Business Cycle Entrainment: A System Dynamics Approach

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Abstract:

Models in the system dynamics tradition often consider that firms can be aggregated in a single sector. The implicit assumption is that individual firms move in phase with each other. After careful development of a simple model of two firms coupled through market share, a set of simulation tests suggests that firms can entrain even when they are *different* and are driven by *different random* inputs. However, the market share coupling may be unable to promote entrainment between firms when they differ *substantially*. When the market share mechanism is not present, even *similar* firms driven by the *same sine* input do not show entrainment. Given that firms within an industry are commonly linked through market share, if they do not differ substantially from each other, it would not seem unusual to observe entrainment among them. Therefore, the simplifying assumption of modeling an industry as an aggregated firm often makes sense.

Key Words:
Business Cycle, Entrainment, Aggregation Assumption, System Dynamics, Model Analysis, Model Formulation.

1. Motivation

System dynamics has long been used to model oscillation cycles in the economy. In this tradition, it is common to aggregate all firms into a single industry sector. But implicit in this modeling strategy is the assumption that individual firms are entrained, that is, moving in phase. It is not obvious, however, why such entrainment exists, or what are the mechanisms underlying it. In reality, within a single industry firms are quite different, they have different types of plant and equipment with different lifetimes and acquired with different lead times. Thus, one could argue that even if individual firms were oscillatory, their parameters and initial conditions would differ enough that they would fluctuate with random periods and phases (Sterman and Mosekilde, 1994). In that case, aggregate industry output would be essentially constant, though individual firms' output may be oscillating.

But this is not what takes place. The economy as a whole follows aggregate business cycles, the short-term business cycle (3-7 years), the construction or Kuznets cycle (15-25 years), and the long-wave or Kondratieff cycle (40-60 years). It is interesting that, given the diversity of

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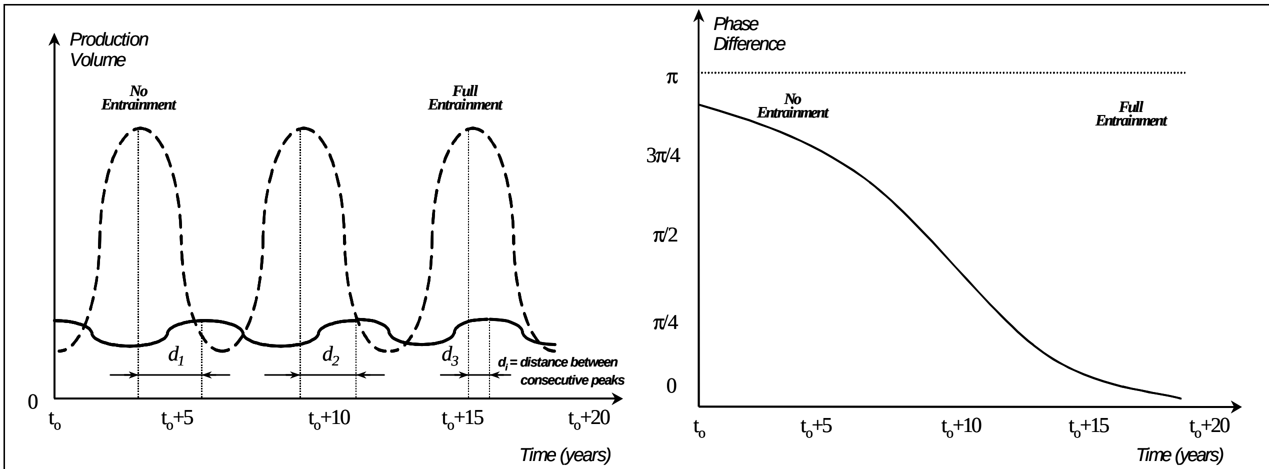
possible periods of oscillation that could be generated by firms with different characteristics, we observe only a few. Why should the cycles of different firms move in phase with one another to create such cycles? And, if firms become entrained, what are the mechanisms that contribute to this taking place? Sterman and Mosekilde (1994) provide some insight on how different cyclical modes interact and synchronize amplifying their individual effects. Homer (1980) considered the mechanisms that would allow two different firms' production to entrain following the business cycle. In this context, this study is much closer to the latter.

Consider two distinct hypothetical firms, which oscillate with different phases and periods, independently starting their operations. Assume that over time the firms become more entrained. There may be several reasons for this to take place. For instance, there could be coupling mechanisms between the firms, such as market share or pricing. So, in Figure 1 (a), for production volume, the distance between consecutive peaks decreases over time as the two firms becomes more entrained. In addition, the change in the degree of entrainment over time can also be seen in Figure 1 (b), through the ratio between periods and the phase difference between firms' output. These will be the reference modes for production volume and degree of entrainment.



(a) Production Volume (b) Degree of Entrainment

Figure 1. Reference Modes

The graph for production shows two firms with different production capacities: one small, one large. Initially, the two firms are perfectly out of phase: The peak in production of the large sized firm corresponds to the trough in production of the small firm. Additionally, the large firm has a longer period of oscillation. Over time, the two firms become more in phase as their peaks occur closer together. The second graph shows the degree of entrainment for the two firms over time. Initially, the two firms are almost 180^o out of phase. Over time, the phase difference is reduced to zero, indicating that the firms become entrained.

The business cycle has received considerable attention in the economics literature and economists developed a number of theories to explain entrainment in the economy. Such theories used external shocks, such as government monetary or fiscal policy and changes in aggregate demand, to explain the causes of entrainment (refer to Zarnowitz, 1985 for a survey). Alternatively, Forrester (1977) suggested that entrainment could arise from the endogenous interaction of nonlinear oscillators, that is, the firms' own interactions. While it may be easier to justify the phenomenon through an exogenous shock, an endogenously generated explanation seems not only challenging but also quite surprising.

These views are located at polar extremes of a scale measuring the nature of the shocks that impact the system. Considering the scale in more detail, in one extreme the same continuous sinusoidal exogenous shock would drive the behavior of the two firms; in another extreme, different random shocks would allow the endogenous nonlinear interaction of the two firms to generate similar continuous sinusoidal patterns in production volumes.

It was mentioned that distinctive firm attributes – like size, production capacity, capacity lead-time and lifetime, raw material and final goods desired inventory coverage, delivery delay, price and product attributes, market share – cause firms to fluctuate with different periods and phases. If firms' attributes are similar, causing them to oscillate with similar periods, entrainment is likely to come about more easily. In contrast, when the firms differ a lot, their periods of oscillation also differ and entrainment may be more difficult to obtain.

Often, managers' decision rules can amplify exogenous shocks that may be constantly impacting the firm. Alternatively, some decision rules may dampen the effect of exogenous shocks. If firms do not oscillate much in response to shocks, in the case where they behave like a highly damped system, it can be difficult to achieve entrainment.

So, in order to study why the cycles of different firms move in phase with one another and what mechanisms might contribute to this, I develop a simple model of two firms interacting in the market, subject to (a) different set of parameters, and (b) different types of shocks. These two dimensions will allow us to test how similar and distinct firms are prone to behave under a number of different shocks exciting the system. In addition, I do not consider firm size in terms of production volume as a parameter influencing entrainment. This does not seem to be the case in industry where firms of different sizes are entrained, while this can only be assured after testing, I'll forego such tests in this study.

We could also consider the level of coupling as an important characteristic of the system, but for a first analysis we assume it fixed. Weak coupling between the two firms in the model takes place only by the firms' market share interaction. Stronger coupling could take place if firms' decisions to adjust own prices or quantity produced depended on competitors' prices and quantities. But here I assume that each firm's pricing and production decisions take into consideration only internal inputs. Thus only weak coupling links the two firms. Ability to obtain entrainment with the more conservative case of weak coupling will provide a stronger result than if a stronger coupling is assumed.

The next section describes a two-firm system dynamics model competing for market share. Firms produce goods according to their share of demand and capacity availability. Capacity can be acquired but becomes available only after a capacity acquisition delay.

2. The model

In order to explore the entrainment hypotheses, I formulate the simplest possible model that can generate the characteristic behavior and is robust to the standard system dynamics modeling practice. The essential characteristic behavior is that the firms' response to a shock such as a step input in demand is a damped oscillation. Therefore the model should behave like a second-order damped oscillator. To assure robustness I adopt standard system dynamics formulations whenever possible. Occasionally, non-standard formulations are used in the model; where this takes place I develop the standard formulations in the appendix and explore its limitations in exploring the problem at hand.

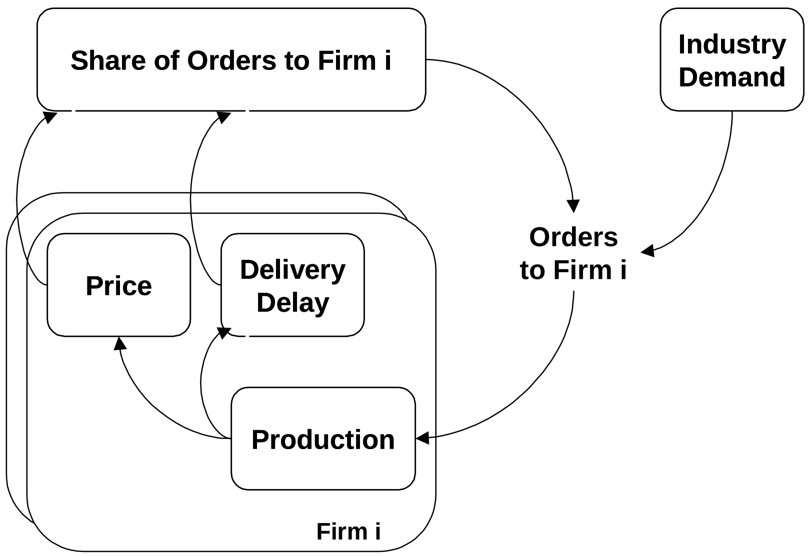


Figure 2. Model Architecture

Figure 2 shows the overall architecture of the model. The model represents a manufacturing (or commodity) industry with exogenous demand. The industry is composed of two firms, the minimum level of disaggregation necessary to test the entrainment hypothesis. The two firms have identical structure, although they may take on different parameters in order to explore entrainment. The share of demand received by each firm will depend on two product attributes: price and availability, where availability is given by delivery delay.

