The Analysis of Newly Gained Feedback Loops after Introducing the Grade-Salary Incitement to HR Management System

1. Introduction

When we do feedback analysis on a complex system, we always wonder how many feedback loops were added when it changed. In this paper, we post a theorem on getting all newly gained feedback loops by the multiplication operation of the 0-diagonal branch-vector matrixes. Then we post a matrix algorithm operated easily on it, which is very useful for the feedback analysis on HR management so as some other management systems.

2. Matrix algorithm of the feedback loops

Given a rate variable fundamental in-tree model

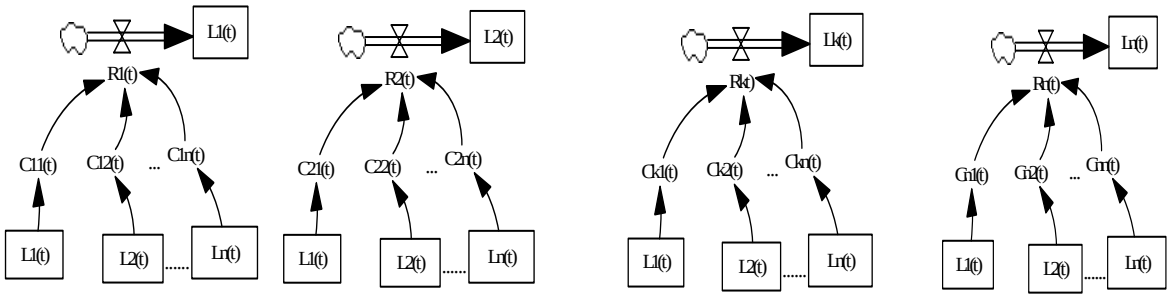


Figure 1 rate variable fundamental in-tree model $T_1(t),T_2(t),\ldots,T_n(t)$

Getting network flow diagram $G_{12,\ldots,n}(t)$ from rate variable fundamental in-tree model.（also signed G(t)）

$$G_{12\ldots n}(t)=\bigcup_{i=1}^{n} Ti(t)$$

Rate variable fundamental in-tree model $T_1(t)$, $T_2(t)$, …,$T_n(t)$ and network flow diagram $G_{12\ldots n}(t)$ are “if and only if” relationship.

Definition 1 Matrix

$$A_{n\times n}=\begin{pmatrix} 0 & (R_1(t),C_{12}(t),L_2(t)) & \cdots & (R_1(t),C_{1n}(t),L_n(t)) \\ (R_2(t),C_{21}(t),L_1(t)) & 0 & \cdots & (R_2(t),C_{2n}(t),L_n(t)) \\ \vdots & \vdots & \cdots & \vdots \\ (R_n(t),C_{n1}(t),L_1(t)) & (R_n(t),C_{n2}(t),L_2(t)) & \cdots & 0 \end{pmatrix}$$

is called the diagonal-0 branch-vector matrix of rate variable fundamental in-tree model $T_1(t),T_2(t),\ldots,T_n(t)$.

Set a_{ij} is element of matrix $A_{n\times n}$, if $a_{ij}=(R_i(t),C_{ij}(t),L_j(t))$, (i, j=1,2,…, n), then $A_{n\times n}=(a_{ij})_{n\times n}$.

Definition 2 multiplication of branch-vector variables:

$(R(t),A_{ij}(t),L_j(t)) \times (R_t(t),A_{tp}(t),L_p(t))=$

$$\begin{cases} (R_i(t), A_{ij}(t), L_j(t), R_j(t), A_{jp}(t), L_p(t)), t = j \text{ and no same variable between } A_{ij}(t) \text{ and } A_{tp}(t) \\ (R_t(t), A_{tp}(t), L_p(t), R_p(t), A_{pj}(t), L_j(t)), r = p \text{ and no same variable between } A_{ij}(t) \text{ and } A_{tp}(t) \\ 0, else \end{cases}$$

Definition 3 Given 0-diagonal branch-vector matrix $A_{n\times n} = (a_{ij})_{n\times n}$. Multiply the corresponding elements of upper triangle by which of lower triangle from the 1st low to n-1 low, then sum them. Such formula is called the multiplication of elements of upper-lower triangle, signed F_2.

