DESIGNING A MANUFACTURING POLICY USING THE REFERENCE APPROACH AND ALTERNATIVE SYSTEM DYNAMICS SUPPORT METHODS

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ABSTRACT. The purpose of this paper is to compare the efficiency and robustness of policies obtained using alternative system dynamics support methods. The comparison shows the need for creating new methods which combine the efficiency of the optimization methods with the robustness of the modal methods. One of these hybrid methods is the recently developed reference approach which exhibits the best efficiency and robustness.

1. THE ALTERNATIVE POLICIES

A system dynamics support method consists of two parts: model building and the design of the best policy to reach a desired dynamics. The current methods can be grouped in three families: heuristic methods (Forrester, 1961; Coyle, 1977), modal methods (Mohapatra and Sharma, 1985) and sequential optimization methods (Keloharju, 1983; Macedo, 1989a). All of these methods build the model using the philosophy and tools proposed by Forrester (1961); however, they differ in the techniques used to design the policy.

The purpose of this paper is to compare the efficiency and robustness of four policies obtained with the alternative system dynamics support methods (table 1). These policies were conceived to stabilize the oscillatory dynamics of the variables FOR(t), PSR(t), INV(t) in the appendix model[1]. This model is the result of applying some rescaling to Coyle's model (1977, chap. 8).

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1 The first five equations of the appendix model generate the growth of the sales rate SR(t) from 100 to 140 units/week in approximately 5 weeks.

Table 1. The alternative policies, for $0 \leq t \leq 30$ weeks (see appendix for symbols).

1) MOHAPATRA'S POLICY (Mohapatra and Sharma, 1985):

$$FOR(t) = \beta(t) * ASR(t) + \tau(t) * [DINV(t) - INV(t)] + [RBL(t) - OBL(t)]/TACL + [DPLA(t) - PLA(t)]/TAL$$

$$PSR(t) = ASR(t) + [DINV(t) - INV(t)]/TAIP + [DPLA(t) - PLA(t)]/TALP$$

where:

$DPLA(t) = WPLD * ASR(t)$; $\beta(t) = 1$; $DINV(t) = WINVD * ASR(t)$;
$\tau(t) = 0.25$, $0 \leq t \leq 30$;
$WPLD = 6$; $TALP = 12$; $TAL = 12$; $TACL = 6$; $TAIP = 4$; $WINVD = 6$

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2) REFERENCE APPROACH POLICY (Macedo, 1989b):

$$FOR(t) = \beta(t) * ASR(t) + \tau(t) * [DINV(t) - INV(t)]$$

$$PSR(t) = \alpha(t) * APL(t)$$

where:

$$\alpha(t) = \alpha_t^* + 0.1[OBL(t) - OBL_t^*] - 0.2[INV(t) - INV_t^*] - 0.15[PLA(t) - PLA_t^*] + 0.2[ASR(t) - ASR_t^*] - 0.2[APL(t) - APL_t^*] + 0.3[SR(t) - SR_t^*]$$

$$\beta(t) = \beta_t^* - 0.2[OBL(t) - OBL_t^*] - 0.15[INV(t) - INV_t^*] - 0.15[PLA(t) - PLA_t^*] - 0.4[ASR(t) - ASR_t^*] - 0.3[AOR(t) - AOR_t^*] - 0.4[APL(t) - APL_t^*] + 0.2[SR(t) - SR_t^*]$$

$$\tau(t) = \tau_t^* - 0.05[ASR(t) - ASR_t^*]$$

$DINV(t) = WINVD * ASR(t)$, $0 \leq t \leq 30$; $WINVD = 6$
$OBL_t^*, INV_t^*, PLA_t^*$: Figure 1; $ASR_t^*, AOR_t^*, APL_t^*$: Figure 2;
SR_t^*: Figure 3; $\alpha_t^*, \beta_t^*, \tau_t^*$: Figure 4