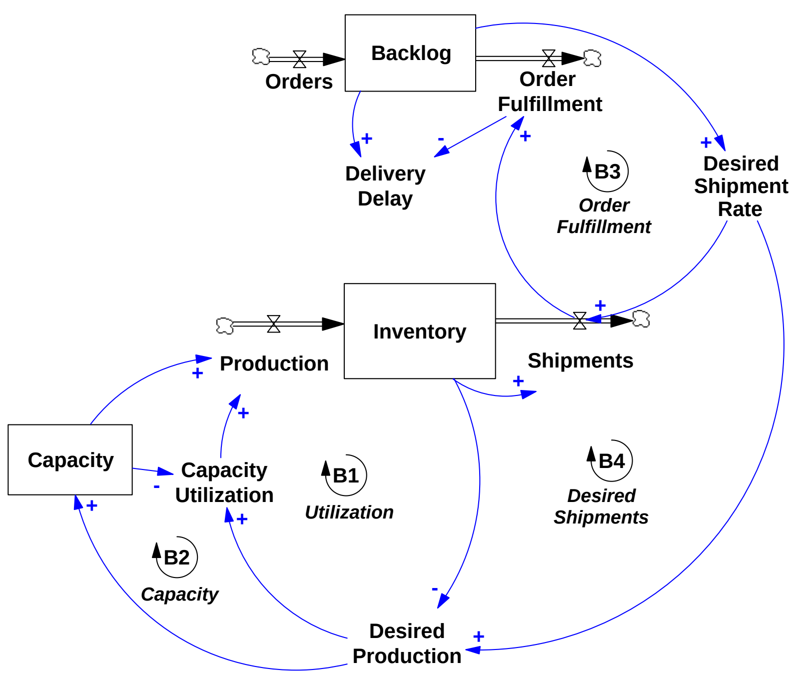


Figure 3. Causal diagram for order fulfillment and production sectors

The single firm model is divided into three sectors: order fulfillment, production and pricing. Figure 3 shows the causal diagram for the order fulfillment and production sectors of the model. The model has three main stocks: the backlog of unfilled orders, the finished goods inventory, and production capacity. There are four main balancing loops (as well as several minor balancing loops) for each single firm: the utilization loop, the capacity loop, the order fulfillment loop, and the desired shipments loop. The diagram does not show all the variables in each of the causal loops. The following sections describe the single firm model in greater detail.

2.1. Order Fulfillment Sector

Customer orders are assumed to be exogenous. Orders are accumulated in a backlog until they can be filled. Backlog is depleted by the order fulfillment rate. Orders are filled when the firm ships the manufactured product. The ratio of backlog and order fulfillment rate determines the product delivery delay. The initial value of backlog is given by the desired backlog, the product of the incoming orders and the target delivery delay.

Backlog (Units) = INTEG(Orders – Order Fulfillment Rate, Desired Backlog)
Order Fulfillment Rate (Units/Week) = Shipments
Delivery Delay (Week) = Backlog/ Order Fulfillment Rate
Desired Backlog (Units) = Long Term Orders *Target Delivery Delay
Target Delivery Delay (Week) = 2

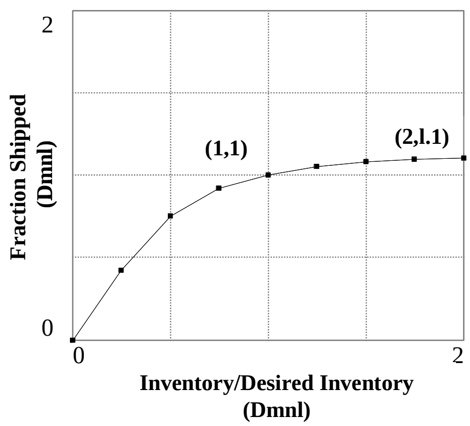
2.2. Production Sector

The product of capacity and capacity utilization determines production. Production accumulates in an inventory. Shipments deplete the inventory. For simplicity, we ignore the work-in-process inventory. The product of the desired shipment rate and the fraction shipped determines shipments. This formulation differs from the standard formulation, but it is more convenient for this exploring this problem.[1] The desired shipment rate is determined by the ratio of backlog and the target delivery delay; and, the fraction shipped is the output of a table function that depends on the ratio of the inventory to the desired inventory. The table for fraction shipped assumes that the firm will send only a fraction of the desired shipment rate when the inventory is below the desired level. But, it will ship more than desired, and run down backlog, when it has too much inventory. Desired Inventory is set as the product of smoothed orders and the desired inventory coverage. Managers allow a very long time to smooth orders before they adjust their desired inventory and backlog.

Inventory (Units) = INTEG(Production – Shipments, Desired Inventory)
Shipments (Units/Week) = Desired Shipment Rate * Fraction Shipped
Fraction Shipped (Dmnl) = Table for Fraction Shipped (Inventory/Desired Inventory)
Table for Fraction Shipped (Dmnl) = [(0,0)-(2,1.2)],(0,0),(0.25,0.42),(0.5,0.75),(0.75,0.92),(1,1),
(1.25,1.05),(1.5,1.08),(1.75,1.095),(2,1.1)
Desired Shipment Rate (Units/Week) = Backlog/Target Delivery Delay
Desired Inventory (Units) = Long Term Orders * Desired Inventory Coverage
Long Term Orders (Units/Week) = SMOOTH(Orders, Time to Smooth Orders)
Time to Smooth Orders (Week) = 150
Desired Inventory Coverage (Week) = 8

[1] Appendix A shows the details of the standard formulation for the order fulfillment ratio and explains why it has not been used here.

Table for Fraction Shipped



Desired production is given by the sum of desired shipment rate and a term for inventory adjustment, and it is constrained to be non-negative, through a max function between zero and the sum above. The inventory adjustment term reflects the firm's willingness to produce more (less) when inventory is below (above) the desired level to correct the discrepancy over time. [2]

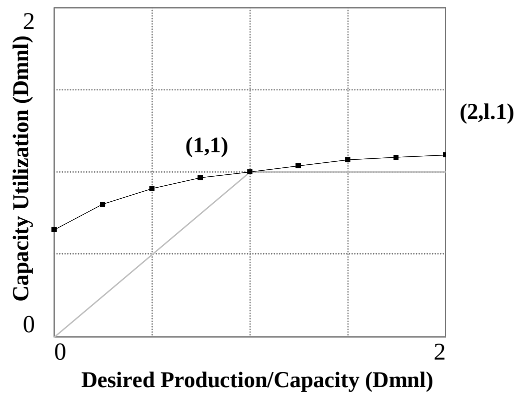
Desired Production (Units/Week) = MAX(0, Desired Shipment Rate + Inventory Adjustment)
Inventory Adjustment (Units/Week) = (Desired Inventory – Inventory)/ Inventory Adjustment Time
Inventory Adjustment Time (Week) = 10

Production is determined by capacity utilization multiplied by capacity. Capacity utilization depends on desired production and capacity. To production managers, capacity is fixed, thus, variations in production are accommodated by changing capacity utilization. When desired production is high relative to available capacity, managers increase capacity utilization to meet desired production. When desired production is equal to capacity, capacity utilization is equal to one, the normal operating point. Finally, when desired production is low relative to capacity, utilization is also low. This assumes that production managers can implement changes in capacity utilization relatively fast, compared to other delays in the system. It also assumes that production managers prefer to keep utilization high, and build inventory, instead of shutting down production lines. Thus, the utilization curve lies above the 45^o reference line. Even when desired production is zero, utilization is still different than zero. Finally, we assume that the maximum capacity utilization can be 10% higher than the normal operating utilization point.

Production (Units/Week) = Capacity*Capacity Utilization
Capacity Utilization (Dmnl) = Table for Capacity Utilization (Desired Production/Capacity)
Table for Capacity Utilization (Dmnl) = [(0,0)-(2,2),(0,0),(1,1),(2,1)],(0,0.65),(0.25,0.8),(0.5,0.9),
(0.75,0.96),(1,1),(1.25,1.04),(1.5,1.07), (1.75,1.1),((2,1.11)

[2] Appendix B derives the standard formulation for desired production and explains why it has not been used here.

Table for Capacity Utilization



Capacity is determined by a third order capacity acquisition delay of desired capacity. An information delay is used since we don’t represent the physical flows of capacity orders, capacity in transit, arrivals, and discards. For simplicity, we assume that desired capacity equals desired production. Thus, we assume that managers respond rationally in setting new capacity to the desired levels. We further assume that the information on desired production is readily available to managers, or more realistically, that the delays involved in obtaining such information are negligible when compared to other delays in the system. The equations for these variables follow:

Capacity (Units/Week) = SMOOTH3I(Desired Capacity, Capacity Acquisition Delay, Initial Capacity)
Capacity Acquisition Delay (Week) = 50
Desired Capacity (Units/Week) = Desired Production

2.3. Pricing Sector

Price is determined by a hill-climbing formulation. Indicated price is anchored in the actual price and it is adjusted by the effect of demand/supply balance on price. We assume that the firm will not lower prices below the fixed unit costs, limiting the indicated price to the adjusted anchor and unit costs.

Price ($/Units) = INTEG(Change in Price, Initial Price)
Change in Price ($/Units/Week) = (Indicated Price – Price)/Price Adjustment Time
Indicated Price ($/Units) = MAX(Unit Costs, Price*Average Effect on Price)
Price Adjustment Time (Week) = 10
Unit Costs ($/Units) = 100
Initial Price ($/Units) = 150

To capture the balance between demand and supply we need to consider the stocks of backlog and inventory. Backlog provides a measure of demand and inventory provides a measurement of supply. When demand goes up, prices should also go up. When supply goes up, prices should go down. But why do we need to consider the stocks of backlog and inventory? Why couldn’t we use instead the flows of orders and shipments as proxies for demand and supply? Well, assume that we have a large increase in customer orders; shipments won’t change right away even if the firm has plenty of inventories to meet that order. A formulation based on the flows would suggest that prices go up as demand is greater than supply, but that should not be the case, since the firm has plenty of inventories. The stocks capture the state of the system, a high backlog adequately represents that the firm has not been able to meet its orders.

Since high backlogs indicate a high demand, or a shortage in supply, prices go up. We normalize this increase by comparing it to the desired backlog level, which allows the firm to ship orders at the target delivery delay. On the other hand, a high inventory, indicating that supply is high compared to demand, pushes prices down. We normalize the inventory by the desired inventory level. Finally, we assume that prices are equally sensitive to inventory and backlog effects.

Figure 5 shows the causal diagram for the pricing sector of the single firm model. The remaining formulations for this sector follow.