Theorem 1 Given $T_1(t),T_2(t),\ldots,T_{k-1}(t)$, we can form archetype $G_{12\ldots(k-1)}(t)= \bigcup_{i=1}^{n} Ti(t)$, then we can calculate the new feedback loops obtained from $G_{12\ldots(k-1)}(t) \ \vec{\cup}\ T_k(t)$:

Build 0-diagonal matrix $A_{k\times k} = (a_{ij})_{k\times k} =$

$$\begin{pmatrix} 0 & a_{12} & \ldots & a_{1k} \\ a_{21} & 0 & \ldots & a_{2k} \\ . & . & & . \\ . & . & \ldots & . \\ . & . & & . \\ a_{k1} & a_{k2} & \ldots & 0 \end{pmatrix} \quad (1)$$

Take row k from $A_{k\times k}$ to build branch-vector row-matrix $X_{1\times k}$, and build 0-diagonal branch-vector matrix $\overline{A}_{k\times k}$

$X_{1\times k} = (a_{k1},a_{k2},\ldots,a_{k(k-1)},0)$ (2)

$$\overline{A}_{k\times k} = \begin{pmatrix} 0 & a_{12} & \ldots & a_{1(k-1)} & a_{1k} \\ a_{21} & 0 & \ldots & a_{2(k-1)} & a_{2k} \\ . & . & & . & . \\ . & . & \ldots & . & . \\ . & . & & . & . \\ a_{(k-1)1} & a_{(k-1)2} & \ldots & 0 & a_{(k-1)k} \\ 0 & 0 & \ldots & 0 & 0 \end{pmatrix} \quad (3)$$

Operate one-power branch-vector matrix multiplication for $\overline{A}_{k\times k}$

$X_{1\times k} \bullet \overline{A}_{k\times k} = (a_{k2}a_{21}+\ldots+a_{k(k-1)}a_{(k-1)1},a_{k1}a_{12}+a_{k3}a_{32}+\ldots+a_{k(k-1)}a_{(k-1)2},\ldots,a_{k1}a_{1k}+a_{k2}a_{2k}+\ldots+a_{k(k-1)}a_{(k-1)k})$

if and only if $a_{ki_1}a_{i_1k} \neq 0$ （i_1=1,2,…,k-1）, the relevant two-order branch-vectors of a_{ki1},a_{i1k} build the two-order new feedback

loops which are produced by $G_{12...(k-1)}(t) \vec{\cup} T_k(t)$。

Operate two-power branch-vector matrix multiplication for $\overline{A}_{k\times k}$

$(X_{1\times k} \bullet \overline{A}_{k\times k}) \bullet \overline{A}_{k\times k} = X_{1\times k} \bullet (\overline{A}_{k\times k})^2$

$(X_{1\times k} \bullet \overline{A}_{k\times k}) \bullet \overline{A}_{k\times k}$ is a row matrix, its elements are composed by the sum equation of $a_{ki1}a_{i1i2}a_{i2i3}$, $a_{ki1}a_{i1i2}a_{i2k} \neq 0$，if and only if its corresponding branch-vector chains form the three-order new added feedback loops which are produced by $G_{12...(k-1)}(t) \vec{\cup} T_k(t)$.

So on, operate (k-1) power branch-vector matrix multiplication $X_{1\times k} \bullet (\overline{A}_{k\times k})^{k-1}$

$X_{1\times k} \bullet (\overline{A}_{k\times k})^{k-1}$ is row matrix，if and only if $a_{ki1}a_{i1i2}a_{i(k-1)k} \neq 0$， its corresponding branch-vector chains build the k-order new added feedback loops which are produced by $G_{12...(k-1)}(t) \vec{\cup} T_k(t)$.