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3) COYLE'S POLICY (Coyle, 1977, chapter 8):

$$FOR(t) = \beta(t) * ASR(t) + \tau(t) * [DINV(t) - INV(t)] + [DPLA(t) - PLA(t)]/TAL$$

$$PSR(t) = \alpha(t) * APL(t)$$

$$DFF(t) = AOR(t) + [WIFA(t) - DWIFA(t)]/TAWIFA$$

where:

$WIFA(t) = PCR(t) - DFF(t)$; $DWIFA(t) = COVER * APL(t)$;
$DFF(t) = AOR(t) + [WIFA(t) - DWIFA(t)]/TAWIFA$;
$DPLA(t) = WPLD * ASR(t)$; $DINV(t) = WINVD * ASR(t)$;
$PCR(t) = DELAY3[PSR(t), PDEL]$; $\alpha(t) = 1$; $\beta(t) = 1$; $\tau(t) = 1/12$, $0 \leq t \leq 30$;
$TAL = 12$; $TAWIFA = 2$; $COVER = 2$; $WPLD = 6$; $WINVD = 6$; $PDEL = 4$

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4) KELOHARJU'S POLICY (Macedo, 1989b):

$$FOR(t) = \beta(t) * ASR(t) + \tau(t) * [DINV(t) - INV(t)]$$

$$PSR(t) = \alpha(t) * APL(t)$$

where:

$DINV(t) = WINVD * ASR(t)$; $\alpha(t) = 1.752$; $\beta(t) = 1.135$; $\tau(t) = 0$, $0 \leq t \leq 30$;
$WINVD = 6$

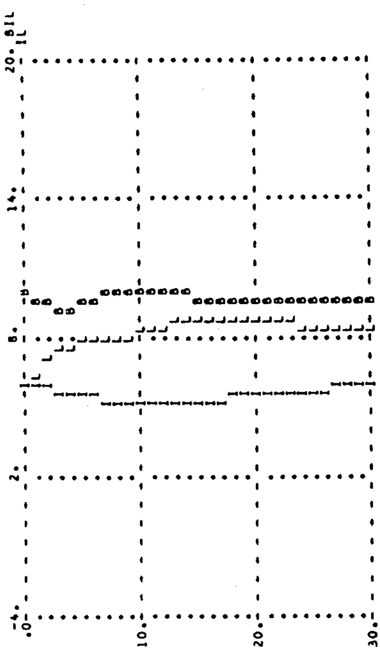

Figure 1. Desired dynamics of the level variables OBL$_t$* (B), INV$_t$* (I), PLA$_t$* (L).

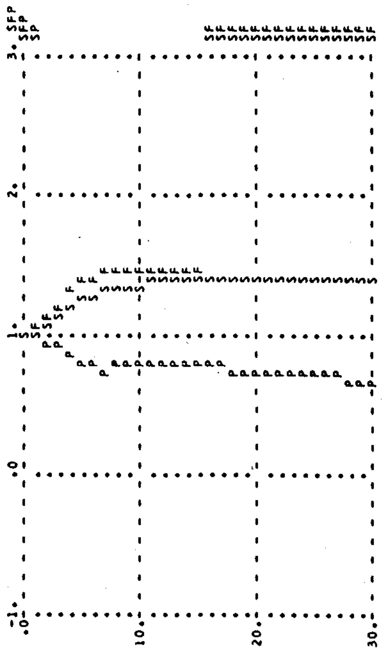

Figure 2. Desired dynamics of the level variables ASR$_t$* (S), AOR$_t$* (F), APL$_t$* (P).

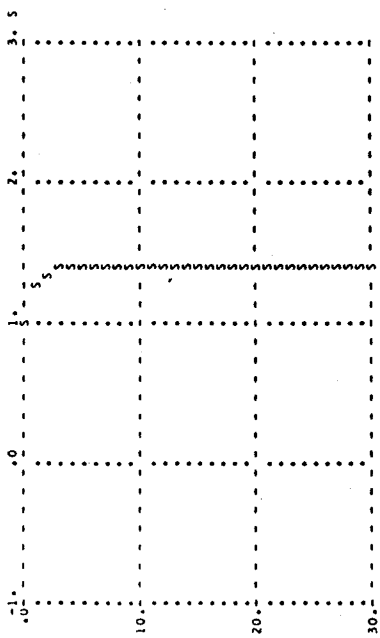

Figure 3. Dynamics of the exogeneous SR$_t$* (S).