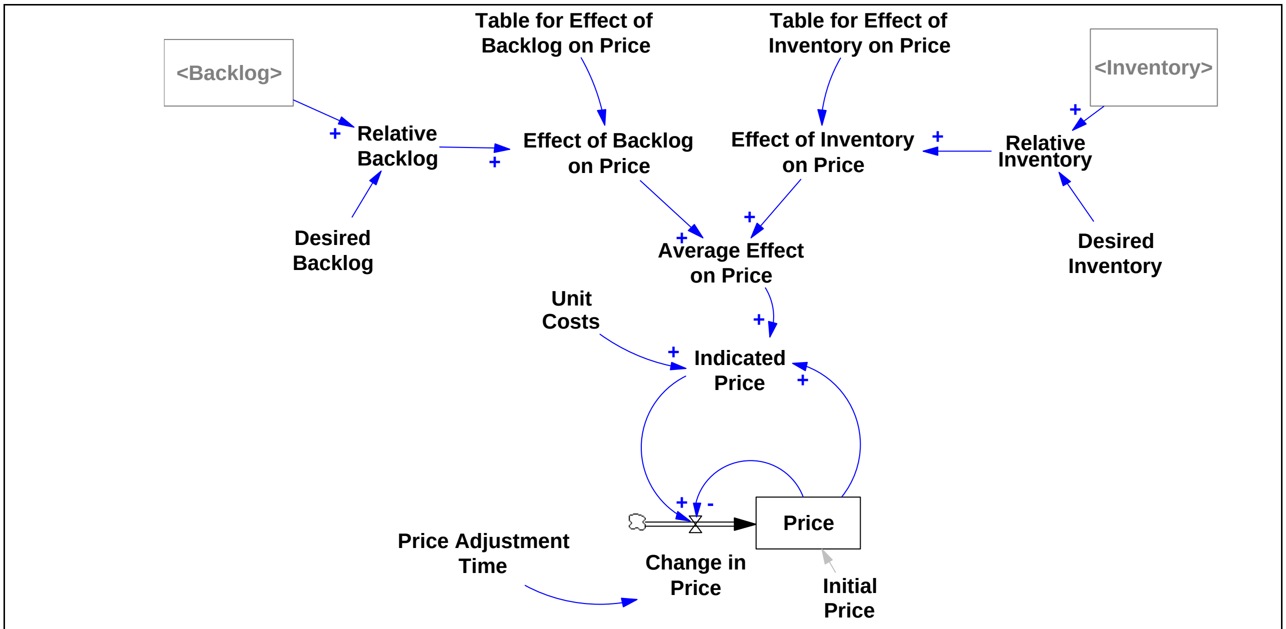
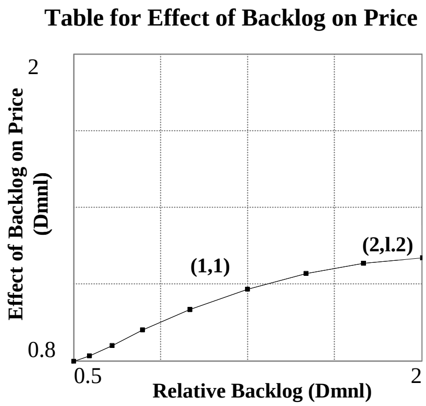
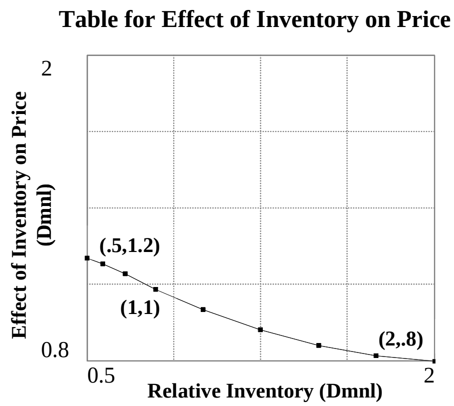


Figure 4. Model structure for the pricing sector

Average Effect on Price (Dmnl) = (Effect of Backlog on Price + Effect of Inventory on Price)/2
Effect of Backlog on Price (Dmnl) = Table for Effect of Backlog on Price(Relative Backlog)
Effect of Inventory on Price (Dmnl) = Table for Effect of Inventory on Price(Relative Inventory)
Table for Effect of Backlog on Price (Dmnl) = [(0.5,0.8)-(2,2)],(0.5,0.8),(0.57,0.82),
(0.667,0.86),(0.8,0.92),(1,1),(1.25,1.08),(1.5,1.14),(1.75,1.18),(2,1.2)
Table for Effect of Inventory on Price (Dmnl) = [(0.5,0.8)-(2,2)],(0.5,1.2),(0.57,1.18),
(0.667,1.14), (0.8,1.08),(1,1),(1.25,0.92),(1.5,0.86),(1.75,0.82),(2,0.8)
Relative Backlog (Dmnl) = Backlog/Desired Backlog
Relative Inventory (Dmnl) = Inventory/Desired Inventory





2.4. Industry Production and Market Shares

To build the two-firm model, pricing and production are replicated to a new firm. Total industry production is just the sum of each firm's production. The product of each firm's market share and customer orders determines individual firm's orders. Customer orders are exogenous and are used to introduce the exogenous test inputs. Market shares will depend on the attractiveness of each firm's product relative to the total attractiveness of products (a traditional Us/(Us+Them) formulation). Attractiveness of each firm's product depends on delivery delay and price. Attractiveness is simply the product of the effects of price on attractiveness and effect of delivery delay on attractiveness. These effects are given by a table function, where lower delivery delays and lower prices are preferred. [3] The relative price and relative delivery delays of both firms are used as inputs to the table functions. Prices and delivery delays are compared to reference values to establish the relative inputs.

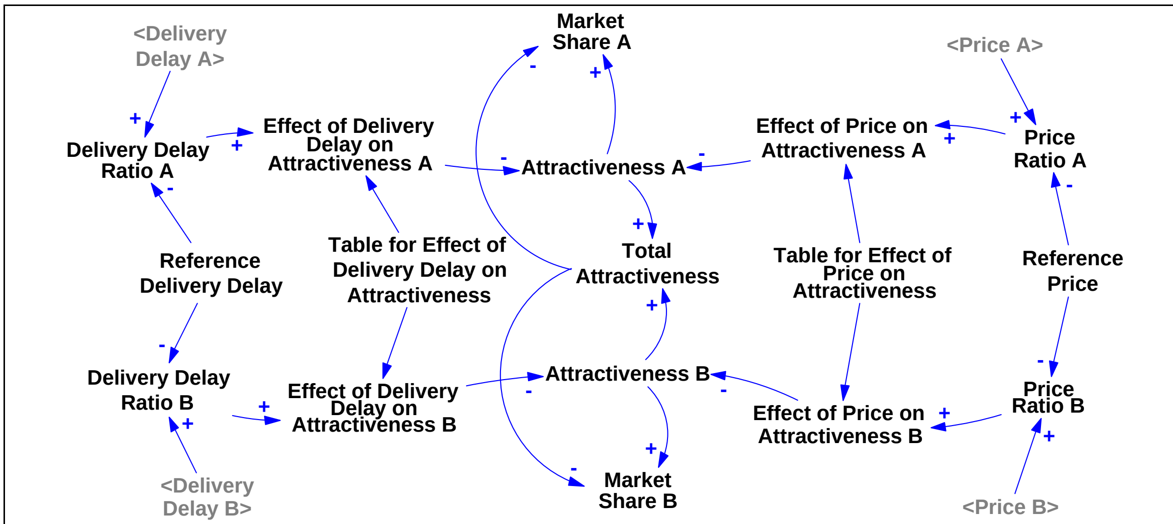
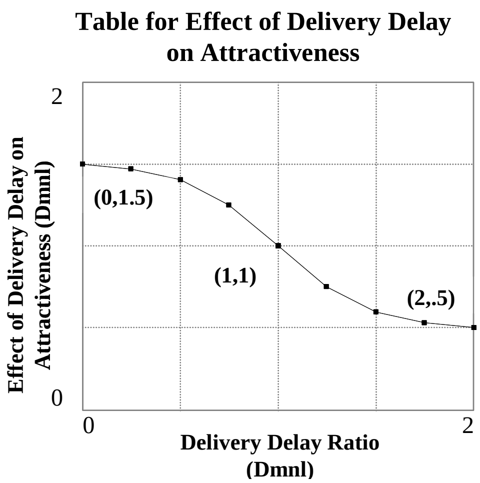


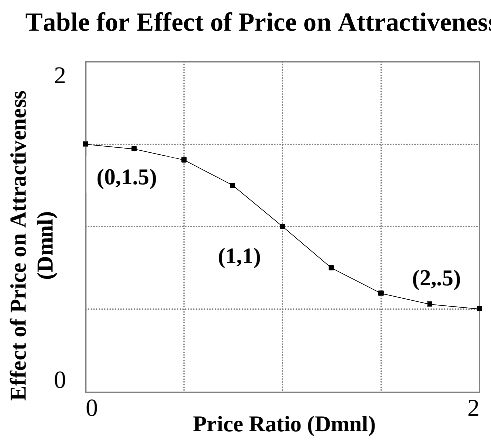
Figure 5. Model structure for the market share sector

Orders [firm] (Units/Week) = Market Share [firm] * Customer Orders

[3] I assume that relative price and delivery delay changes have the same impact on attractiveness. Appendix D determines the magnitude of such impacts and elaborates the notion of substitution effects between the two products.

Market Share [firm] (Dmnl) = Attractiveness [firm] /Total Attractiveness
Total Attractiveness (Dmnl) = SUM (Attractiveness [firm!])
Attractiveness [firm] (Dmnl) = Effect of Delivery Delay on Attractiveness [firm] *Effect of Price on Attractiveness [firm]
Effect of Delivery Delay on Attractiveness [firm] (Dmnl) = Table for Effect of Delivery Delay on Attractiveness (Delivery Delay Ratio[firm])
Effect of Price on Attractiveness [firm] (Dmnl) = Table for Effect of Price on Attractiveness (Price Ratio [firm])
Table for Effect of Delivery Delay on Attractiveness (Dmnl) = [(0,0)-(2,2)],(0,1.5),(0.25,1.47),(0.5,1.4),(0.75,1.25),(1,1),(1.25,0.75),(1.5,0.6),(1.75,0.53),(2,0.5)
Table for Effect of Price on Attractiveness [firm] (Dmnl) = [(0,0)-(2,2)],(0,1.5),(0.25,1.47),(0.5,1.4),(0.75,1.25),(1,1),(1.25,0.75),(1.5,0.6),(1.75,0.53),(2,0.5)
Delivery Delay Ratio [A] (Dmnl) = Delivery Delay [A]/Reference Delivery Delay
Delivery Delay Ratio [B] (Dmnl) = Delivery Delay [B]/Reference Delivery Delay
Price Ratio [A] (Dmnl) = Price [A] / Reference Price
Price Ratio [B] (Dmnl) = Price [B] / Reference Price
Reference Delivery Delay (Week) = INITIAL (Target Delivery Delay)
Reference Price ($) = INITIAL (Initial Price)





2.5. Base Case Run

Figures 6, 7, 8 and 9 show the base runs for the case of two similar firms. The graphs show forty simulated years in which a step input of amplitude of 10 units/week (10% increase) is introduced to total customer orders. Initially, customer orders are set at 100 units/week with each firm retaining a 50% market share. The model starts in equilibrium and the shock is introduced at time 100 weeks. The two firms differ in two parameters: capacity acquisition delay and time to adjust inventory. Firm B has a 25% shorter capacity delay (50 weeks) and 20% shorter inventory correction time (9 weeks).

