Discrete Versus Continuous Formulation: A Case Study Using Coyle's Aircraft Carrier Survivability Model

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Introduction

The issue of discrete versus continuous formulation of system dynamics models has provided for lively, thought provoking discussion among modeling practitioners for more than thirty years. As early as 1961, Forrester argued that models of feedback systems should be formulated as continuous models based on the philosophy that real systems are continuous. Forrester used the example of a contract signing, normally a discrete event, and explained how contracts can be viewed as continuous when one considers that contracts go through a process of negotiations. As the expectations associated with a contract signing increase, the parties begin preparations so that the contract obligations can be fulfilled (1961; p. 65). Thus, a contract signing represents a continuous process. The debate of discrete versus continuous has continued into the 1990's with Richardson (1991) maintaining that the decision to model a problem discretely or continuously depends on the conceptual distance from which one views a problem. Richardson (1991) makes a strong argument for a continuous perspective when he writes, "Only from a more distant perspective in which events and decisions are deliberately blurred into patterns of behavior and policy structure will the notion that 'behavior is a consequence of feedback structure' arise and be perceived to yield powerful insights" (p. 346).

Although the concept of formulating a system dynamics model in terms of continuous flows and interactions of variables has always been a basic premise of the system dynamics approach (Forrester, 1961: p. 64), the ability to develop discrete models has been built into system dynamics software in the form of clip, min, max, and if-then-else functions. Discrete functions are used most often by system dynamicists to test policies which are turned on during a simulation run. In 1992, Coyle used a model which relied heavily on discrete formulations to analyze different policies of aircraft carrier survivability in enemy waters. Coyle argued that this approach was necessary since aircraft carriers are costly entities and need to be measured and treated as integers (p. 194). Coyle's choice to use a discrete model is supported by Pidd's (1992) argument that discrete models are appropriate when the tracking of individual entities is important.

This paper examines two versions of Coyle's aircraft carrier model -- Coyle's original model which combined discrete and continuous formulations and a version developed by the author which converts Coyle's discrete formulations into continuous formulations. A comparison of the two models is used to explore the differences in equation formulation, model behavior, and policy implications.

Equation Formulation

Coyle's original model was written in the DYSMAP programming language and contained 18 discrete equations and 52 continuous equations. The author's continuous model was developed in two stages. The first stage involved converting Coyle's model from DYSMAP to STELLA, where the second stage dealt with writing the discrete equations as continuous equations.

The discrete equations used by Coyle take the form of clip, min, max, and if-then-else functions or, in a few instances, combinations of these functions. Two methods were considered in converting the discrete equations to continuous equations. The first involved using standard continuous formulations to replace all of the discrete formulations. For example, the formulation for a min function can be written discretely as:

$$\text{Rate} = \min(x,y) \qquad (1)$$

or continuously as:

$$\text{Rate} = x * f(y/x) \qquad (2)$$

where the function of f(y/x) would be similar to that in Figure 1 while the continuous formulation of a max function, max(x,y), would be the same as equation 2. It is important to note, however, that the structure of a continuous version of a discrete min or max function would require the addition of one constant and four auxiliary variables.

Figure 1

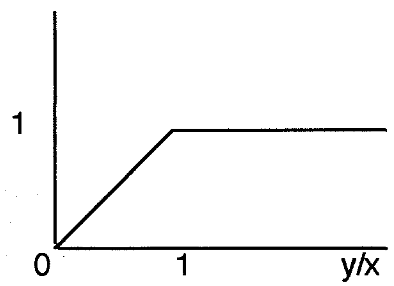
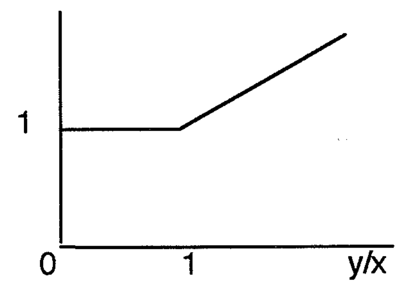