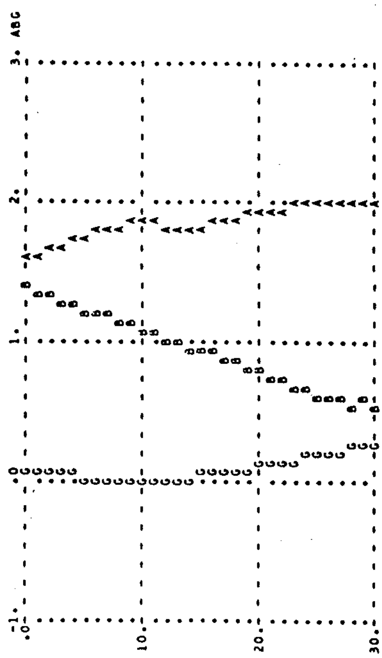

Figure 4. Structural changes α_t* (A), β_t* (B), τ_t* (G).

In the heuristic methods the policy is conceived based on the understanding of the model dynamics. This is achieved using intensive simulation and interpreting the results with a loops diagram. In the modal methods, the policy is designed by moving the eigenvalues of the linearized model and is expressed as a function of the level variables. Sequential optimization methods converge to the best policy by solving a sequence of optimization models.

In Keloharju's optimization method. the policy is defined as a set of relationships that link the control variables[2] of the system dynamics model to the rest of its variables. These relationships are defined heuristically and exogeneously using artificial parameters. Next, the policy is inserted into the system dynamics model and the resultant model is optimized. As a result of the optimization, the values of the parameters and the policy become defined. In this way several policies are tried (and several optimization models solved) until the one that generates the best value for an objective function is found.

In the reference approach, the structure of the policy is generated endogeneously from the parameters that control the behavior of the system dynamics model (these parameters become the control variables). The reference approach expresses the policy as a sum of structural and tactical changes. The structural changes are the parametric changes to be introduced in the system dynamics model. The tactical changes are the nonlinear relationships to be created between the control variables and the level variables of the system dynamics model.

The reference approach uses a hierarchy of two sequential optimizations. In the first, a sequence of reference models are optimized with the goal of obtaining the structural changes capable of pushing an *approximate* system dynamics model to a desired zone[3]. In the second, a sequence of control models are optimized with the goal of reducing the gap between the behavior of the *original* system dynamics model and the behavior of the *approximate* system dynamics model *which is located in the desired zone.* This second sequential optimization produces the tactical changes.

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2 The control variables of a system dynamics model are those that control its problematic behavior.

3 The desired zone is the zone which presents the following two properties. First, the variables of the system dynamics model show the desired dynamics. Second, the pattern of these dynamics does not change when small parametric variations occur.

In this paper a stabilization problem was chosen because Mohapatra's method is limited to the design of stabilization policies (this is not the case for the other methods analyzed). In addition, we optimize the same policy structure with the reference approach and Keloharju's method. This is necessary because these two methods are conceptually different.

2. RESULTS AND DISCUSSION

The efficiency of each policy is evaluated as follows. The policy is first introduced into the system dynamics model (appendix). This generates trajectories for the problematic variables FOR(t), PSR(t), INV(t). Those trajectories are then characterized by three relative coefficients: overshoot (maximum difference between the transient behavior and the dynamic equilibrium of the variable), settling time (time required by the variable to reach and remain within 90% and 110% of its dynamic equilibrium) and accumulated deviation (sum of the percentage deviations of the variables with respect to their dynamic equilibriums).

