedge fund has great positions that guarantee a statistical arbitrage, the hedge fund might collapse before these positions converge and make a profit.

Model Boundary

The model has a hedge fund, a dealer, and market (investors). It has both financing functions as well as psychological ones such as “flight to quality” and “flight to liquidity” feedbacks. Balance sheets of a hedge fund and a dealer are modeled as well as decisions of a hedge fund on taking leverage and how to deal with a margin call. In the model, a hedge fund can either decide to post more collateral from available cash or close out positions once a dealer imposes a margin call. The dealer is modeled. During the “liquidity crunch,” a dealer is risk-averse and is not willing to hold a lot of inventory of illiquid asset. Investors are modeled. Investors can decide to hold cash, liquid and illiquid assets. Different types of investors are modeled: momentum, imitators and noise. Price is endogenously determined in the model based on demand supply balance.

Time Horizon

The time horizon is 100 days, or over 3 1/3 years for the model. I am using the data for Long Term Capital Management case that has data for four years, from inception of the fund from June, 1994 to its collapse, September, 1998. “Liquidity crunch” is modeled over 10 days, which is similar to what is found in the LTCM case.

Reference Mode

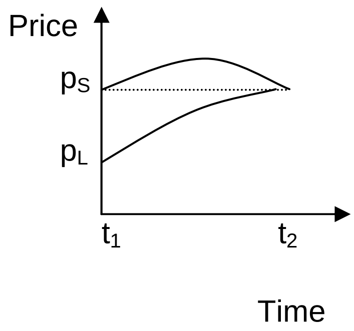


Figure 1. Successful Arbitrage

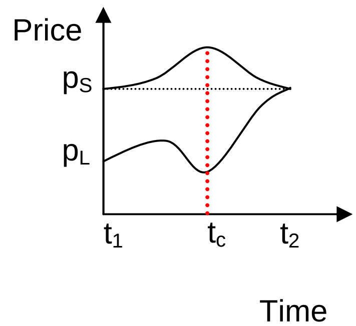


Figure 2. Collapse Before Profits Are Realized

A hedge fund will engage in the arbitrage position at time t_1. The profit will be realized at time t_2 when prices of L and S assets converge. However, it is possible that before converging prices diverge at time t_c. Given hedge fund' exposure, leverage decisions and hedge fund manager's decision how to deal with margin calls, a hedge fund can either fail or realize long-term profits while carrying short-term losses. The goal of the paper is to understand which decisions and effects lead to the collapse of a hedge fund, and whether it is possible to prevent the collapse. The collapse is measured by the negative total equity of a hedge fund.

8. Part 1 – Two Agents

Dynamic Hypotheses

"If you aren't in debt, you can't go broke and can't be made to sell, in which case "liquidity" is irrelevant. But a leveraged firm may be forced to sell, lest fast accumulating losses put it out of business. Leverage always gives rise to this same brutal dynamic, and its dangers cannot be stressed too often." (Lowenstein, 2000).

Let's consider the following financial entities: a hedge fund, a dealer, and a bank. A hedge fund is interested in obtaining leverage. It goes to a dealer, and can borrow money from a dealer at the maximum 50% margin (value of leverage divided by the total value of positions). Therefore, for example, if a hedge fund has $10 million worth of security L, it can borrow another $10 million worth of security S from a broker and sell it to the market. The dealer earns the transaction fee as well as charges the hedge fund for the loan. Now, the dealer delivers \$10 million of security L to the bank and sells \$10 million of security S to the market. If the value of security S goes up in value, the hedge fund has to put more collateral or sell security L and vice versa. A dealer is in the business of extending credit. The dealer will require more collateral if a value of a long position goes down or the value of the short position goes up, even if the trade might be profitable in the future. Say, the value of security S goes up by 20% from \$100 to \$120. The value of security L is maintained at \$100. Therefore, to maintain this position, the hedge fund should have equity of \$110, and it only has \$80. Therefore, a hedge fund has to put an additional \$30 worth of collateral.

Let, L be the value of leverage, A is the total value under management of a hedge fund, its assets. E is the equity of a hedge fund. Therefore,

$A=E+L$ (eq. 5)

L/A should be at most 50%. In this case, E is the same as collateral. So, if A decreases to A_1, then the new collateral to be posted is $L/0.5- A_1$.

The collateral to be posted is $\max(0, L/0.5- A_1)$ (eq. 6)

Dealers do not win if the value of positions hedge fund is holding goes up; however, they lose if the value of positions goes down. They might end up responsible for the value to be paid back to the banks. A broker dealer makes money by providing credit. He does not want to lose money. The stock and flow diagram of the interactions between a hedge fund, a dealer, a commercial bank, and a market is shown in Figure 3.

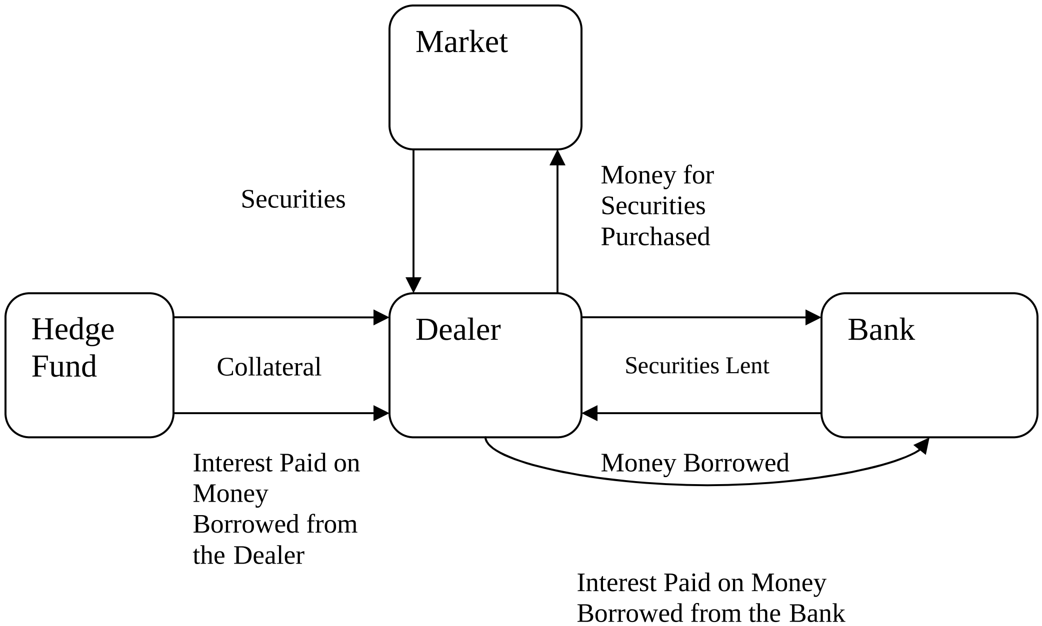


Figure 3. Hedge fund, Dealer, Commercial Bank, and Market Interactions

If a dealer calls a hedge fund with a margin call, a hedge fund can either sell the assets, thus reducing the margin, as depicted by the balancing loop B1 Cover Margin By Closing Positions, or by using additional cash (proceeds from other trades) to cover the margin, as depicted by the balancing loop B2 Cover Margin By Available Cash in Figure 4.

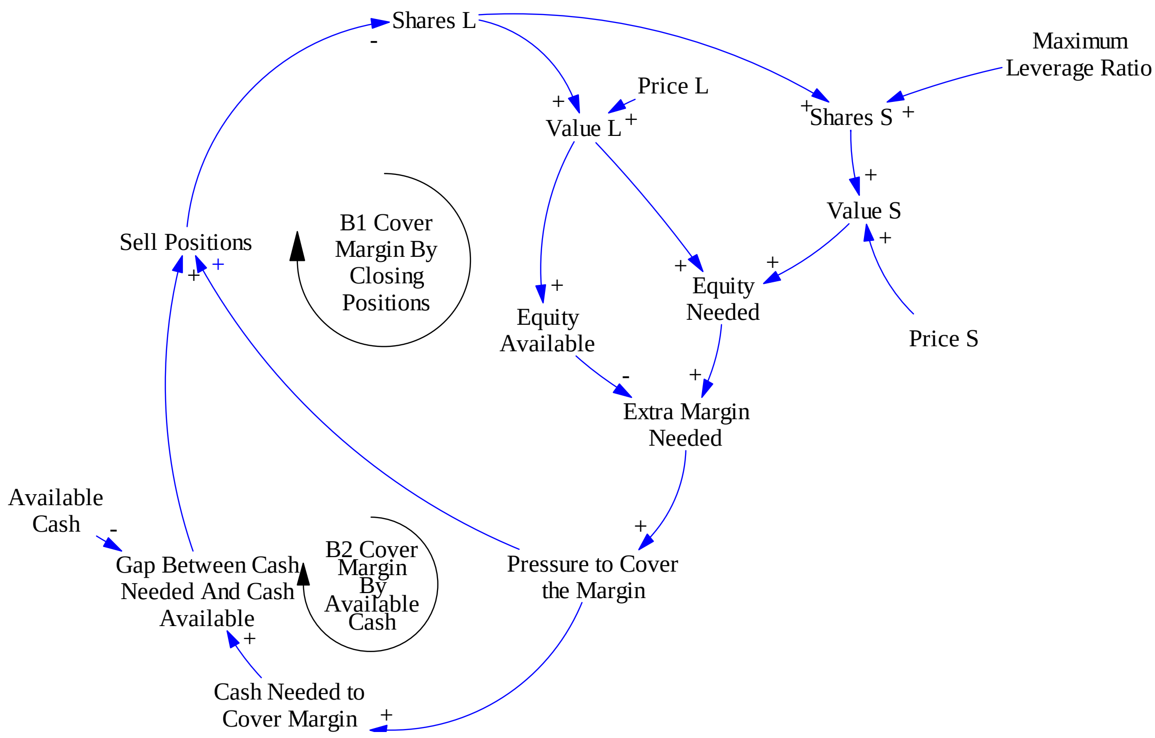


Figure 4. Hedge Fund Decisions How to Deal With Margin

Hedge Fund Strategy

A hedge fund maximizes its profits from the statistical arbitrage strategy:

$Max(-S_t^{H,S} P_t^S - S_t^{H.L} P_t^L)$

subject to:

1) The prices P_t^S and P_t^L will converge at time t^f

2) $Cash_t^H \geq 0$

3) Meeting Margin Calls

Assumptions

1. A hedge fund's strategy is a statistical arbitrage. Therefore, a hedge fund tries to buy an undervalued security (long) and short sell an overvalued security (short). The hedge fund is trying to have the same amount of shares of both long and short securities.
2. A typical hedge fund does not typically have cash and will invest all its available cash into long securities. It will use long securities as a collateral to borrow short securities.
3. A dealer has unlimited inventory and will always take the other side of the hedge fund order.
4. There is no price impact – selling and buying a security does not change the price of the security.
5. A hedge fund starts with no exposure to the long and short positions.
6. When arbitrage opportunities go away, a hedge fund liquidates both short and long positions. Therefore, in this model, both position forming and unwinding are modeled.
7. Federal and house requirements are 50%.
8. No minimum dollar requirements under regulation T, house and NYSE requirements.
9. No interest charge in a margin account, and no interest credit in a short account.

Variable	Value	Units	Description
L^{max}	2	Dmnl	Maximum Allowable Leverage
m	0.5	Dmnl	House Margin Requirement
m^t	0.5	Dmnl	Regulation T Margin Requirement
$f^{F,Cash}$	0.8	Dmnl	Fraction of Free Cash Invested
$Cash_0^H$	40,000	$	Initial Hedge Fund Cash
$P_0^{F,S}$	100	$/Share	Initial Price of Security S is Fundamental Price of Security S
$P_0^{F,L}$	100	$/Share	Initial Price of Security L is Fundamental Price of Security L
$S_0^{H,L}$	0	Share	Initial Number of Shares of Security L a Hedge Fund Has
$S_0^{H,S}$	0	Share	Initial Number of Shares of Security S a Hedge Fund Has
$S_0^{D,L}$	400	Share	Initial Number of Shares of Security L a Dealer Has
$S_0^{D,S}$	400	Share	Initial Number of Shares of Security S a Dealer Has
$S_0^{I,L}$	400	Share	Initial Number of Shares of Security L an Investor Has
$S_0^{I,S}$	400	Share	Initial Number of Shares of Security S an Investor Has
$Cash_0^I$	40,000	$	Initial Cash an Investor Has
$w_0^{A,S}$	0.5	Dmnl	Actual Equity Weight of Security S by Investor
$w_0^{A,L}$	0.5	Dmnl	Actual Equity Weight of Security L by Investor

Table 3. Assumptions and Initial Conditions

Formulations

Hedge Fund Cash

$$Cash_t^H = \int (Income\ from\ Other\ Investments_t^H + Cash\ Increase_t^H - Cash\ Decrease_t^H)dt$$

(eq. 7)

$$Cash\ Increase_t^H = Sell\ Rate_t^{H,L} * P_t^L + Fraction\ Reinvested * Sell\ Rate_t^{H,S} * P_t^S \quad \text{(eq. 8)}$$

$$Cash\ Decrease_t^H = Buy\ Rate_t^{H,L} * P_t^L + Buy\ Rate_t^{H,S} * (P_t^S - \overline{P_t^S}) \quad \text{(eq. 9)}$$

Total Cost Basis

$$Total\ Cost\ Basis_t^{H,L} = \int (Increase\ in\ Total\ Value_t^{H,L} - Decrease\ in\ Total\ Value_t^{H,L})dt$$

(eq. 10)

$$Increase\ in\ Total\ Value_t^{H,L} = Buy\ Rate_t^{H,L} * P_t^L \quad \text{(eq. 11)}$$

$$Decrease\ in\ Total\ Value_t^{H,L} = Sell\ Rate_t^{H,L} * \overline{P_t^L} \quad \text{(eq. 12)}$$

$$\overline{P_t^L} = \begin{cases} \dfrac{\text{Total Cost Basis}_t^{H,L}}{S_t^{H,L}} & \text{If } S_t^{H,L} \neq 0 \\ 0 & \text{If } S_t^{H,L} = 0 \end{cases} \quad \text{(eq. 13)}$$

$$Total\ Cost\ Basis_t^{H,S} = \int (Increase\ in\ Total\ Value_t^{H,S} - Decrease\ in\ Total\ Value_t^{H,S})dt$$

(eq. 14)

$$Increase\ in\ Total\ Value_t^{H,S} = Sell\ Rate_t^{H,S} * P_t^S \quad \text{(eq. 15)}$$

$$Decrease\ in\ Total\ Value_t^{H,L} = Buy\ Rate_t^{H,S} * \overline{P_t^S} \quad \text{(eq. 16)}$$

$$\overline{P_t^S} = \begin{cases} \dfrac{\text{Total Cost Basis}_t^{H,S}}{-S_t^{H,S}} & \text{If } S_t^{H,L} \neq 0 \\ 0 & \text{If } S_t^{H,L} = 0 \end{cases} \quad \text{(eq. 17)}$$

Margin and Available Cash

Hedge fund cash is divided into a free cash $Cash_t^{H,F}$ that can be used to buy new assets and into a cash pledged as a collateral in order to cover a margin call $Cash_t^{H,P}$.

$$Cash_t^{H,F} = \int (Income\ from\ Other\ Investments_t^{H,F} + Cash\ Increase_t^{H,F} - Cash\ Decrease_t^{H,F} + Margin\ Refund_t^{H,F} - Margin\ Call_t^{H,F})dt \qquad \text{(eq. 18)}$$

$$Cash\ Increase_t^{H,F} = Sell\ Rate_t^{H,L} * P_t^L + Fraction\ Reinvested * Sell\ Rate_t^{H,S} * P_t^S \qquad \text{(eq. 19)}$$

$$Cash\ Decrease_t^{H,F} = Buy\ Rate_t^{H,L} * P_t^L + Buy\ Rate_t^{H,S} * (P_t^S - \overline{P_t^S}) \qquad \text{(eq. 20)}$$

Cash Decision = 1 if a hedge fund decides to finance *Margin Needed* $_t$ with money made from other trades.

Cash Decision = 0 if a hedge fund decides to finance *Margin Needed* $_t$ by closing long position.

$$Margin\ Call_t^{H,F} = \begin{cases} \text{Cash Decision} * \text{MIN}\left(\dfrac{\text{Margin Needed}_t}{\text{Time to Cover Margin}}, \dfrac{\text{Cash}_t^{H,F}}{\text{Minimum Payment Time}_t}\right) & \text{If Margin Needed}_t > 0 \\ 0 & \text{If Margin Needed}_t \le 0 \end{cases} \qquad \text{(eq. 21)}$$

$$Margin\ Refund_t^{H,F} = \begin{cases} \text{Cash Decision} * \text{MIN}\left(-\dfrac{\text{Margin Needed}_t}{\text{Time to Cover Margin}}, \dfrac{\text{Cash}_t^{H,P}}{\text{Minimum Payment Time}_t}\right) & \text{If Margin Needed}_t \le 0 \\ 0 & \text{If Margin Needed}_t > 0 \end{cases} \qquad \text{(eq. 22)}$$

$$Margin\ Needed_t = Margin\ Required_t - Margin_t \qquad \text{(eq. 23)}$$

$$Margin\ Required_t = -(1+m) * S_t^{H,S} P_t^S \qquad \text{(eq. 24)}$$

$$Margin_t = S_t^{H,L} P_t^L * (1-m) + Total\ Cost\ Basis_t^{H,S} + \text{Cash}_t^{H,P} \qquad \text{(eq. 25)}$$

Hedge Fund Balance Sheet

$$Equity_t^H = Cash_t^H + S_t^{H,L} P_t^L + Total\ Cost\ Basis_t^{H,S} - S_t^{H,S} P_t^S \qquad \text{(eq. 26)}$$

$$Profit_t^H = Equity_t^H - Equity_0^H \qquad \text{(eq. 27)}$$

Desired Shares

$S_t^{H,D,L}$ is the desired long shares by a hedge fund. $S_t^{H,D,S}$ is the desired short shares by a hedge fund.

$$\text{Decision to Get Into Arbitrage}_t = \begin{cases} 1 & \text{If } P_t^L < P_t^S \\ 0 & \text{If } P_t^L \geq P_t^S \end{cases} \qquad \text{(eq. 28)}$$

If Decision to Get Into Arbitrage$_t$=0 THEN $S_t^{H,D,L}$=0 (eq. 29)

If Decision to Get Into Arbitrage$_t$=1 AND Extra Margin Needed$_t \leq 0$ THEN

$$S_t^{H,D,L} = MAX\left(S_t^{H,L} + MAX\left(\frac{Cash_t^{H,F} - (1 - f^{F,Cash})Cash_0^{H,F}}{P_t^L}, 0\right), 0\right) \qquad \text{(eq. 30)}$$

If Decision to Get Into Arbitrage$_t$=1 AND Extra Margin Needed$_t > 0$ THEN

$$S_t^{H,D,L} = \text{Cash Decision} * MAX\left(\frac{Cash_t^{H,F}}{P_t^L} + S_t^{H,L} - \frac{L^{max}\text{Margin Needed}_t}{P_t^L}, 0\right) + (1 - \text{Cash Decision}) * MAX\left(S_t^{H,L} - \frac{L^{max}\text{Margin Needed}_t}{P_t^L}, 0\right) \qquad \text{(eq. 31)}$$

In this model, the desired amount of short stock has two different formulations. First, given the maximum allowed leverage provided by a dealer for a hedge fund, a hedge fund can borrow the stock S and short sell it:

$$S_t^{H,D,S} = -MIN\left(\frac{L^{max} * S_t^{H,L} * P_t^L - S_t^{H,L} * P_t^L}{P_t^S}, S_t^{H,D,L}\right) \qquad \text{(eq. 32)}$$

However, in order to execute a perfect arbitrage (buying x shares of stock L and short selling x shares of stock S), a hedge fund can desire to have as much shares of stock S as it desires of having a stock L.

$$S_t^{H,D,S} = -S_t^{H,D,L} \qquad \text{(eq. 33)}$$

Results

Scenario 1_0. Equilibrium

The model is put into equilibrium when prices of long and short securities equal their fundamental prices.

Fundamental prices of short and long security = $P^{F,S} = P^{F,L}$ = 100 $/share.

Under these conditions, a hedge fund is not going to get into the arbitrage position.

Scenario 1_1. Successful arbitrage – Keep Within Margin Requirements

Fundamental prices of short and long security = $P^{F,S} = P^{F,L}$ = 100 $/share.

Prices are exogenous, no feedback. A hedge fund manager sells short shares of security S such that he is within margin requirements.

$P^{F,L}$ = 100+ STEP(-20,20)+STEP(20,30) $/share

$P^{F,S}$ = 100+STEP(20,20)-STEP(20,30) $/share

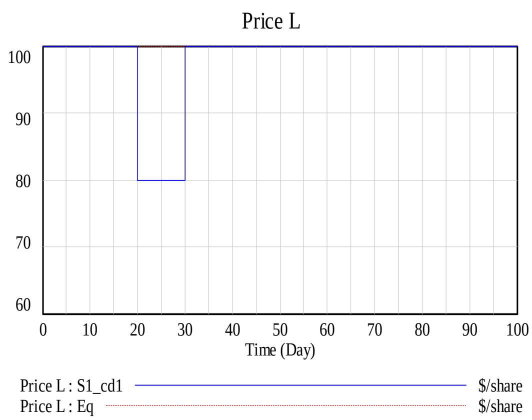


Figure C1_1_1. Price of Security L

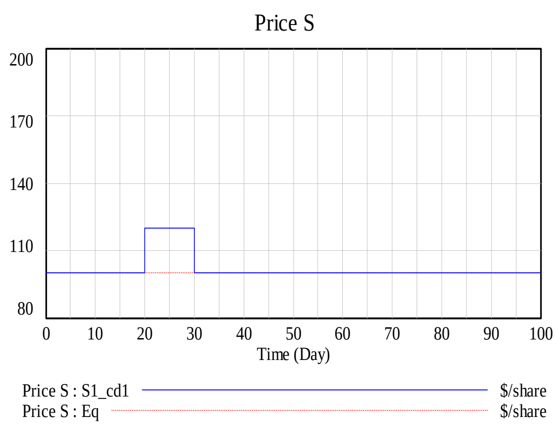


Figure C1_1_2. Price of Security S

Therefore, from time = 0 to time = 20 days, prices are the same, then an arbitrage opportunity window exists from 20 to 30 days, and then prices of both assets converge again.

In this scenario, a hedge fund decides to short-sell as much as allowed within margin requirements, so the hedge fund manager will not face a margin call.

S1_cd1big – a run with Cash Decision=1 and Income from Other Investments = STEP(1000,10)-STEP(1000,20)

S1_cd1 – a run with Cash Decision=1

S1_cd0 – a run with Cash Decision=0

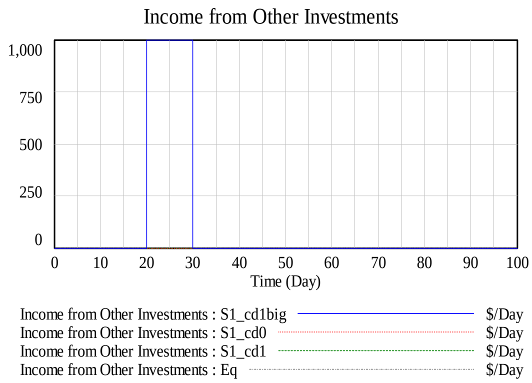


Figure C1_1_3. Income from Other Investments

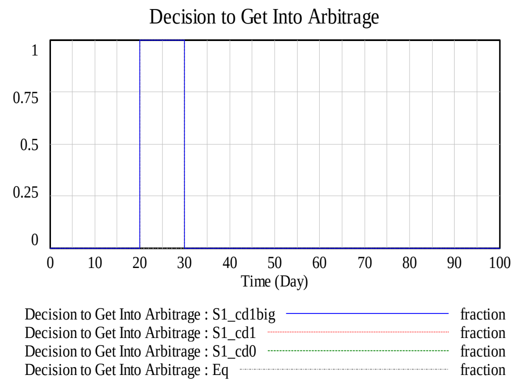


Figure C1_1_4. Decision to Get Into Arbitrage

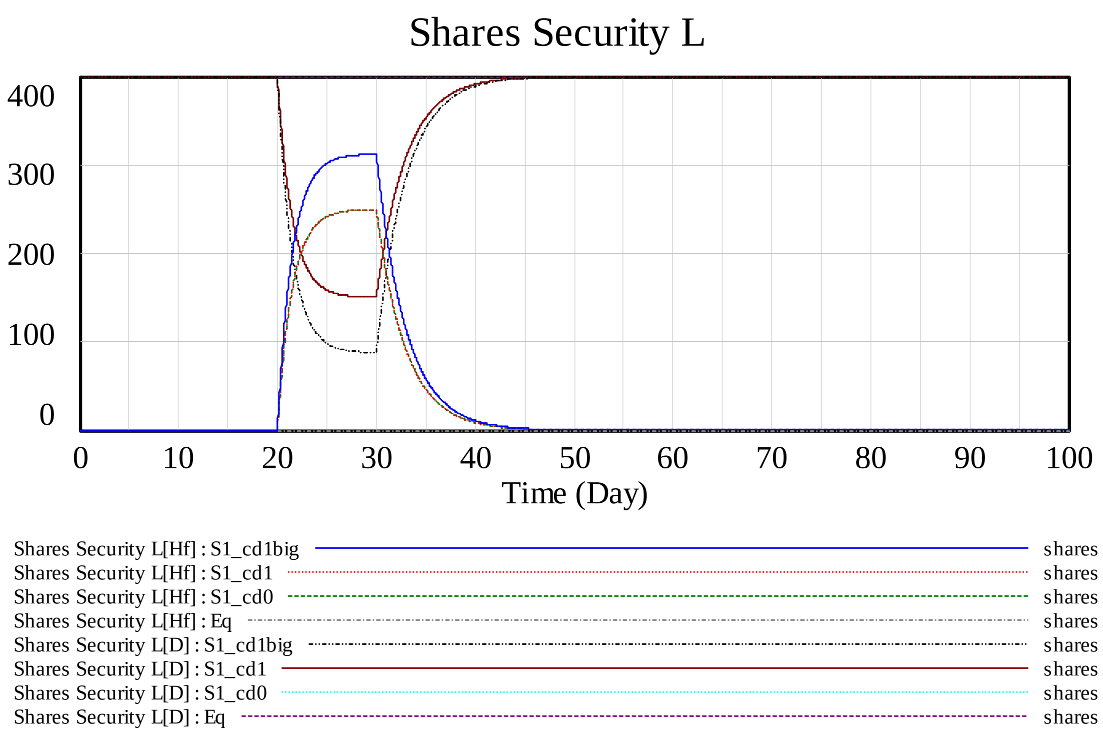


Figure C1_1_5. Shares of Security L

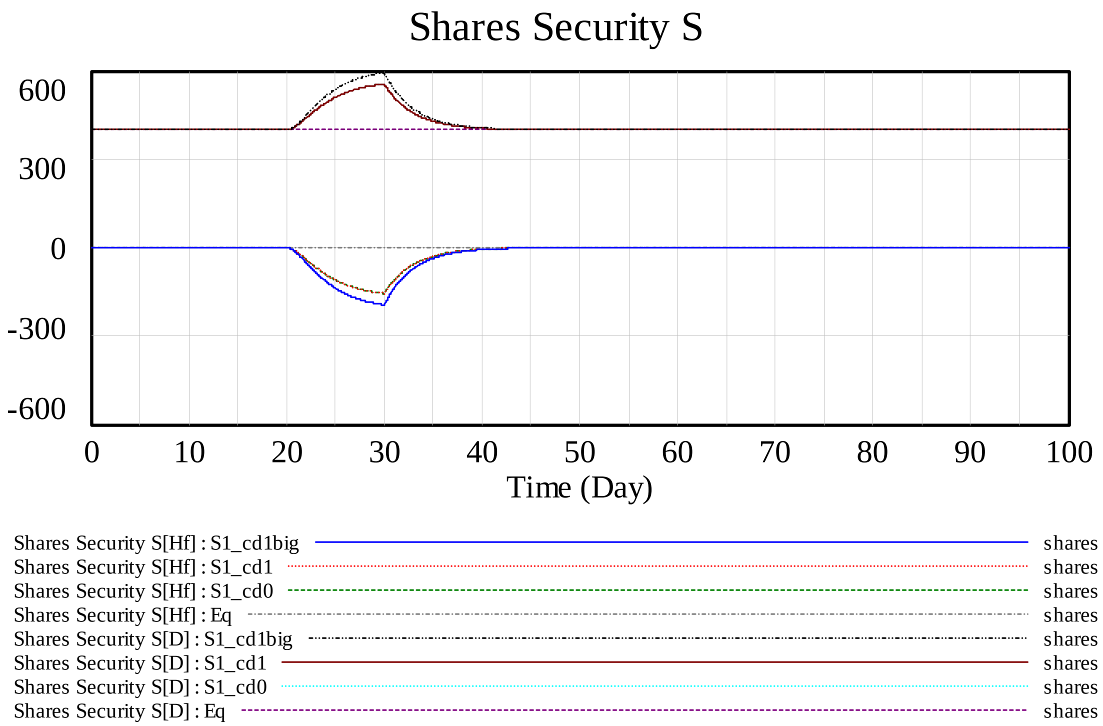


Figure C1_1_6. Shares of Security S

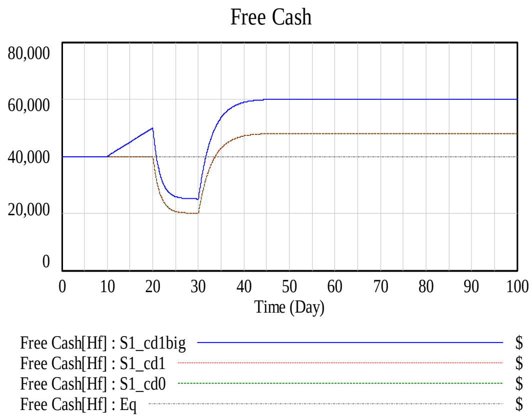


Figure C1_1_7. Free Cash

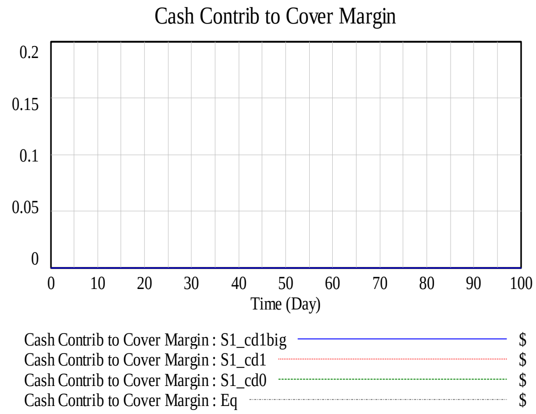


Figure C1_1_8. Cash Contribution to Cover Margin

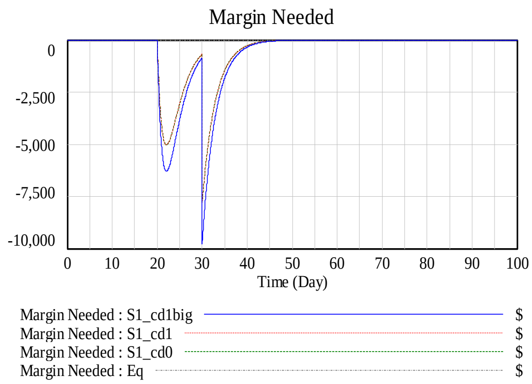


Figure C1_1_9. Margin Needed

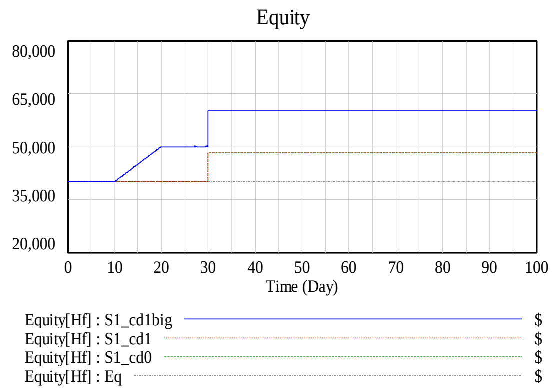


Figure C1_1_10. Equity

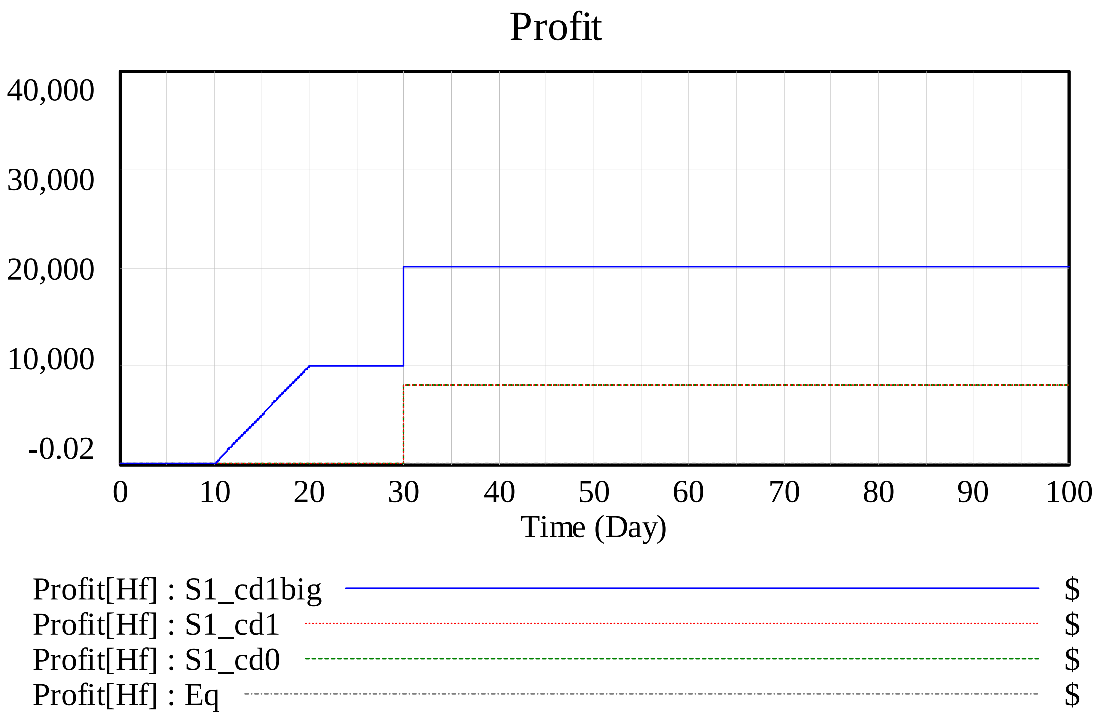


Figure C1_1_11. Profit

In all runs, after prices for long and short securities converge at time = 30 days, profits become positive. Between time = 20 and 30 days, in runs S1_cd1 and S1_cd0 profits are not realized; however, for run S1_cd1big, profits are positive ($10,000 due to accumulation of Income from Other Investments). In all runs, decisions to short are made within margin requirements. Therefore, a dealer never makes a margin call to the hedge fund. As a result, behavior of S1cd1 and S1cd0 is identical. The difference in behavior of runs S1cd1 and S1cd1big is intuitive. If a hedge fund has more cash available, it will be able to buy more long shares and short more short shares, thus making more profit. Therefore, the profit for the run S1cd1big is higher than for other two runs. Final equity is also higher for this run due to the income from other investments.