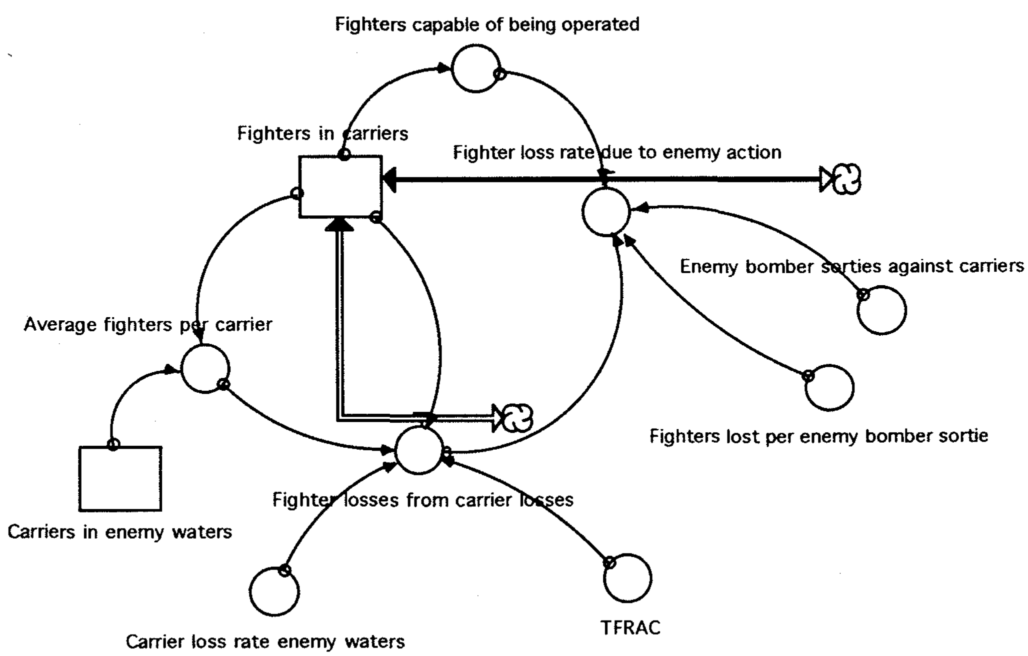
Figure 2



The second approach which can be used to change a discrete equation to a continuous equation is to understand the meaning of the discrete equation and rewrite it as a continuous equation. This method was used to convert Coyle's discrete equations to continuous equations as it limited the need to add additional variables not in the Coyle's original model.

An interesting example of the issues which need to be addressed when converting a discrete equation to a continuous equation can be found in Coyle's rate variable for Fighter loss rate due to enemy action. A STELLA diagram of this structure is contained in Figure 3.

Figure 3



The discrete STELLA equation for Fighter loss rate due to enemy action is as follows:

Fighter loss rate due to enemy action =
Max(0,min(Fighters capable of being operated / DT, Enemy bomber sorties against carriers * Fighters lost per enemy bomber sortie)
- Fighter losses from carrier losses)

The max part of this equation tells us that the rate, fighter loss rate due to enemy action, cannot go negative. The actual number of fighters lost is thus any positive number generated by the min function.

The min function indicates that the number of fighters lost is either the number of fighters remaining in the level, i.e., Fighters capable of being operated / DT, or the number shot down by enemy bombers minus those fighters lost due to carriers sinking. There are two problems with this formulation. The first problem concerns the formulation for Fighters capable of being operated / DT. This formulation raises a basic question: what are the units since dividing by DT has no real meaning? Despite this obvious question, dividing by DT does serve a purpose in this discrete formulation. It eliminates the number of fighters remaining in the level in one DT when the number of fighters remaining in the level is less than the number of fighters that could be shot down. The second problem with the discrete formulation listed above concerns the subtraction of the rate Fighter losses from carrier losses. This is necessary since the rates Fighter loss rate due to enemy action and Fighter losses from carrier losses are both discretely formulated outflows from the same level. This requires that some decision be built into the formulation which prioritizes the outflows when the number of fighters demanded by the two outflows is greater than the number of fighters left in the level. By subtracting Fighter losses from carrier losses from Fighter loss rate due to enemy action, Coyle gives higher priority to Fighter losses from carrier losses. This formulation is completely biased since one outflow is always favored over the other. From a philosophical perspective, Coyle is arguing that fighter aircraft in the air, which cannot be transferred to other aircraft carriers, are eliminated when their aircraft carrier sinks. The more likely scenario is that fighters continue to fight until they either get shot down by enemy fighters or crash into the ocean when they run out of fuel.

The continuous formulation eliminates these problems through appropriate first order controls. The continuous formulation is as follows:

Fighter loss rate due to enemy action =
Enemy bombers sorties against carriers * Fighters lost per enemy bomber sorties *
Effect of fighters in carriers on flrfea

This formulation states that the number of fighters shot down by the enemy is determined by multiplying the number of enemy bomber sorties against carriers by the average number of fighters lost per enemy bomber sortie times the effect of fighters in carriers on flrfea. The variable, effect of fighters in carriers on flrfea, is a first order control in the form of a nonlinear table function that gradually shuts the rate down as the fighters in carrier level approaches zero. This formulation serves two purposes. First, it eliminates division by DT, and second it eliminates the bias related to having Fighter losses from carrier losses dominate the outflow Fighter loss rate due to enemy action. The outflows are completely unbiased in the continuous formulation. This is accomplished through the addition of the two new first order control variables in the form of nonlinear table functions. The new variables are each a function of the level, Fighters in carriers, and their table functions are identical.

Conclusions

In Coyle's aircraft carrier survivability model, the transformation from a discrete set of equations to a continuous set of equations eliminated issues of bias and division by DT in a number of rate equations. Although the two models behaved differently the difference was not significant. The three primary policies which Coyle tested, sending aircraft carriers into enemy waters singly, in pairs, and in pairs with double fighter effectiveness, had similar results for both models.

The results of this exercise, which show no significant behavioral differences between the discrete and continuous models, lend support to the argument that modelers need to focus on three basic issues when faced with the decision of whether to use discrete or continuous formulation. First, how would the model's consumer draw graphs over time of key variables? Do they draw them continuously or discretely? Second, is the formal model going to be a black box or an open box to the model consumers? If it is a black box, the equation formulation does not need to be understandable to the model consumers. If it is an open box, then the equation formulation must be in terms easily understandable to the model's consumers. Third, does system behavior change substantially when specific variables reach a threshold level? Or do systems, as Forrester (1961) argued, gradually build up pressure to react in a specific way to changes in system variables? Forrester (1961) also argued that discontinuous variables should be explored only when they have a significant influence on behavior (p. 66). It is obvious, however, both from this work with Coyle's model and from the literature that social scientists, and feedback thinkers in particular, will continue to choose discrete or continuous formulations based on their own philosophical perspective.

References

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