Table 2 shows the reference approach policy as the most efficient followed by Keloharju's policy which presents the second lowest value for the accumulated deviation. The higher efficiency of these two policies is due to the use of the optimization algorithms. These tools simultaneously perform an analysis in both time and variables space. This is not the case for Mohapatra's and Coyle's policies which utilize partial analysis. Hence, they "wait" to observe the growth of the sales rate before increasing the production start rate (the overshoot time of PSR is bigger for Coyle and Mohapatra's policies than for the other two policies in table 2).

Keloharju's policy shows low overshoot coefficients (table 2) which means high efficiency, however its settling times are not as good as the reference approach policy. This is due to the fact that the control variables $\alpha(t)$, $\beta(t)$, $\tau(t)$ cannot vary with time in the optimization algorithm used by Keloharju's method.

In order to evaluate the robustness of each policy an aggregate reaction coefficient is calculated for each one. This coefficient is the accumulated deviation of the original trajectories of the

Table 2. Efficiency and robustness coefficients of the alternative policies (see appendix for symbols).

Efficiency coefficient	*ALTERNATIVE POLICIES*			
	MOHAPATRA	*REFERENCE APPROACH*	*COYLE*	*KELOHARJU*
Overshoot:				
INV	0.43 (t=7)[a]	0.125 (t=11)	0.83 (t=17)	0.32 (t=16)
FOR	0.39 (t=8)	0.098 (t=4)	0.373 (t=12)	0.187 (t=0)
PSR	0.42 (t=8)	0.15 (t=1)	0.42 (t=18)	0.25 (t=0)
Settling time:				
INV	18 weeks	17 weeks	>30 weeks	27 weeks
FOR	16 weeks	4 weeks	23 weeks	>30 weeks
PSR	16 weeks	3 weeks	27 weeks	>30 weeks
Accumulated deviation	17.476	4.2724	29.681	12.807
Robustness coefficient				
Aggregate reaction	20.303	19.76	36.567	25.008

a: t in parenthesis indicates the time (weeks) when the overshoot is evaluated.

FORMULAE FOR TABLE 2:

Overshoot of variable x = $[\max|(x(t)-x_E)/x_E|, 0 \leq t \leq 30 \text{ weeks}]$

Settling time of variable x = $[t/\ 0.9x_E \leq x(t) \leq 1.1x_E, 0 \leq t \leq 30 \text{ weeks}]$

$$\text{Accumulated deviation} = \int_0^{30} \sqrt{[(INV(t)-INV_E)/INV_E]^2} + \sqrt{[(FOR(t)-FOR_E)/FOR_E]^2} + \sqrt{[(PSR(t)-PSR_E)/PSR_E]^2}\,dt$$

$$\text{Aggregate reaction coefficient} = \int_0^{30} \sqrt{[(INVG(t)-INV(t))/INV(t)]^2} + \sqrt{[(FORG(t)-FOR(t))/FOR(t)]^2} + \sqrt{[(PSRG(t)-PSR(t))/PSR(t)]^2}\,dt$$

INV(t), FOR(t), PSR(t): Values of INV, FOR, PSR at time t when GOAL=1.4

INVG(t), FORG(t), PSRG(t): Values of INV, FOR, PSR at time t when GOAL=1.7

INV_E, FOR_E, PSR_E: Dynamic equilibriums of INV, FOR, PSR

x(t): Value of variable x at time t (x = INV, FOR, PSR)

x_E: Dynamic equilibrium of variable x (x = INV, FOR, PSR) problematic variables FOR(t), PSR(t), INV(t) (obtained simulating the appendix model with GOAL=1.4) with respect to their modified trajectories (obtained simulating the appendix model with GOAL=1.7).

Table 2 shows the reference approach policy as the most robust, closely followed by Mohapatra's policy. These policies exhibit lower aggregate reaction coefficients than Coyle's and Keloharju's policies because they are expressed as a function of the level variables. In fact, they can capture small parametric variations because these variations necessarily affect a level variable via the feedback loops. While Mohapatra's method uses modal analysis to express the policy as a function of the level variables, the reference approach uses an equivalent tool, an optimal control model.