Scenario 2. Successful arbitrage – Short as Much as Long

Fundamental prices of short and long security = $P^{F,S} = P^{F,L}$ = 100 \$/share.

Prices are exogenous, no feedback.

$P^{F,L}$ = 100+ STEP(-20,20)+STEP(20,30) \$/share

$P^{F,S}$ = 100+STEP(20,20)-STEP(20,30) \$/share

Therefore, from time = 0 to time = 20 days, prices are the same, then an arbitrage opportunity window exists from 20 to 30 days, and then prices of both assets converge again.

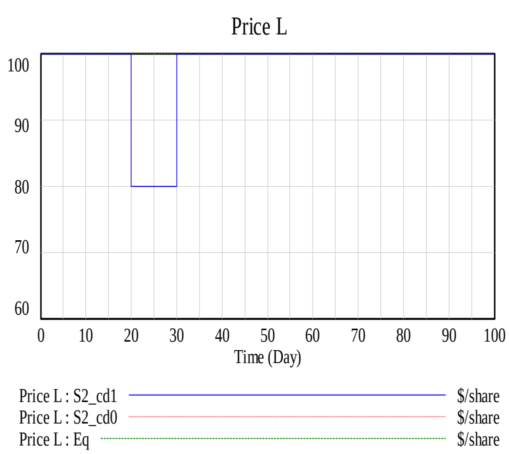


Figure C1_2_1. Price of Security L   Figure C1_2_2. Price of Security S

In this scenario, a hedge fund decides to short-sell as many shares as he desires to long, in order to make a perfect arbitrage.

S2_cd1big – a run with Cash Decision=1 and Income from Other Investments = STEP(1000,10)-STEP(1000,20)

S2_cd1 – a run with Cash Decision=1

S2_cd0 – a run with Cash Decision=0

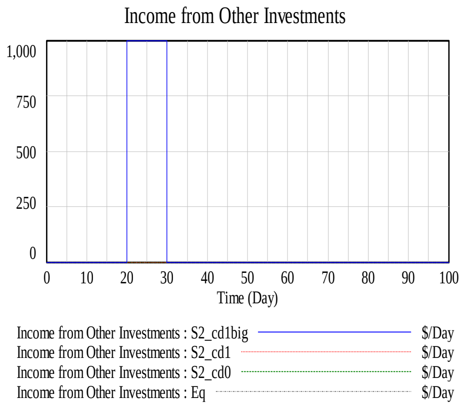


Figure C1_2_3. Income from Other Investments

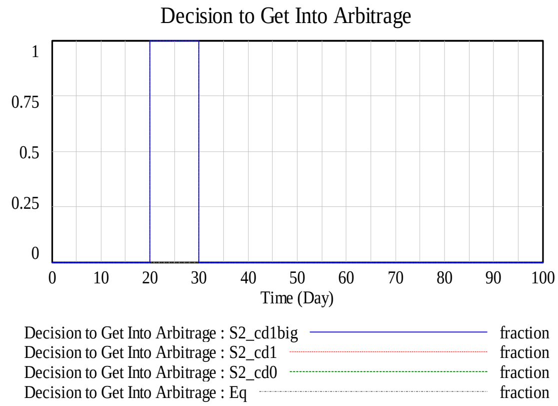


Figure C1_2_4. Decision to Get Into Arbitrage

An arbitrage window exists from time = 20 to 30 days.

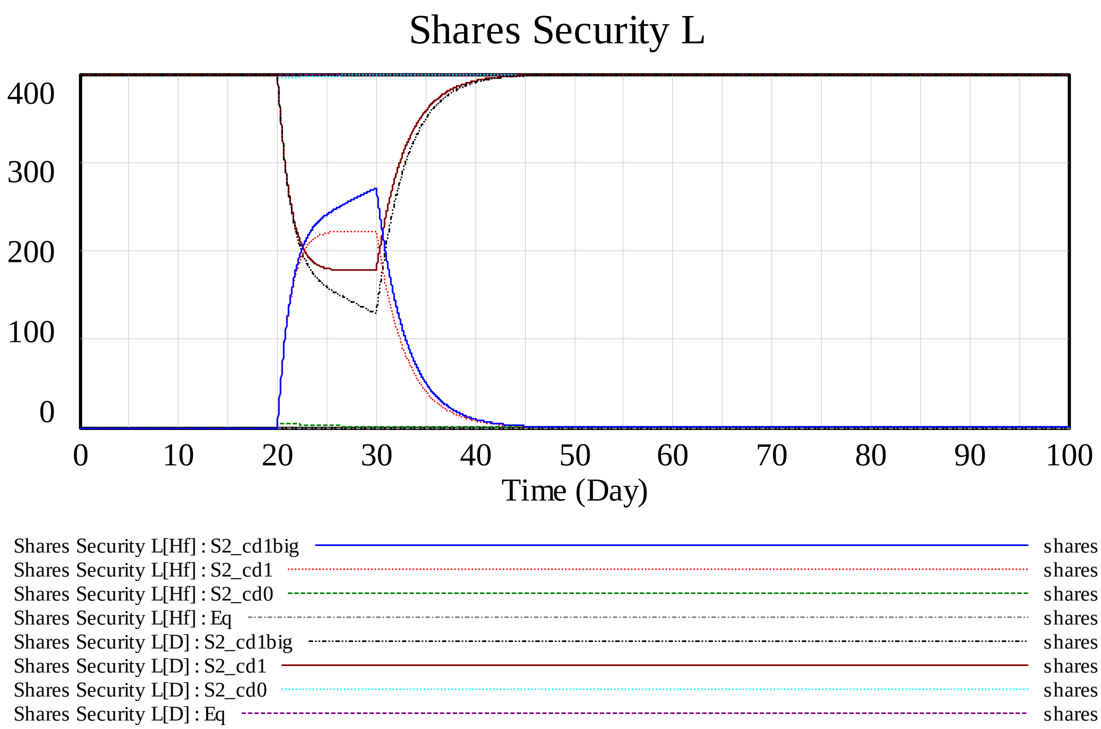


Figure C1_2_5. Shares of Security L

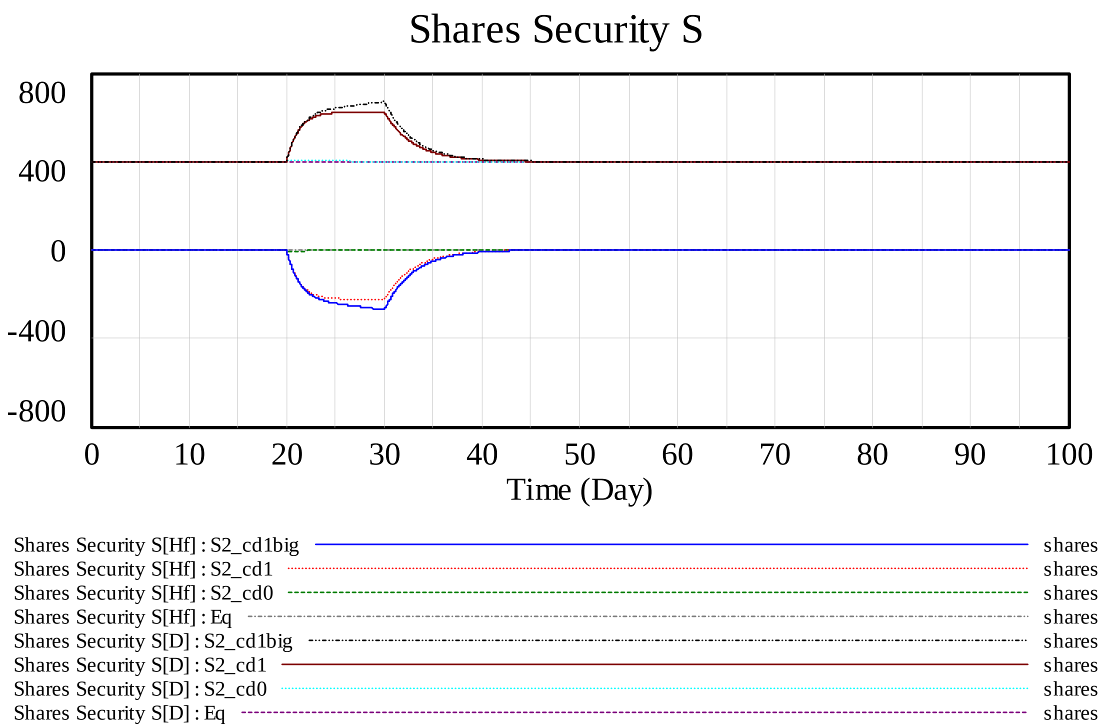


Figure C1_2_6. Shares of Security S

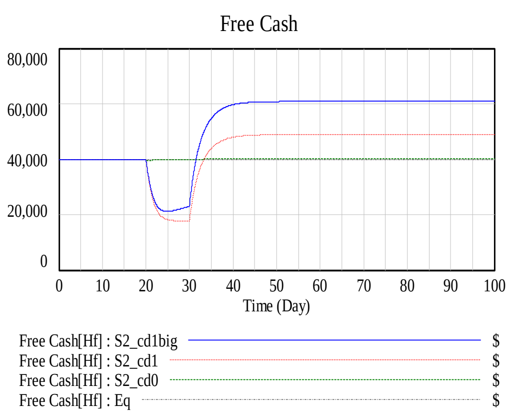


Figure C1_2_7. Free Cash

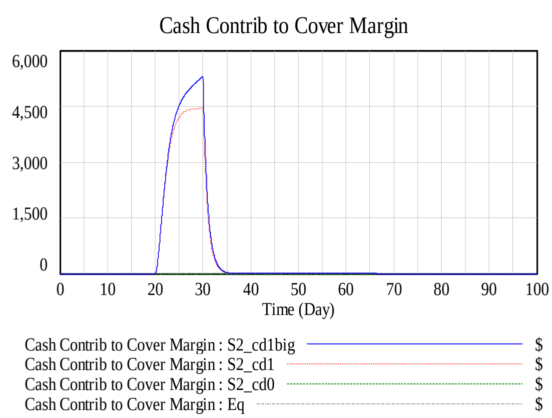
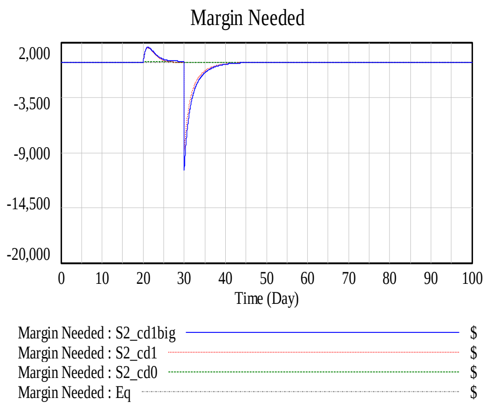


Figure C1_2_8. Cash Contribution to Cover Margin



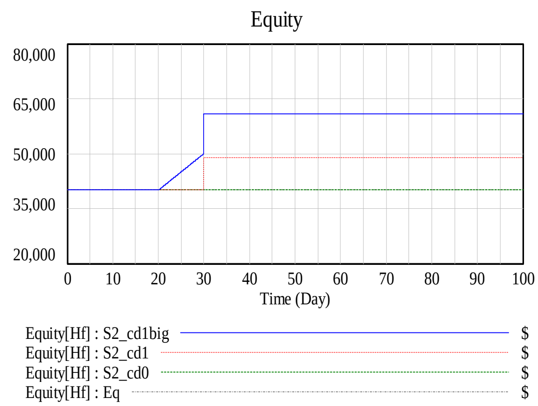


Figure C1_2_9. Margin Needed Figure C1_2_10. Equity

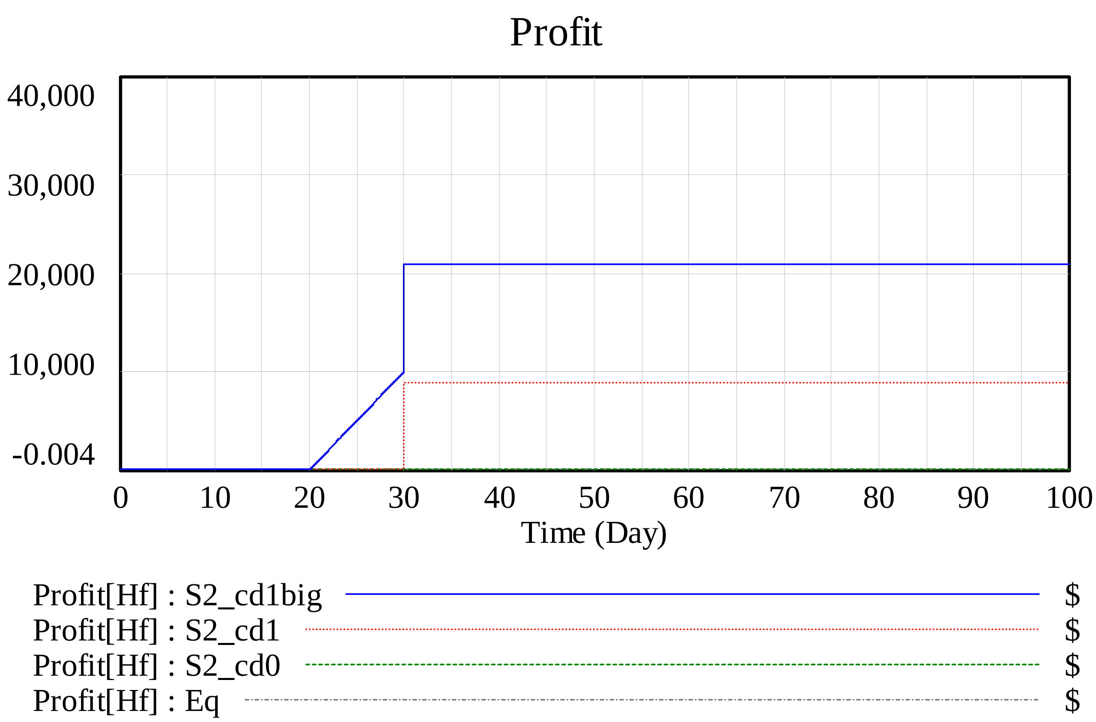


Figure C1_2_11. Profit

Due to the price difference between long and short securities, the hedge fund manager faces a margin call by a broker. In case of S2_cd1 and S2_cd1big, margin is covered by available cash. However, in case of S2_cd0, margin is covered by selling long shares. Therefore, the amount of long and short shares in S2_cd0 is much smaller than in other runs. At time = 30 days, the hedge fund becomes lucky and positions converge. Due to a relative small amount of short and long securities in S2_cd0, the profits in this run are much smaller than for S2_cd1 run ($39 compared to $8889). Also, in S2_cd1big run, profits ($20,840) are larger than in S2_cd1 run because of the extra income that is used to buy additional long shares and borrow short shares that would be realized later at a profit.

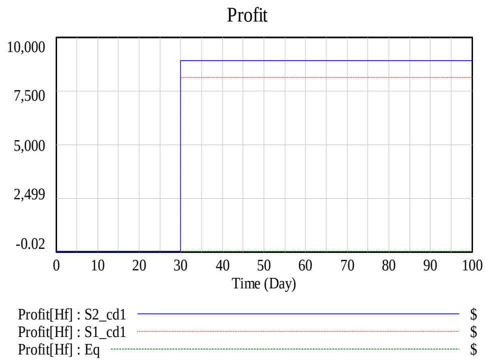


Figure C1_2_12. Profits Compared, Cash Decision = 1

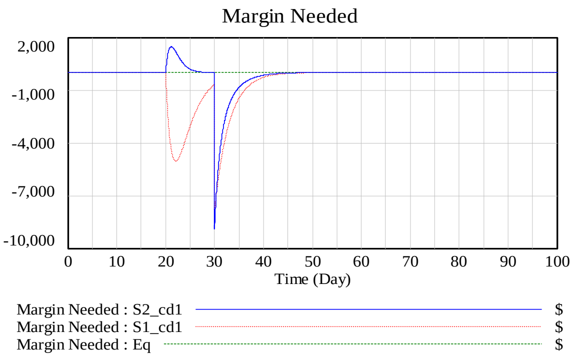


Figure C1_2_13. Margin Needed Compared Cash Decision = 1

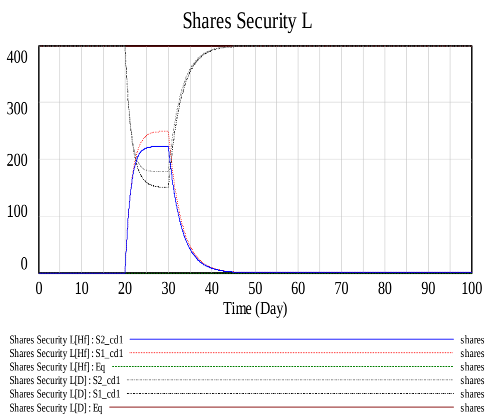


Figure C1_2_14. Shares of Security L Compared, Cash Decision = 1

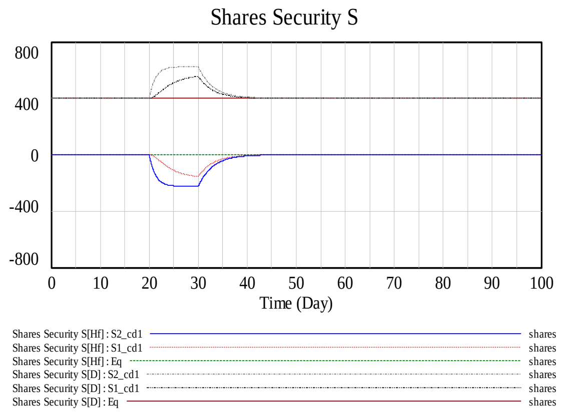


Figure C1_2_15. Shares of Security S Compared, Cash Decision = 1

The above graphs compare behavior in Scenario 1 with behavior Scenario 2 for the case where cash decision = 1. In both cases profits are always positive as the arbitrage strategy was properly conceived and was realized at time = 30 days. However, in case S2_cd1, a hedge fund manager was not constrained to operate within margin requirements by selling as many shares of expensive security S as the number of shares of a cheaper security L.

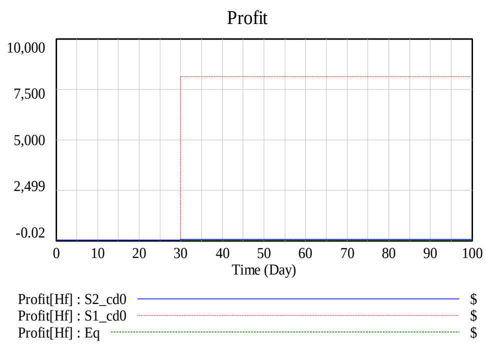


Figure C1_2_16. Profits Compared, Cash Decision = 0

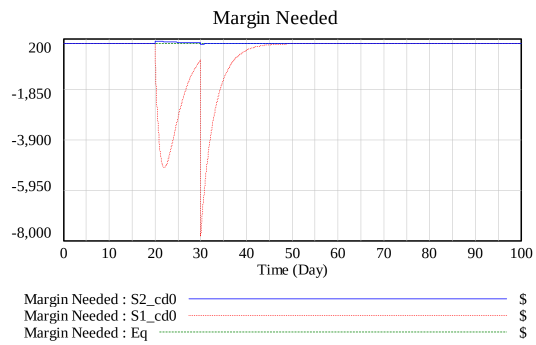


Figure C1_2_17. Margin Needed Compared Cash Decision = 0

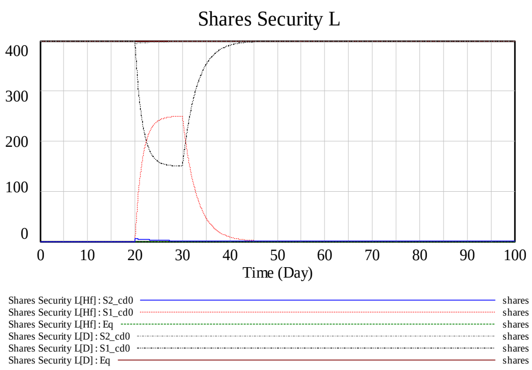


Figure C1_2_18. Shares of Security L Compared, Cash Decision = 0

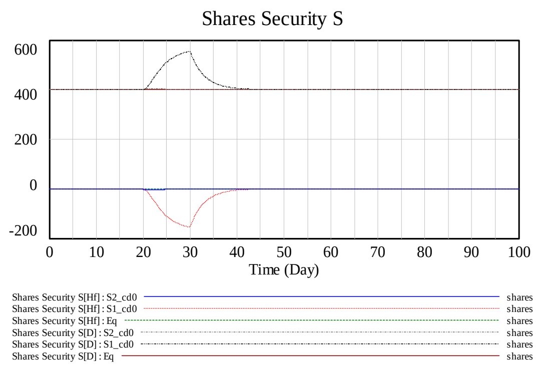


Figure C1_2_19. Shares of Security S Compared, Cash Decision = 0

The above graphs compare behavior in Scenario 1 with behavior Scenario 2 for the case where cash decision = 0. In S2_cd0 case, due to the decision of a hedge fund manager to keep as many shares of security S as security L, Margin Needed is positive, compared to run S1_cd0. Therefore, in S2_cd0, the hedge fund has to dispose of the both long and short securities compared to S1_cd0, which leads to a much smaller long term profit for run S2_cd0, compared to run S1_cd0. In this case, it pays to stay longer in the position by constraining to keep positions within margin limits, waiting for the arbitrage opportunities to realize.

Also, as can be seen from scenarios 1 and 2, given that positions will converge at some time, it is better to use money from available cash and finance the margin calls than selling securities.

Scenario 3. Successful arbitrage – Keep Within Margin Requirements

Fundamental prices of short and long security = $P^{F,S} = P^{F,L}$ = 100 \$/share.

Prices are exogenous, no feedback.

$P^{F,L}$ = 100 \$/share

$P^{F,S}$ = 100+STEP(40,20)-STEP(40,30) \$/share

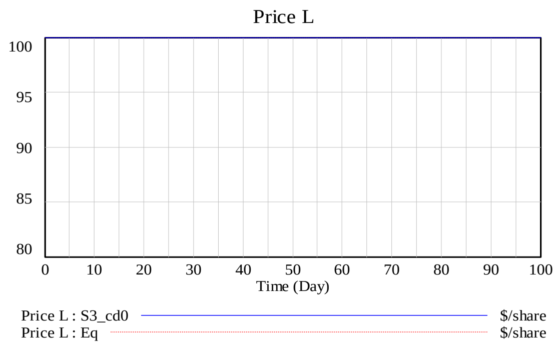


Figure C1_3_1. Price of Security L

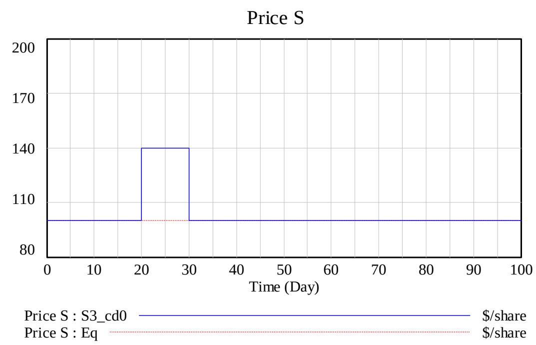


Figure C1_3_2. Price of Security S

Therefore, from time = 0 to time = 20 days, prices are the same, then an arbitrage opportunity window exists from 20 to 30 days, and then prices of both assets converge again.

In this scenario, a hedge fund manager decides to short-sell as much as allowed within margin requirements, so he would not have a margin call.

S3_cd0 – a run with Cash Decision=0

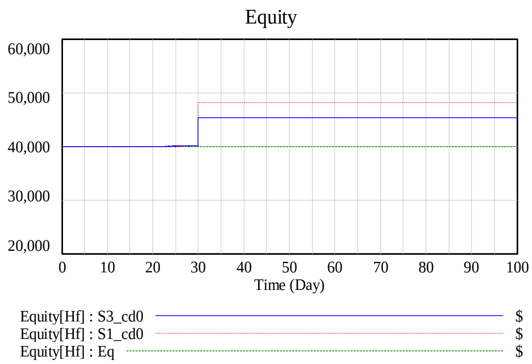


Figure C1_3_3. Equity

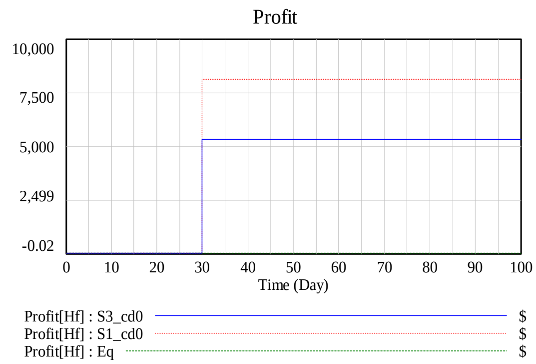


Figure C1_3_4. Profit

Here, scenarios 1 and 3 are compared, cash decision = 0. In both scenarios, between time = 20 and 30 days, the spread between a short and a long position equals 40 $/share. However, in the scenario 1, the spread is distributed equally between long and short positions; whereas, in the scenario 3, the price of security L stays at its fundamental value, and the price of security S increases by 40% between time = 20 and 30 days. In the absence of margin requirements and laws regulating leverage, the results from two scenarios should be exactly the same. However, due to leverage requirements and proportional changes in prices, the results are different.

The total equilibrium profits in S1_cd0 are larger than in S3_cd0. In S1cd0, profits come from both short and long positions; whereas, in S3_cd0, profits only come from the convergence of the short position to the equilibrium value. Due to the maximum leverage ratio of 2, the amount of borrowed short positions (in $ amount) can be no more than the amount of long positions (in $ amount).

Proof for these scenarios: Assume that shares are bought and disposed of instantaneously. In Scenario 1, given x dollars of cash available to invest in long assets and the maximum leverage ratio of 2, profit is: $\frac{x}{80}20+\frac{x}{120}20=\frac{35}{84}x$. In Scenario 3, the profit is: $\frac{x}{140}40=\frac{24}{84}x$, where profit in Scenario 1 is always bigger than profit in Scenario 3 for any x. The intuition for the result is the following: It is better to be exposed to an asset that will have a bigger proportional increase (in the case of long) or decrease (in the case of short).

Scenario 4. Successful arbitrage – Short as Much as Long

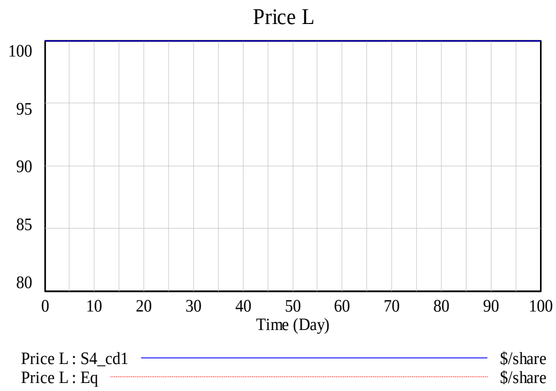
Fundamental prices of short and long security = $P^{F,S}$ = $P^{F,L}$ = 100 $/share.

Prices are exogenous, no feedback.

$P^{F,L}$ = 100 $/share

$P^{F,S}$ = 100+STEP(40,20)-STEP(40,30) $/share

Therefore, from time = 0 to time = 20 days, prices are the same, then an arbitrage opportunity window exists from 20 to 30 days, and then prices of both assets converge again.