Figure 6 shows that backlogs seem to be perfectly in phase. This takes place despite individual differences in the two firms. The backlog plot shows the damped oscillatory response to the step input. The periodicity of fluctuations is about 3 years for both firms and the average amplitude of fluctuations is in the order of 10%, compared to an individual increase in firms' orders of about 5%. The product of the change in orders (5 units/week) and the target delivery delay (2 weeks) provides the change in the desired backlog value (10 units).

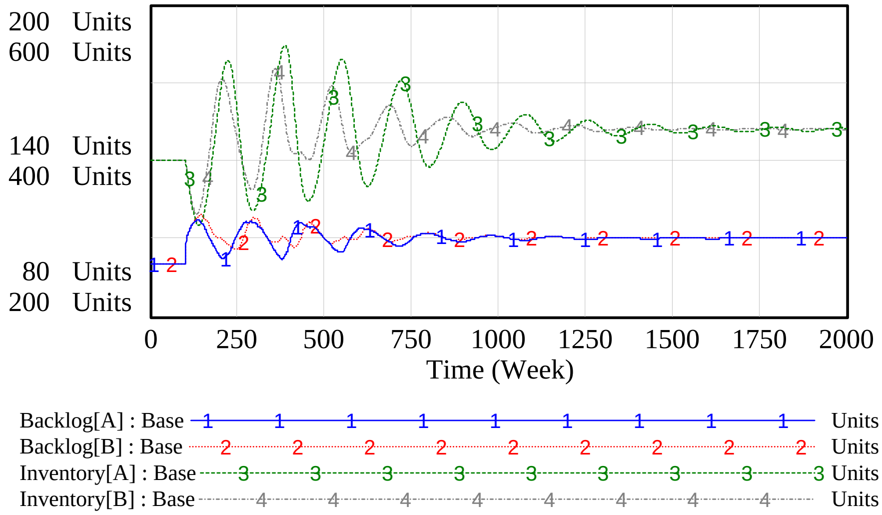


Figure 6. Base Run: Backlog and Inventory

Figure 7 shows capacity behavior for the two firms. The plots show that firms' capacities are entrained, but firm A's peaks lag behind firm B's peaks. This is consistent with Graham's (1977) and Homer's (1980) finding that exogenously-induced entrainment in a system where firm's had different periods of oscillation manifested itself in terms of phase differences, instead of different periods. Also, "disturbances are transmitted more quickly to the system with the shorter natural period; so its fluctuations tend to lead those of the slower system" (Homer, 1980, p. 52).

Firm A's longer capacity acquisition delay and inventory adjustment time allows it to smooth out some variation from the backlog, which permits firm A to have lower amplitudes in capacity (about 15% between the first peak and trough). Although not shown oscillations in production

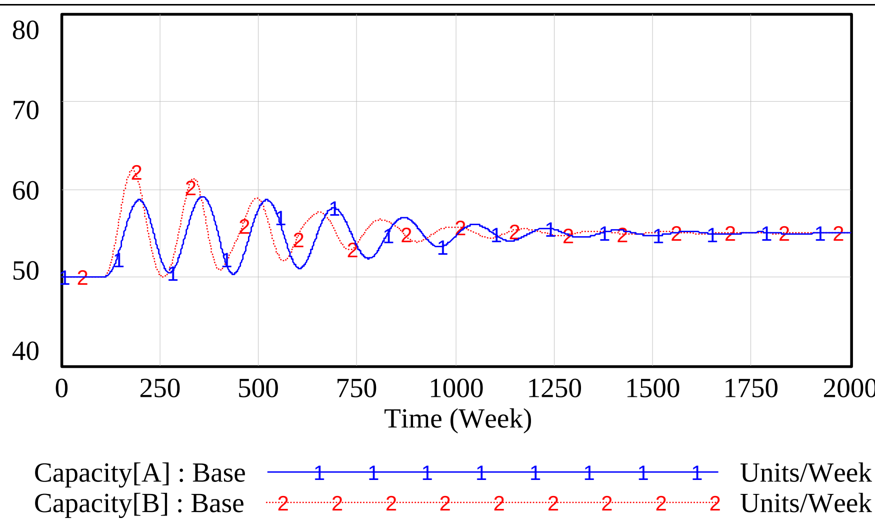


Figure 7. Base Run: Capacity and Production

Figure 8 shows the behavior of prices and delivery delays. Delivery delays are entrained but firm B's delivery delay leads firm A's. The maximum delivery delay amplitude reaches almost 10%. Prices for the two firms are almost perfectly in phase, but as the oscillations dampen out firm B's peaks lead those of firm A's. In addition, final prices settle at a higher level than initially as the net effect of excess backlogs surpass the net effect of excess inventories.

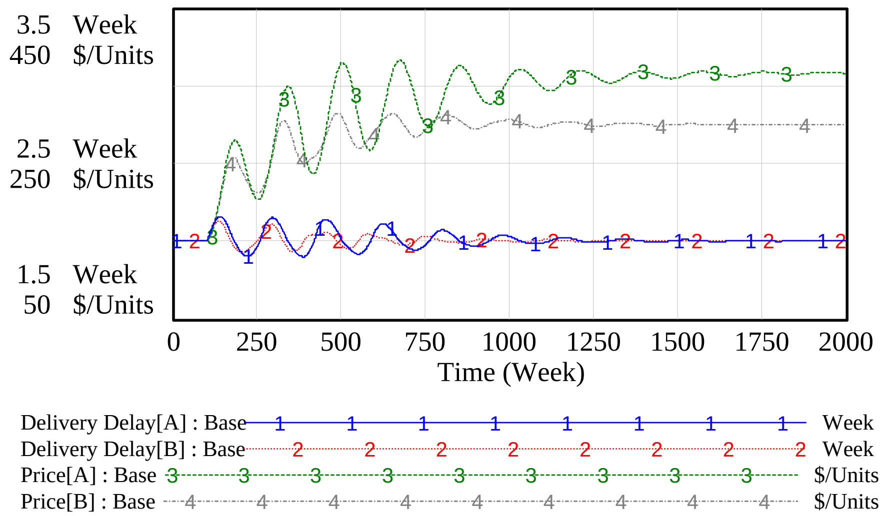


Figure 8. Base Run: Delivery Delay and Price

Figure 9 shows that market shares for the two firms are perfectly (180^{o}) out-of-phase. Even though this occurs, backlogs are perfectly entrained (see figure 6). Its oscillations have maximum amplitude of about 10%, comparable to that of backlogs. Analogously to Homer (1980) the market share fluctuations may "equalize the loads on the two production sectors" producing the entrainment in the observed variables.

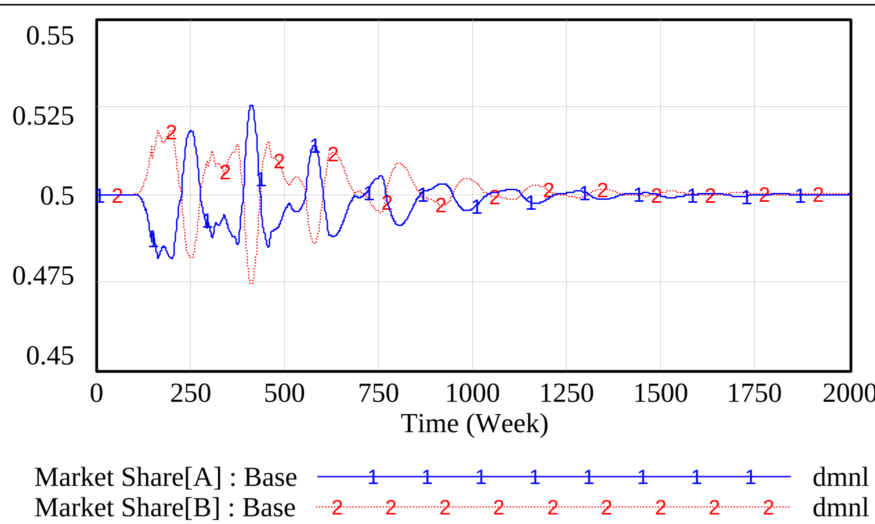


Figure 9. Base Run: Market Share

3. Hypotheses

I assume that firms interact only through the market share coupling. Firms sell products on the same market, but they do not base own production or pricing decisions on observations of competitors' quantities or prices. I will seek to show that the following hypotheses cannot be disconfirmed by our tests:

1. Entrainment takes place more easily when firms are driven by the same sine input.
2. Entrainment takes place more easily when firms' attributes are similar to each other, and firms' have similar natural periods of oscillation.
3. Entrainment takes place more easily when firms' outputs oscillate strongly (the system has low damping).

If individually these hypotheses hold, their combination may also hold:

4. Entrainment takes place more easily when firms are driven by the same sine input, their attributes are similar to each other and their outputs oscillate strongly.
5. Entrainment is less likely to take place, if at all, when firms are driven by different random inputs, their attributes are different to each other and their outputs are highly dampened.

A firm's natural period of oscillation will depend on its specific characteristics, such as the time to acquire new capacity. When firms have similar characteristics, it is likely that they oscillate with similar periods. While some characteristics impact the natural frequency of oscillation, others may impact the level of damping on the system. This is the case for the decision rule for capacity utilization. [4] When the systems do not oscillate much, it becomes hard to observe entrainment. Hence I hypothesize that entrainment can be more easily obtained when firms are similar and have less damping.

Additionally, firms are likely to behave in a different way when subjected to different exogenous inputs. A system perturbed by an exogenous input with a strong frequency of oscillation (sine input), is likely to respond with the same frequency. A system perturbed by a random input – a signal composed by a number of different frequencies – will amplify the frequency of oscillation characterized by the system natural frequency (Forrester, 1977). Hence firms subjected to the same sine input are more likely to entrain than firms subjected to different random inputs. The table below provides a framework to investigate the entrainment hypotheses, given the nature of inputs, firm's characteristics and firm's decision rules.

Table 1 – A framework to investigate business cycle entrainment.

Set B: Different Random Input

Set A: Same Sine Input		**Firm Characteristics**	
		Similar	***Different***
Decision Rules	***High Cap Utilization***	EE*	
	Low Cap Utilization		DE*

Note: EE* – Easy to obtain Entrainment; DE** - Difficult to obtain Entrainment.

[4] Appendix C explores the impact of capacity utilization on the oscillatory behavior of each firm.

4. Entrainment Analysis

4.1. Framework for Simulation Tests

Following the proposed framework, I generate two sets of tests (A & B), each with a different exogenous input. In each set of tests, firms can differ in two dimensions: (1) firm characteristics and (2) decision rule for capacity utilization. Firm's characteristics change if managers use different inventory correction times and face different capacity acquisition delays, the main drivers of the firm's natural periods of oscillation. Managers can also respond to changes in production with different capacity utilization decision rules. Table 2 presents the general framework for testing firm entrainment under different test inputs. The two sets of tests evaluate production behavior when firms are subjected to the same sine input and different random inputs, respectively.

Table 2 – Framework for simulation tests.