3. Creating an SD model of the HR management of an organization

3.1 the main problem and its significance

The HR management system of an organization is a complex system which is composed of organizational structure design, salary management, train management, performance evaluation, etc. For the content of this HR management system, there are different points of view from different aspects. The HR management system in this article refers the activities to improve organizational performance by managing the internal human resource of the organization. In China, many organizations use a new salary system called grade-salary incitement to prompt the performance of both employees and the organization. They pay employees differently based on their performance instead of the same salary in the past. In most organizations, this incitement improved the performance of organization by making employees to be more active in their work, but in a few organizations, it did not work.

We'll discuss what kind of feedback dynamic complex relationship existed among different variables. How do they act mutually? Are there any positive feedback structures existing in the system to boost the organizational performance and personnel performance? At the same time, are there any negative feedback structures to restrain the organizational performance? Thus we can make the certain management policies for advancing organizational performance.

3.2 Variable definition of the system

Based on the above condition and analysis, we design the flow rate variables, flow level variables and its adjusting parameters for the internal HR management system model of an organization as the following:

$L_1(t)$, $R_1(t)$——Personnel performance (sum of all personnel performance evaluation, unit: grade) and its change (unit: grade per year)

$L_2(t)$, $R_2(t)$, $k_1(t)$——Cost of personnel salary (Unit: Yuan), its change (Unit: Yuan per year), and relevant adjust parameters

$L_3(t)$, $R_3(t)$——Fee of training (Unit: Yuan), its change (Unit: Yuan per year)

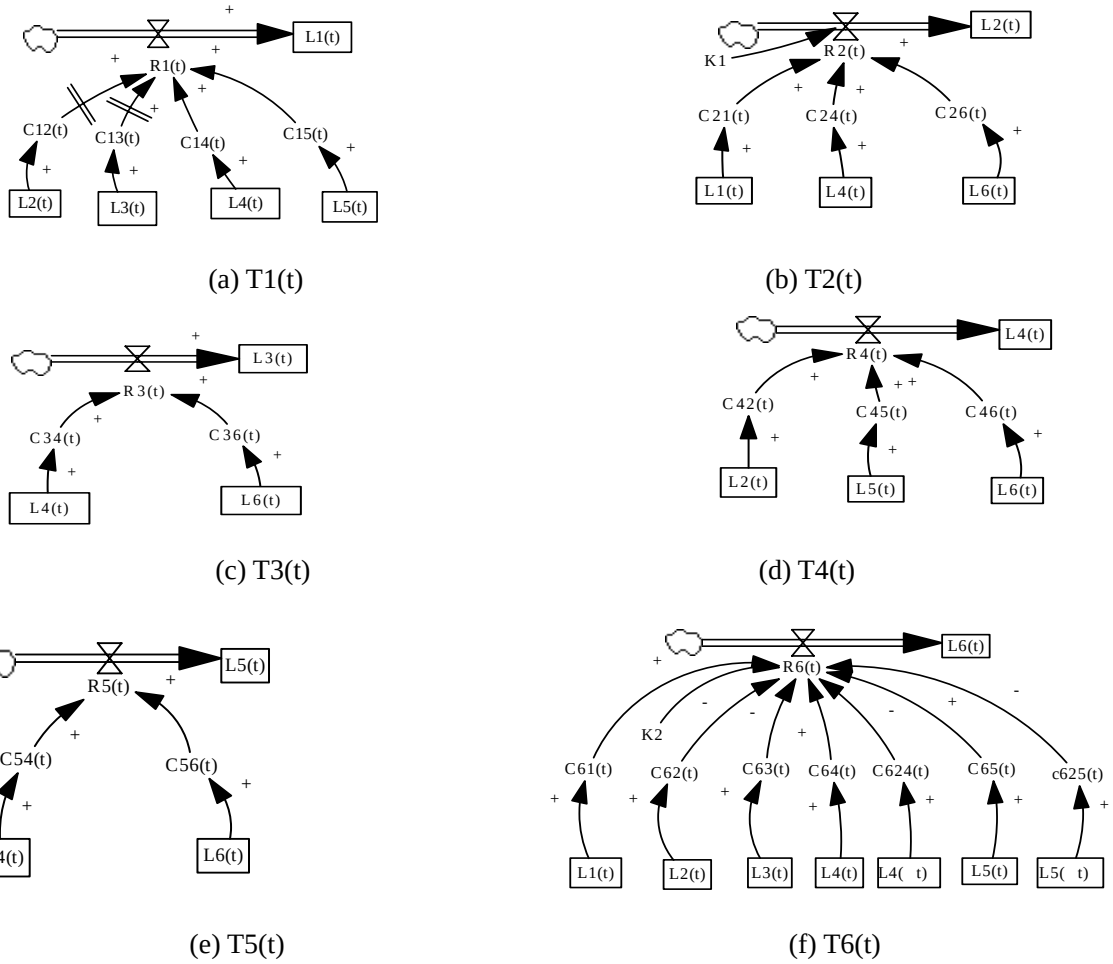
$L_4(t)$, $R_4(t)$——Total amount of employees and its change per year