3. CONCLUSION

This paper demonstrates the necessity of developing new methods which combine the simultaneous analysis capabilities of the optimization methods with the robustness of the modal methods. One of these hybrid methods is the reference approach which, for the case analyzed in this paper, presents the best efficiency and robustness.

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APPENDIX: THE LABORATORY SYSTEM DYNAMICS MODEL

NOTE	*EQUATIONS OF THE DYNAMIC EXOGENEOUS*	
L	$EXOG_k$	$= EXOG_j + dt \ast TAUX_{jk}$
	$EXOG_0$	$= 1$
R	$TAUX_{kl}$	$= FPT \ast (GOAL - EXOG_k)$
	FPT	$= 0.6$
	$GOAL$	$= 1.4$
NOTE	*EQUATIONS OF THE MODEL*	
R	SR_{kl}	$= EXOG_k$
L	ASR_k	$= ASR_j + dt \ast (SR_{jk} - ASR_j)/TASR$
	ASR_0	$= 1$
	$TASR$	$= 4$
L	INV_k	$= INV_j + dt \ast (DFF_{jk} - SR_{jk})$
	INV_0	$= ASR_0 \ast WINVD$
	$WINVD$	$= 6$
A	$DINV_k$	$= WINVD \ast ASR_k$
R	FOR_{kl}	$= \beta \ast ASR_k + \tau \ast (DINV_k - INV_k)$
	β	$= 1$
	τ	$= 1/TAI$
	TAI	$= 4$
L	AOR_k	$= AOR_j + dt \ast (FOR_{jk} - AOR_j)/TAOR$
	AOR_0	$= ASR_0$
	$TAOR$	$= 4$
L	OBL_k	$= OBL_j + dt \ast (FOR_{jk} - PSR_{jk})$
	OBL_0	$= 6 + TABL \ast ASR_0$
	$TABL$	$= 4$
A	RBL_k	$= TABLE(TRBL, AOR_k, 0.5, 1.5, 0.25)$
T	$TRBL$	$= 4/5.25/6/6.5/6.75$
A	IPL_k	$= (OBL_k - RBL_k)/TABL$
L	APL_k	$= APL_j + dt \ast (IPL_j - APL_j)/PAT$
	APL_0	$= AOR_0$
	PAT	$= 3$
R	PSR_{kl}	$= \alpha \ast APL_k$
	α	$= 1$
R	DFF_{kl}	$= PLA_k/PDEL$
	$PDEL$	$= 6$
L	PLA_k	$= PLA_j + dt \ast (PSR_{jk} - DFF_{jk})$
	PLA_0	$= ASR_0 \ast WPLD$
	$WPLD$	$= 6$
L	t_k	$= t_j + dt$
	t_0	$= 0$

$k = kl = t;\ j = jk = t-1;\ 0 \le t \le 30$

SYMBOLS USED (hu: hundred of units; hu/w: hundred of units/week; w: weeks):

AOR : Average order rate at factory (hu/w)
APL : Actual production level (hu/w)
ASR : Average sales rate (hu/w)
DFF : Delivery rate from factory (hu/w)
DINV : Desired inventory of finished products (hu)
EXOG : Auxiliary variable which defines the sales rate SR
FOR : Factory order rate (hu/w)
GOAL : Sales rate objective (hu/w)
INV : Inventory of finished products (hu)
IPL : Indicated production level for backlog control (hu/w)
OBL : Order baklog (hu)
PAT : Time to adjust to planned production level (w)
PDEL : Production process delay (w)
PLA : Goods in production pipeline (hu)
PSR : Production start rate (hu/w)
RBL : Required level of backlog (hu/w)
SR : Sales rate (hu/w)
TAI : Time to adjust inventory (w)
TAOR : Time to average order rate (w)
TABL : Time to adjust backlog (w)
TASR : Time to average sales rate (w)
TAUX : Growth rate of EXOG
WINVD: Weeks of average sales rate desired in inventory (w)
α : Fraction of APL in PSR (no dimension)
β : Fraction of ASR in FOR (no dimension)
τ : Fraction of (DINV-INV) in FOR (1/w)