Set B: Different Random Input

Set A: Same Sine Input		**Firm Characteristics**	
		Similar	***Different***
Decision Rules	***High Cap Utilization***	Test 1	Test 2
	Low Cap Utilization	Test 3	Test 4

4.2. Test Implementation

To differentiate the firms' natural periods of oscillations, I use different time constants for inventory correction (TAI) and capacity acquisition (CAD).[5]

- Similar firm characteristics

  Time to adjust inventory: TAI[A] = 12 weeks; TAI[B] = 10 weeks; and,
  Capacity acquisition delays: CAD[A] = 60 weeks; CAD[B] = 50 weeks.

- Different firm characteristics

  Time to adjust inventory: TAI[A] = 12 weeks; TAI[B] = 6 weeks; and,
  Capacity acquisition delay: CAD[A] = 60 weeks; CAD[B] = 30 weeks.

To differentiate the managers' decision rules for capacity utilization, I use two different tables for capacity utilization:[6]

- High Capacity Utilization

  Managers maintain capacity utilization high even when desired production is low.

- Low Capacity Utilization

  Managers maintain capacity utilization low when desired production is low.

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[5] It is obviously possible to change one parameter at a time to obtain the natural periods of oscillation. But, since the goal is simply to obtain different natural periods of oscillation, we change the two parameters that can impact the periods, to test entrainment under the most general case creating the difference in the period of oscillation.

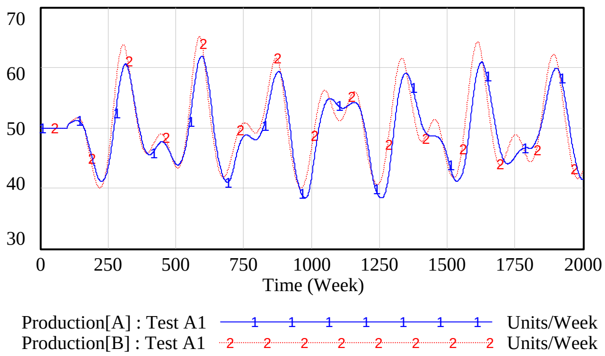
[6] Appendix C shows the decision functions for high and low capacity utilization.

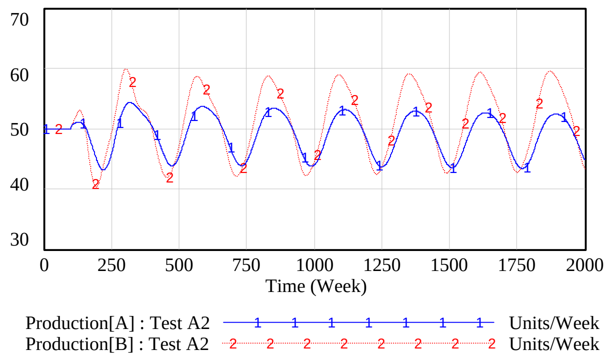
4.3. Hypotheses Testing

From the hypotheses, I expect entrainment to become harder to obtain as firms move from the same sine input to different random inputs. In addition, within each set of tests, I expect firms with similar periods of oscillation and low damping (test 1) to be more entrained than firms with different periods of oscillation and high damping (test 4). It is not clear a priory how tests 2 and 3 compare.

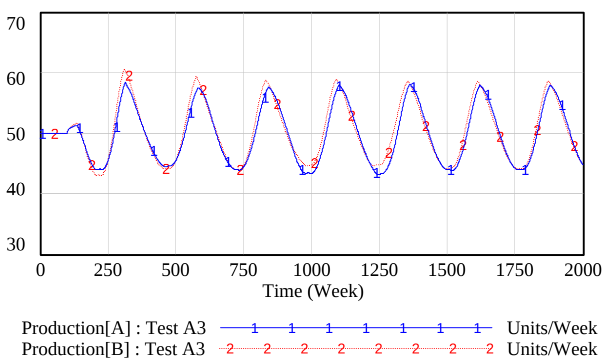
4.3.1. Set A: same sine input [7]

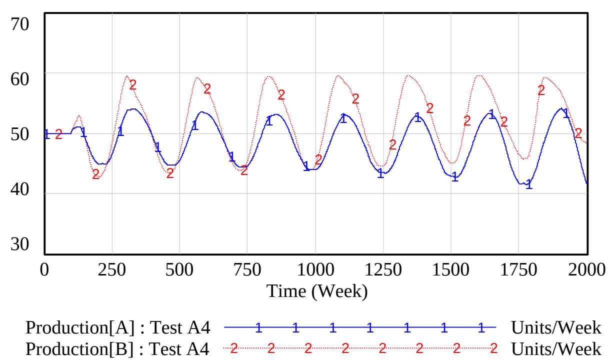
The results for the set of test A (figure 10) shows production for forty simulated years in which a sine input of amplitude of 10 units/week (10% increase) is introduced to total customer orders.[8] The model starts in equilibrium, with each firm retaining a 50% market share, and the shock is introduced at time 100 weeks.





(a) Test A1: high utilization & similar periods (b) Test A2: high utilization & different periods





(c) Test A3: low utilization & similar periods (d) Test A4: low utilization & different periods

Figure 10. Set A: Same sine input

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[7] This test input was introduced through an exogenous sinusoidal input to customer orders. Further details about this test input can be found in Appendix E.

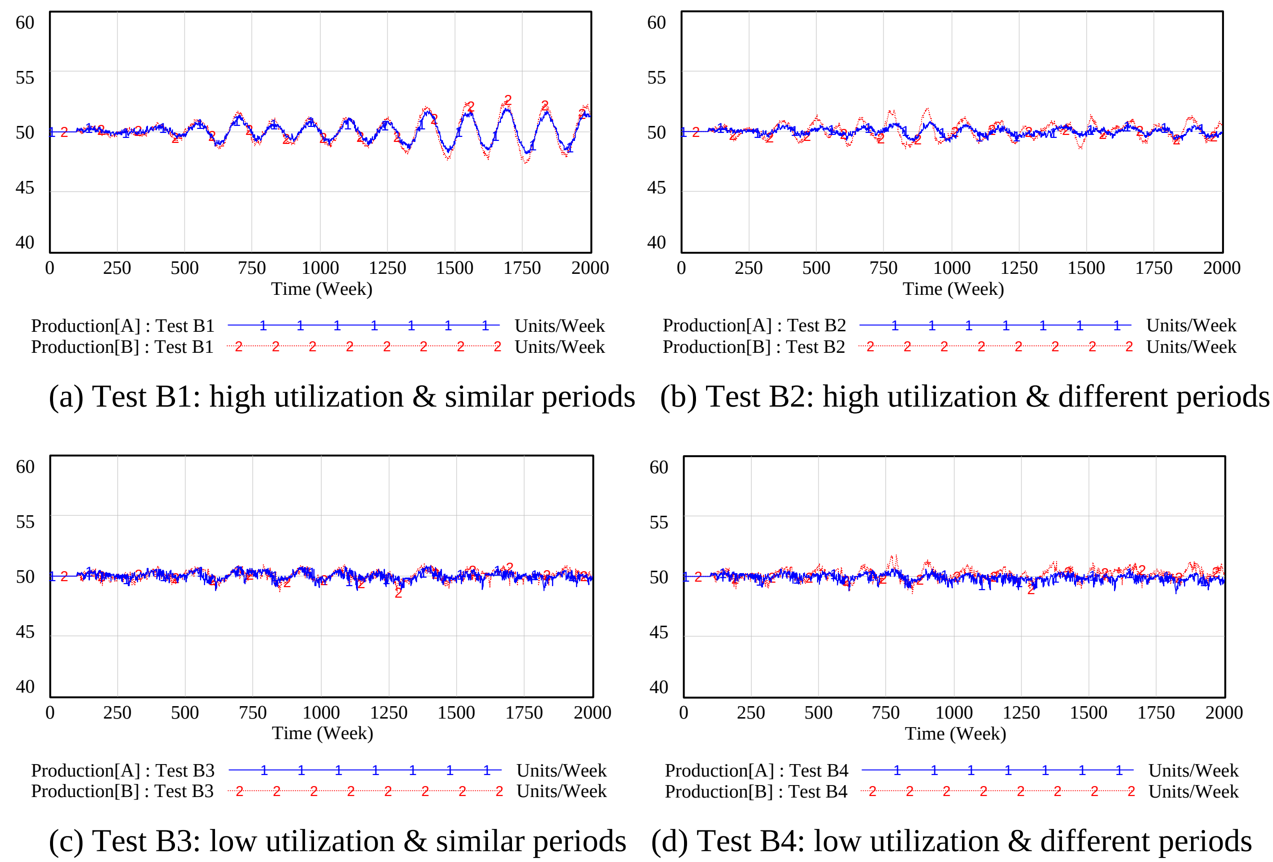
[8] Although we only show the behavior of production for the two firms, this is sufficient to give us an idea of the level of entrainment between them.

First, all plots look particularly similar, production oscillates with a period of five years. Second, Firms with different characteristics show different amplitudes for production, with higher amplitude to the firm with shorter natural period. Third, firms exhibited strong entrainment in all cases.

Comparing the results for tests A1 and A4, we cannot disconfirm the hypothesis that firms with high capacity utilization and similar periods of oscillation are more entrained than firms with low capacity utilization and different periods of oscillation.

4.3.2. Set B: different random input[9]

The results for the set of test B (figure 11) shows production for forty simulated years in which a different normal random input of amplitude of range 10 units/week (10% increase) is introduced to customer orders. The model starts in equilibrium, with each firm retaining a 50% market share, and the shock is introduced at time 100 weeks.



(a) Test B1: high utilization & similar periods (b) Test B2: high utilization & different periods

(c) Test B3: low utilization & similar periods (d) Test B4: low utilization & different periods

Figure 11. Set B: Different Random Inputs

First, there is a qualitative difference between this set of graphs and the previous ones. The period of oscillation is shorter, on the range of three years. Oscillations are filtered from the range of frequencies introduced in the random shock and amplified by the production system.

[9] The different random input was introduced through two different exogenous normally distributed input to orders to firm A and B. Further details about this test input can be found in Appendix E.

The frequency reflects the ability of the system to generate its own behavior and not the frequency introduced by an exogenous shock. In this set of tests, individual firm's characteristics play a bigger role in production output (capacity utilization for instance played a smaller role in the earlier set of tests). Since the only difference between the two sets was the nature of the input, this suggests that the sine input has greater effect in driving the system to behave in a particular way.

Second, the simulation results corroborate the analytical finding that firms with high capacity utilization (tests B1 and B2) exhibit a stronger oscillatory behavior (low damping) than firms using low capacity utilization (tests B3 and B4).

Third, firms still exhibited strong entrainment. But this effect seemed to be more pronounced in firms with high capacity utilization decision rules. In part due to the fact that it becomes harder to observe entrainment when we cannot see the oscillation pattern. Firms with distinct parameters exhibited some variation in amplitude, even though it still retained a similar pattern of oscillation. Here too, firms with shorter time constants had higher amplitudes.