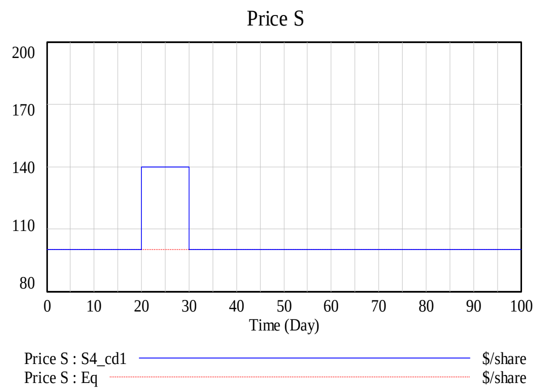
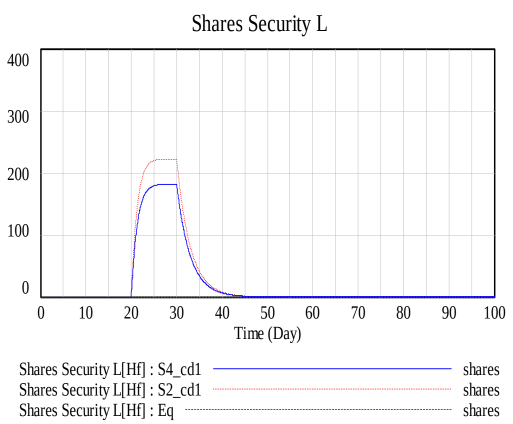


Figure C1_4_1. Price of Security L

Figure C1_4_2. Price of Security S

In this scenario, a hedge fund decides to short-sell as many shares as he desires to long, in order to make a perfect arbitrage.

S4_cd1 – a run with Cash Decision=1



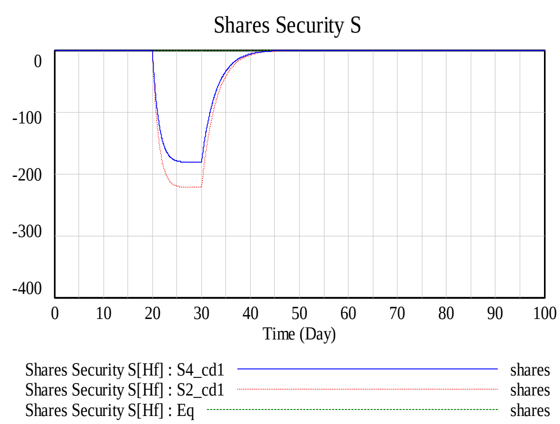
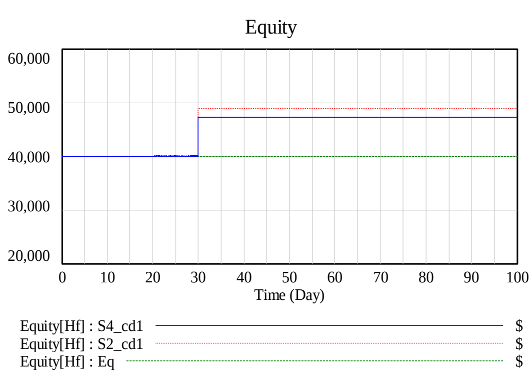


Figure C1_4_3. Shares of Security L

Figure C1_4_4. Shares of Security S



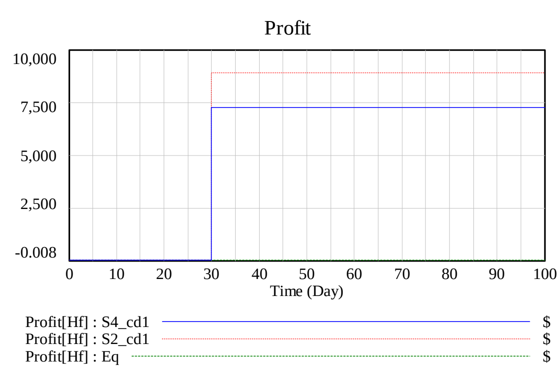


Figure C1_4_5. Equity

Figure C1_4_6. Profit

Profits for scenario 4 are a little bit smaller than profits in scenario 2. This is due to different amount of shares of both securities L and S in both scenarios. In scenario 4, security L is more expensive (100 $/share) than security L in scenario 2 (80 $/share); therefore, given the same amount of cash, a hedge fund can afford to buy less of security L, and therefore, short less of security S leading to smaller final arbitrage profits. Note, that if the number of long and short securities were the same in both scenarios, profits would be the same. For the proof, look at the section: Necessary Condition for a Hedge Fund to Fail.

Scenario 5. Collapse – Keep Within Margin Requirements

Fundamental prices of short and long security = $P^{F,S} = P^{F,L}$ = 100 $/share.

Prices are exogenous, no feedback.

$P^{F,L}$ = 100 $/share

$P^{F,S}$ = 100+ STEP(20,20)+STEP(260,30)-STEP(280,60) $/share

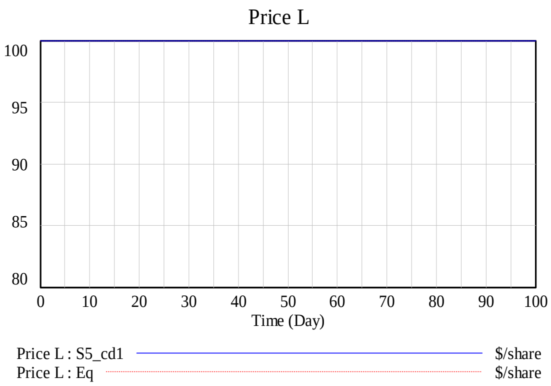


Figure C1_5_1. Price of Security L

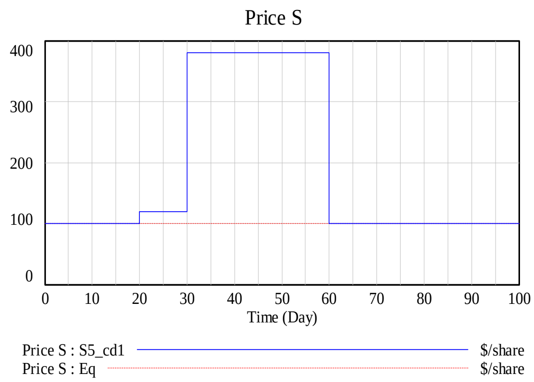


Figure C1_5_2. Price of Security S

Therefore, from time = 0 to time = 20 days, prices are the same, then an arbitrage opportunity window exists from 20 to 30 days. From time 30 to time 60 days, instead of converging, prices diverge even more before converging at time 60 days.

In this scenario, a hedge fund decides to short-sell as much as allowed within margin requirements, so a hedge fund manager would not have a margin call.

S5_cd1big – a run with Cash Decision=1 and Income from Other Investments = STEP(1000,10)-STEP(1000,20)

S5_cd1 – a run with Cash Decision=1

S5_cd0 – a run with Cash Decision=0

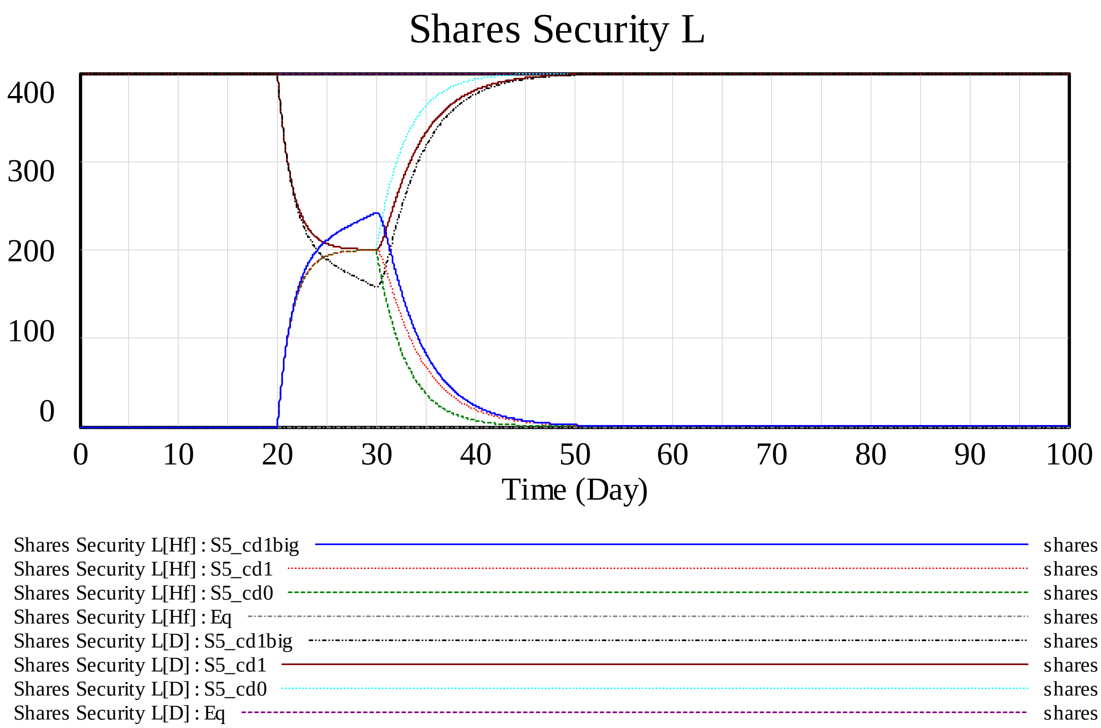


Figure C1_5_3. Shares of Security L

In S5_cd1big run, more shares can be bought because of higher available cash.

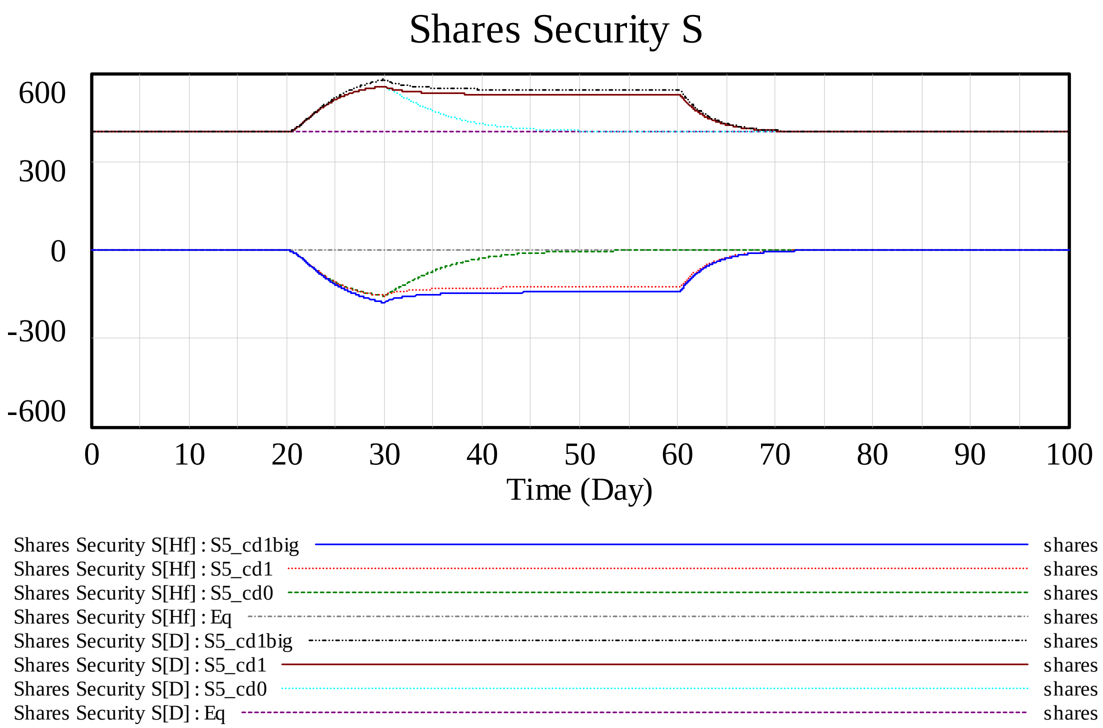


Figure C1_5_4. Shares of Security S

From time = 20 to 30 days, Margin Needed is negative; therefore, for runs S5_cd1 and S5_cd0, the number of shares sold short and long is the same. However, from time = 30 days, the Margin Needed becomes positive. Therefore, in run S5_cd0, to cover the margin, long shares are forced to be sold. In order not to increase the margin, shorts have to be covered. On the other hand, in run S5_cd1, the fund chooses to cover margin call with available cash. However, as soon as the available cash runs out, the hedge fund manager is forced to sell long shares and cover the shorts. However, in this simulation, cash runs out before all shares of security S can be bought off. There is a lag for unwinding the positions in the run S5_cd1 compared to S5_cd0 because in the first case, the manager covers the margin with cash before unwinding the shares, and in the second case, the manager unwinds the shares as soon as possible.

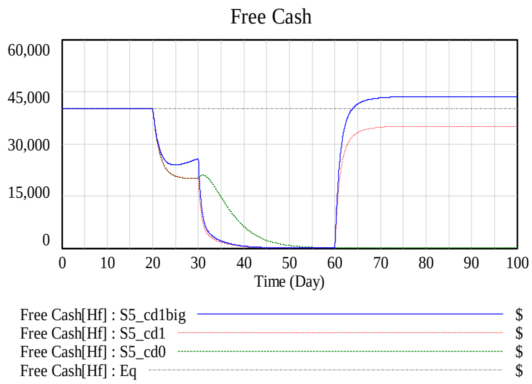


Figure C1_5_5. Free Cash

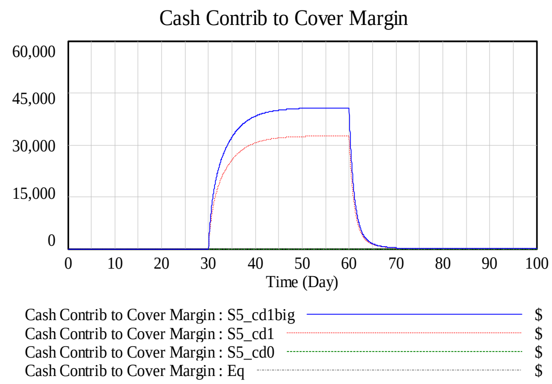


Figure C1_5_6. Cash Contribution to Cover Margin

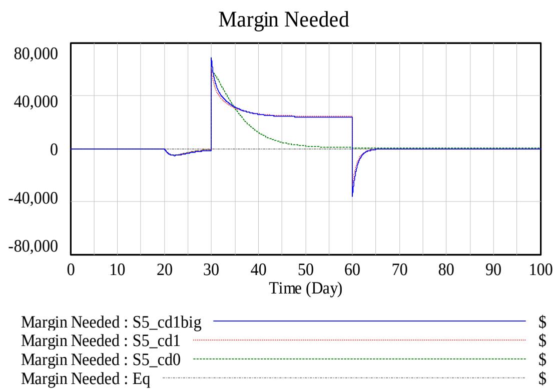


Figure C1_5_7. Margin Needed

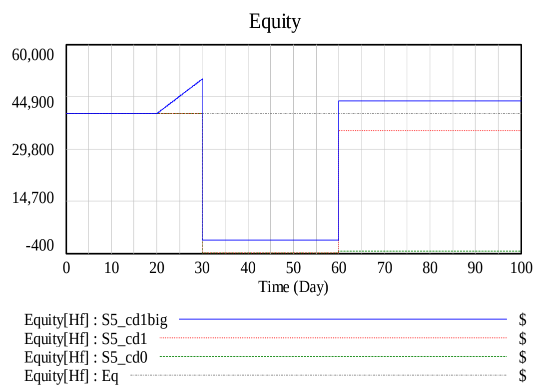


Figure C1_5_8. Equity

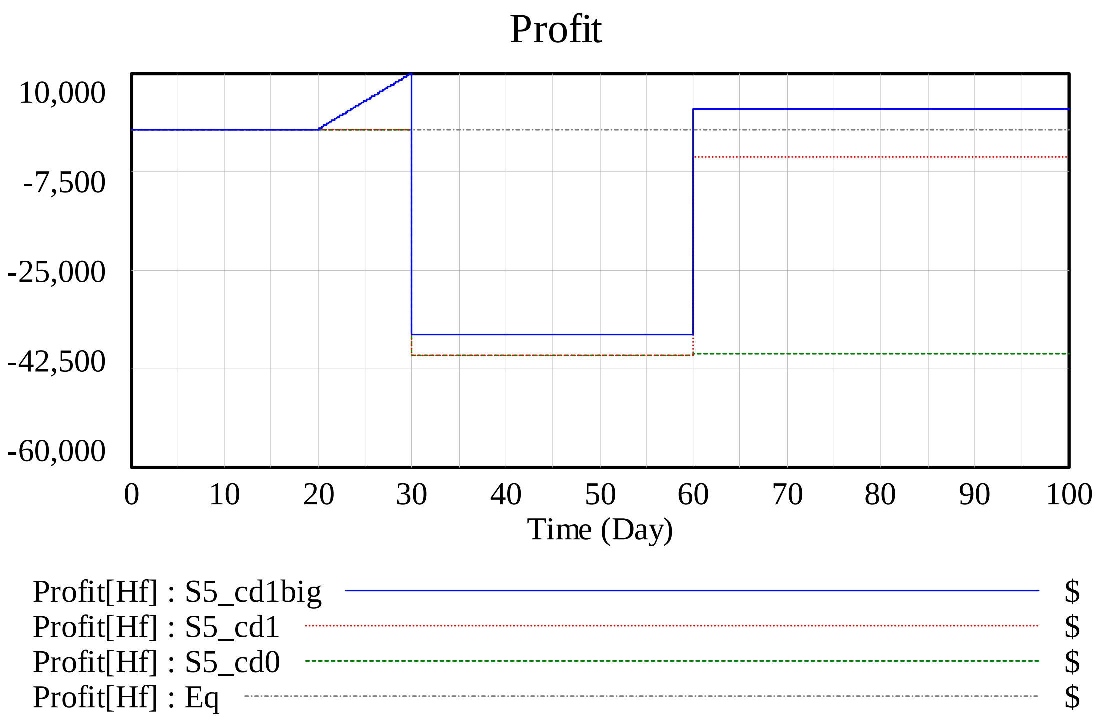


Figure C1_5_9. Profit

For runs S5_cd1 and S5_cd0, we have seen a collapse in the hedge fund, as indicated by negative equity. In scenario S5_cd1big, a hedge fund has an outside stream of cash to make sure that equity does not fall below zero. The profits for cases S5_cd1 and S5_cd0 are the same before time = 60 days because the number of both long and short sells is the same at time = 30 days before the divergence in prices. At time= 60 days, positions converge, thus making profits for a hedge fund. By time = 60 days, almost no positions are opened in scenario S5_cd0 compared to scenario S5_cd1. Compared to other funds, a hedge fund has an outside income of cash in scenario S5_cd1big, thus, a hedge fund does not fail in this case.

However, note, that formulation of $P^{F,S}$ is chosen as to be near a break-point for a hedge fund to have positive to negative equity. The break-point for an increase of price of security S is 258 \$/share, and here 260 \$/share is chosen.

$P^{F,L}$ = 100 \$/share

$P^{F,S}$ = 100+ STEP(20,20)+STEP(**260**,30)-STEP(280,60) \$/share

In this case, for a hedge fund to fail, the following condition should hold: *Cash Hf* $_0$ + $S_0^{H,L}(\Delta x + \frac{\Delta y P_0^L}{P_0^S}) < 0$ (for derivation, look at Necessary Condition for a Hedge Fund to Fail section of the paper). In this case, *Cash Hf* $_0$ = \$40,000. $S_0^{H,L}$ = 200 shares. Δx =0. $S_0^{H,S}$ = -155 shares. Therefore, Δy =258 \$/share.

In a separate simulation, not shown here, the same conditions as depicted in this scenario are used except that

$P^{F,S}$ = 100+ STEP(20,20)+STEP(**500**,30)-STEP(**520**,60) \$/share

A new price for a short asset is much higher than the break-point price. For this case, the losses for the run S5_cd1big are much bigger than losses for the run S5_cd1 because of a greater exposure of the hedge fund to short shares in that case. Starting time = 30 days, a hedge fund has to cover the margin and unwind the positions at a loss.

Scenario 6. Collapse – Short As Much as Long.

Fundamental prices of short and long security = $P^{F,S} = P^{F,L}$ = 100 \$/share.

Prices are exogenous, no feedback.

$P^{F,L}$ = 100 \$/share

$P^{F,S}$ = 100+ STEP(20,20)+STEP(215,30)-STEP(235,60) \$/share

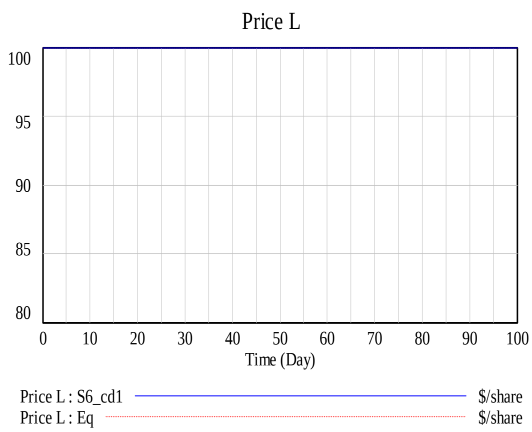


Figure C1_6_1. Price of Security L

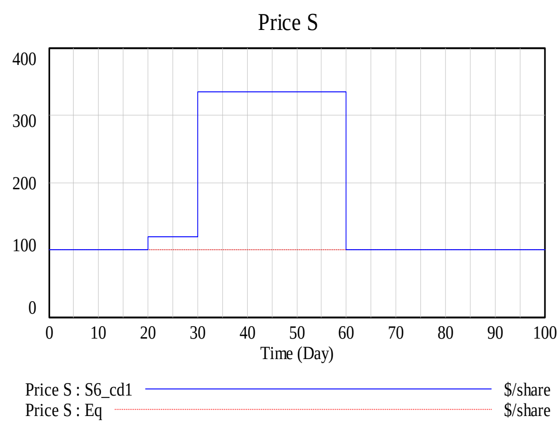


Figure C1_6_2. Price of Security S

Therefore, from time = 0 to time = 20 days, prices are the same, then an arbitrage opportunity window exists from 20 to 30 days. From time 30 to time 60 days, instead of converging, prices diverge even more before converging at time 60 days.

In this scenario, a hedge fund decides to short-sell as many shares as he desires to long, in order to make a perfect arbitrage.

S6_cd1big – a run with Cash Decision=1 and Income from Other Investments = STEP(1000,10)-STEP(1000,20)

S6_cd1 – a run with Cash Decision=1

S6_cd0 – a run with Cash Decision=0

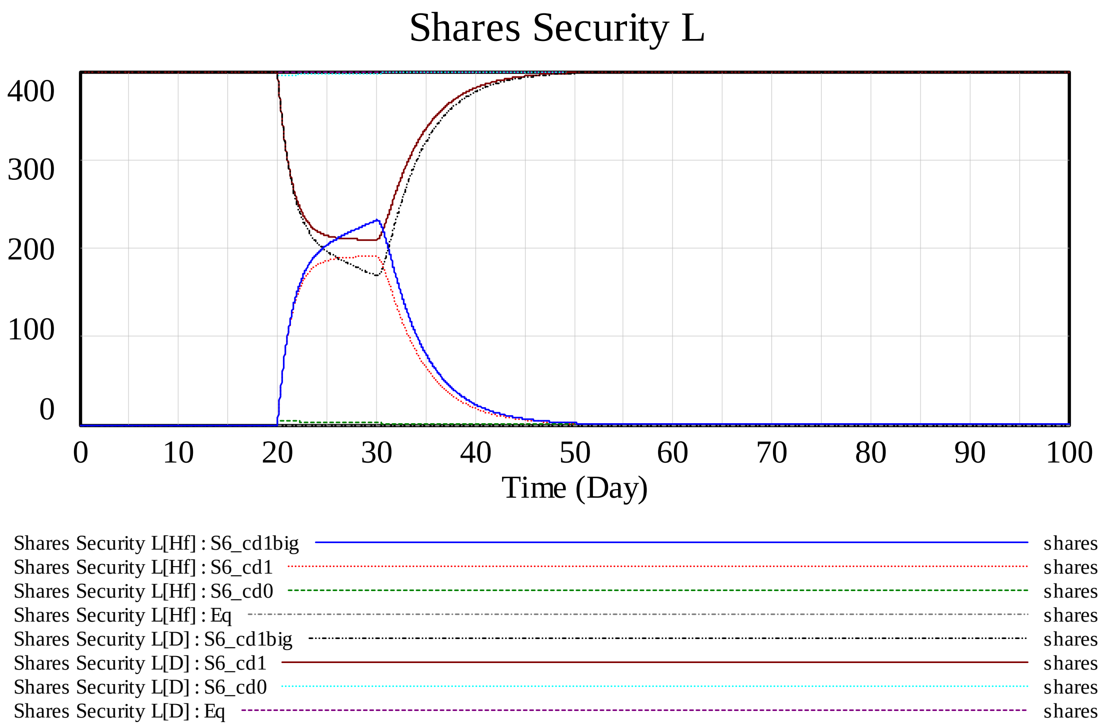


Figure C1_6_3. Shares of Security L

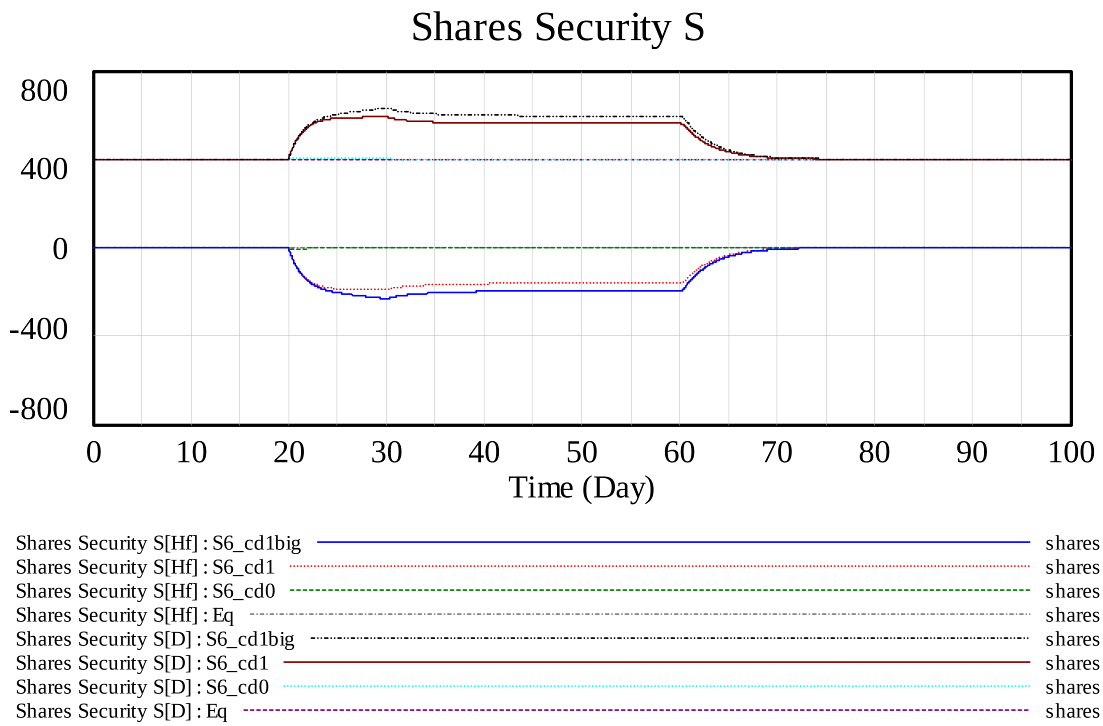


Figure C1_6_4. Shares of Security S

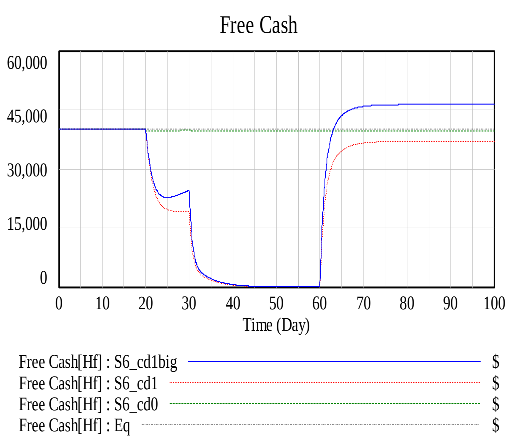


Figure C1_6_5. Free Cash

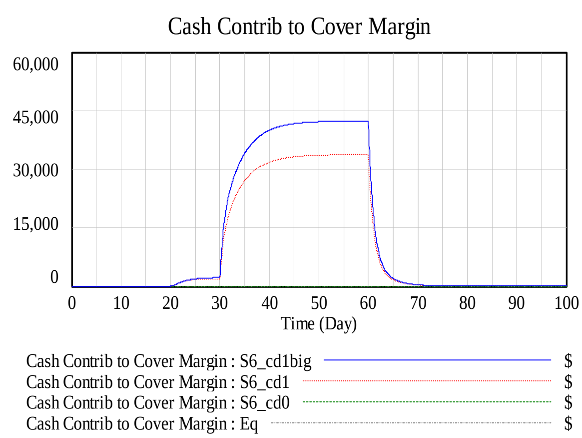


Figure C1_6_6. Cash Contribution to Cover Margin

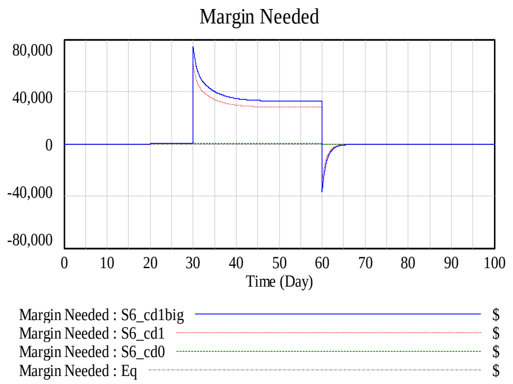


Figure C1_6_7. Margin Needed

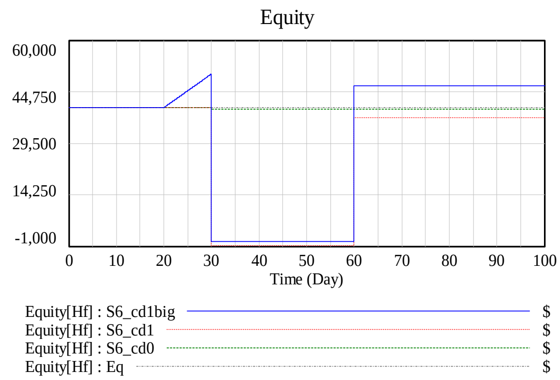


Figure C1_6_8. Equity

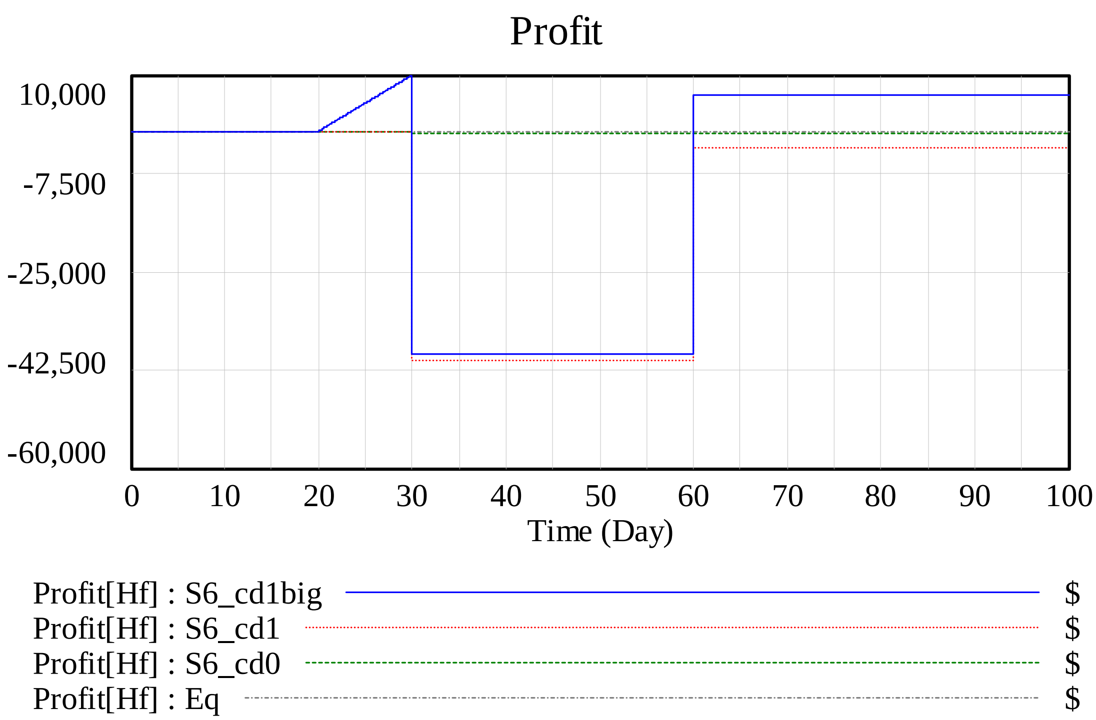


Figure C1_6_9. Profit

In this scenario, a hedge fund manager would like to have as many shares of asset S as of asset L. Due to price differences, Margin Needed is greater than 0 in all three runs from time = 30 to 60 days. Profits and Final Equity are negative for run S6_cd1, small and positive for run S6_cd1big and large and positive for run S6_cd0. In S6_cd0, the manager is precautious, and is not throwing “good money after bad.” In this scenario, he minimizes potential losses in case of position divergence (this is what exactly happened in the Scenario 6) compared with position convergence (what a hedge fund manager counts on). In this case, compared to previous scenarios, it pays off to be cautious.