$L_5(t)$, $R_5(t)$——Total amount of branches and its change per year

$L_6(t)$, $R_6(t)$, $k_2(t)$——Organizational performance, its change (unit: per year) and relevant adjust parameters

3.3 Create the rate variable fundamental in-tree model

Based on our experience of research on HR management theory and investigation, especially our consultant practice of HR management in many organizations, by investigation and statistic, we found that there are some cause and effect relationships among the above variables of flow level and flow rate as the following figures. By applying the theory of SD, we can create rate variable fundamental in-trees as figure 2.



(a) T1(t) (b) T2(t)

(c) T3(t) (d) T4(t)

(e) T5(t) (f) T6(t)

Figure 2 Rate variable fundamental in-tree model of HR management of an organization

In above model, six level variables, personnel performance $L_1(t)$, salary $L_2(t)$, fee of training $L_3(t)$, total amount of employees $L_4(t)$, total amount of branches $L_5(t)$, are controlling the corrected flow rate variables $R_1(t)$, $R_2(t)$, …, $R_6(t)$ through auxiliary variables $C_{ij}(t)$. The linear relationship between every pair variables is:

In in-tree $T_1(t)$, the changing of personnel performance $R_1(t)$, controlled by salary, fee of training, total amount of employees and total amount of branches.

In in-tree $T_2(t)$, the changing of salary $R_2(t)$, controlled by personnel performance, total amount of employees and organizational performance.

In in-tree $T_3(t)$, the changing of fee of training $R_3(t)$, controlled by total amount of employees and organizational performance. That is because in the organizations of our research field, they do training plan based on their capital and amount of employees. When organizational performance is good, more capital can be used in training.

In in-tree $T_4(t)$, the changing of total amount of employees $R_4(t)$, controlled by salary, total amount of branches and organizational performance.

In in-tree $T_5(t)$, the changing of total amount of branches $R_5(t)$, controlled by total amount of employees and organizational performance.

In in-tree $T_6(t)$, the changing of organizational performance $R_6(t)$, controlled by personnel performance, salary, fee of training, total amount of employees and total amount of branches.

In the software of System Dynamics “Vensim”, the in-tree model is the same as flow chart model. From the six in-trees, we get flow chart as figure 3 easily.