Fourth, it is impressive that even under different random shocks and different decision rules, the firms still exhibit some entrainment. This takes place despite the fact that, firms in B2 and B4 do not seem as entrained as firms in tests B1 and B3.

Finally, comparing set (A) with set (B), the firms seem to be more closely entrained in set (A). This does not disconfirm the hypothesis that it was easier to obtain entrainment with same sine inputs.

4.4. Preliminary results

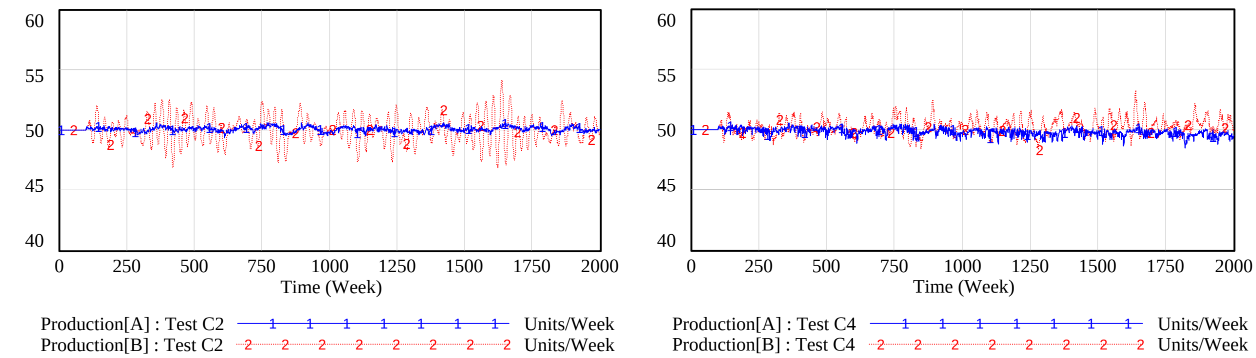
The set of tests above suggests three important conclusions. First, the results don't contradict the hypothesis that firms are more likely to entrain if they adopt similar parameters for their decision rules. Similar firms seem to be more in phase than different firms. Second, they don't contradict that firms are more likely to entrain when they are subject to the same sine inputs. Firms subjected to the same sine input seem to be largely driven by the exogenous input. Third, the results also do not seem to contradict the hypothesis that firms are more likely to entrain if they oscillate strongly (low damping). Managers using high capacity utilization decisions increase the firms' tendency to oscillate allowing a greater potential for entrainment.

More importantly, the result in test B4 suggests that firms still entrain even though they are different, driven by different random shocks, and coupled only through the market share mechanism. This suggests that (a) even an indirect coupling mechanism, such as market share, is sufficient to promote firms to entrain, given that they are not completely different; and, (b) if firms become sufficiently different they may not entrain at all. To test these two hypotheses, I run two additional sets of tests.

4.4.1 Set C: Different Firms

In set C, firms differ a lot from one another (TAI [Firm] = 12,2 weeks; CAD[Firm] = 60,10 weeks) and are subjected to different random shocks. This test will help us identify the impact of firm difference on firm entrainment.

Figure 12 shows forty production years for two very different firms. Results for test D2 and D4, seems to confirm the hypothesis that when firms are sufficiently different entrainment does not take place. But it is not possible to compare the peaks and valleys of both production curves to investigate whether they are entrained. Both tests show large differences in the periods of oscillation and amplitude for the two firms. Production in firm B is characterized by high frequency/high amplitude oscillation; production in firm A shows a high frequency/low amplitude oscillation.



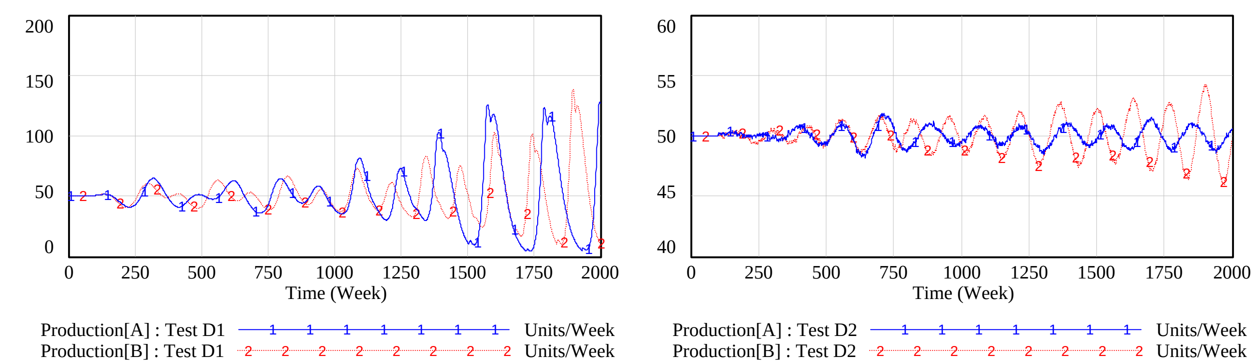
(a) Test C2: high utilization & different periods (b) Test C4: low utilization & different periods

Figure 12. Set C: Different Firms Under Different Random Inputs

4.4.2 Set D: Independent Firms

In set D, the market share mechanism is removed, so firms do not interact with one another.[10] In this test, similar firms operate with a high capacity utilization, to induce oscillatory behavior. Firms are subjected to the same sine input (test D1) and different random inputs (test D2).

Figure 13 shows forty years of production, when firms are independent. The plots suggest that independent firms do not entrain. This suggests that the indirect market share coupling and the feedbacks from market share are the only mechanisms responsible for entrainment in the model.



(a) Test D1: uncoupled firms & same sine input (b) Test D2: uncoupled firms & diff. random inp

Figure 13. Set D: Independent Firms Under Different Inputs

The independent firm set of tests is similar to the situation where firms are monopolies. In such case, production will be independent in each of the two firms and individual

[10] To implement this test, we run the two firms independently in a version of the model where there is no feedback from market share.

characteristics and different random shocks will cause firms to behave differently. In the case where the two firms compete for market share, they behave as a duopoly where market share is determined by two product dimensions: delivery delay and price. Through their market share interaction, firms will set production to reduce the delivery delay and price. Therefore, even though the companies are different and they are driven by different random shocks their production will be influenced by each other through the market share mechanism. The next section provides a closer look into such claims.

5. Feedback Analysis

First, an important characteristic of the model for coupled firms is that the market is shared among them. So when market share is high for firm A it is low for its competitor, firm B. Figure 14 (base run) shows that while orders and order fulfillment, for the two firms, are perfectly out of phase, backlogs are almost perfectly entrained.

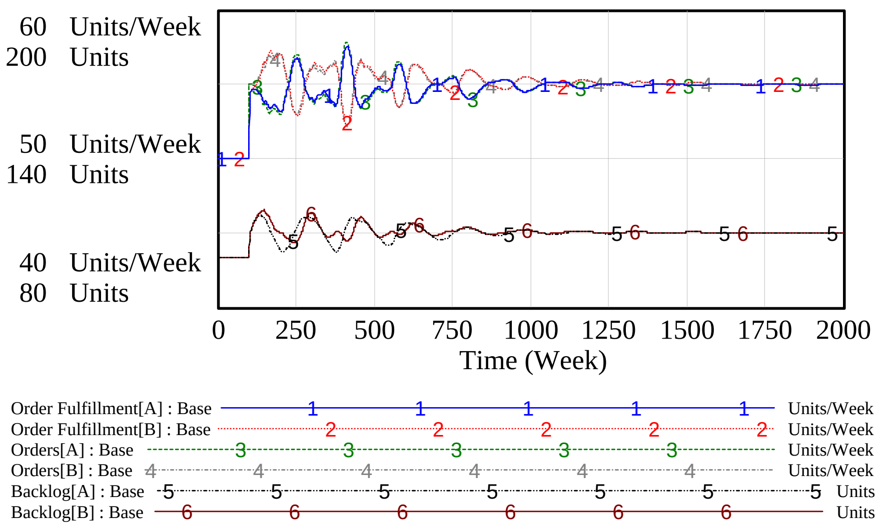


Figure 14. Orders, order fulfillment and backlogs for both firms.

Note also that firm A, with a longer natural period of oscillation, always ships more than ordered when it has low market share; and ships less than ordered when it has a high market share. Since it has a long natural period, firm A will take longer to adjust to shifts in demand. Hence it will not cut back capacity so aggressively in downturns, and not increase capacity as fast during upturns. The net result for firm A is that *backlogs increase in upturns*, since it cannot fill orders as quickly, and *backlogs decrease during downturns*, since it still has plenty of capacity available to meet demand.

The situation is the opposite for firm B. With a short natural period of oscillation, it always ships less than ordered when it has low market share; and ships more than ordered when it has a high market share. With a short natural period, firm B will quickly adjust to shifts in demand. Hence it will cut back capacity aggressively in downturns, and increase capacity rapidly during upturns, causing it to overshoot. The net result for firm B is that *backlogs decrease in upturns*, since it fills orders more rapidly than they come in, and *backlogs increase during downturns*, since its little available capacity does not allow it to meet demand. Finally, as firms compete in same market place, upturns for firm A correspond to downturns to firm B and viceversa. Hence the out-of-phase market shares for each firm permit that backlogs entrain as a response to each firm's internal production characteristics.

Figure 15 shows the causal loop diagrams responsible for entrainment in the model. There are two reinforcing loops reflecting the influence of long-term delivery delay and price response to supply availability. As orders for a product increase, the company builds capacity allowing it to produce more. Over time, production grows permitting the firm to deliver products with lower delivery delays and prices. Cheap and available products boost attractiveness for the firms' products, further increasing orders. The positive loops are destabilizing as they differentiate the behavior of the two firms – promoting the increase in market share for a firm and simultaneously decreasing the market share for its competitor. In this context, the reinforcing loops increase the amplitude of the business cycle.

On the other hand, two balancing loops counteract the long-term effects mentioned above with short-term adjustments to the imbalances in supply and demand. As orders for a product increase, the company's inventories erode, increasing delivery delay and prices to customers. Expensive and scarce products reduce attractiveness. Customers turn to competitors, decreasing orders for the firms' products. The balancing loops take place through the substitution between products, allowing the firms to compensate for each other's changes in production and providing the mechanism by which they can entrain.

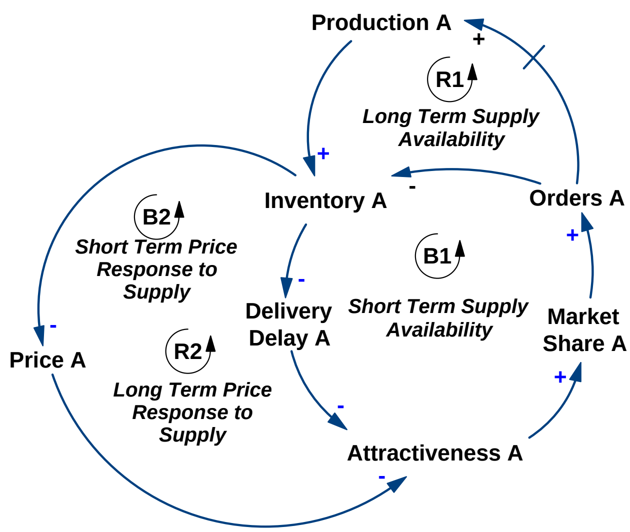


Figure 15. Causal loop diagram for the coupling mechanisms

It is possible to test the influence of the feedback processes by cutting links on the diagram above. In the following tests, I consider the substitution effects from delivery delay and prices separately.

