The Application of System Dynamics in Solving a Dynamic Input-Output Model with Delays

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ABSTRACT

Since the dynamic input-output method was put forward by W.Leontief, some results have been obtained to a greater or less degree in various fields of quantative economy, which have played an important role in the application of the method. Yet, insolving the delay-having dynamic input-output models, whether the matrix converse exists or not has not had sufficient mathematical proofs. Having taken these problems into consideration, our paper attempted to solve the problem of multiyear delay-having dynamic input-output model with the application of the properties of system dynamics in structure and time sequence, the properties of BOXLIN and SUM functions, and has combined these two models, the combination of which is possible in the sense of economy. The DIOSD (Dynamic Input-Output and System Dynamics Model) not only has

the advantage of man-and-machine conversation as well as screen display, but also we can put the DIOSD completely into the SD model with the consideration of the overall system structure. Therefore, we can make full use of the advantages of the dynamic input-output model in economy planning and forcasting, and also provide an efficient tool for its future application.

1. The Delay-Having Dynamic I/O:

In order to make all sectors to increase outputs, the purpose of the dynamic input-output model is to study the investment in the accumulative item of the static model. And so the investment products (capital outlays and newly-added liquid assets) are divided from the accumulative item. The rest of accumulative item (LI) and the consumpation (C) as well as import and export difference (W) amount to final net required portion. The balance equation is:

$$X_t = A_t X_t + \sum_{m=1}^{k} B_{tm} R_{tm} (X_{t+m} - X_{t+m-1})$$

$$+ (LI_t + C_t + W_t) \qquad \langle 1 \rangle$$

here $X_t = (X_j^t)_{nx1}$ refers to the total output vector of year t;

$A_t = (a_{ij}^t)_{nxn}$ refers to the direct consumptive matrix of year t;

$B_{tm} = (b_{tm}^{ij})_{nxn}$ refers to the investment coefficient matrix of year t with year m delay;

$R_{tm} = (r_{tm}^{j})_{nxn}$ is a diagonal matrix. It is the ratio that year t+m' increment invested by year t hold the total year t+m's increments $(X_{t+m} - X_{t+m-1})$;

k is the longest delay-period taken from statistical forecast of all the sectors. We write ⟨1⟩ as concrete form and reduce it to investment products:

$$V_i^t = X_i^t - \sum_{j=1}^{n} a_{ij}^t X_j^t - (LI_i^t + C_i^t + W_i^t)$$
$$= \sum_{m=1}^{k} \sum_{j=1}^{n} b_{tm}^{ij} r_{tm}^{j} (X_j^{t+m} - X_j^{t+m-1}) \quad \langle 2\rangle$$

sum <2>:

$$\sum_{j=1}^{n} \{ \sum_{i=1}^{n} \sum_{m=1}^{k} b_{tm}^{ij} r_{tm}^{j} (X_j^{t+m} - X_j^{t+m-1}) - V_j^t \} = 0$$

As we know, $\sum_{i=1}^{n} \sum_{m=1}^{k} b_{tm}^{ij} r_{tm}^{j} (X_j^{t+m} - X_j^{t+m-1})$

is the part of investment products which are invested by all sectors in year t in order to increse the outputs of sector j during year t to year t+k. Thus we have:

$$\sum_{i=1}^{n} \sum_{m=1}^{k} b_{tm}^{ij} r_{tm}^{j} (X_j^{t+m} - X_j^{t+m-1}) = q_j^t \sum_{i=1}^{n} V_j^t \quad \langle 3\rangle$$

here, q_j^t refers to policy-making parameters, $\sum_{j=1}^{n} q_j^t = 1$, $0 \leqslant q_j^t \leqslant 1$. In particular, when $q_j^t = V_j^t / (\sum_{i=1}^{n} V_i^t)$, <3> is

that the part of investment products owned by sector j is equal to the investment product(V_j^t) input by sector j in year t.

According to above points of view, we have designed DIOSD model by using the BOXLIN and SUM (see figure) and have been tentatively applying it to the personal flow and fund distribution of Education Department in Zhejing Province.