Most of smaller hedge funds, not LTCM, close out their positions as soon as they are faced with a margin call. Therefore, as in 1998, lots of smaller funds managed to close off their positions and survive. In the case of large hedge funds like LTCM which had available cash from other positions, these hedge funds decided to finance the margin calls with cash before closing their positions. Only after it had no cash left, LTCM was forced to sell its positions at a loss due to divergence in the prices of long and short assets.

In the case of a future position convergence, it is better to use cash to optimize exposure to the position that due to future position convergence will become profitable. However, even if positions eventually converge, they might diverge, as in the case of Scenario 6 and lead to a collateral collapse.

In this case, $P^{F,S} =$ 100+ STEP(20,20)+STEP(215,30)-STEP(235,60) \$/share, where 215 \$/share is very close to a break-point of 210 \$/share. The break-point for a hedge fund to collapse is calculated in the section: Necessary Condition for a Hedge Fund to Fail. For this case, in order for a hedge fund to fail, the following condition should hold:

$Cash\ Hf_0 + \Delta x S_0^{H,L} - \Delta y S_0^{H,S} = Cash\ Hf_0 + (\Delta x + \Delta y) S_0^{H,L} < 0$

$Cash\ Hf_0 =$ \$40,000. Δx =0, and $S_0^{H,L} = -S_0^{H,S}$ =190.39 shares. Therefore, Δy should be at least 210 \$/share.

Note, if a price of a short asset S is going to go much higher than the break-point, then both equity and profits are negative for run S6_cd1big. The losses are augmented in run S6_cd1big compared to run S6_cd1 as a hedge fund manager is using more available cash to buy more assets and cover margin with cash.

Scenario 7. Success: Use Short Proceeds to Obtain More Positions

Fundamental prices of short and long security = $P^{F,S} = P^{F,L}$ = 100 \$/share.

Prices are exogenous, no feedback.

$P^{F,L}$ = 100+ STEP(-20,20)+STEP(20,30) \$/share

$P^{F,S}$ = 100+STEP(20,20)-STEP(20,30) $/share

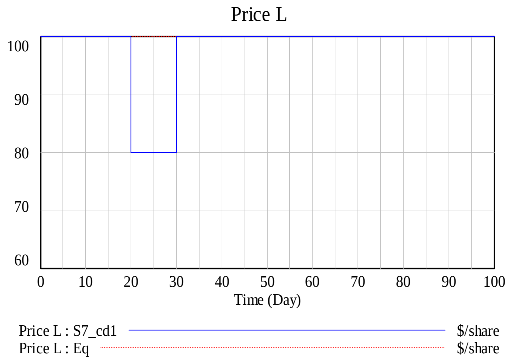


Figure C1_7_1. Price of Security L

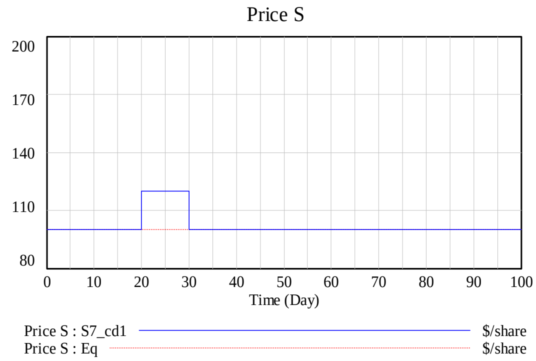


Figure C1_7_2. Price of Security S

Therefore, from time = 0 to time = 20 days, prices are the same, then an arbitrage opportunity window exists from 20 to 30 days, and then prices of both assets converge again.

In this scenario, a hedge fund decides to short-sell as many shares as he desires to long, in order to make a perfect arbitrage.

Also, in this scenario, proceeds from short selling are used to increase hedge fund's cash, and therefore, can be used to obtain more shares. In the previous scenarios, that cash was set aside and could not be used to buy more of security L.

$Cash\ Increase_t^{H,F} = Sell\ Rate_t^{H,L} * P_t^L + Sell\ Rate_t^{H,S} * P_t^S * Fraction\ Reinvested$

Fraction Reinvested = 0.5

The fact that proceeds from the sale of the short security can be used to buy a long security produces a positive feedback loop.

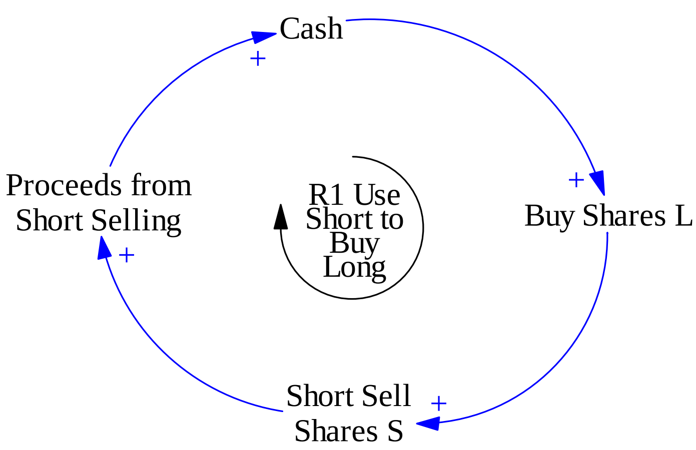


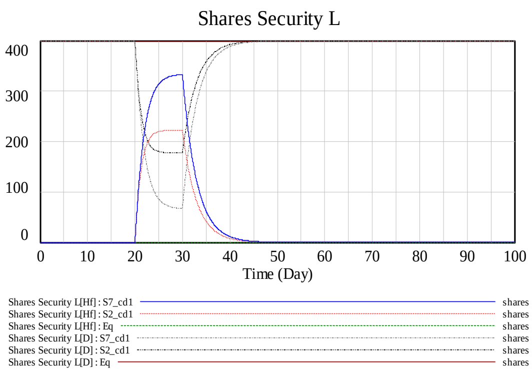
Figure 5.   Using Short Proceeds to Go Long

Hedge fund's cash is used to buy shares of L. Due to margin requirements, each dollar of security L can be used as a collateral to short sell a dollar of security S. If the proceeds from short selling are transferred back into the cash position of the hedge fund, more shares of L can be bought, more shares of S can be short sold, and so on.

S7_cd1big – a run with Cash Decision=1 and Income from Other Investments = STEP(1000,10)-STEP(1000,20)

S7_cd1 – a run with Cash Decision=1

S7_cd0 – a run with Cash Decision=0



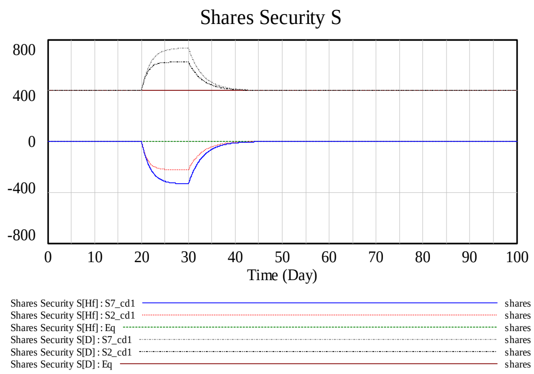


Figure C1_7_3. Shares of Security L

Figure C1_7_4. Shares of Security S

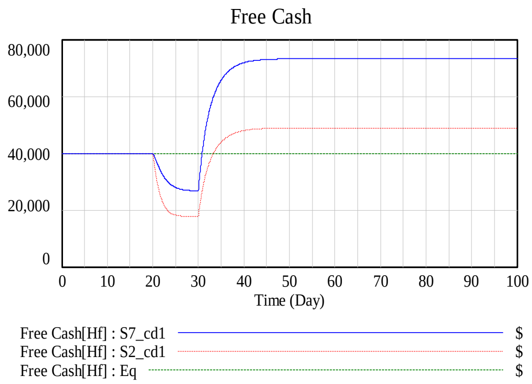


Figure C1_7_5. Free Cash

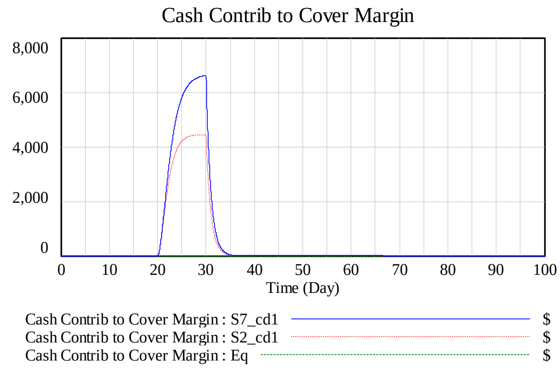


Figure C1_7_6. Cash Contribution to Cover Margin

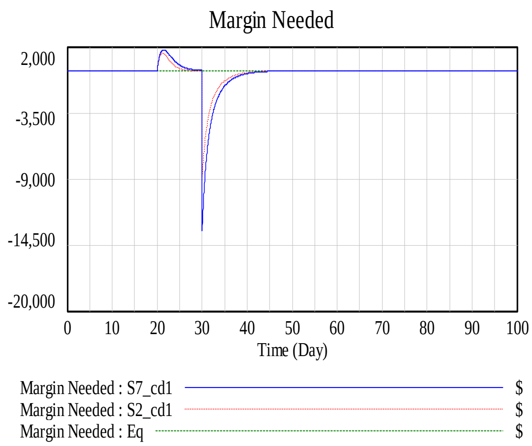


Figure C1_7_7. Margin Needed

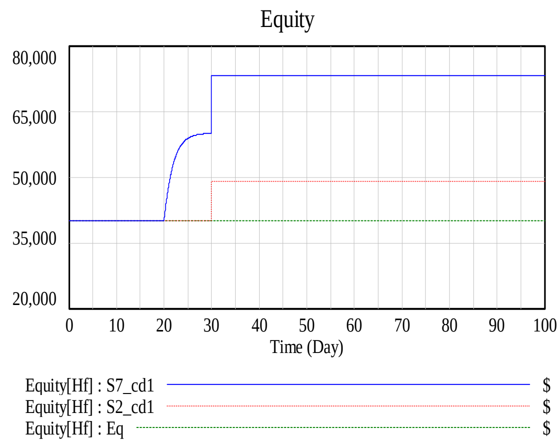


Figure C1_7_8. Equity

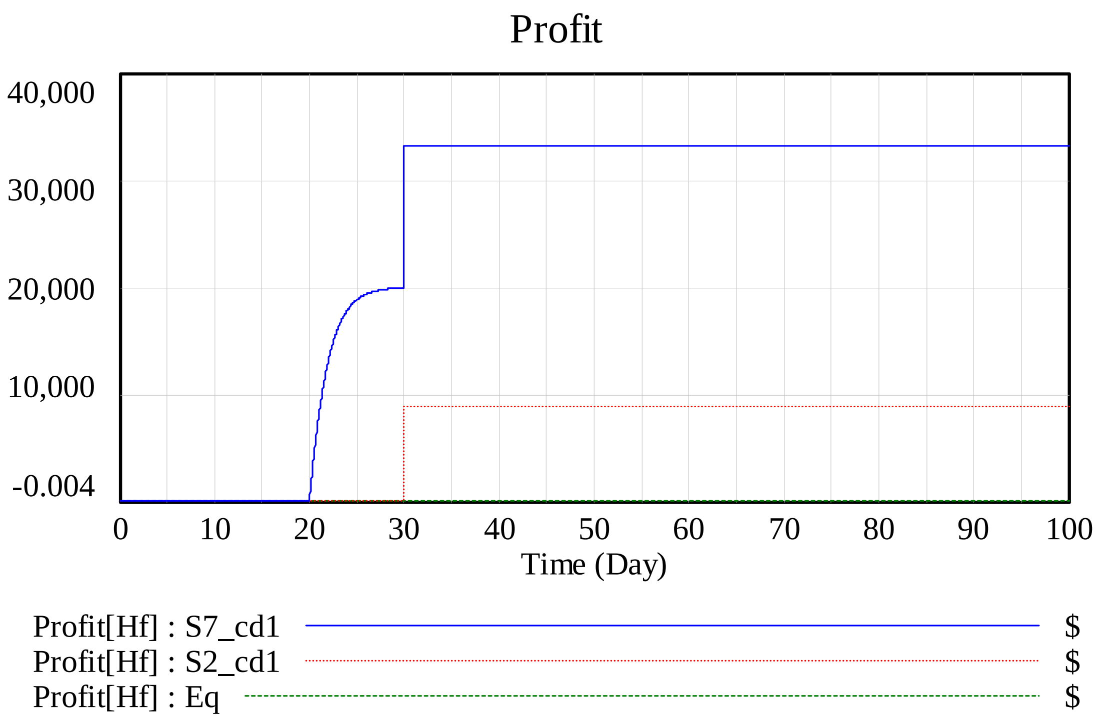


Figure C1_7_9. Profit

Here scenarios 2 and 7 are compared. The scenarios are the same except the reinforcing loop "Use Short to Buy Long" is absent from the scenario 2 and is present in the scenario 7. As can be seen from figures above, due to the reinforcing loop, the hedge fund is more exposed to both long and short positions. The hedge fund manager uses free cash to cover the margin calls when the positions diverge. However, due to a lucky position convergence, the hedge fund is making money in both scenarios, more in scenario 7 where it has more position exposure. Therefore, the reinforcing loop amplifies the gains. The analogy for the difference in behavior in runs S7_cd1big and S2_cd1big is similar to the differences in behavior in runs S7_cd1 and S2_cd1. The behavior in runs S2_cd0 and S7_cd0 is virtually the same due to a very small position exposure in both cases.

According to securities regulations in the USA, it is illegal to reinvest any proceeds from the short sale. Therefore, in the USA, the Fraction Reinvested = 0.

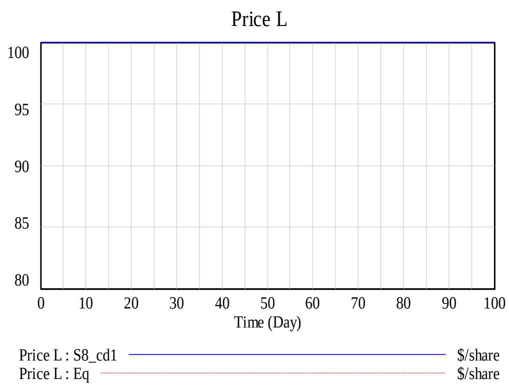
Scenario 8. Failure: Use Short Proceeds to Obtain More Positions

Fundamental prices of short and long security = $P^{F,S} = P^{F,L}$ = 100 \$/share.

Prices are exogenous, no feedback.

$P^{F,L} = 100$ \$/share

$P^{F,S} = 100 + STEP(20,20) + STEP(215,30) - STEP(235,60)$ \$/share



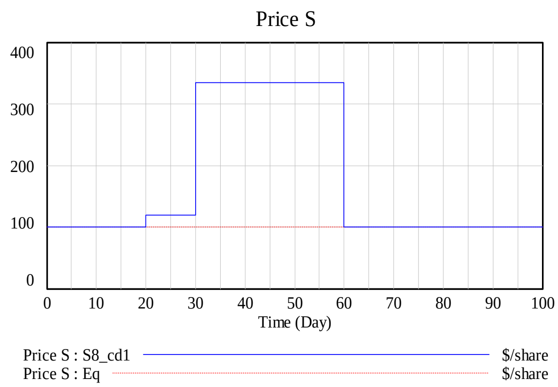


Figure C1_8_1. Price of Security L

Figure C1_8_2. Price of Security S

Therefore, from time = 0 to time = 20 days, prices are the same, then an arbitrage opportunity window exists from 20 to 30 days. From time 30 to time 60 days, instead of converging, prices diverge even more before converging at time 60 days.

In this scenario, a hedge fund decides to short-sell as many shares as he desires to long, in order to make a perfect arbitrage.

Also, in this scenario, like in scenario 7, proceeds from short selling are used to increase hedge fund's cash, and therefore, can be used to obtain more shares. In previous scenarios, that cash was set aside and could not be used to buy more of security L.

$Cash\ Increase_t^{H,F} = Sell\ Rate_t^{H,L} * P_t^L + Sell\ Rate_t^{H,S} * P_t^S * Fraction\ Reinvested$

Fraction Reinvested = 0.5

S8_cd1big – a run with Cash Decision=1 and Income from Other Investments = STEP(1000,10)-STEP(1000,20)

S8_cd1 – a run with Cash Decision=1

S8_cd0 – a run with Cash Decision=0

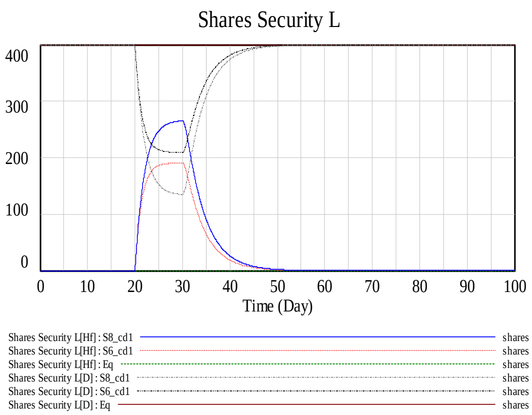


Figure C1_8_3. Shares of Security L

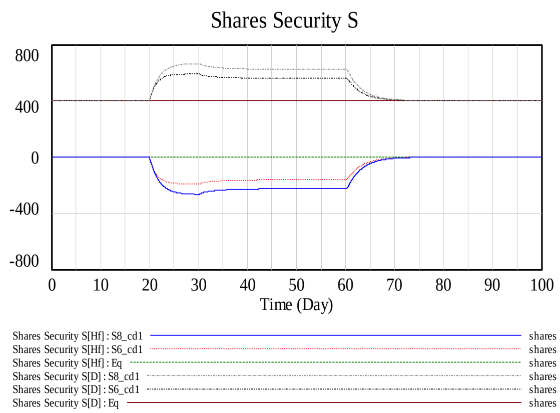


Figure C1_8_4. Shares of Security S

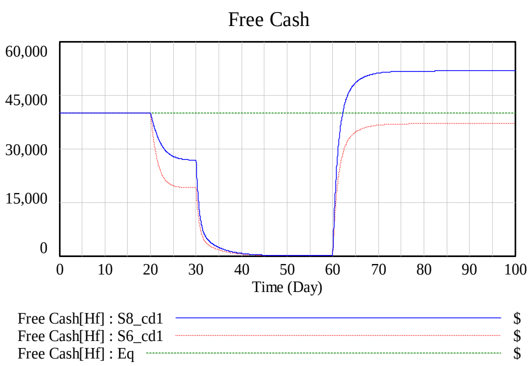


Figure C1_8_5. Free Cash

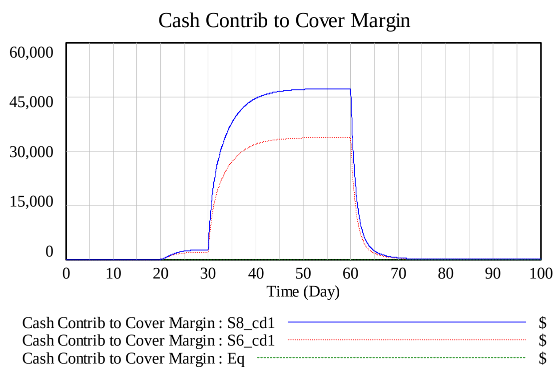


Figure C1_8_6. Cash Contribution to Cover Margin

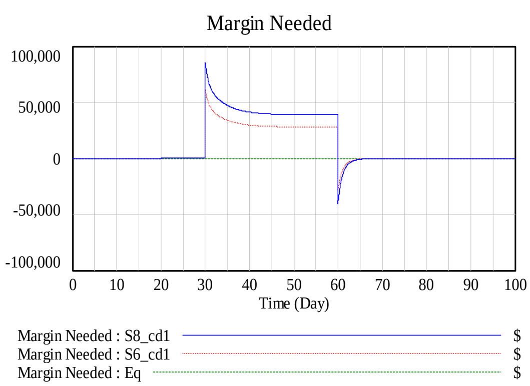


Figure C1_8_7. Margin Needed

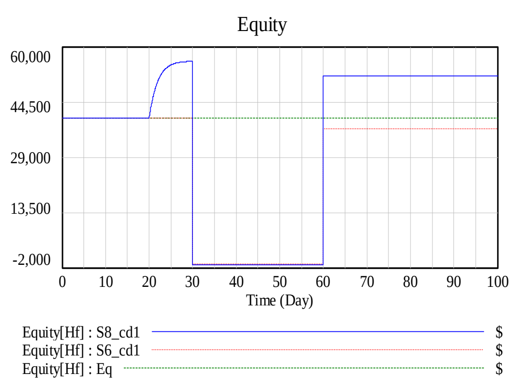


Figure C1_8_8. Equity

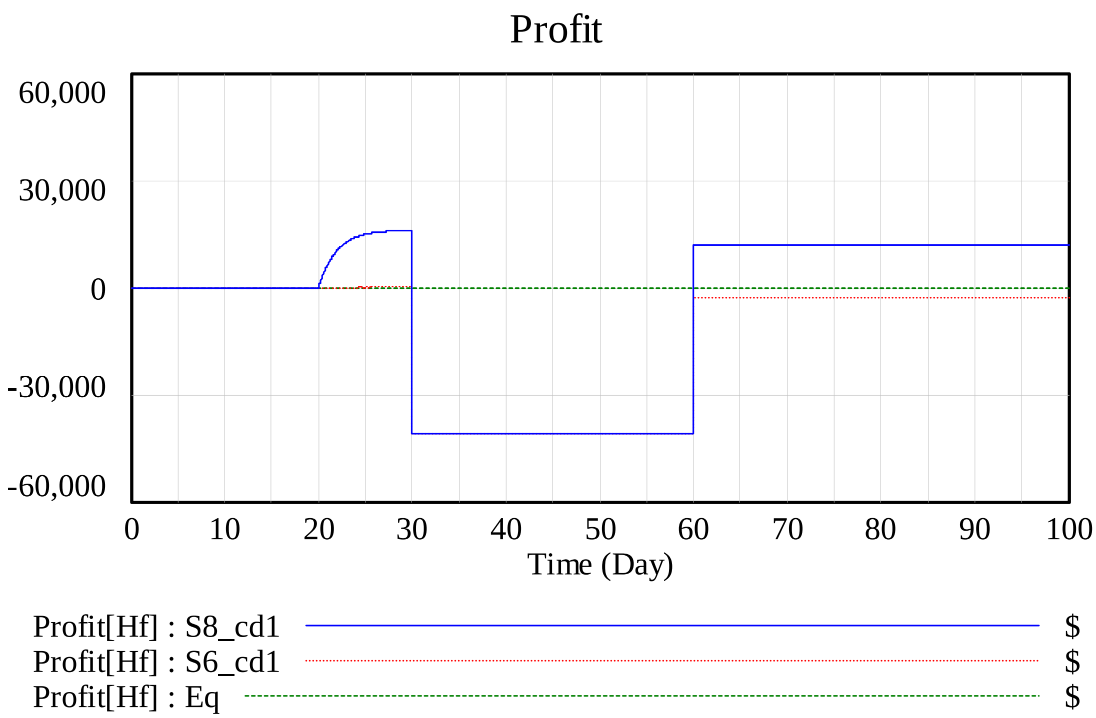


Figure C1_8_9. Profit

Scenarios 6 and 8 are compared. The scenarios are the same except for the reinforcing loop "Use Short to Buy Long" that is absent from scenario 6 and is present in scenario 8. In both cases, we observe the failure of a hedge fund. However, the process is reinforced in scenario 8, as the proceeds are used to buy more security L and short sell more security S. Therefore, the total position exposure in scenario 8 is bigger than in scenario 6. In both cases, instead of converging, the positions greatly diverge. Free cash in both cases is 0. In both cases, the fund collapses, but the collapse is more pronounced in the scenario 8 compared to scenario 6 due to a higher position exposure (note, in this scenario, the difference is not too big due to price of security S being close to break-point for both cases). The analogy for the difference in behavior in runs S8_cd1big and S6_cd1big is similar to the differences in behavior in runs S8_cd1 and S6_cd1. The behavior for runs S8_cd0 and S6_cd0 is virtually the same due to a very small position exposure in both cases.

According to securities regulations in the USA, it is illegal to reinvest any proceeds from the short sale. Therefore, in the USA, the Fraction Reinvested = 0.

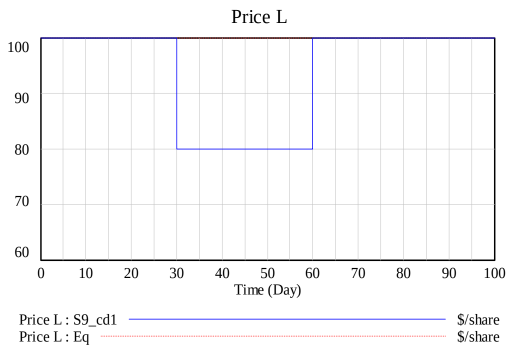
Scenario 9. Divergence Before Convergence

Fundamental prices of short and long security = $P^{F,S} = P^{F,L}$ = 100 \$/share.

Prices are exogenous, no feedback.

$P^{F,L}$ = 100+STEP(-20,30)+STEP(+20,60) \$/share

$P^{F,S}$ = 100+STEP(20,20)-STEP(20,30)+STEP(40,30)-STEP(40,60) \$/share



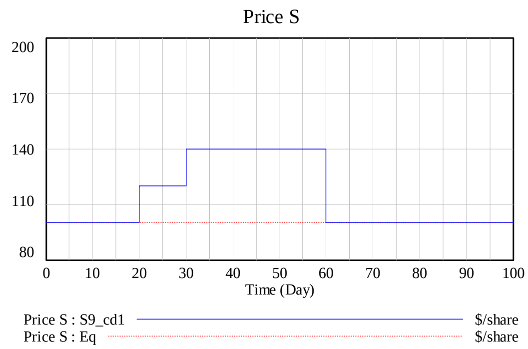


Figure C1_9_1. Price of Security L   Figure C1_9_2. Price of Security S

Therefore, from time = 0 to time = 20 days, prices are the same, then an arbitrage opportunity window exists from 20 to 30 days. From time 30 to time 60 days, instead of converging, prices diverge even more before converging at time 60 days. Note, the divergence is not as extreme as in the scenarios 5,6 and 8.

In this scenario, a hedge fund manager decides to short-sell as much as allowed within margin requirements, so he would not have a margin call.