5.1. Price Insensitivity

This run represents a situation where customers are insensitive to price, hence the product demand balances through shortage and availability of products. This would be equivalent as firms selling products with fixed prices or prices that were too small to matter.

Figure 16 shows that entrainment still takes place even when prices are not responsible for the substitution between products. Since the link from prices to attractiveness is not present, feedback loops R2 and B2 are cut in this simulation. Entrainment takes place due to the effects of loops R1 and B1. So, the substitution effect due to delivery delay is sufficient to generate entrainment between the two firms.

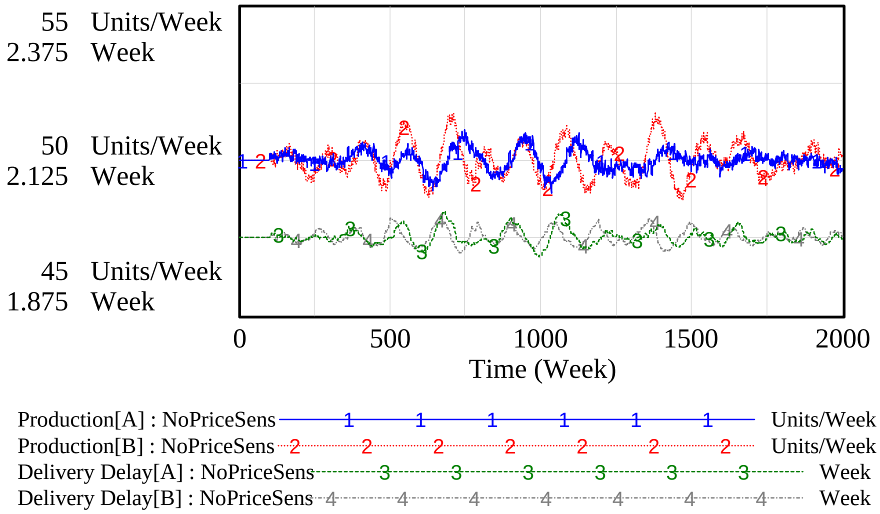


Figure 16. Production and Delivery Delay for Price Insensitive Market

5.2. Delivery Delay Insensitivity

This run represents a situation where customers are insensitive to product availability; hence the product demand balances through high and low product prices. An equivalent condition would take place if both products were readily available or delivery delays were fixed for the two firms.

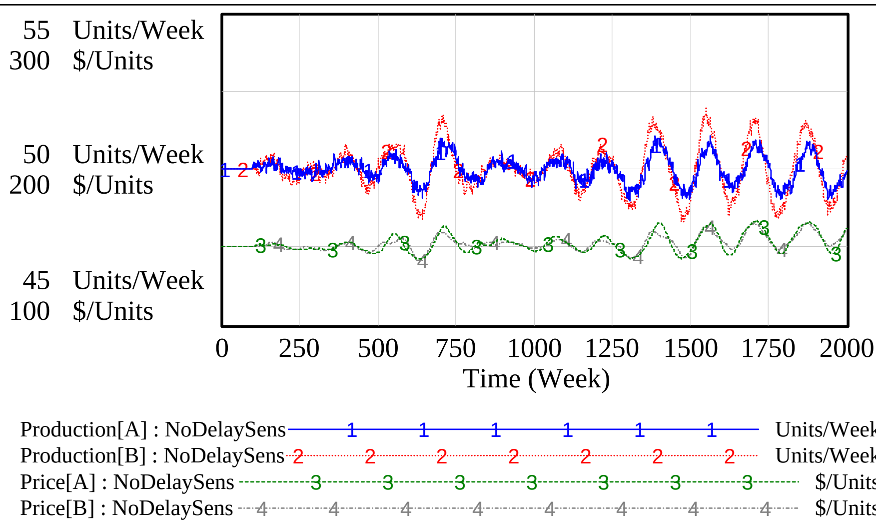


Figure 17. Production and Prices for Delivery Delay Insensitive Market

Figure 17 shows that entrainment still takes place even when delivery delays do not contribute to the substitution between products. Since the link from delivery delay to attractiveness is not present, feedback loops R1 and B1 do not operate in this simulation. Entrainment takes place due to the effects of loops R2 and B2. So, the substitution effect due to price is sufficient to generate entrainment between the two firms.

6. Conclusion

Models in the system dynamics tradition often consider that firms can be aggregated in a single sector. The implicit assumption is that individual firms move in phase with each other. Through a set of simulation tests of a simple model with two distinct firms and a market share coupling mechanism, firms displayed entrainment even though they *differed* from one another and were driven by *different random* inputs.

A series of simulation tests was not able to disconfirm that when firms are driven by the same sinusoidal inputs they are more likely to get entrained than when they are subjected to different random shocks. Additionally, the results suggest that *similar* firms using *high* capacity utilization decision rules are more likely to entrain than *different* firms with *low* capacity utilization rules, but for the majority of tests I ran they still were able to entrain. The market share coupling may be unable to promote entrainment between firms when they differ *substantially*.

Moreover, when the market share mechanism was excluded from the model, even *similar* firms driven by the *same sine* input did not move in phase with each other. This result suggests that, in our model, the market share mechanism is sufficient to cause different firms to entrain. Additionally, the balancing loops, that adjust short-term imbalances in supply and demand, allow the firms to compensate for each other’s changes in production and provide the mechanism by which they can entrain. When the substitution effects of product quality and price are not present firms cannot offset the discrepancies between own product attributes and competitor’s, so entrainment never takes place. The tests also suggest that price and delivery delay substitution effects are sufficient to promote business cycle entrainment.

Given that firms within an industry are commonly linked through market share, it does not seem unusual to observe lots of entrainment among firms. Therefore, the traditional system dynamics simplifying assumption of modeling an industry as one aggregated single firm usually makes sense.

6. References

Forrester, J.W. 1961. *Industrial Dynamics*. Cambridge, MA: Productivity Press.

——— .1977. Growth Cycles. *De Economist* 125: 525-543.

Graham, Alan K. 1977. *Principle on the Relationship between Structure and Behavior of Dynamic Systems*. Unpublished Ph.D. Dissertation. Massachusetts Institute of Technology.

Haxholdt, C., C. Kampmann, E. Mosekilde, and J. Sterman. 1995. Mode-locking and Entrainment of Endogenous Economic Cycles. *System Dynamics Review*, 11 (3). 177-198.

Homer, J. 1980. The Role of Consumer Demand in Business Cycle Entrainment. Working paper D-3227-1, System Dynamics Group, MIT E60-375.

Sterman, J.D. 2000. *Business Dynamics: Systems Thinking and Modeling for a Complex World*. Chicago: Irwin-McGraw Hill.

Sterman, J.D., and E. Mosekilde. 1994. Business Cycles and Long Waves: A Behavioral Disequilibrium Perspective. In *Business Cycles: Theory and Empirical Methods*, ed. W. Semmler, 13-51. Dordrecht: Kluwer Academic Publishers.

Zarnowitz, V. 1985. Recent Work on Business Cycles in Historical Perspective: A Review of Theories and Evidence. *Journal of Economic Literature* 23, 523-580.

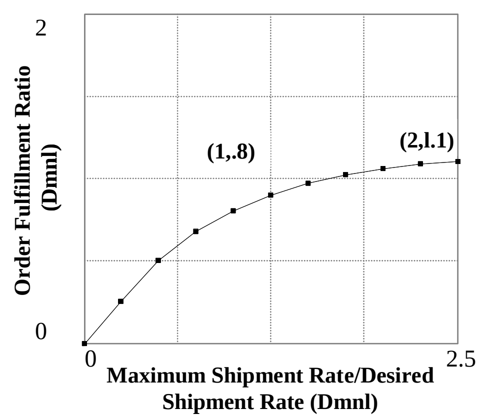
Appendices

Appendix A — Standard formulation for Shipments

The product of the desired shipment rate and the order fulfillment ratio determines shipments. While the desired shipment rate is determined by the ratio of backlog and the target delivery delay, the order fulfillment ratio depends on the ratio of the maximum to desired shipment rate. The latter is determined by the ratio of inventory and minimum order processing time. The table for order fulfillment assumes a firm with several products in stock. When the maximum shipment rate is small relative to the desired shipment rate, the firm only fills a fraction of the inventory it can ship. When the maximum shipment rate equals the desired shipment rate, the firm is able to fill about 80% of its orders. As the maximum shipment rate increases relative to the desired, the firm fills a greater fraction of its orders. Finally, as the maximum shipment rate increases further, managers can choose to ship faster than the desired shipment rate to run down backlog and decrease its delivery delay.

Shipments (Units/Week) = Desired Shipment Rate * Order Fulfillment Ratio
Order Fulfillment Ratio (Dmnl) = Table OF(Maximum Shipment Rare/Desired Shipment Rate)
Table for Order Fulfillment (Dmnl) = [(0,0)(2.5,2)],(0,0),(0.25,0.25),(0.5,0.5),(0.75,0.68),(1,0.8),
(1.25,0.9),(1.5,0.97),(1.75,1.02),(2,1.06),(2.25,1.085),(2.5,1.1),(10,1.1)
Maximum Shipment Rate (Units/Week) = Inventory/Minimum Order Processing Time

Table for Order Fulfillment Ratio



The combined assumption of partial fulfillment – when maximum shipment rate equals desired shipment rate, the order fulfillment ratio differs from one – and the possibility of running down backlogs – when maximum rate is high relative to desired, the fulfillment ratio can go above one – makes it more complicated to find the equilibrium to this model. In order to determine the equilibrium level of initial inventory (desired inventory) I start from the desired shipments' equation:

$$S^* = \frac{B}{TDD}; B^* = TDD \cdot CO$$

Initially, backlog is set at the same level as the desired backlog, such that:

$$S^* = \frac{B^*}{TDD} = CO \quad (1)$$

For the backlog to be in equilibrium, the inflow must equal the outflow. Hence customer orders must equal order fulfillment. In addition, the latter is equal to shipments.

$$\dot{B} = 0 \Rightarrow CO = OF = S \quad (2)$$

Also, from the equation for shipments and the result (1) for desired shipments, one obtains:

$$S = OFR \cdot S^* = OFR \cdot CO \quad (3)$$

Comparing result (3) to result (2), one must deduce that the order fulfillment ratio equals one ($OFR = 1$). Using the definition of the order fulfillment ratio and taking its inverse:

$$OFR = f(\frac{MS}{S^*})$$

$$f^{-1}(1) = \frac{I^* / MOT}{CO} \quad (4)$$

The result (4) relates the inverse order fulfillment function ($f^{-1}(.)$) to desired inventory (I^*) the maximum order processing time (*MOT*) and customer orders (*CO*), from which one can obtain the equilibrium value for the desired inventory.

$$I^* = f^{-1}(1) \cdot MOT \cdot CO$$

From the table for the order fulfillment ratio and using interpolation, I obtain the value of $f^{-1}(1)$ equal to 1.65, leading to a desired inventory level of 330 units. Alternatively, the formulation using the fraction shipped provides a simple formulation and an easy way to obtain the desired inventory. But the drawback of that formulation is that the table for the fraction shipped doesn’t have such an intuitive meaning and may be harder to explain.