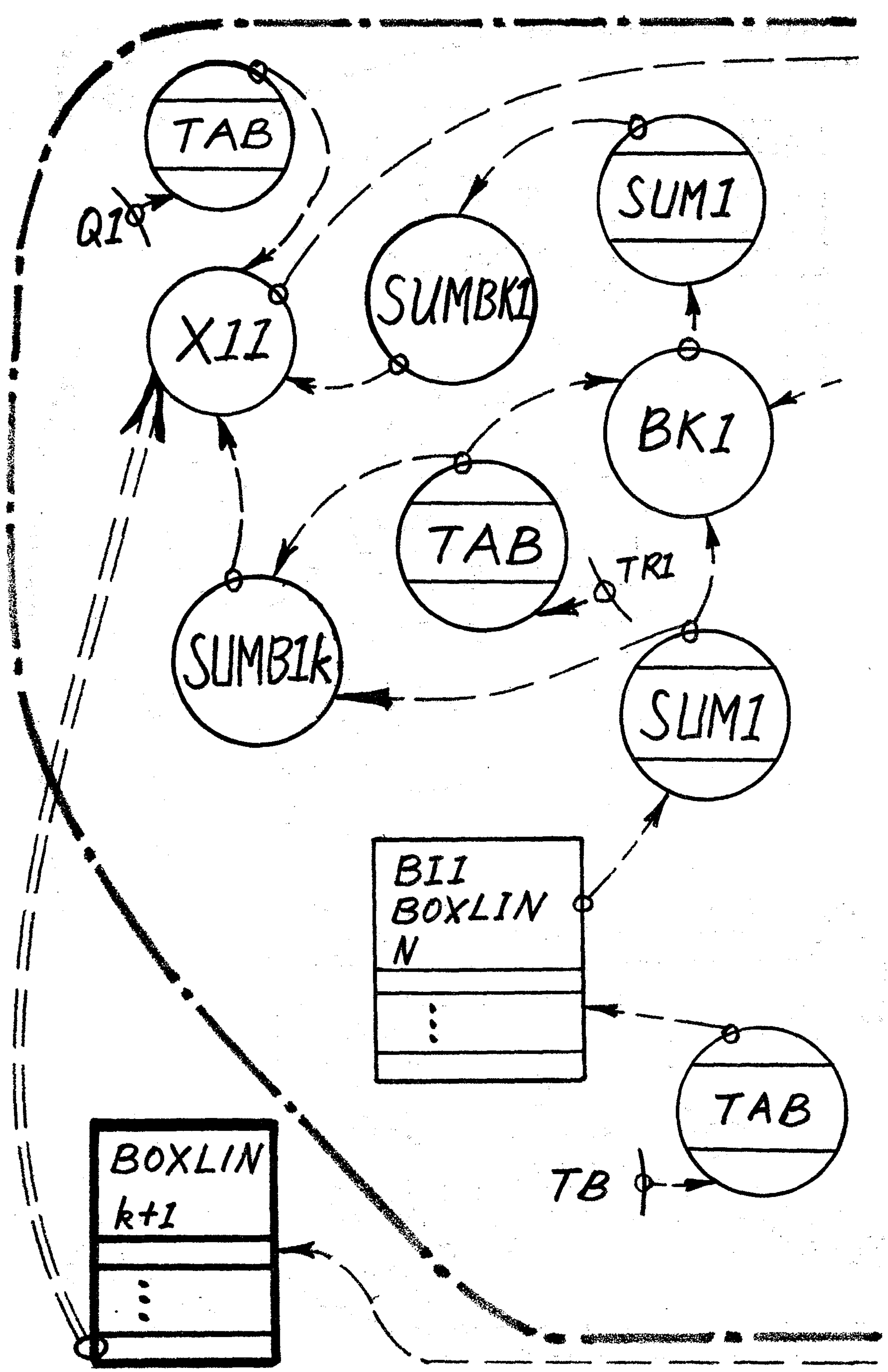
3.Note and Analysis of DIOSD:

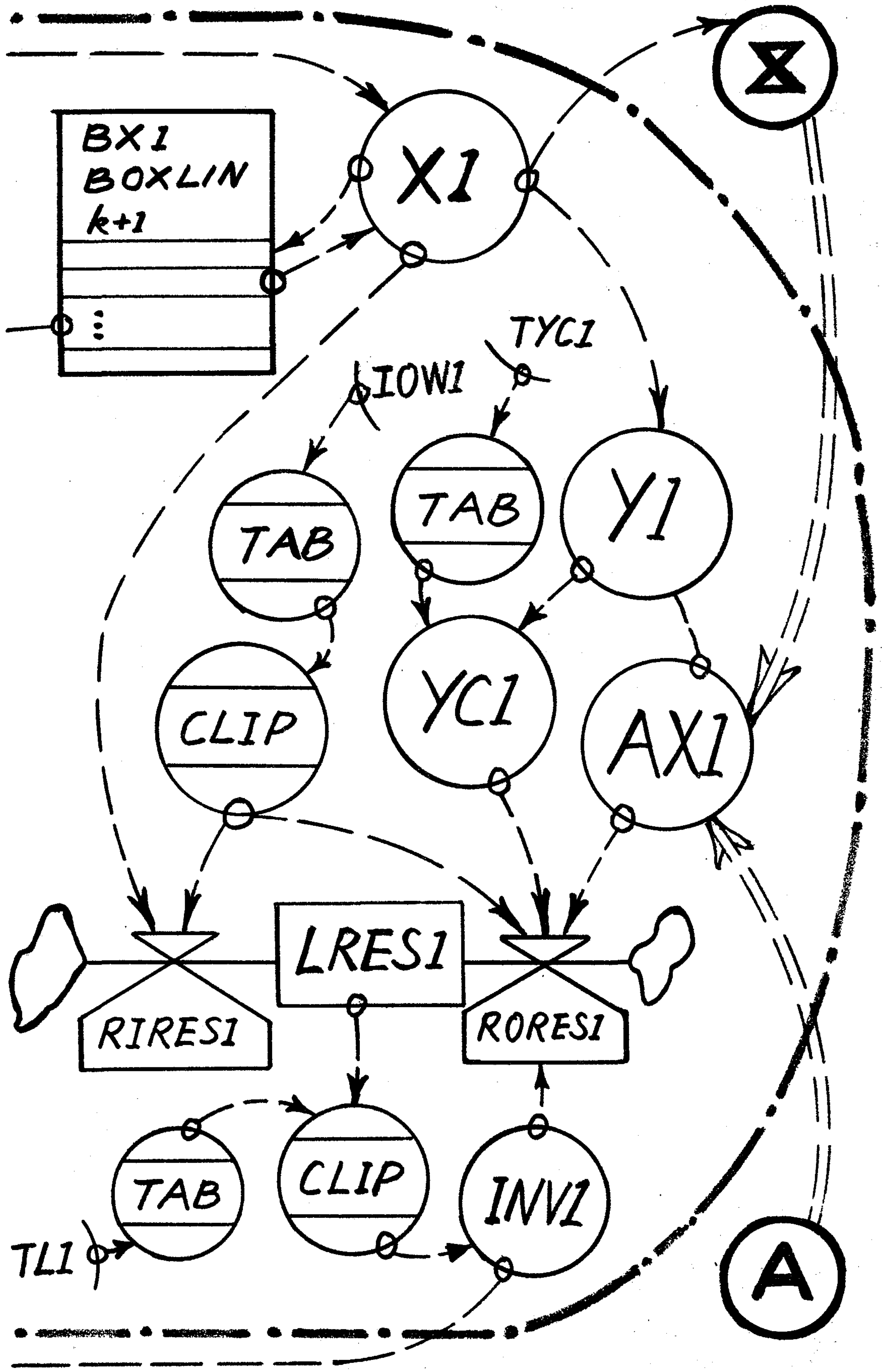
briefly, the chart of DIOSD is for one sector. The n sectors are the same. The main variations are explained as follows, here j=1:

LRESj: refers to the reserve of the sector. The initial yaer is t+k;

IOMj, AXj, YCj: are import and export difference, middle products and consumption;

INVj: is the investment products of the sector which are controled by expective reserve table TLj;


TAB
Q1
X11
SUMBK1
SUM1
BK1
TAB
TR1
SUMB1k
SUM1
BII
BOXLIN
N
TAB
TB
BOXLIN
k+1


BX1
BOXLIN
k+1
X1
TYC1
LOW1
TAB
TAB
Y1
CLIP
YC1
AX1
LRES1
RIRES1
RORES1
TAB
CLIP
INV1
TL1
A

TB, TRj, Qj: are the table of $\{b_{tm}^{ij}\}$, $\{r_{tm}^{j}\}$, $\{q_j^t\}$, i=1,2,...,n, m=1,2,...,k;

SUMBlk: refers to $\sum_{i=1}^{n} b_{tk}^{ij} \cdot r_{tk}^{j}$;

SUMBKj: refers to $\sum_{m=1}^{k-1}\sum_{i=1}^{n} b_{tm}^{ij} r_{tm}^{j} (X_j^{t+m} - X_j^{t+m-1})$;

Xjj: refers to

$(q_j^t(\sum_{i=1}^{n} V_i^t)$-SUMBKj)/SUMBjk;

We believe that the following points are worthy of note:

1> The DIOSD model shall have matrix sequences A_t and B_{tm} beforehand. Its accuracy decides the value of dynamic imput-output model;

2> We assume $r_{tk}^{j} \neq 0$ of the longest delay k in the DIOSD model, j=1,2,...,n. But it won't be just consistent in practical application. So each sector shall have its own k(j), j=1,2,...,n, to ensure $r_{tk(j)}^{j}=0$.

When m>k(j), $r_{tm}^{j}=0$. So we can get the liner formulation of

$$X^{t+k(j)}=f(X^{t},X^{t+1},\ldots,X^{t+k(j)-1})$$

in the same way. What we shall do is to make some changes in the usage of train variation of model;

3> DIOSD model always has many equations. So it needs high requirement for inner storage of computer. Especially , when DIOSD is combined with practical model, the micro-computer is not enough to work;

4> Though the determination of q_{j}^{t} is available theoretically, it is often regarded as policy-making variation in practical problem, because b_{tm}^{ij} itself exists some errors.