Fraction Reinvested = 0

S9_cd1big – a run with Cash Decision=1 and Income from Other Investments = STEP(1000,10)-STEP(1000,20)

S9_cd1 – a run with Cash Decision=1

S9_cd0 – a run with Cash Decision=0

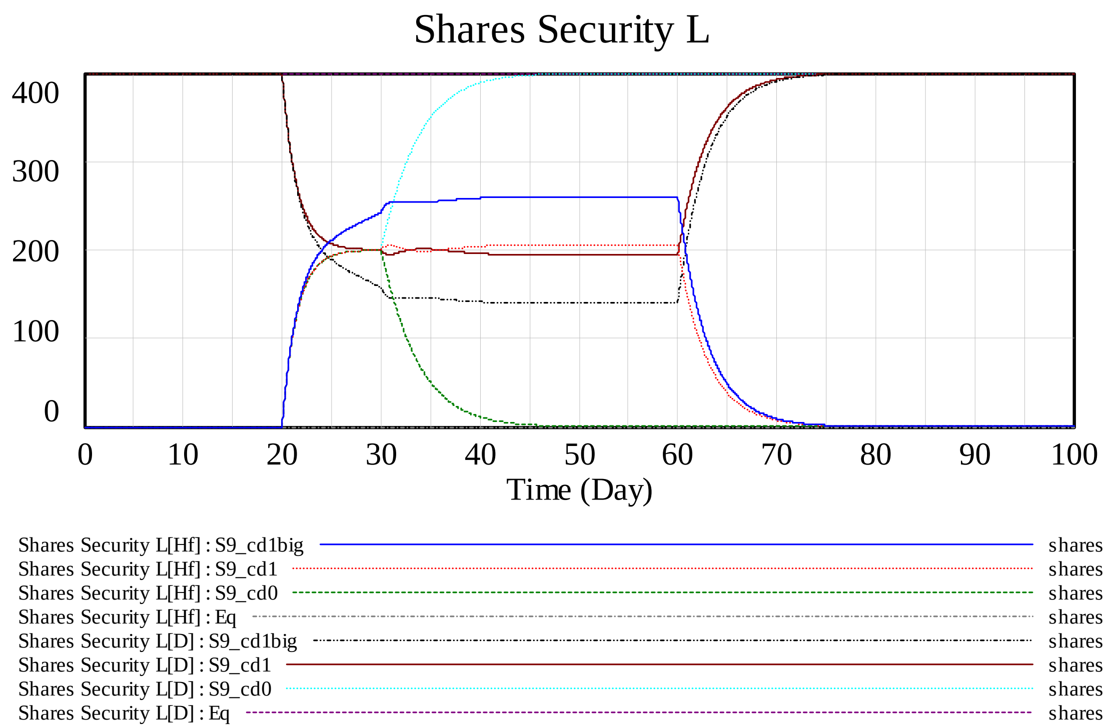


Figure C1_9_3. Shares of Security L

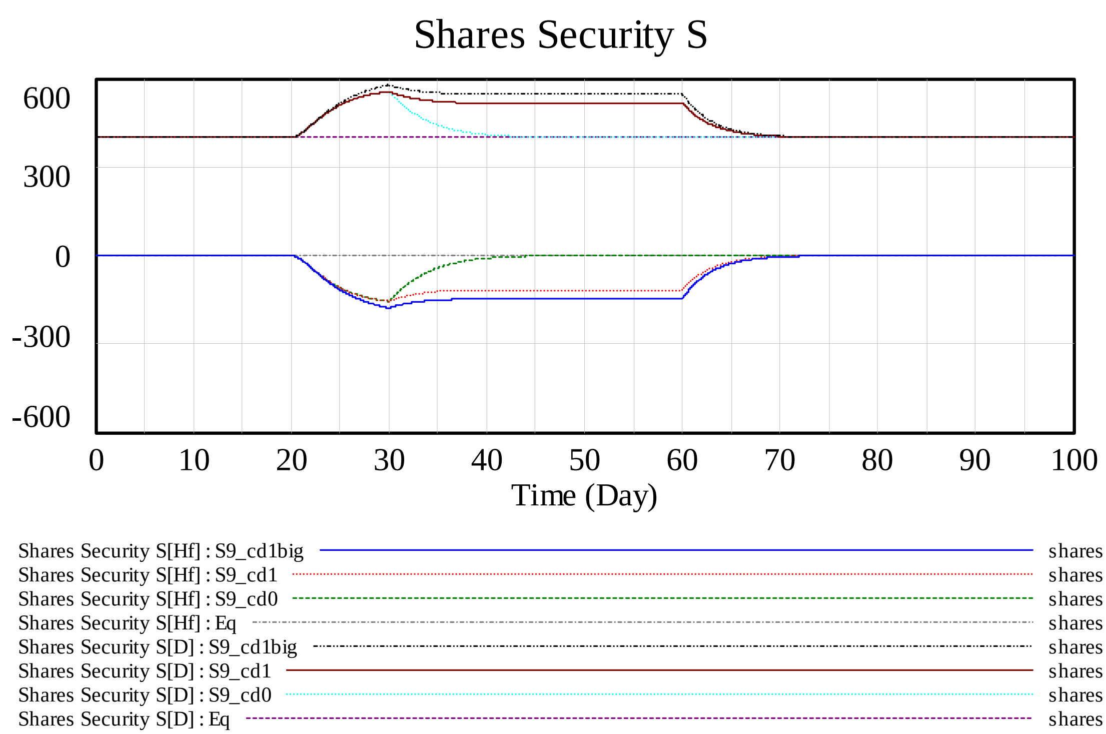


Figure C1_9_4. Shares of Security S

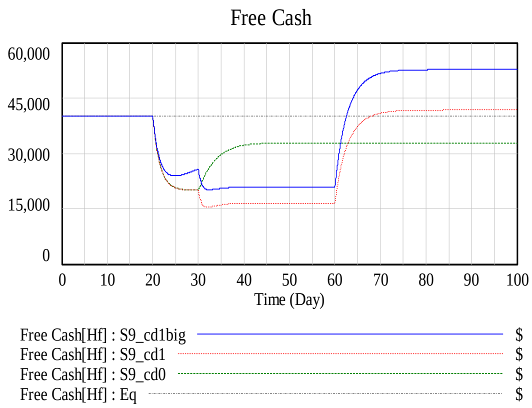


Figure C1_9_5. Free Cash

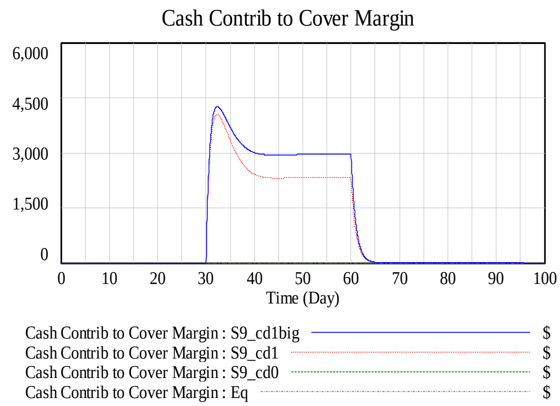


Figure C1_9_6. Cash Contribution to Cover Margin

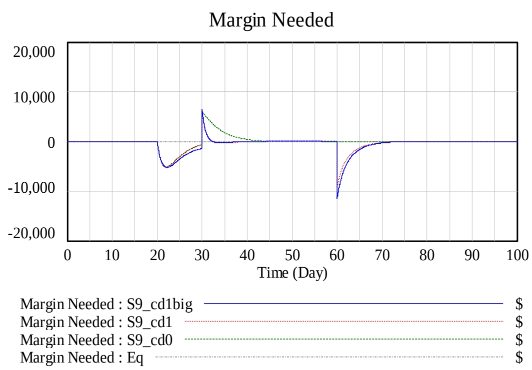


Figure C1_9_7. Margin Needed

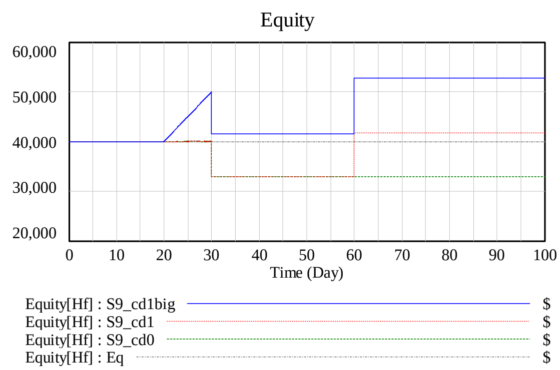


Figure C1_9_8. Equity

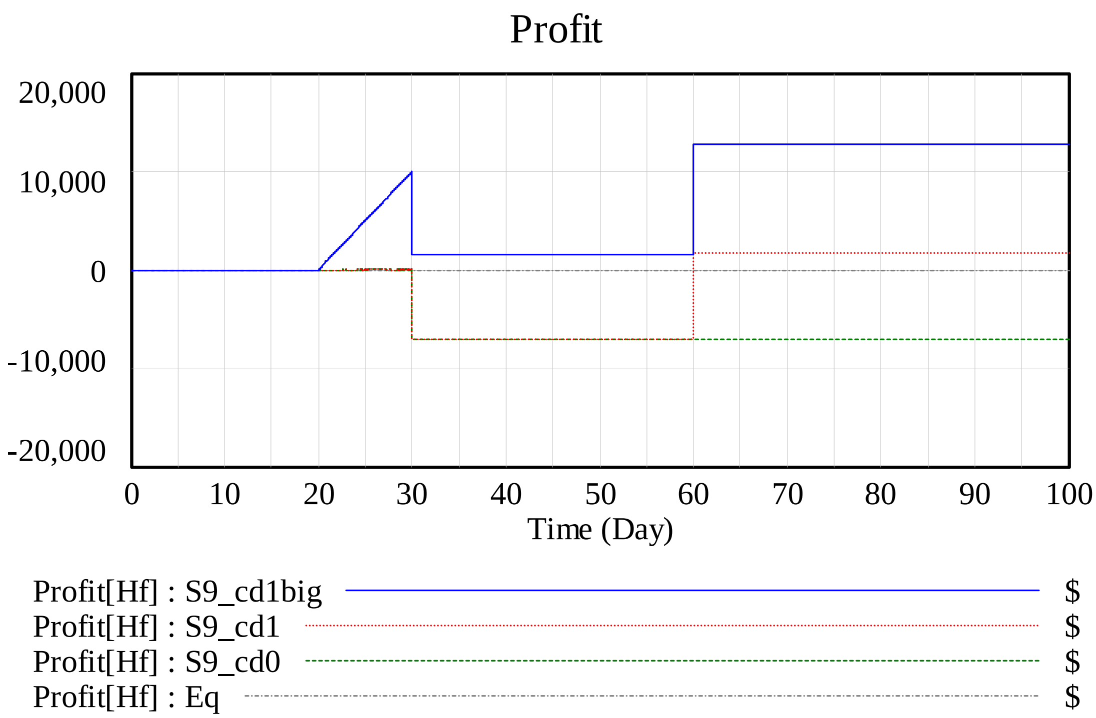


Figure C1_9_9. Profit

As prices diverge before converging after time = 30 days, a hedge fund manager uses available cash to cover margin and buys more of the long asset in S9_cd1 and S9_cd1big runs. In comparison, in S9_cd0, a hedge fund manager unwinds both long and short positions in order to reduce the margin. As a result, final profits and equity in the S9_cd1big are larger than in S9_cd1 which are in return larger than profits in the S9_cd0 case. However, in all three cases, the hedge fund survives.

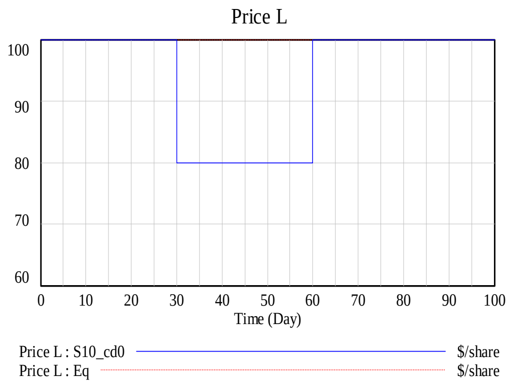
Scenario 10. Divergence Before Convergence: Different Strategies Compared

Fundamental prices of short and long security = $P^{F,S} = P^{F,L}$ = 100 \$/share.

Prices are exogenous, no feedback.

$P^{F,L}$ = 100+STEP(-20,30)+STEP(+20,60) \$/share

$P^{F,S}$ = 100+STEP(20,20)-STEP(20,30)+STEP(40,30)-STEP(40,60) \$/share



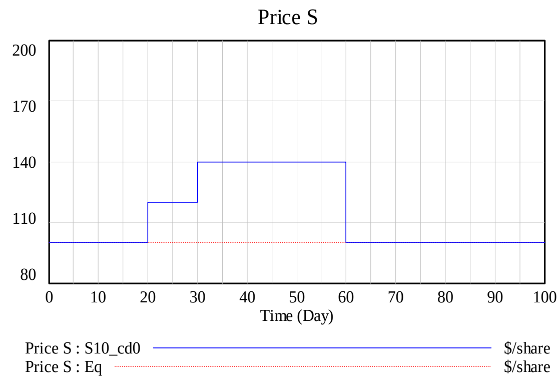


Figure C1_10_1. Price of Security L Figure C1_10_2. Price of Security S

Therefore, from time = 0 to time = 20 days, prices are the same, then an arbitrage opportunity window exists from 20 to 30 days. From time 30 to time 60 days, instead of converging, prices diverge even more before converging at time 60 days. Note, the divergence is not as extreme as in the scenarios 5,6 and 8.

In this scenario, a hedge fund manager decides to short-sell as many shares as it goes long. This scenario is compared to scenario 9 where a hedge fund manager decides to short-sell as much as allowed within margin requirements, so he would not have a margin call.

S10_cd1 – a run with Cash Decision=1

S10_cd0 – a run with Cash Decision=0

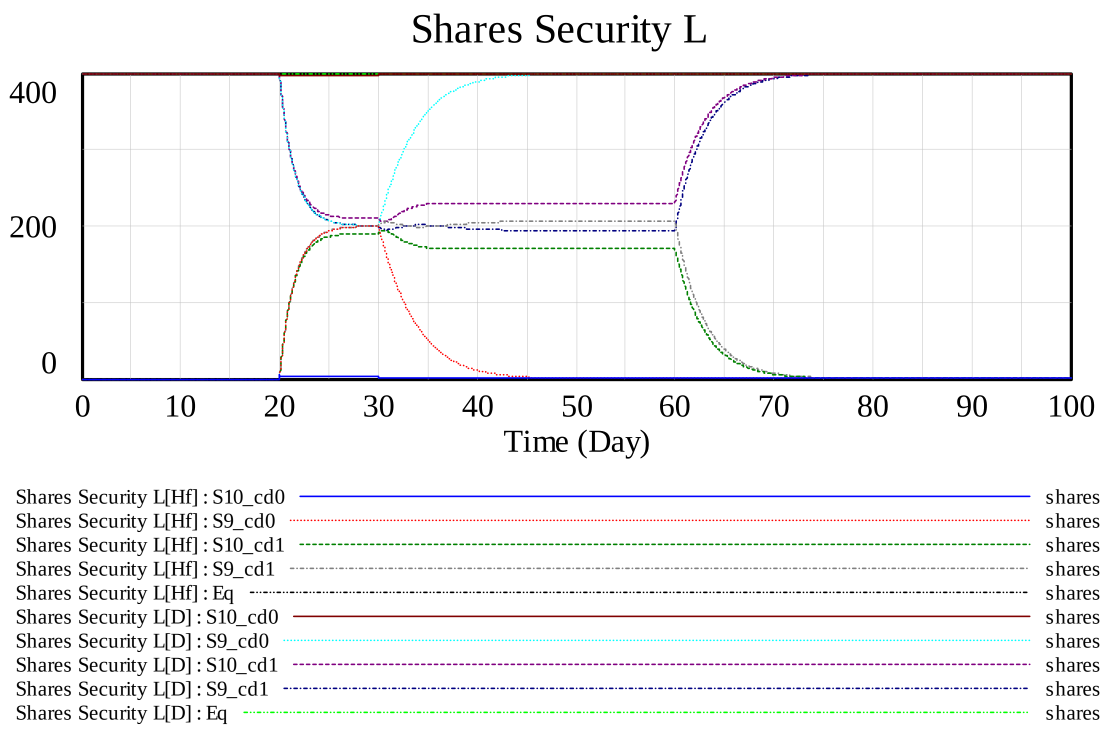


Figure C1_10_3. Shares of Security L

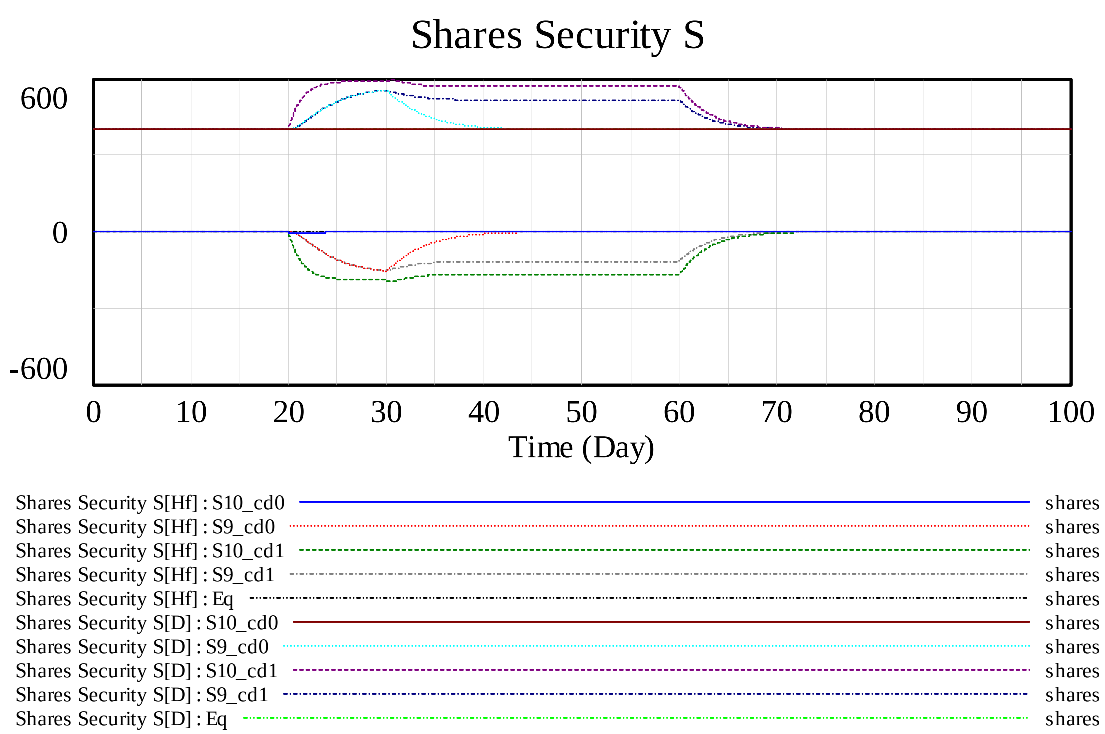


Figure C1_10_4. Shares of Security S

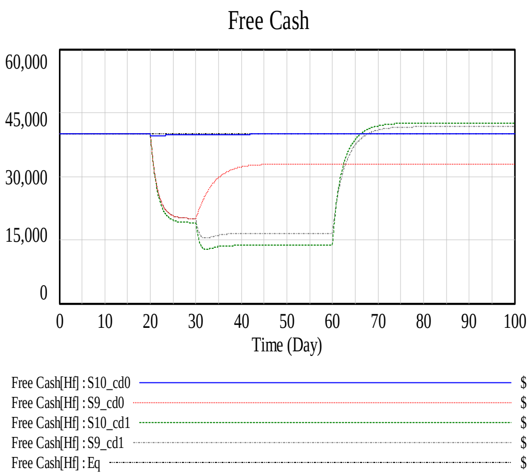


Figure C1_10_5. Free Cash

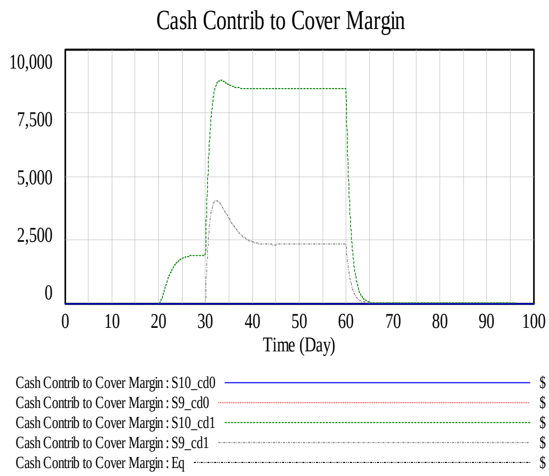


Figure C1_10_6. Cash Contribution to Cover Margin

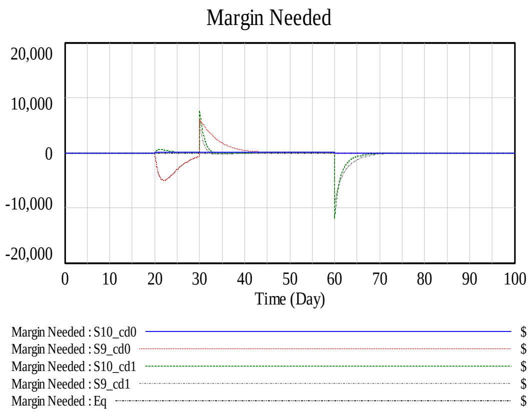


Figure C1_10_7. Margin Needed

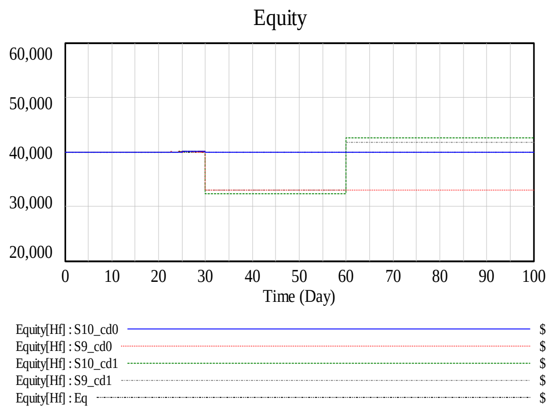


Figure C1_10_8. Equity

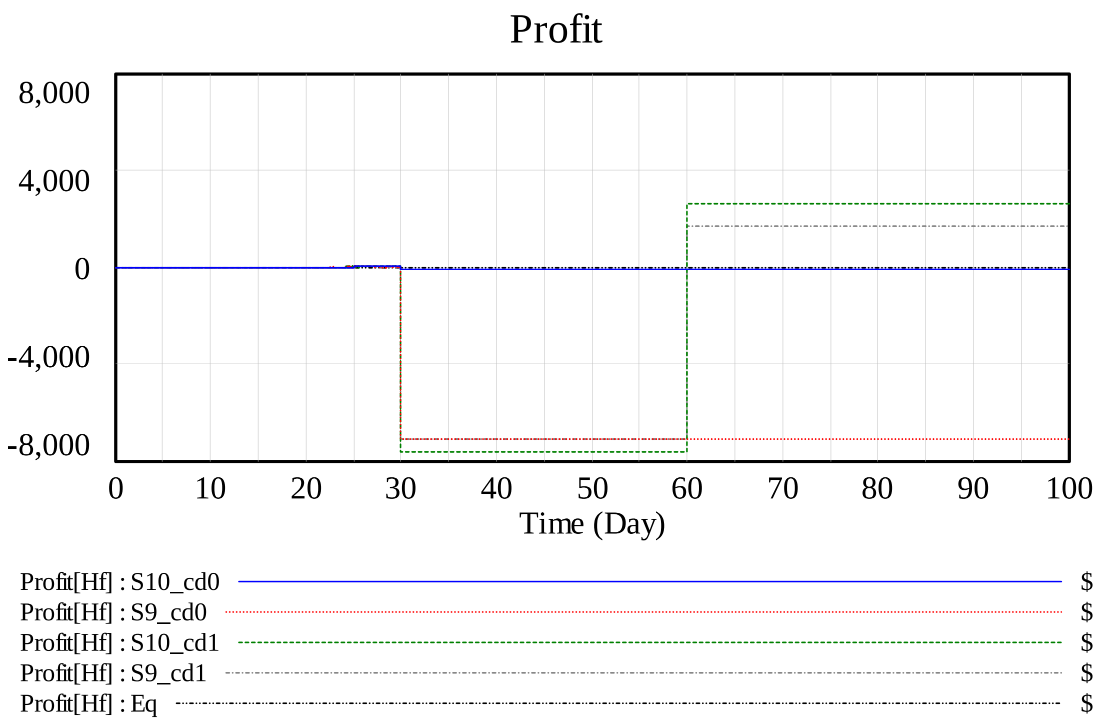


Figure C1_10_9. Profit

Here S10_cd0 run is compared to S9_cd0 run, and S10_cd1 run is compared to S9_cd1 run. As prices diverge even more at time = 30 days before converging at time = 60 days, a hedge fund manager tries to cover new margin either by selling security L or by using free money. A hedge fund does not increase its positions as the spread widens further. In this case, a hedge fund is better off by using a strategy of shorting as many shares of S as going long on L.

Scenario 11. An Increase in Maximum Allowed Leverage

Fundamental prices of short and long security = $P^{F,S} = P^{F,L}$ = 100 \$/share.

Prices are exogenous, no feedback.

$P^{F,L}$ = 100+STEP(-20,30)+STEP(+20,60) \$/share

$P^{F,S}$ = 100+STEP(20,20)-STEP(20,30)+STEP(40,30)-STEP(40,60) \$/share

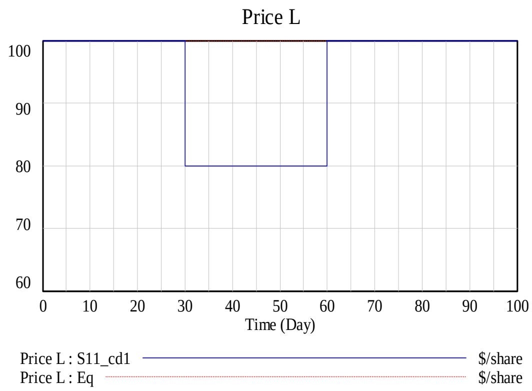


Figure C1_11_1. Price of Security L

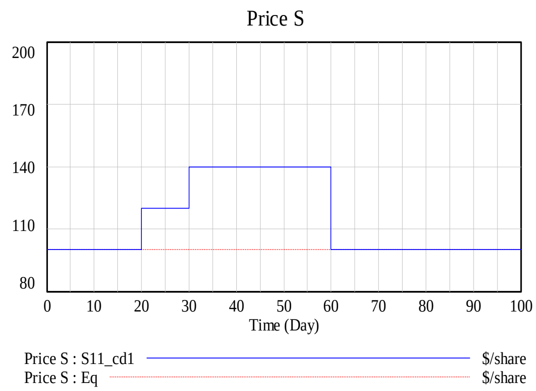


Figure C1_11_2. Price of Security S

Therefore, from time = 0 to time = 20 days, prices are the same, then an arbitrage opportunity window exists from 20 to 30 days. From time 30 to time 60 days, instead of converging, prices diverge even more before converging at time 60 days. Note, the divergence is not as extreme as in the scenarios 5,6 and 8.

Maximum Allowed Leverage = 10 (instead of the usual 2)

In this scenario, a hedge fund manager decides to short-sell as much as allowed within margin requirements, so he would not have a margin call.

S11_cd1 – a run with Cash Decision=1

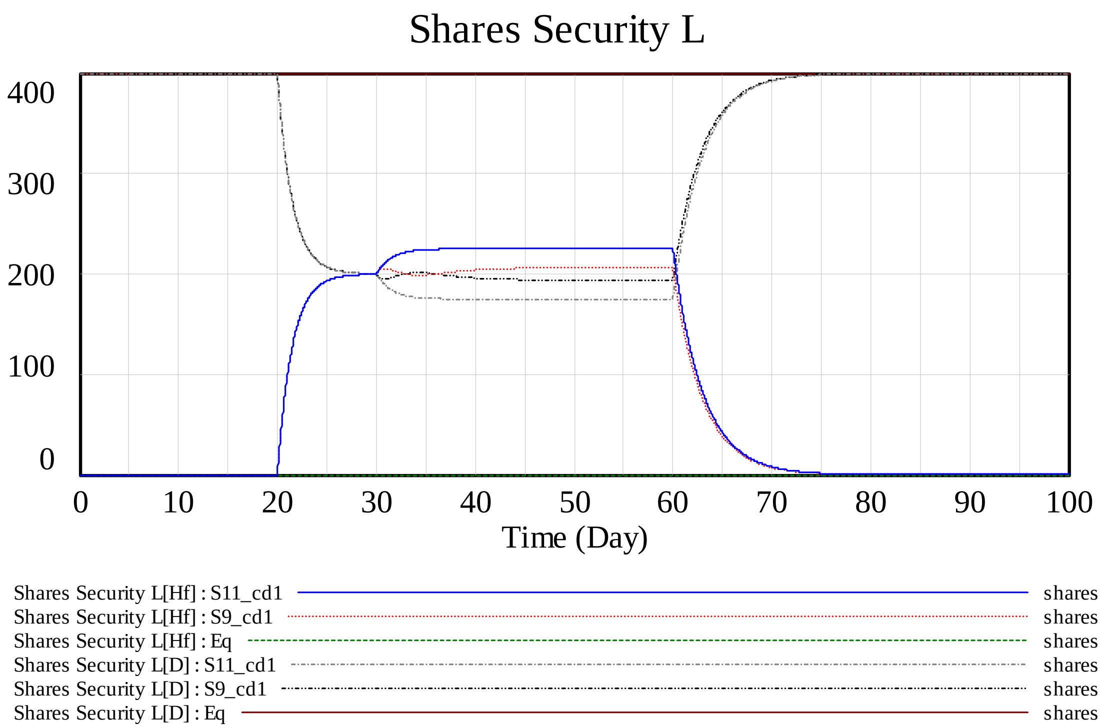


Figure C1_11_3. Shares of Security L

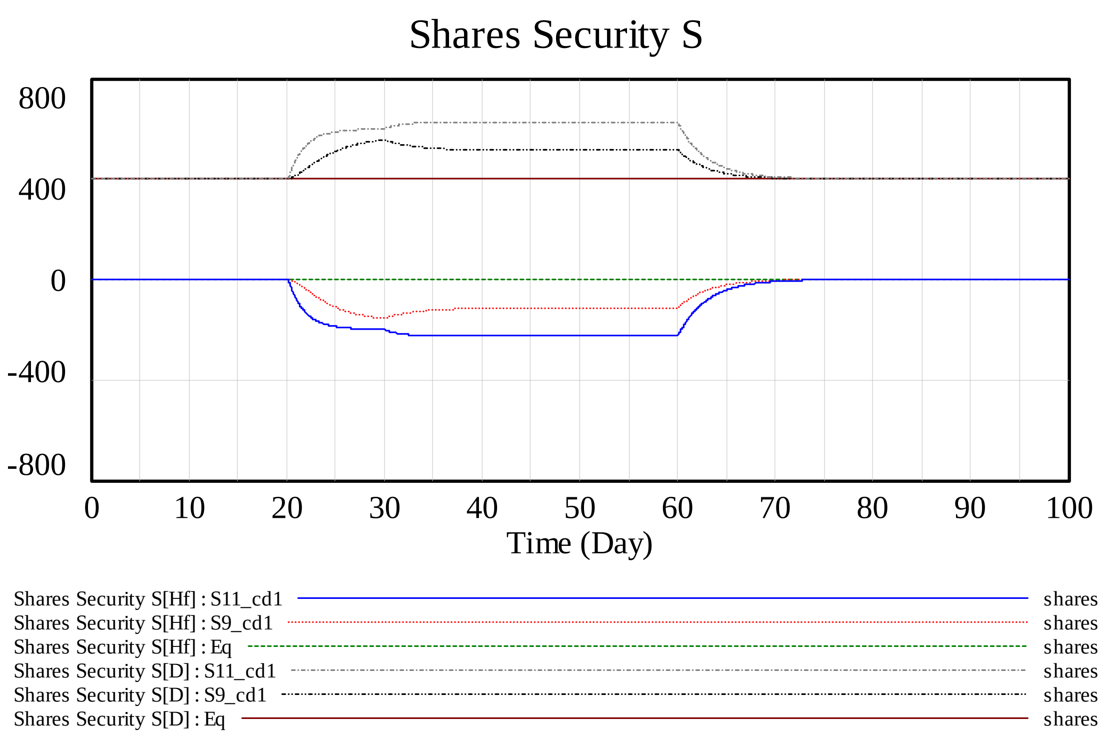


Figure C1_11_4. Shares of Security S

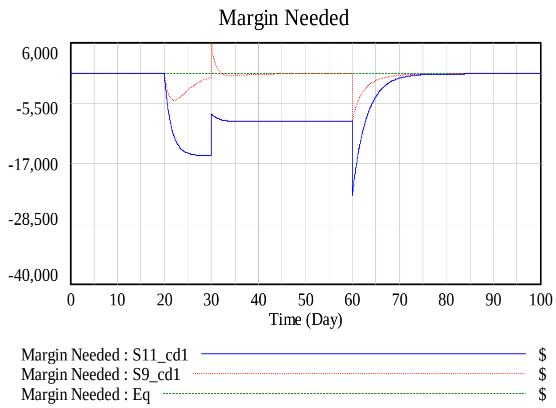


Figure C1_11_5. Margin Needed

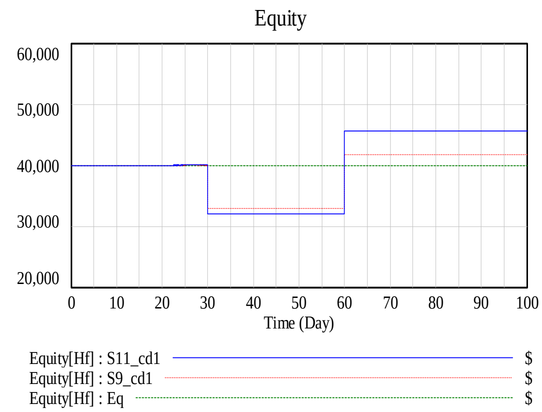


Figure C1_11_6. Equity

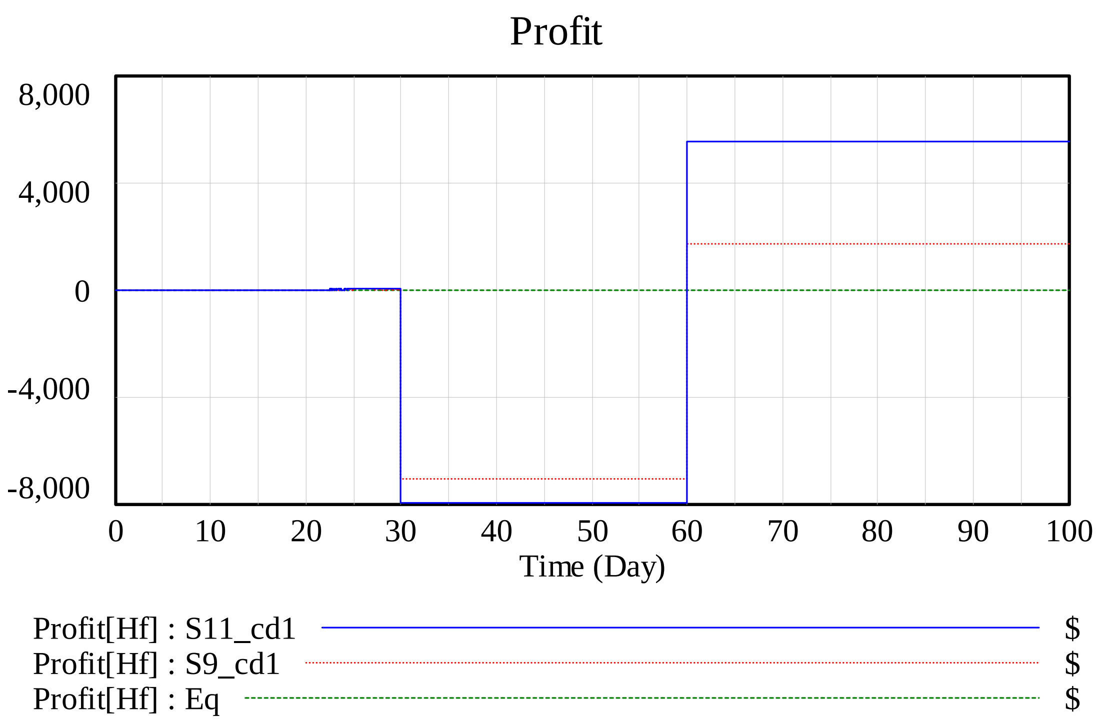


Figure C1_11_7. Profit

Given a higher Maximum Allowed Leverage, a hedge fund can actually borrow more of an asset S and obtain higher Equity and Profits as can be seen by comparing runs S10_cd1 and S9_cd1. The runs are exactly the same except for the amount of Maximum Allowed Leverage. In S10_cd1, the Maximum Allowed Leverage is 10, and in S9_cd1, the Maximum Allowed Leverage is 2.