Appendix B — Desired Production using a backlog adjustment formulation

One must account for three inputs in the formulation for desired production: the inventory adjustment, the backlog adjustment and the expected order rate. Also one must impose that desired production be positive.

Desired Production Rate (Units/Week) = MAX(0, Expected Orders + Inventory Adjustment – Backlog Adjustment)

Rewriting the equation above in terms of its components we obtain:

$$P^* = EO + \frac{I^* - I}{TAI} - \frac{B^* - B}{TAB}$$

And, in equilibrium, the desired backlog equals the product of shipments by the target delivery delay, which equals expected orders times target delivery delay.

$$B^* = Ship \cdot TDD = EO \cdot TDD$$

Replacing the last identity for desired backlog, I obtain:

$$P^* = EO + \frac{I^* - I}{TAI} - \frac{EO \cdot TDD - B}{TAB}$$

If the Time to Adjust Backlog (*TAB*) equals the Target Delivery Delay (*TDD*) the equation above is reduced to:

$$P^* = \frac{I^* - I}{TAI} + \frac{B}{TDD} = IA + S^*$$

Where the desired production is simply the sum of the inventory adjustment and desired shipments. This is what I have used instead of the standard formulation. A problem with the standard formulation may arise when $TAB < TDD$:

$$P^* = \frac{I^* - I}{TAI} + \frac{B}{TAB} - EO \cdot \left(\frac{TDD - TAB}{TAB}\right) = IA + \frac{B}{TAB} - \beta \cdot EO \qquad \text{, with } \beta > 0$$

Since β is positive, an increase in expected orders leads to a decrease in desired production. Although this is not reasonable, it can take place in the standard formulation, if the modeler fails to set the time to adjust backlog above the target delivery delay ($TAB > TDD$). In general, as $TAB > TDD$ we get:

$$P^* = \frac{I^* - I}{TAI} + \frac{B}{TAB} + EO \cdot \left(\frac{TAB - TDD}{TAB}\right) = IA + \frac{B}{TAB} + \alpha \cdot EO \qquad \text{, with } \alpha > 0$$

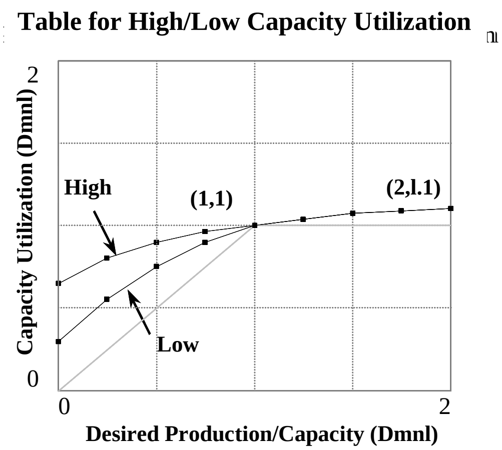
While formulating desired production in terms of the sum of the inventory adjustment and the desired shipment rate is simple and elegant, it carries some potential drawbacks. For example, if orders vary widely from week to week, so will the desired shipment rate and ultimately desired production. But a production scheme that is too oscillatory may not be followed factory managers. When possible, factory managers will try to schedule its operations smoothly, avoiding costly change of set-ups, reduced batch sizes, etc. If the factory does not follow the schedule proposed by desired production, our model representation would be inadequate and unrealistic. At times though, factories may be forced to alter production constantly to meet production quotas. If this is the case, the formulation proposed could be a good representation of the firm.

The standard formulation with the adequate time to adjust backlog (*TAB* > *TDD*) will prevent this from happening. First, by adjusting backlog over some period (*TAB*) longer than what was previously being adjusted (*TDD*). Second, by smoothing orders through its smoothed input: the expected order rate.

__Appendix C — Impact of high (low) capacity utilization on oscillatory behavior__

The model has a tendency to behave as a second-order damped oscillator. An important characteristic of the "Capacity Utilization Function" is that it can influence the dampening ratio of the oscillating behavior of the model, i.e., the response to a step input in demand.

Firms have stronger (weaker) oscillations when they maintain their capacity utilization high (low) even when desired production (relative to capacity) is low. Below, we explain why this takes place.



First we linearize the capacity utilization function. A good approximation of the curve at an operating point where DP/C < 1 is given by a line crossing the y-axis at a constant value y_0.

$$f(x) = \alpha \cdot x + y_0$$

Where α is the slope of the line. Now, considering that the x axis is given in terms of P^*/C and that y_0 is equal to 1- α [11], we have:

$$f(\frac{P^*}{C}) = \alpha \cdot \frac{P^*}{C} + (1-\alpha)$$

Simplifying, we obtain:

$$f(\frac{P^*}{C}) = 1 + \alpha \cdot (\frac{P^*}{C} - 1)$$

Now let us consider the equation for production, which is determined by the product of capacity and capacity utilization:

$$P = C \cdot CU$$

$$P = C \cdot f(\frac{P^*}{C})$$

Now, substituting the linearized formula for the capacity utilization function, we get:

[11] The slope α is determined by the ratio of the y rise and the x length. Since the length ranges from 0 to 1 and y goes from y_0 to 1, we get that α = 1- y_0.

$$P = C \cdot \left[1 + \alpha \cdot (\frac{P^*}{C} - 1) \right]$$

$$P = C + \alpha \cdot (P^* - C)$$

When capacity utilization is low, the slope of the linearized function is close to one ($\alpha \approx 1$), then production is approximately the same as desired production. The influence of capacity in determining production is very small. And, the oscillatory behavior due to the delay in capacity acquisition only plays a limited role.

On the other hand, when capacity utilization is high, the slope of the linearized function is much smaller than one ($\alpha << 1$), then production is approximately the same as capacity. Since the slope differs from one, desired production still has an influence in determining production, but the influence of capacity in determining production is strong. Therefore, the strong influence of capacity in determining production ensures that the oscillatory behavior takes place, due to the capacity acquisition delay.

Appendix D — Substitution Effects for Price and Delivery Delay

This section borrows largely on Jack Homer's 1980 paper on the role of consumer demand in business cycle entrainment. By construction relative price and delivery delay changes impact firm's attractiveness with the same magnitude. Low quantities (prices and delivery delays) are preferred to high ones. When price for a product goes up, its attractiveness to customers goes down and the firm looses market share. Analogously, a decrease in product availability drives customers to the competitor firm.

To answer just how strong this effect is, one must compute the price and delivery delay substitution effects. In this model, the elasticities of substitution are embedded in the table functions for the effect of price (delivery delay) on attractiveness and they are fixed. Homer (1980) presents an alternative formulation where the price elasticity of substitution can be adjusted by changing the value of a model parameter. The price (delivery delay) elasticities are evaluated at the "normal" operating point where $p_1/p_r = p_2/p_r = 1$ and $DD_1/DD_r = DD_2/DD_r = 1$, such that $Att_A = Att_B = 1$. Using the equation for market share and the fact that the equation for the table function can be approximated by a straight line with a slope of –1, in the vicinity of the operation point, we can derive own- and cross-price (delivery delay) elasticities of substitution.

The own-price and cross-price elasticity of substitution are given below:

$$\varepsilon_{PAA} = \frac{dMS_A / MS_A}{dP_A / P_A} = -1/2 \qquad \varepsilon_{PAB} = \frac{dMS_A / MS_A}{dP_B / P_B} = +1/2$$

The own-delivery delay and cross-delivery delay elasticity of substitution are given below:

$$\varepsilon_{DDAA} = \frac{dMS_A / MS_A}{dDD_A / DD_A} = -1/2 \qquad \varepsilon_{DDAB} = \frac{dMS_A / MS_A}{dDD_B / DD_B} = +1/2$$

Hence, an increase of 1% in price of product A will have a 0.5 % decrease in the market share of product A. Adding the own-price and cross price elasticities, allows me to evaluate the impact of the proportional increase in both prices. And since the sum is zero, it indicates that there will be no change in market share if both prices are increase by the same amount.

Appendix E — Test Inputs

All test inputs were introduced through the exogenous customer orders. Total customer orders were set at 100 units/week and each firm has 50% of the market initially. The model is originally set in equilibrium and at time 100 weeks the shock is introduced to the system.

Same step input:

The step input, used in the base case scenario, was set with amplitude of 10 units/week (10% increase) for each firm. In order to introduce this test, we set the switch for step input equal to one and all other switches to zero. The following equation was used for its implementation:

Step Input (Units/Week) = STEP (10,100)

Same sine input:

The sine input was set with amplitude of 10 units/week (10% magnitude) and period of five years. In order to introduce this test, we set the switch for sine input equal to one and all other switches to zero. The following equation was used for its implementation:

$$\text{Sine Input (Units/Week)} = A \cdot \sin\left(\frac{2\pi \cdot t}{260}\right)$$

Same random input:

The random input consists of a normally distributed random noise with a maximum value of 10 units/week (10% increase), a minimum value of –10 units/week (10% decrease), an average of zero, a standard deviation of 5 units/week (5% change) and a noise seed of 1. In order to introduce this test, we set the switch for same random input to one and all other switches to zero. The following equation was used for its implementation:

Random Input (Units/Week) = RANDOM NORMAL (-10, 10, 0, 5, 1)

The random noise will randomly extract values from a normal distribution with mean and standard deviation set above. A more realistic random test would use “pink” noise, where subsequent draws are auto correlated. The test imposed here is therefore more stringent than that with a “pink” noise. If firms are able to entrain under a more stringent condition, they are more likely to entrain under a more realistic and less stringent test.

Different random input:

The different random shocks to each firm retained the distribution assumption from the earlier test but use half the maximum value, minimum value, and standard deviation values. Also, the shocks differ in the noise seeds for each firm (1, 2). In order to introduce this test, we set the switch for different random input to one and all other switches to zero. The following equation was used for its implementation:

Random Shock A (Units/Week) = RANDOM NORMAL (-5, 5, 0, 2.5, 1)
Random Shock B (Units/Week) = RANDOM NORMAL (-5, 5, 0, 2.5, 2)