Necessary Condition for a Hedge Fund to Fail

The necessary condition for a hedge fund to fail if its equity becomes negative. Mathematically, for a hedge fund to fail, the following inequalities must hold:

Equity = Assets + Liabilities < 0 (eq. 34)

Cash Hf $_t$ + Market Value of L Position $_t$ + Credit Balance of S Position $_t$ – Market Value of S Position $_t$ (eq. 35)

Market Value of L Position $_t$ $= P_t^L S_t^{H,L}$ (eq. 36)

Credit Balance of S Position $_t$ = Total Basis Security S $_t$ $= -P_0^S S_0^{H,S}$ (eq. 37)

Market Value of S Position $_t$ $= -P_t^S S_t^{H,S}$ (eq. 38)

Therefore, for a hedge fund to fail, the following inequality should hold:

Cash Hf $_t$ $+\ P_t^L S_t^{H,L} - P_0^S S_0^{H,S} + P_t^S S_t^{H,S} < 0$ (eq. 39)

Cash Hf $_t$ $+\ (P_0^L + \Delta x) S_t^{H,L} - P_0^S S_0^{H,S} + (P_0^S - \Delta y) S_t^{H,S} < 0$ (eq. 40)

Arbitrage Window

Here a special case for a necessary condition for a hedge fund to fail is explored. In this case, it is assumed that either a spread converged or diverged at time no earlier than time=t.

Assume $S_t^{H,L} = S_0^{H,L}$ and $S_t^{H,S} = S_0^{H,S}$, where initial time is when an arbitrage position is conceived. This assumption makes sense, given no exogenous cash inflows or outflows, a hedge fund is not going to unwind it position unless at time t, it is hit with a margin call or the spread converged. This assumption follows directly from the main assumption in this case that either a spread converged or diverged at time no earlier than time=t.

Therefore, for a hedge fund to fail:

Cash Hf $_t$ $+\ (P_0^L + \Delta x) S_0^{H,L} - P_0^S S_0^{H,S} + (P_0^S - \Delta y) S_0^{H,S} < 0$ (eq. 41)

Cash Hf $_t$ $+\ (P_0^L + \Delta x) S_0^{H,L} - \Delta y S_0^{H,S} < 0$ (eq. 42)

Assume a hedge fund uses its cash only to buy security L. A hedge fund does not have any other expenditures and is not buying back security S. This assumption follows directly from the main assumption in this case that either a spread either converged or diverged at time no earlier than time=t.

Therefore, for a hedge fund to fail:

$$Cash\ Hf_0 - P_0^L S_0^{H,L} + (P_0^L + \Delta x)S_0^{H,L} - \Delta y S_0^{H,S} < 0 \qquad \text{(eq. 43)}$$

$$Cash\ Hf_0 + \Delta x S_0^{H,L} - \Delta y S_0^{H,S} < 0 \qquad \text{(eq. 44)}$$

Case 1: Short as Much as Long

In this case,

$$S_0^{H,L} = -S_0^{H,S} \qquad \text{(eq. 45)}$$

$$Cash\ Hf_0 + \Delta x S_0^{H,L} - \Delta y S_0^{H,S} = Cash\ Hf_0 + (\Delta x + \Delta y)S_0^{H,L} < 0 \qquad \text{(eq. 46)}$$

So, for example, final equity and profits are the same for scenarios 2 and 4 depicted above. In scenario 2, P_0^L =80 \$/share, P_0^S =120 \$/share. In scenario 4, P_0^L =100 \$/share, P_0^S =140 \$/share, and in both scenarios, $P_t^L = P_t^S$ =100 \$/share. In scenario 2, Δx =20 \$/share, and Δy =20 \$/share. In scenario 4, Δx =0 \$/share, and Δy =40 \$/share. Therefore, $Cash\ Hf_0 + \Delta x S_0^{H,L} - \Delta y S_0^{H,S} = Cash\ Hf_0 + (\Delta x + \Delta y)S_0^{H,L}$ is the same for two scenarios. In both scenarios, hedge funds have the same equity and final profits.

Case 2: Keep Within Margin Requirements

In this case,

$$-S_0^{H,S} = \frac{P_0^L S_0^{H,L}}{P_0^S} \qquad \text{(eq. 47)}$$

$$Cash\ Hf_0 + \Delta x S_0^{H,L} - \Delta y S_0^{H,S} = Cash\ Hf_0 + \Delta x S_0^{H,L} + \frac{\Delta y P_0^L S_0^{H,L}}{P_0^S} = Cash\ Hf_0 +$$

$$S_0^{H,L}(\Delta x + \frac{\Delta y P_0^L}{P_0^S}) < 0 \qquad \text{(eq. 48)}$$

So, for example, final equity and profits are different for scenarios 1 and 3 depicted above. In scenario 1, P_0^L =80 \$/share, P_0^S =120 \$/share. In scenario 3, P_0^L =100 \$/share, P_0^S =140 \$/share, and in both scenarios, $P_t^L = P_t^S$ =100 \$/share. In scenario 1, Δx =20 \$/share, and Δy =20 \$/share. In scenario 3, Δx =0 \$/share, and Δy =40 \$/share.

Therefore, in scenario 1, *Cash Hf* $_0$ + $S_0^{H,L}(\Delta x + \frac{\Delta y P_0^L}{P_0^S})$ = *Cash Hf* $_0$ + $20\frac{S_0^{H,L}P_0^L}{P_0^L}$ +

$20\frac{S_0^{H,L}P_0^L}{P_0^S}$

In scenario 3, *Cash Hf* $_0$ + $S_0^{H,L}(\Delta x + \frac{\Delta y P_0^L}{P_0^S})$ = *Cash Hf* $_0$ + $40\frac{S_0^{H,L}P_0^L}{P_0^S}$

Since $P_0^S > P_0^L$, equity and final profits in scenario 1 will always be higher than those in scenario 3, even though $\Delta x + \Delta y$ is the same for both scenarios. The result is due to an understanding that only proportional, and not absolute increases and decreases in prices of underlying assets matter in calculation of equity and profits.

9. Part 1 – Three Agents

Dynamic Hypotheses

If arbitrage spreads widen, as happened in May, 1998 for LTCM, people start liquidating, therefore, further depressing the price of an illiquid asset L and increasing the price of a liquid asset S. In this case, a preference for liquidity increases. As preference for liquidity increases, that forces more people to liquidate illiquid positions and buy liquid ones. People were willing to buy Treasuries at any price as long as they got out of the risky bonds and obtained the less risky instruments. Everybody on the street started talking about "flight to quality" or buying Treasury bonds. That lead to losses of LTCM. Owing to its loss of capital, Long-Term's leverage had become very high, because losses accumulate faster as leverage increases. Therefore, they wanted to sell something. At that point, leverage was very high, and the fund's partners were looking forward to sell several positions and raise more money before the end of the month. LTCM knew it had to reduce its positions, but couldn't with markets under the stress. There was no liquidity in the market. Everybody wanted to be out at the same time – something that models missed. When losses mount, leveraged investors such as Long-Term are forced to sell, lest their losses overwhelm them. When a firm has to sell without buyers, prices are very low. In addition, Wall Street players learned more about the fund's positions, and went against them. They wanted to "squeeze" as much as possible from the fund, knowing that if the fund would get help from the government, it will be able to buy back its shorts. Therefore, anybody who held those securities would make money. In September 1998, many banks were exposed to the same positions as LTCM. Therefore, to cut their losses, they unwound those positions, thus, hurting LTCM. Therefore, both cutting the losses and predatory trading led to the collapse of the fund.

As price of illiquid security L goes down, therefore, the net asset value of a hedge fund goes down. That in turn leads for the collateral value to decrease. Lenders either require more collateral, or in case of many leveraged hedge funds, they pressure the hedge fund (through the dealer) to sell the assets. As it usually happens, hedge funds use their own assets as collateral, which leads to this vicious loop: R1 Collateral Collapse described in Figure 6. Note, large hedge funds like LTCM are considered in this causal diagram as only large hedge funds can have market impact.

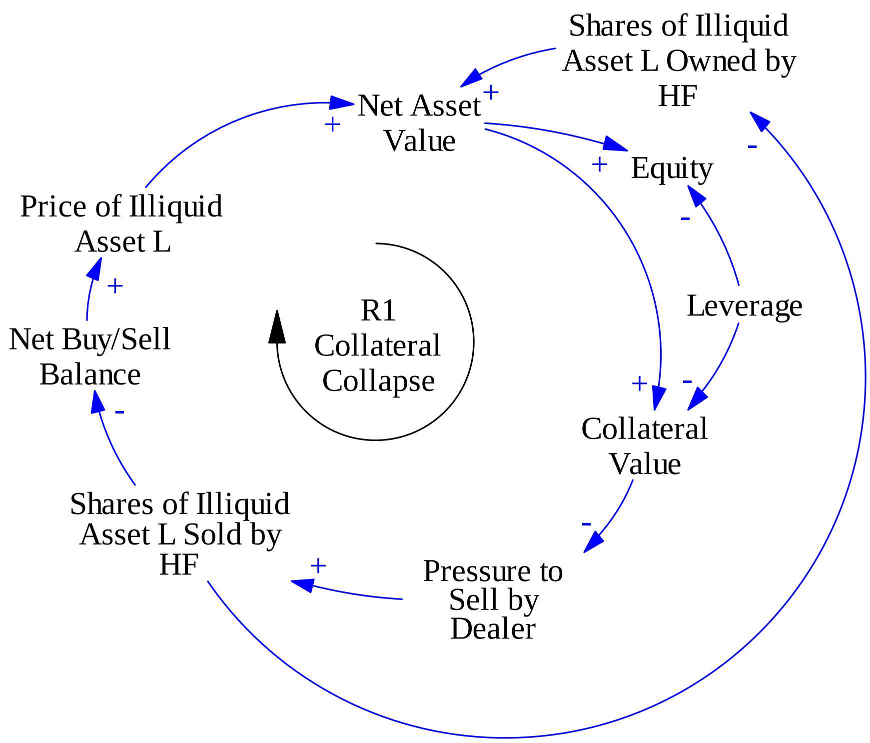


Figure 6. Collateral Collapse Feedback

As is described in the case, the pressure to sell usually decreases price, leading to an increase in volatility (negative trend of an illiquid security L) of an asset leading to more pressure to sell by lenders. This dynamic is described in Figure 7.

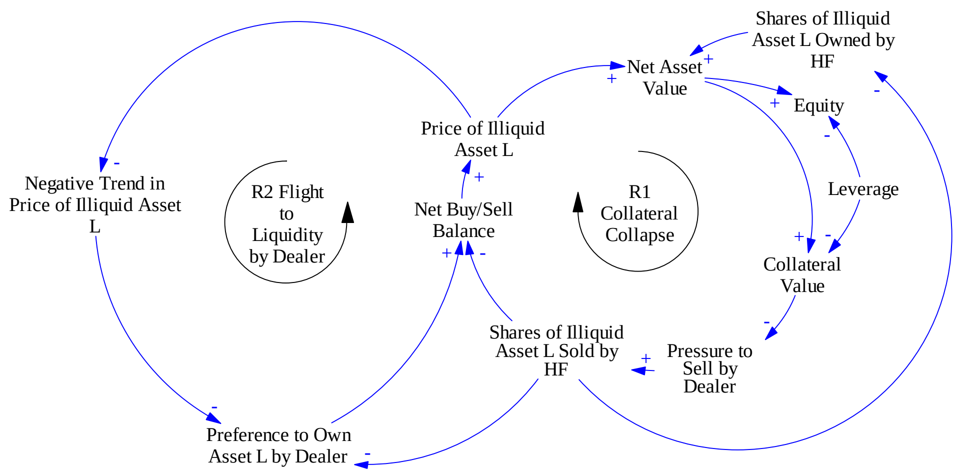


Figure 7: Flight to Liquidity by Dealer Feedback

Dealers do not want to be in a position to be left with an inventory of illiquid security L as prices of that security continue to go down. Therefore, they prefer to be buy less of security L on their own account, further exacerbating the Net Buy/Sell Balance. As is depicted in Figure 7, as price of security L goes down and a hedge fund is forced to sell more shares, preference to owning those shares by a dealer goes down.

Net Assets (A) of a hedge funds equals the sum of Price (P) multiplied by Shares (S) for each position in a fund: $A = \sum_{i=1}^{n} S_i P_i$ (eq. 49)

Equity (E) equals to net Assets (A) minus Leverage (L): $E = A - L$ (eq. 50)

Collateral Value (C) equals to Assets minus Leverage: $C = A - L$. (eq. 51)

In this case, E is the same as collateral. So, if A decreases to A_1, then the new collateral to be posted is L/0.5- $A_{1.}$

The collateral to be posted is max(0, L/0.5- A_1). (eq. 52)

Price is assumed to take the following form: $P_i = \alpha_i - \beta_i \Delta S_i$. (eq. 53)

So price is anchored to some fundamental value α_i, and is adjusted according to $\beta_i \Delta S_i$, where β_i is a illiquidity proxy for the asset, and ΔS_i is the volume of net stock sold. Therefore, if β_i is high, then the price impact of a sell is very large. We expect that to happen for illiquid stocks or during “liquidity crunch.” The “liquidity crunch” or “flight to quality" by a dealer is depicted in R2 Flight to Liquidity by Dealer reinforcing loop and by a momentum investor in R3 Flight to Liquidity by Momentum Investors depicted in Figure 8.

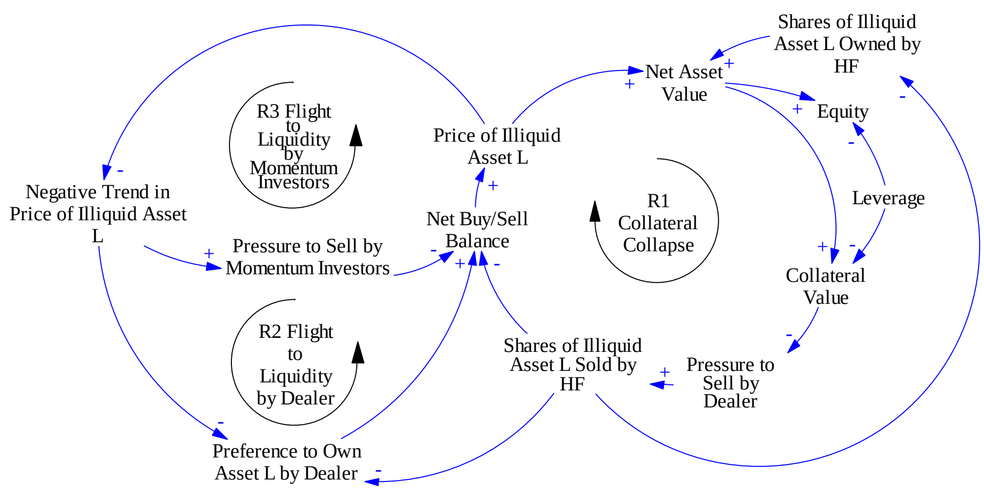


Figure 8: Flight to Liquidity by Momentum Investors

Momentum investors are investors that make decisions on buying and selling stock by following a price of the asset. If price is going down, they extrapolate the trend and decide that the price would go down even more, thus, these investors prefer to own less of that asset. The reverse is true. However, it is important to note that momentum strategy is not solely responsible for collapse of a hedge fund. In LTCM case, there were several types of players in the market: 1) momentum (analyzed in Figure 8), 2) imitators (analyzed in Figure 9), and 3) predators (actively trying to bankrupt LTCM).

Figure 9 depicts a reinforcing loop that leads to a collapse of a hedge fund where investors are imitators. They follow and imitate buy and sell orders of the hedge fund.

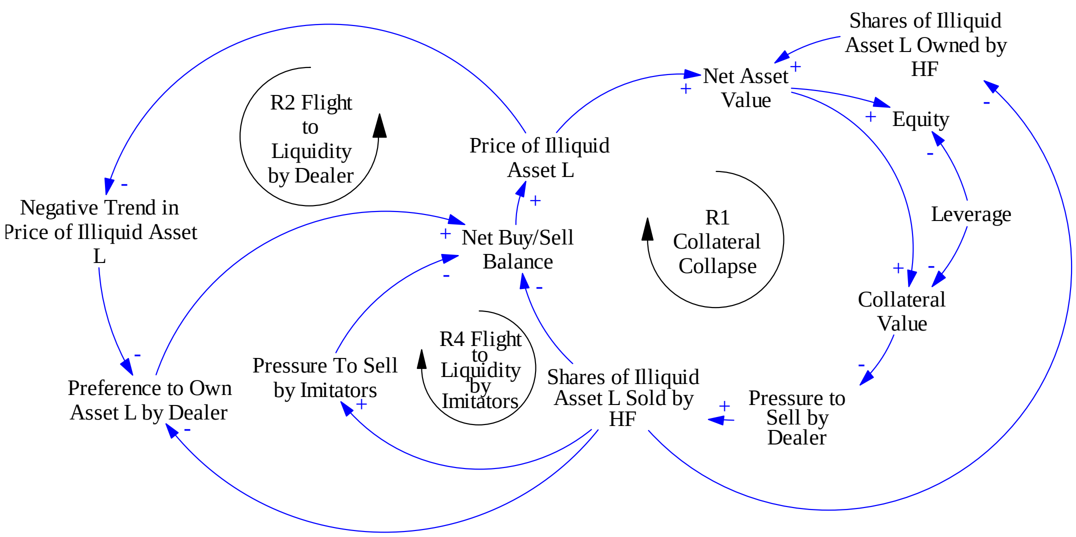


Figure 9: Flight to Liquidity by Imitators

As more shares of illiquid security L are sold by a hedge fund, more shares of security L are sold by imitators, thus leading to a lower price, and leading to collateral collapse.

Note, that for analysis, it is important to differentiate between large and small relative to the marketplace hedge funds. For example, LTCM was a large hedge fund, and its buying and selling significantly contributed to swings in prices in those assets. However, a small hedge fund can decide or even be forced by a dealer to sell its securities, and will have virtually no impact on prices of these securities. For smaller hedge funds the reinforcing loops R1-R4 are not that strong due to a minimal price impact. For smaller hedge funds, mostly balancing loops in Figure 4: Hedge Fund Decisions How to Deal With Margin are in place.

Assumptions

1. A hedge fund's strategy is a statistical arbitrage. Therefore, a hedge fund tries to buy an undervalued security (long) and short sell an overvalued security (short). The hedge fund is trying to have the same amount of shares of both long and short securities.
2. A typical hedge fund does not typically have a lot of cash and will invest most of its available cash into long securities. It will invest most of its available cash

(Fraction of Free Cash Invested = 0.8). It will use long securities and cash as a collateral to borrow short securities.

3. A dealer will always take the other side of the hedge fund order unless there is a "liquidity crunch" time.
4. There are three agents: a hedge fund, a dealer and an investor
5. An investor can take several strategies: momentum, imitator, and noise.
6. Price is endogenous. For a large hedge fund, there is a price impact – selling and buying a security changes the price of the security.
7. A hedge fund starts with no exposure to the long and short positions.
8. When arbitrage opportunities go away, a hedge fund liquidates both short and long positions. Therefore, in this model, both position forming and unwinding are modeled.
9. Federal and house requirements are 50%.
10. No minimum dollar requirements under regulation T, house and NYSE requirements.
11. No interest charge in a margin account, and no interest credit in a short account.

Formulations

Investor Cash

$$Cash_t^I = \int (Income_t^I - Consumption_t^I + Cash\ Increase_t^I - Cash\ Decrease_t^I)dt \tag{eq. 54}$$

$$Cash\ Increase_t^I = Sell\ Rate_t^{I,S} * P_t^S + Sell\ Rate_t^{I,L} * P_t^L \tag{eq. 55}$$

$$Cash\ Decrease_t^I = Buy\ Rate_t^{I,L} * P_t^L + Buy\ Rate_t^{I,S} * P_t^S\ \ Buy\ Rate_t^{I,L} * P_t^L \tag{eq. 56}$$

$$w_t^{A,S} = \frac{S_t^{I,S} P_t^S}{S_t^{I,L} P_t^L + S_t^{I,S} P_t^S} \tag{eq. 57}$$

$$w_t^{A,L} = \frac{S_t^{I,L} P_t^L}{S_t^{I,L} P_t^L + S_t^{I,S} P_t^S} \tag{eq. 58}$$

Pricing

$P_t^S = P_t^{E,S}$ * *Effect of demand supply balance on price Security S* (eq. 59)

$P_t^{E,S} = \int$ (*Change in Expected Price* S_t)*dt* (eq. 60)

Change in Expected Price S_t = $\dfrac{P_t^S - P_t^{E,S}}{\text{Time to Adjust Expected Price Security S}}$ (eq. 61)

$P_t^L = P_t^{E,L}$ * *Effect of demand supply balance on price Security L* (eq. 62)

$P_t^{E,L} = \int$ (*Change in Expected Price* L_t)*dt* (eq. 63)

Change in Expected Price L_t = $\dfrac{P_t^L - P_t^{E,L}}{\text{Time to Adjust Expected Price Security L}}$ (eq. 64)

Momentum Investor

$w_t^{D,S} = 1 - w_t^{D,L}$ (eq. 65)

$w_t^{D,L}$ =*l* * *Table for Desired Equity Weight L(Forecast Price Relative to Current Price* L_t*)*

(eq. 66)

where l is preference for liquidity fraction

Forecast Price Relative to Current Price L_t =*Forecast Price* L_t/*Perceived Price* L_t

(eq. 67)

Forecast Price L_t = *Perceived Price* L_t*(1+*Trend in Price* L_t*(*Price Forecast Horizon+Time to perceive price*)) (eq. 68)

$\text{Perceived Price L}_t = \int$ (*Change in Perceived Price* L_t)*dt* (eq. 69)

Change in Perceived Price L_t = $\dfrac{P_t^L - \text{Perceived Price L}_t}{\text{Time to Perceive Price L}}$ (eq. 70)

Imitator Investor

Desired Sale Rate$_t^{I,L}$ = *Desired Sale Rate*$_t^{H,L}$ (eq. 71)

Desired Sale Rate$_t^{I,S}$ = *Desired Sale Rate*$_t^{H,S}$ (eq. 72)

Desired Buy Rate$_t^{I,L}$ = *Desired Buy Rate*$_t^{H,L}$ (eq. 73)

Desired Buy Rate$_t^{I,S}$ = *Desired Buy Rate*$_t^{H,S}$ (eq. 74)

Results

Scenario 1. Large Hedge Fund and Momentum Investor. Prices Diverge Instead of Converging

There are three players present: investor, dealer and a hedge fund. An investor is a momentum investor. A dealer has a preference for liquidity when the price of illiquid asset L has a high negative trend, and a hedge fund is forced to sell due to margin call. Prices of illiquid security L and liquid security S are endogenous.

A model starts in equilibrium. $P_0^L = P_0^S = 100$ \$/share

$S_0^{D,L} = S_0^{I,L} = 400$ shares. $S_0^{H,L} = 0$ shares. $S_0^{D,S} = S_0^{I,S} = 400$ shares.

$S_0^{H,S} = 0$ shares. $Cash_0^H = \$40,000$ dollars

Inv1_cd1 is a simulation where Cash Decision = 1

Inv1_cd0 is a simulation where Cash Decision = 0

Desired Equity Weight L Inv = Investor Fraction(Preference for Liquidity Fraction*Table for Desired Equity Weight L(Forecast Price Relative to Current Price L))*

Investor Fraction=1

Preference for Liquidity Fraction= 1-STEP(0.1,10)+STEP(0.1,20)-STEP(0.8,20)+STEP(0.8,30) is an exogenous input

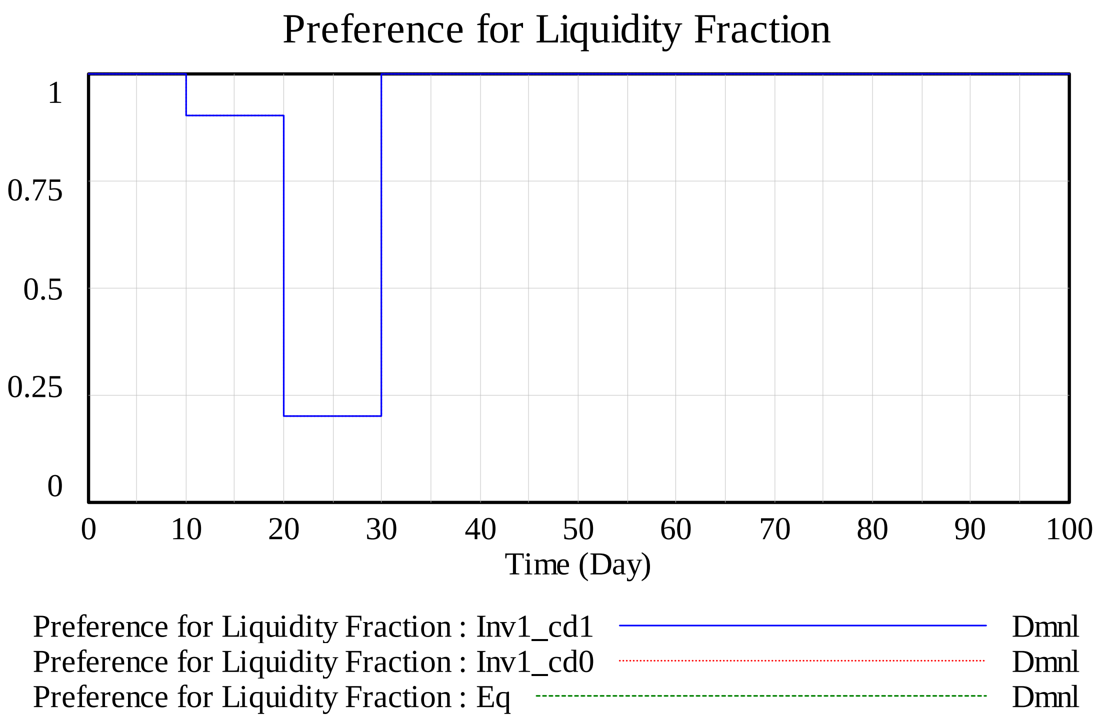
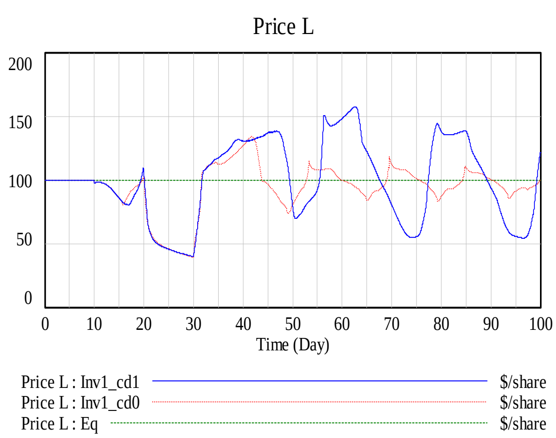


Figure 2_1_1. Preference for Liquidity Fraction



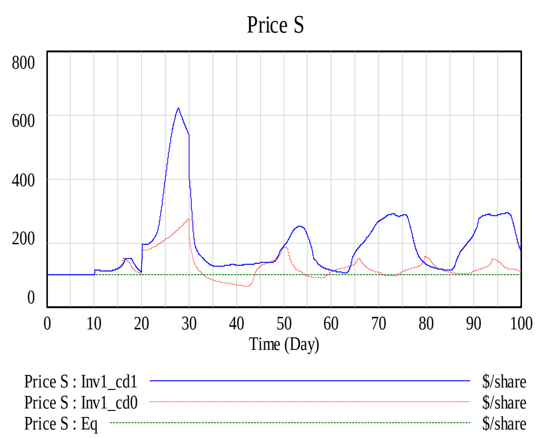


Figure 2_1_2. Price of Security L

Figure 2_1_3. Price of Security S

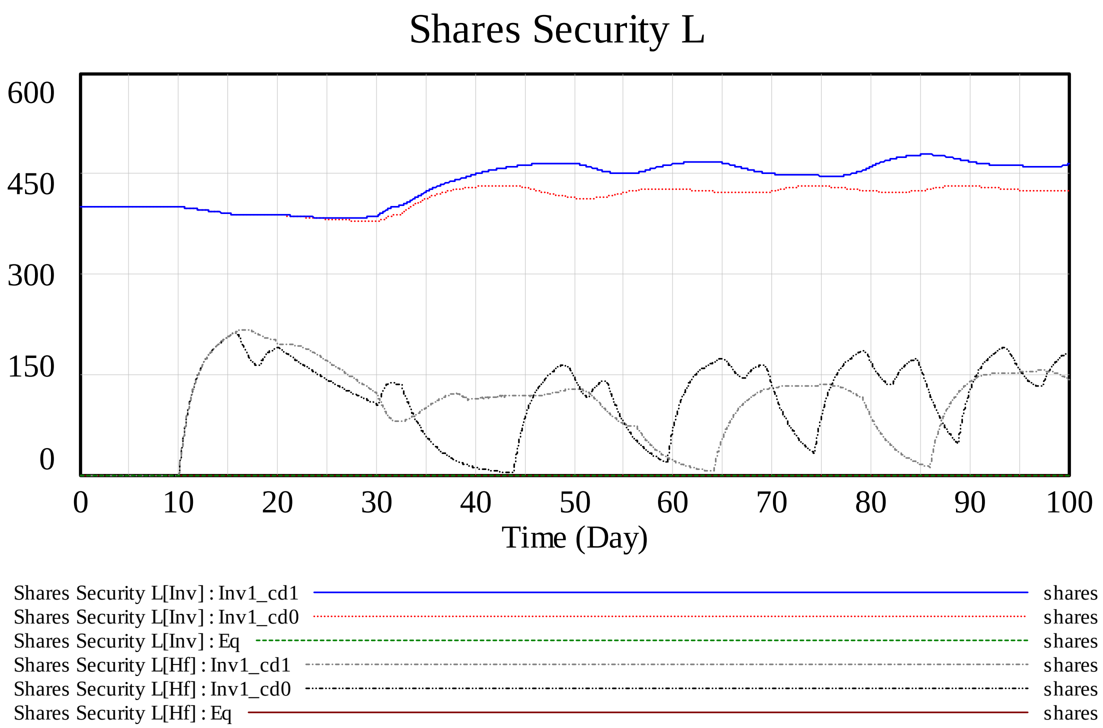


Figure 2_1_4. Shares of Security L

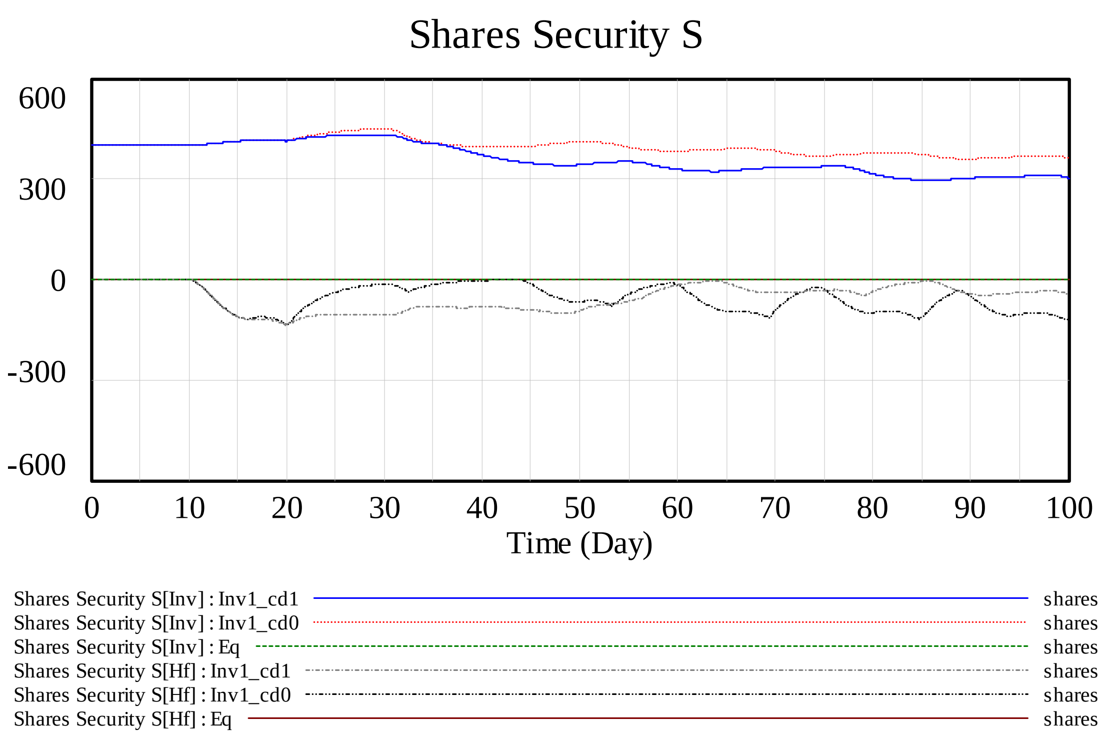


Figure 2_1_5. Shares of Security S

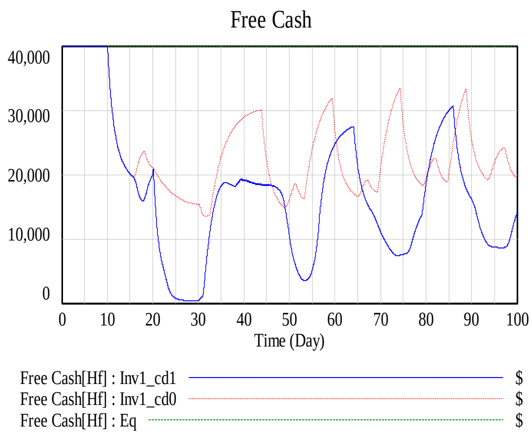


Figure 2_1_6. Free Cash

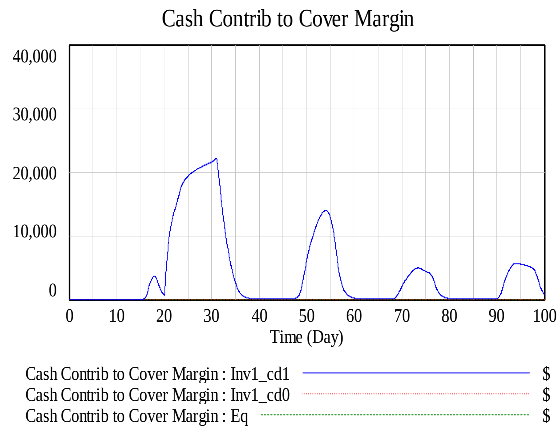


Figure 2_1_7. Cash Contribution to Cover Margin

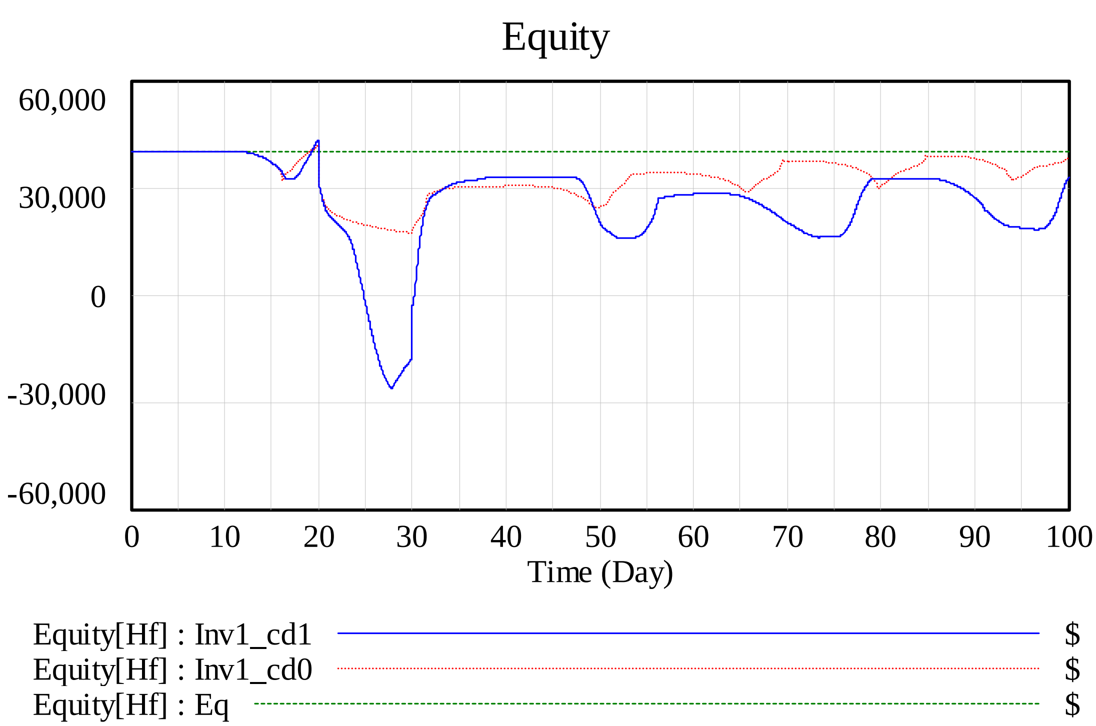


Figure 2_1_8. Equity

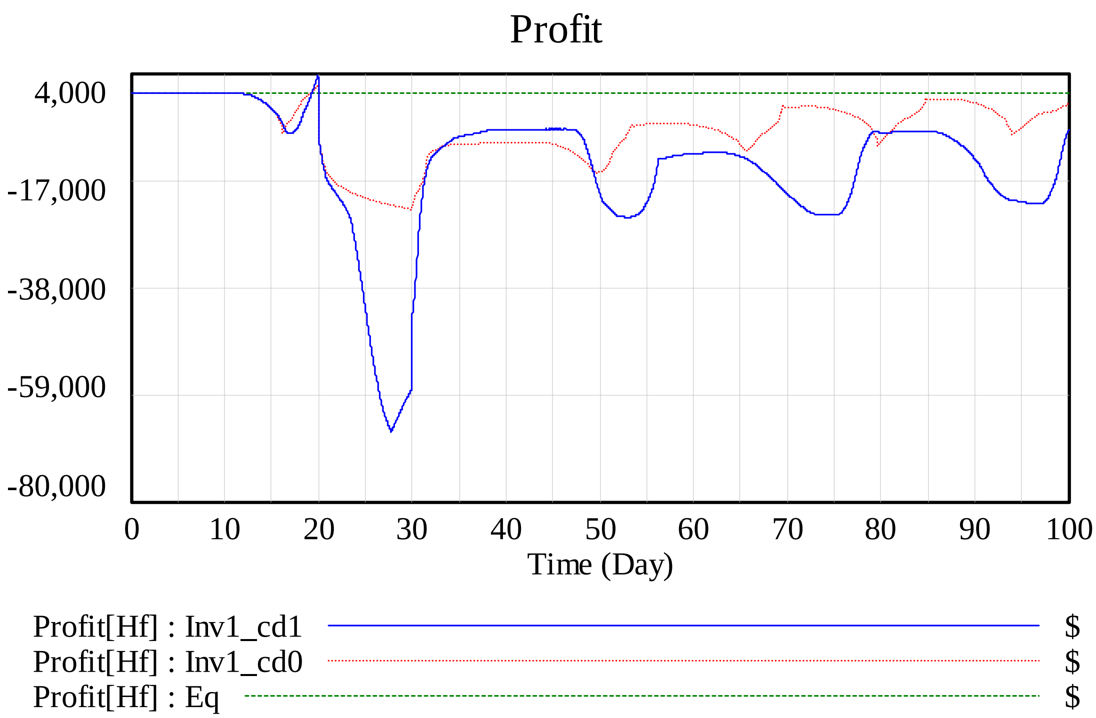


Figure 2_1_9. Profit

From time = 0 to 10 days, no arbitrage opportunities exist. From time = 10 to 20 days, arbitrage opportunities exist for a hedge fund as there is a spread between a cheaper illiquid security L and liquid security S. From time = 20 to 30 days, instead of converging, the spread diverges even more, thus, a hedge fund is hit with a margin call. As a hedge fund is hit with a margin call, hedge fund has to sell the illiquid security L (long) at further depressed prices. In this case, it is better for a hedge fund to unwind its positions as quick as possible instead of “throwing good money after bad,” as LTCM did. This scenario depicts exactly what happened to LTCM. LTCM was a large fund, and by unwinding its positions, it contributed to a further decrease in prices of security L. Investors were afraid that the drop of prices was going to continue and started selling more of security L, thus further reducing the prices. LTCM did not have enough cash from other positions to finance the ever increasing margin calls from brokers. However, it tried to hang on to the positions without selling them. At LTCM hedge fund managers would double up the exposure of a position when spreads widen even further as they firmly believed in their strategy that spreads would narrow in the near future (Lowenstein, 2000).

As can be seen in this scenario, a hedge fund runs out of all available cash by time = 25 days, before spreads converge. After that, the hedge fund manager has to sell securities at even lower prices in order to cover a margin call leading to lower profits, and collapse of a hedge fund. As a result, if a large hedge fund does not have enough funding to finance margin calls through widening of spreads, it is always better for the hedge fund to unwind its positions as soon as a margin call is issued. As a hedge fund is selling its positions, thus decreasing the price, momentum investors demand less of the security. Also, in the times of liquidity crunch, as it happened in the 10 day window, a dealer prefers not to buy the illiquid securities as he is afraid of being left with an inventory that is spiraling down in value.

Scenario 2. Small Hedge Fund and Momentum Investor. Prices Diverge Instead of Converging

There are three players present: investor, dealer and a hedge fund. An investor is a momentum investor. A dealer has a preference for liquidity when price of illiquid asset L has high negative trend, and a hedge fund is forced to sell due to margin call. Prices of illiquid security L and liquid security S are endogenous.

A model starts in equilibrium. $P_0^L = P_0^S = 100$ \$/share

$S_0^{D,L} = S_0^{I,L} = 400$ shares. $S_0^{H,L} = 0$ shares. $S_0^{D,S} = S_0^{I,S} = 400$ shares.

$S_0^{H,S} = 0$ shares. $Cash_0^H = \$1{,}000$ dollars. The difference between Simulation 2 and Simulation 1 is that in simulation 1, a hedge fund is large compared to other market players, and in simulation 2, a hedge fund is small.

Invs_cd1 is a simulation where Cash Decision = 1

Invs_cd0 is a simulation where Cash Decision = 0

Desired Equity Weight L Inv = Investor Fraction(Preference for Liquidity Fraction*Table for Desired Equity Weight L(Forecast Price Relative to Current Price L))*

Investor Fraction=1

Preference for Liquidity Fraction= 1-STEP(0.1,10)+STEP(0.1,20)-STEP(0.8,20)+STEP(0.8,30) is an exogenous input

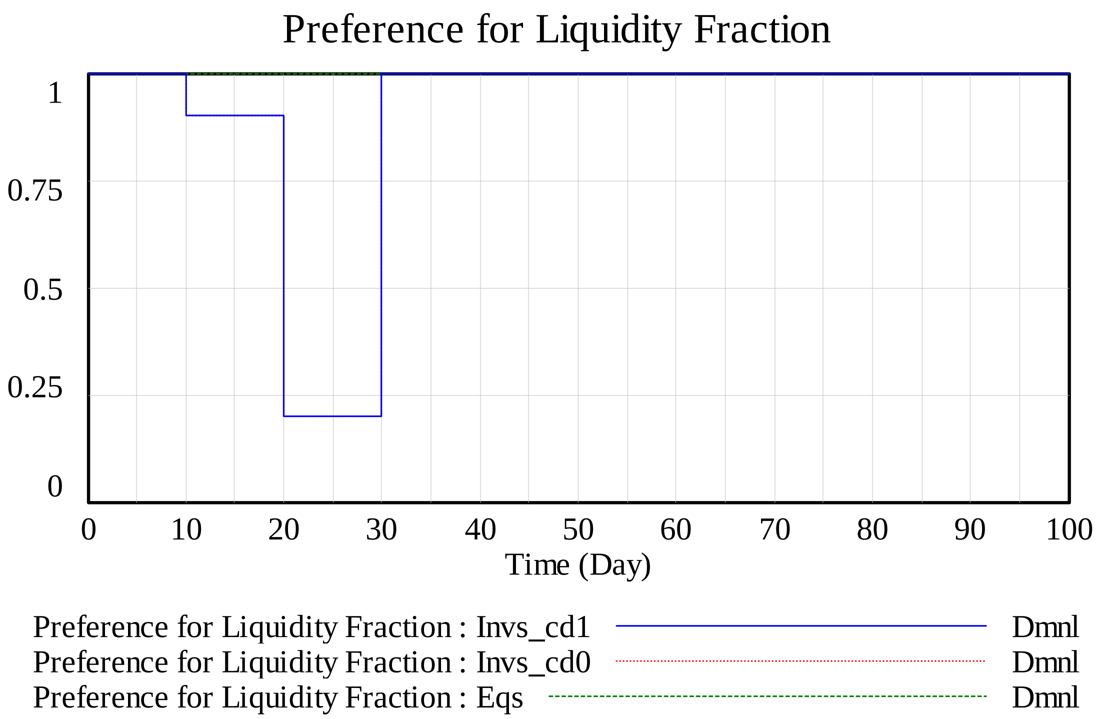


Figure 2_2_1. Preference for Liquidity Fraction

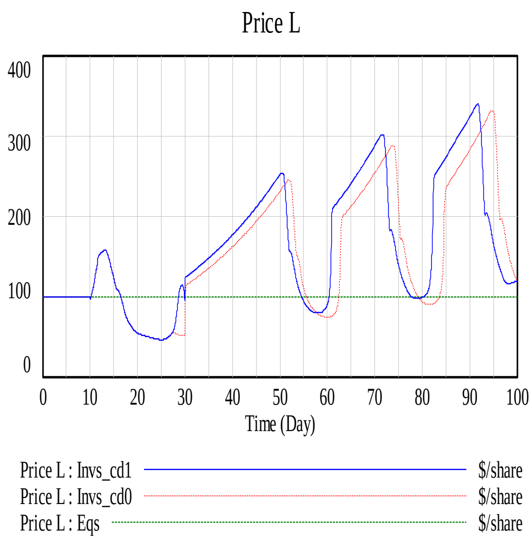


Figure 2_2_2. Price of Security L

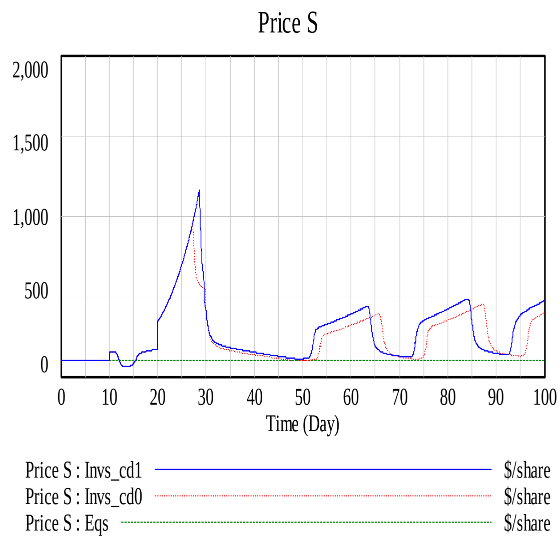


Figure 2_2_3. Price of Security S

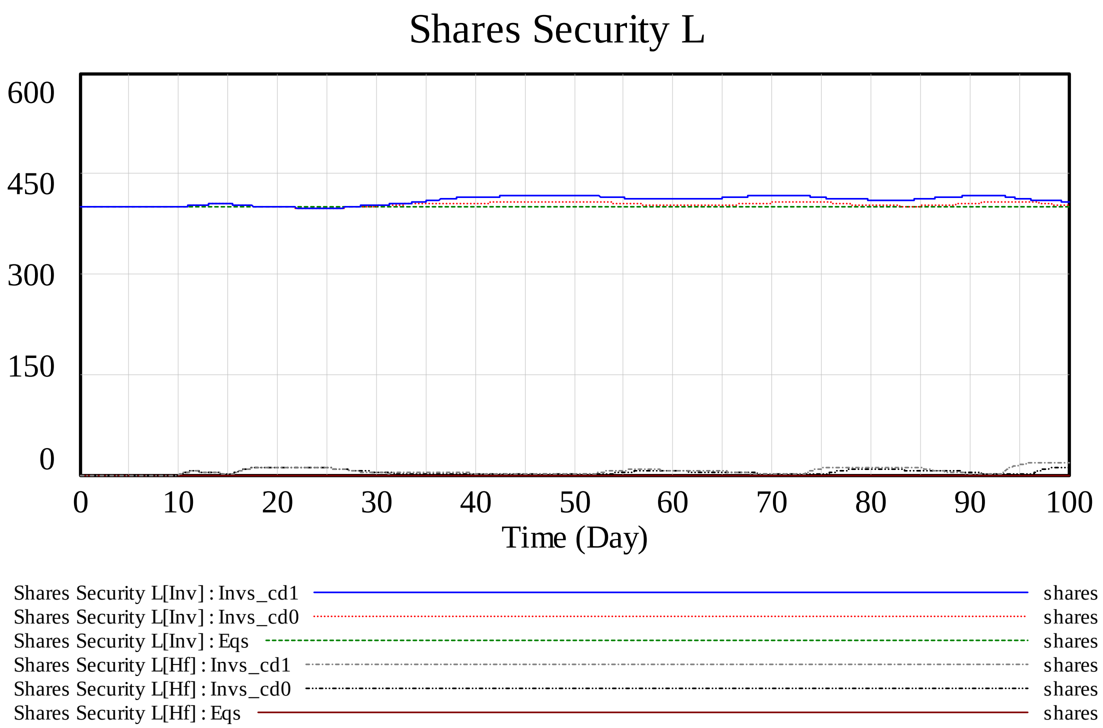


Figure 2_2_4. Shares of Security L

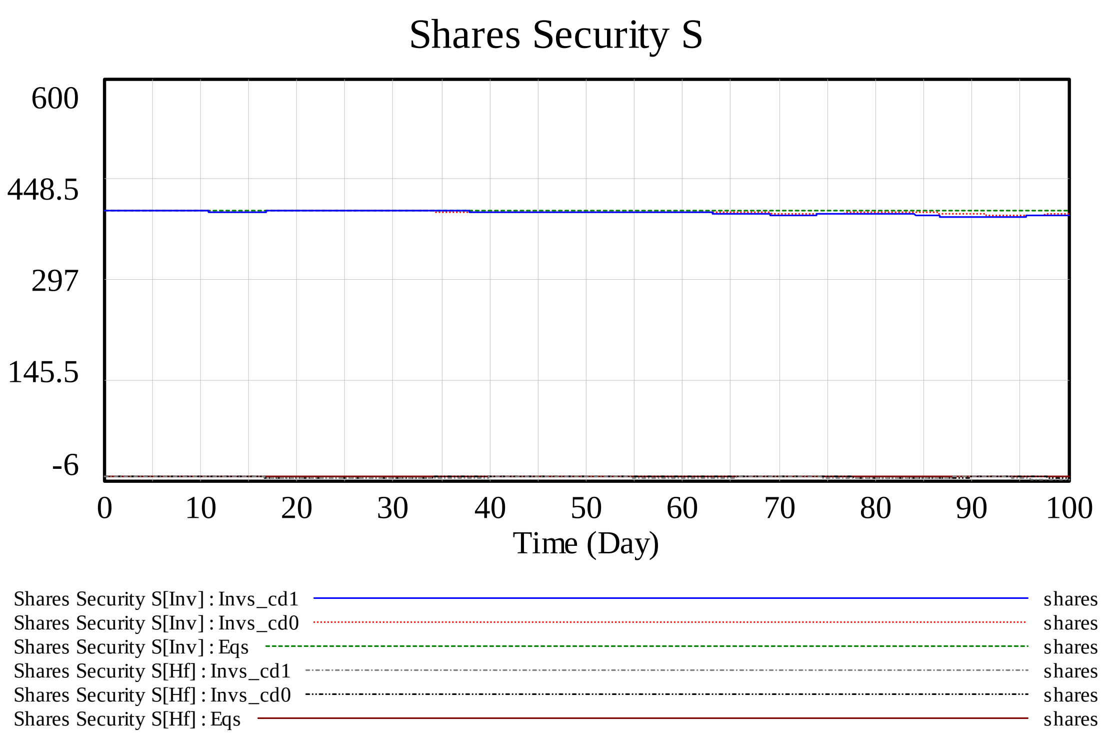


Figure 2_2_5. Shares of Security S

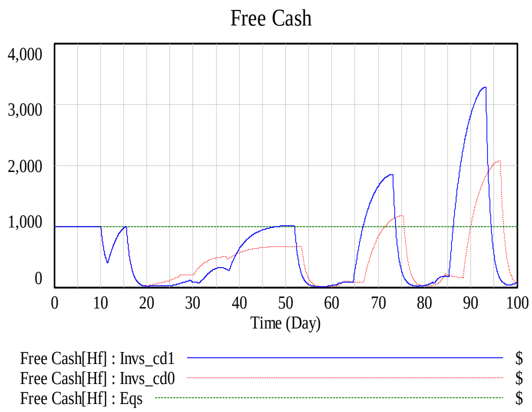


Figure 2_2_6. Free Cash

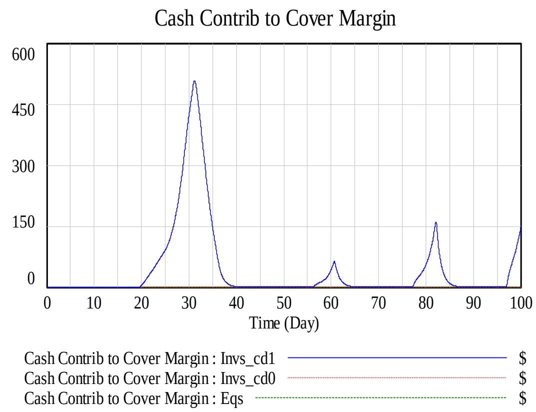


Figure 2_2_7. Cash Contribution to Cover Margin

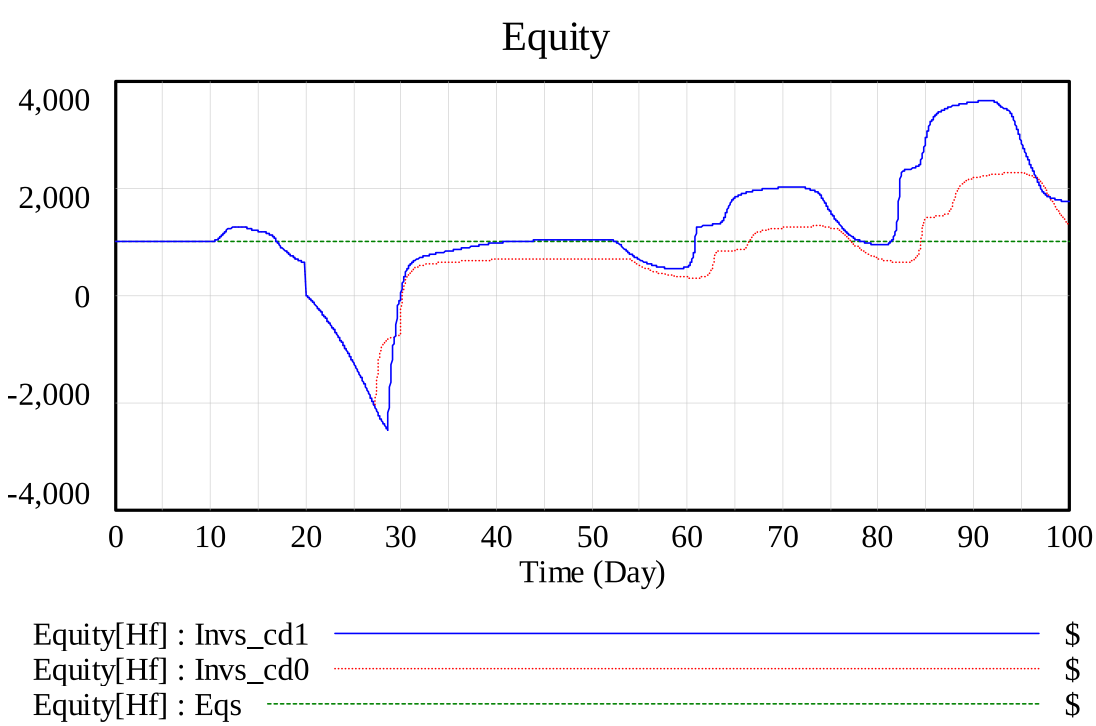


Figure 2_2_8. Equity

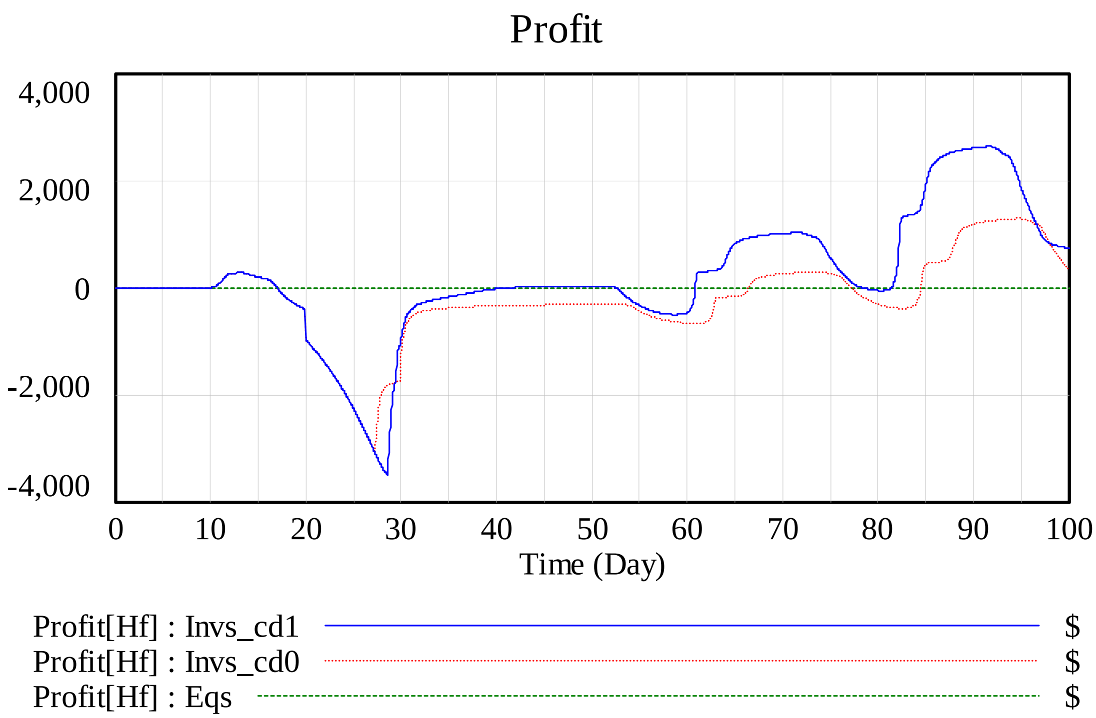


Figure 2_2_9. Profit

From time = 0 to 10 days, no arbitrage opportunities exist. Starting time = 10 days, arbitrage opportunities exist for a hedge fund as there is a spread between a cheaper illiquid security L and liquid security S. From time = 20 to 30 days, instead of converging, the spread diverges even more, thus, a hedge fund is hit with a margin call. As a hedge fund is hit with a margin call, hedge fund has to sell the illiquid security L (long) at further depressing prices. Compared to scenario 1, where a large hedge fund is present, selling security L does not impact the price a lot. Investors are adjusting their desired equity weights for securities L and S. As price of L goes down, the desired equity weight of security L by investors goes down, and the actual equity weight of security L goes down too. Therefore, investors are not selling as much of L as in scenario 1.

Generally, by time = 30 when preference for liquidity went away and spreads converge, a hedge fund is in a similar position in both runs (Invs_cd1 and Invs_cd0). A hedge fund is a little bit better in Invs_cd0 run. Therefore, for a small hedge fund, once it is hit with a margin call, it is better to unwind its positions rather than finance it with available cash (throwing good money after bad), given that a hedge fund does not have enough cash to finance all its margin calls.

Scenario 3. Large Hedge Fund and Imitator Investor. Prices Diverge Instead of Converging

There are three players present: investor, dealer and a hedge fund. An investor is an imitator. A hedge fund is large compared to other players in the market. A dealer has a preference for liquidity when price of illiquid asset L has high negative trend, and a hedge fund is forced to sell due to margin call. Prices of illiquid security L and liquid security S are endogenous.

A model starts in equilibrium. $P_0^L = P_0^S = 100$ \$/share

$S_0^{D,L} = S_0^{I,L} = 400$ shares. $S_0^{H,L} = 0$ shares. $S_0^{D,S} = S_0^{I,S} = 400$ shares.

$S_0^{H,S} = 0$ shares. $Cash_0^H = \$40{,}000$ dollars.

Invf_cd1 is a simulation where Cash Decision = 1

Invf_cd0 is a simulation where Cash Decision = 0

P_t^L = *Expected Price L*Effect of demand supply balance on price Security L*(1-STEP(0.1,10)+STEP(0.1,20)-STEP(0.8,20)+STEP(0.8,30))*

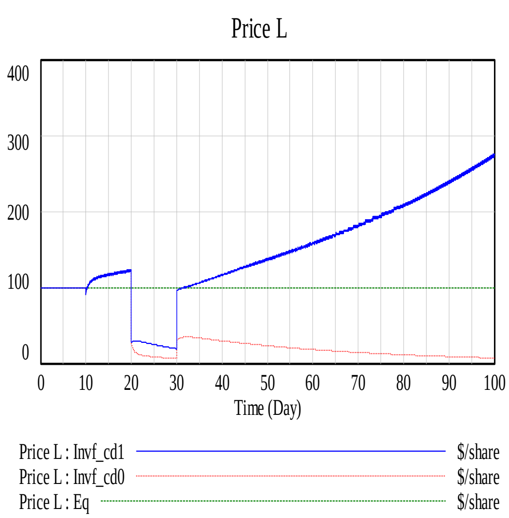


Figure 2_3_1. Price of Security L

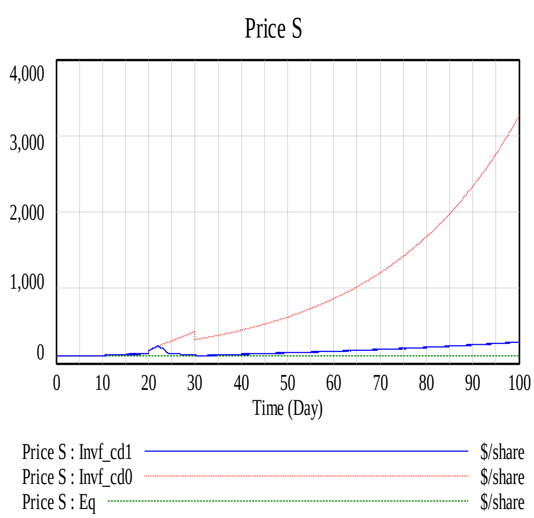


Figure 2_3_2. Price of Security S

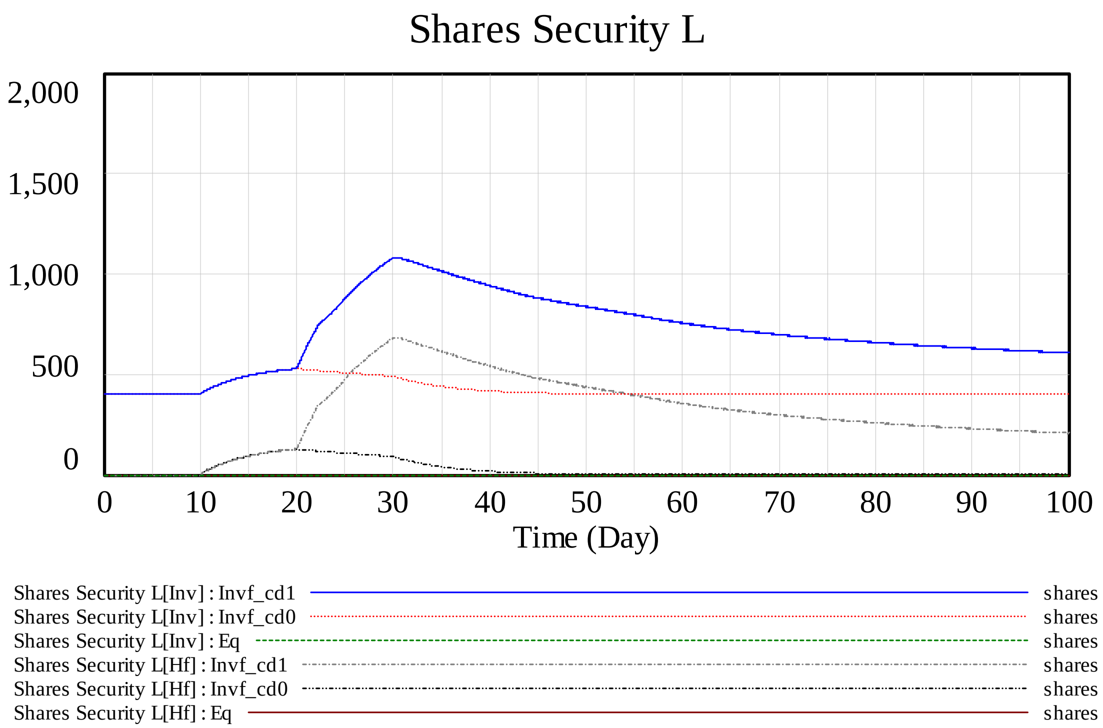


Figure 2_3_3. Shares of Security L

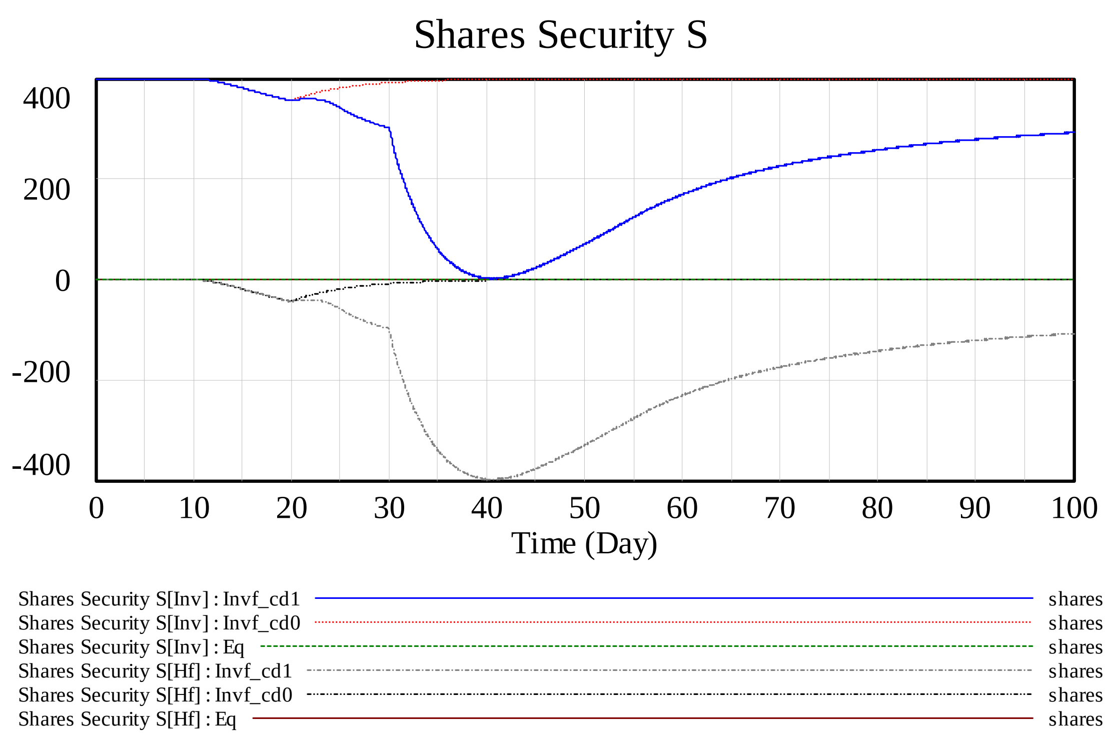


Figure 2_3_4. Shares of Security S

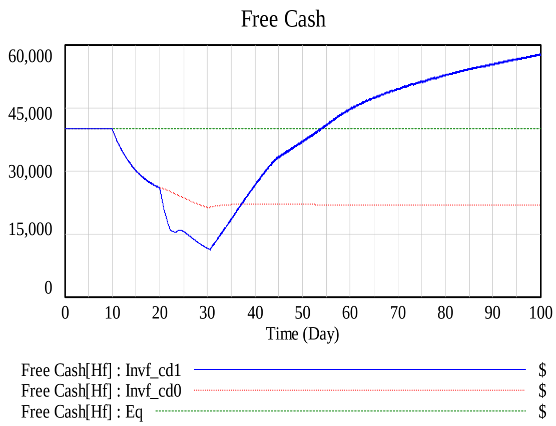


Figure 2_3_5. Free Cash

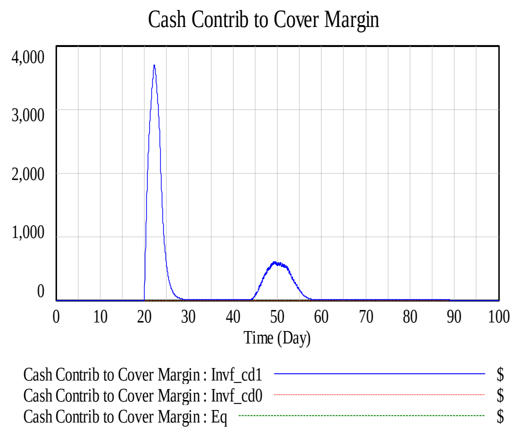


Figure 2_3_6. Cash Contribution to Cover Margin

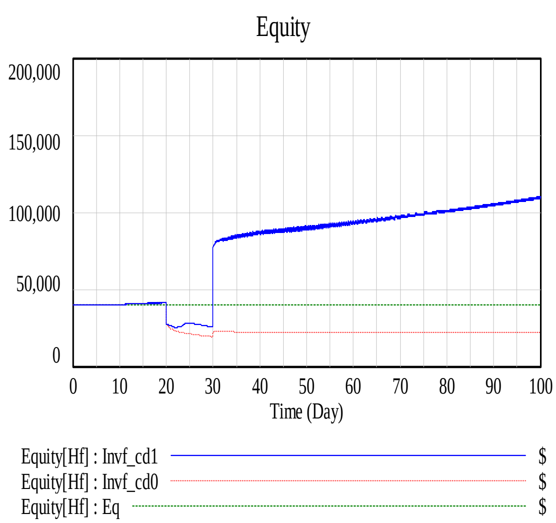


Figure 2_3_7. Equity

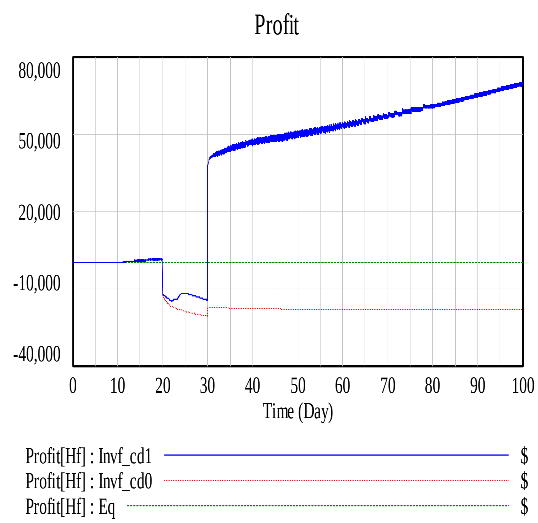


Figure 2_3_8. Profit

From time = 0 to 10 days, no arbitrage opportunities exist. Starting time = 10 days, arbitrage opportunities exist for a hedge fund as there is a spread between a cheaper illiquid security L and liquid security S. From time = 20 to 30 days, instead of converging, the spread diverges even more, thus, a hedge fund is hit with a margin call. As a hedge fund is hit with a margin call, the hedge fund has to sell the illiquid security L (long) at further depressing prices. Compared to scenario 1, here a hedge fund has enough cash to finance margin calls before spreads narrow. In this scenario, Free Cash is always positive. In this case, it is better to use cash, as equity during the times of spread divergence and final equity are higher for a case Invf_cd1 where a hedge fund actually uses cash to finance its margin compared to case Invf_cd0 where a hedge fund unwinds its positions once hit with a margin call.

Scenario 4.   Large Hedge Fund and Imitator Investor. Prices Diverge and Never Converge

There are three players present: investor, dealer and a hedge fund. An investor is an imitator. A hedge fund is large compared to other players in the market. A dealer has a preference for liquidity when price of illiquid asset L has high negative trend, and a hedge fund is forced to sell due to margin call. Prices of illiquid security L and liquid security S are endogenous.

A model starts in equilibrium. $P_0^L = P_0^S = 100$ \$/share

$S_0^{D,L} = S_0^{I,L} = 400$ shares. $S_0^{H,L} = 0$ shares. $S_0^{D,S} = S_0^{I,S} = 400$ shares.

$S_0^{H,S} = 0$ shares. $Cash_0^H = \$40,000$ dollars.

Invf_cd1 is a simulation where Cash Decision = 1

Invf_cd0 is a simulation where Cash Decision = 0

P_t^L = *Expected Price L*Effect of demand supply balance on price Security L*(1-STEP(0.4,10)+STEP(0.4,20)-STEP(0.8,20)+STEP(0.8,30))*

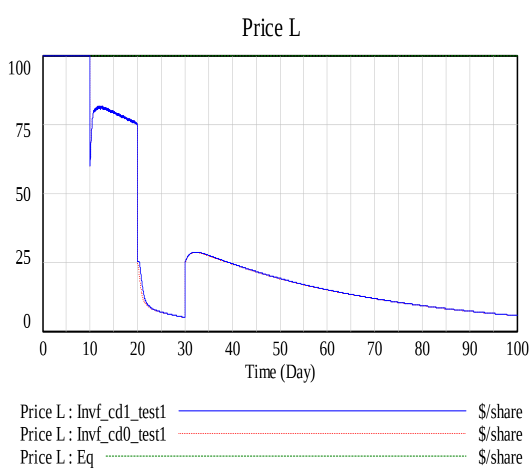


Figure 2_4_1. Price of Security L

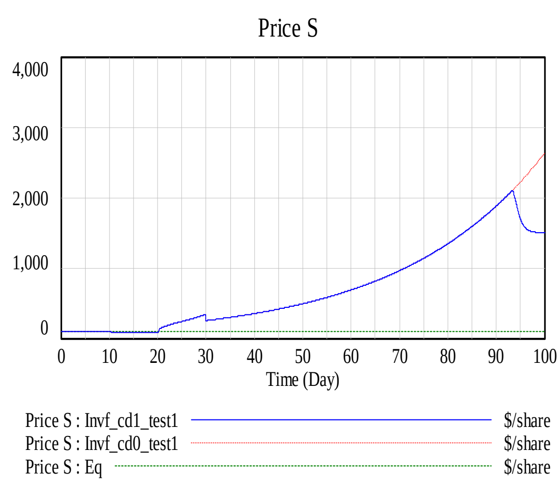


Figure 2_4_2. Price of Security S

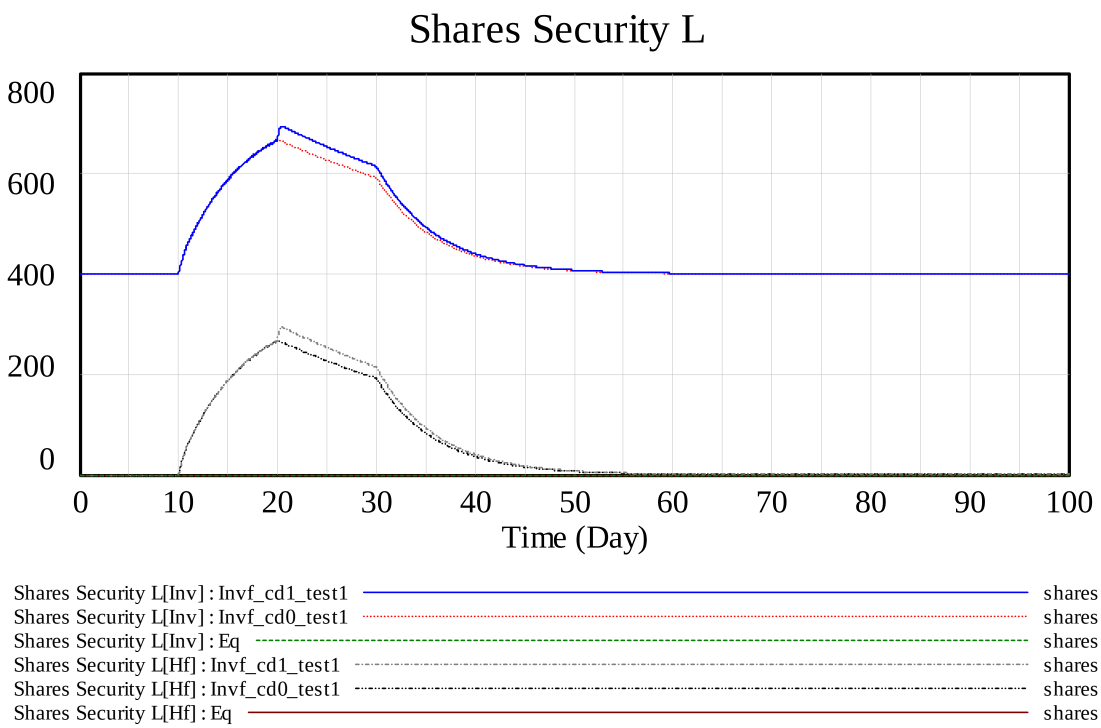


Figure 2_4_3. Shares of Security L

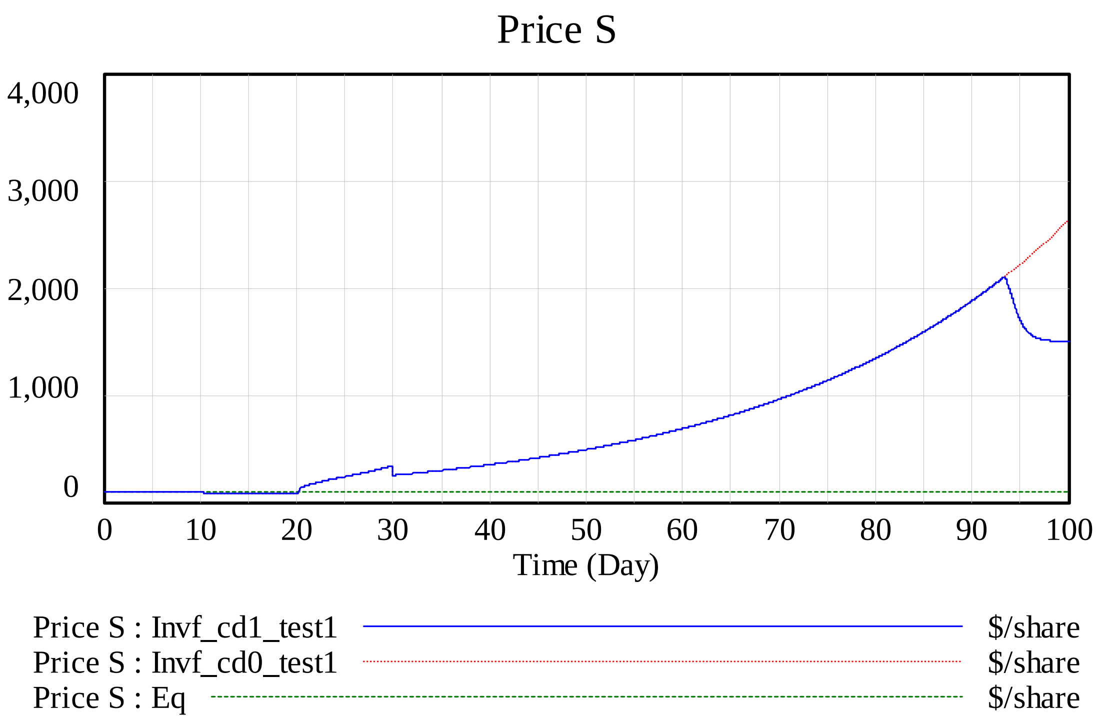


Figure 2_4_4. Shares of Security S

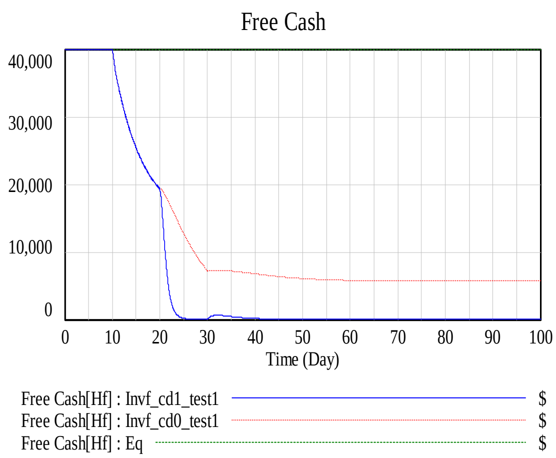


Figure 2_4_5. Free Cash

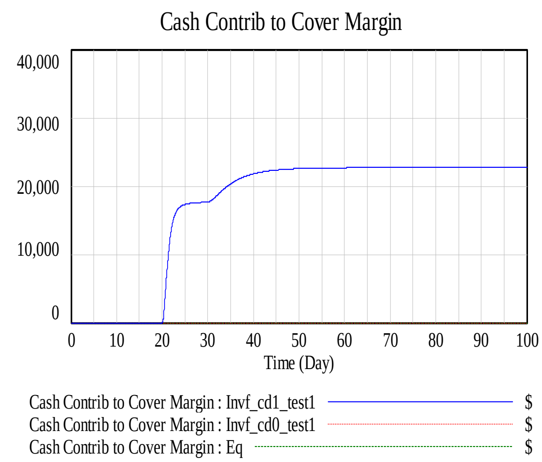


Figure 2_4_6. Cash Contribution to Cover Margin

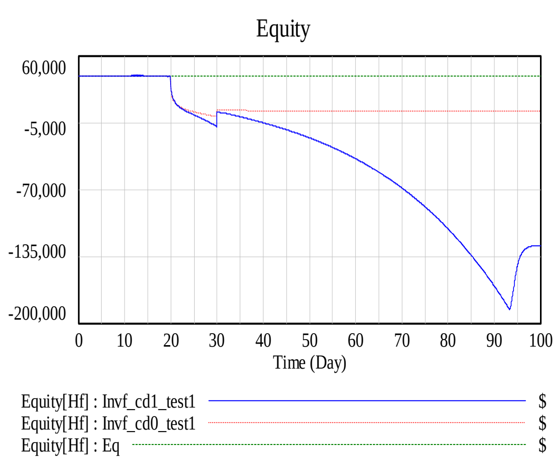


Figure 2_4_7. Equity

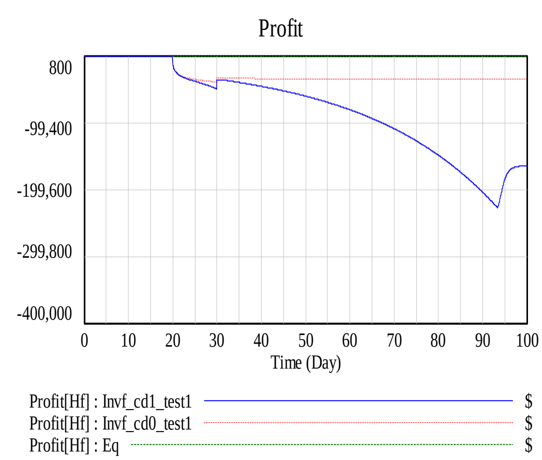


Figure 2_4_8. Profit

From time = 0 to 10 days, no arbitrage opportunities exist. Starting time = 10 days, arbitrage opportunities exist for a hedge fund as there is a spread between a cheaper illiquid security L and liquid security S. From time = 20 days, instead of converging, the spread diverges even more, thus, a hedge fund is hit with a margin call. As a hedge fund is hit with a margin call, hedge fund has to sell the illiquid security L (long) at further depressing prices. Compared to scenario 3, here a hedge fund does not have enough cash to finance margin calls before spreads narrow. In this case, spreads never narrow. As can be seen from the runs, a hedge fund fails if it uses a strategy of using available cash to finance margin calls (Invf_cd1_test1). It survives if once a hedge fund is hit with a margin call, it unwinds its positions. It is true that by closing its positions, investors imitate the hedge fund and close positions, thus depressing prices even more. However, this effect is present in both runs : Invf_cd1_test1 and Invf_cd1_test0. By financing margin with available cash, a hedge fund only introduces a lag in this relationship, and will have to unwind its positions at a later time at lower prices.

Scenario 5. Large Hedge Fund and Noise Investor. Prices Diverge Instead of Converging

There are three players present: investor, dealer and a hedge fund. An investor is a noise trader. A hedge fund is large compared to other players in the market. A dealer has a preference for liquidity when price of illiquid asset L has a high negative trend, and a hedge fund is forced to sell due to margin call. Prices of illiquid security L and liquid security S are endogenous.

A model starts in equilibrium. $P_0^L = P_0^S = 100$ \$/share

$S_0^{D,L} = S_0^{I,L} = 400$ shares. $S_0^{H,L} = 0$ shares. $S_0^{D,S} = S_0^{I,S} = 400$ shares.

$S_0^{H,S} = 0$ shares. $Cash_0^H = \$40{,}000$ dollars.

In this case (to be shown in the next draft), a hedge fund is better off using cash to finance off its margin call instead of unwinding the positions. As hedge fund sells illiquid positions, the price of those positions goes down due to a price impact of the large hedge fund. Therefore, if a hedge fund has enough cash to finance margin before positions converge, a hedge fund is better off following this strategy.

10. Implications for Risk Management

Using Value at Risk Analysis (VAR) explained below, $VAR = 1.65A\delta$, where δ is the standard deviation of the hedge fund performance, and is the squared root of the variance: $\delta^2 = \sum_{i=1}^{n} P^2{}_i S^2{}_i \delta_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} P_i S_i \delta_i^2 P_j S_j \delta_j^2$, where σ_{ij} is the correlation between assets (i) and (j) held by a hedge fund. It is important to note that during "liquidity crunch" positions that were previously not correlated, become dependent. In the context of this model, as β_i or liquidity preference become high for hedge fund positions, then σ_{ij} becomes higher, therefore, inflating the variance of the hedge fund. Hence, Value at Risk of the hedge fund increases.

Value At Risk Analysis (VAR)

VAR describes how risky a stock is.

VAR is the maximum expected loss over a given horizon period at a given level of confidence C i.e., the maximum likely loss.

VAR depends upon two arbitrarily chosen parameters: the horizon period (daily, weekly, monthly, quarterly, etc.) and the level of confidence (90%, 95%, 99%, 99.9%, etc.).

Origins: October 1994, J.P. Morgan, RiskMetric

For example: Results show that 99% quarterly VAR is $.767 million. It means that the probability of losing more than $767,000 over a quarter is less than or equal to .01.

Calculation of VAR: VAR=Market Cap*Standard Deviation*1.65 where

Market Cap ($Billion)

Standard Deviation (s.d. of monthly returns)

1.65 is the one side 5% point (Prob(z<-1.65)=0.05 if z obeys standard normal)

VAR ($Billion, 5%, 1 month)

11. Discussion and Conclusion

A hedge fund manager will conceive an arbitrage trade if the manager sees mispricing in illiquid and liquid assets. The manager would buy a cheaper illiquid asset (security L) and will short a more expensive liquid asset (security S). Typically, a hedge fund manager has two ways of getting into the arbitrage trade. Of course, any combinations of these two options are also present in hedge fund strategies. In the first case, a hedge fund manager would max out on security S and security L positions, subject to being within a limit of an allowable margin. For example, if a hedge fund spent all of its cash and bought $100,000 worth of security L, a hedge fund manager can at most sell $100,000 worth of security S in the case of maximum leverage equals to two. In the second case, a hedge fund manager will short sell as many shares of security S as it would go long on security L. For example, if a hedge fund buys 1,000 shares of security L for $80 ($80,000 total value), a hedge fund manager will go 1,000 shares short on security S that sells for $120 ($120,000). Given a hedge fund cash is 0, a hedge fund manager is going to be hit with a margin call from a dealer.

Once a margin call is issued, a hedge fund has two options: liquidate the conceived positions or use money from other trades to finance this position. In the paper, it is shown that a hedge fund that is not constrained to keep within margin requirements and has cash from other accounts at its disposal, will earn more profits compared to a similar hedge fund that is constrained to keep within margin requirements, given that arbitrage strategy is successful. However, the opposite is true in a case where a hedge fund does not have an access to cash from other accounts. In this case, a hedge fund that keeps within margin requirements will earn more profits compared to a hedge fund that is not constrained to keep within margin requirements, given that arbitrage strategy is successful. In this case, a hedge fund that is not constrained to operate within margin requirements, is hit with a margin call that a hedge fund manager will only be able to fulfill by unwinding the positions. In the end, the value of the position is much smaller compared to the case where a hedge fund manager is constrained to stay within hedge fund margin requirements, resulting in smaller profits.

In a case when a spread widens before narrowing without causing a hedge fund collapse in the interim, a hedge fund is better off by following a strategy of shorting as many shares as it has long compared to a strategy of keeping within margin requirements. This assumes that a hedge fund does not increase its exposure to both S and L securities as spread unexpectedly widen up. This is true for small hedge funds; however, in LTCM, many traders followed an approach that if spreads widen up, they would double up their exposure. If a hedge fund follows a strategy of keeping within margin requirements, a hedge fund is better off by using cash from other accounts to finance a margin due to widening spreads.

In a case that an arbitrage position (strategy: short as much as long) goes against a hedge fund manager, i.e. the spread between positions L and S widen, a hedge fund manager prevents a collapse by unwinding positions as soon as margin call is issued. If a hedge fund manager decides to finance the margin with other money by "throwing good money after bad," a hedge fund will collapse. This conclusion is applicable to all small hedge funds that are not price-makers in the marketplace. If a hedge fund decides to keep within margin requirements, then the likelihood of a hedge fund collapse given that margin is financed with new money or by unwinding positions is the same.

If a large hedge fund does not have enough funding to finance margin calls through widening of spreads, it is always better for the hedge fund to unwind its positions as soon as a margin call is issued. As a hedge fund is selling its positions, thus decreasing the price, momentum investors demand less of the security. Also, in the times of liquidity crunch, a dealer prefers not to buy the illiquid securities as he is afraid of being left with an inventory that is spiraling down in value.

If a large hedge fund has enough cash to finance margin calls before spreads narrow, it is better to use cash to cover margins compared to unwinding positions.

For a small hedge fund, once it is hit with a margin call, it is better to unwind its positions rather than finance it with available cash (throwing good money after bad), given that a hedge fund does not have enough cash to finance all its margin calls.

In conclusion, even if arbitrage opportunities are found in a statistical sense, they might not be exploitable. Moreover, a fund manager who engages in such risk arbitrage might lose all his money before realizing the positions at a profit. The hedge fund collapse happens due to an unmet margin call that arises due to leverage effects. A hedge fund manager can decide how to deal with a margin call. As managers at most of small hedge funds do, they usually decrease the position exposure or close out the position entirely. This will assure that a hedge fund will not lose any money given further position divergence instead of an intended divergence. However, if positions converge, a hedge fund is likely to earn less profit compared to hedge funds that put available cash to cover margins. Large hedge funds, like LTCM, which can use cash from some profitable positions to finance margins in other positions, are likely to use available cash to cover margins instead of closing out the positions. In case of position divergence, they are likely to make more money than other hedge funds; however, they are also more likely to suffer a severe collapse in case of further prolonged position divergence. This is what happened in the LTCM case. The managers were "throwing good money after bad" leading to the collapse of the fund. Meanwhile, lots of smaller hedge funds that followed similar strategies to LTCM, survived due to a timely closure of their levered positions.

As assets go down in value, the firm has to post more collateral. If it is unavailable, this often leads to a hedge fund collapse. However, given that positions are well diversified and not closely correlated, leverage by itself, does not lead to the collapse of a fund. Correlated positions in the absence of leverage might lead to a loss, but are not subject to collateral collapse. Given diversified positions in a fund, a price drop in one asset does not necessarily correspond to a price drop in another asset, even less likely there is a possibility of a cascade in drop in prices of all assets. However, the superimposition of both leverage and induced high correlation between assets can lead to a collapse. This is something that sophisticated hedge funds like LTCM did not take into equation in determining risk exposure. Their decisions were bounded rational. The managers separately managed leverage and diversification of positions, not thinking that two can feed on each other during a period of a crisis leading to the "flight to quality" and "collateral collapse" dynamics.

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13. Appendix

Model Formulation Equations are available upon request.