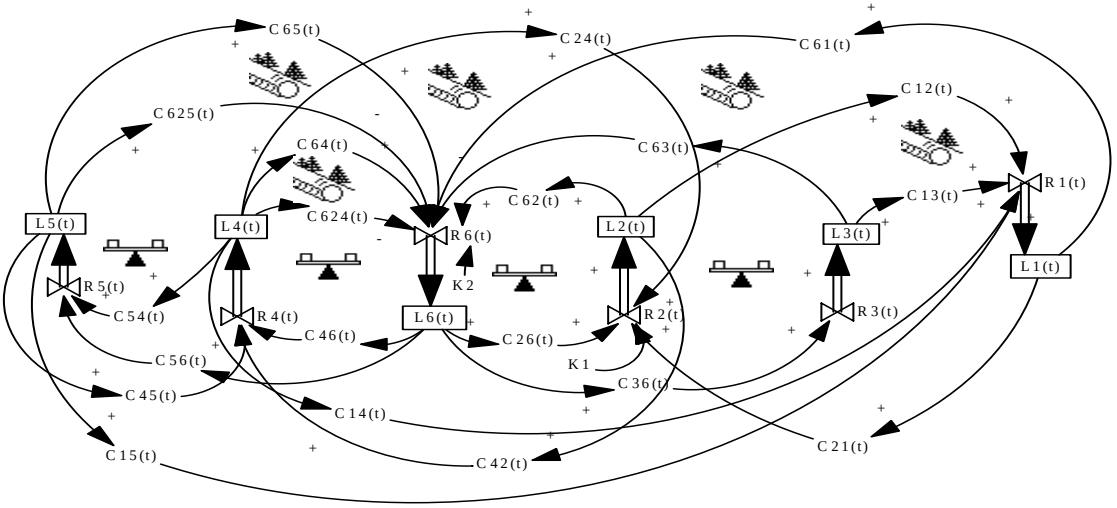


Figure 3 G_{123456} Overall flow chart of the HR management system of organization

3.4 The feedback loops analysis of the HR management of the grade-salary incitement

Divide flow chart $G_{123456}(t)$ into $G_{13456}(t) \overrightarrow{\cup} T_2(t)$, of which the $G_{13456}(t)$ is a sub flow chart of HR management system that doesn’t adopt the salary-grade incitement, as figure 4. By theorem 1, we can get all the newly gained feedback loops generated by $G_{13456}(t) \overrightarrow{\cup} T_2(t)$. By analyzing the features of these newly gained feedback loops, we can summarize the important effect of salary-grade incitement on improving organizational performance, and find out the function points. At the same time, we can analyze its restricting factors and put forward proper management policies.

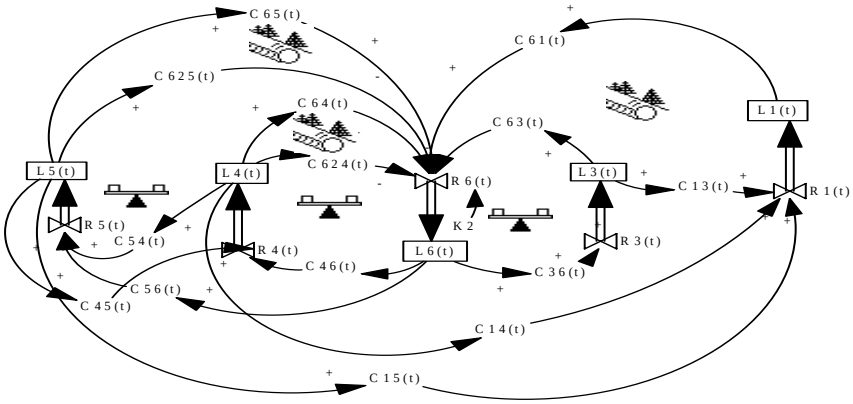


Figure 4 $G_{13456}(t)$

Get the newly gained feedback loops for $G_{13456}(t) \overrightarrow{\cup} T_2(t)$

According to theorem 1, we can get a row matrix by taking out the second row of in-tree model diagonal-0 branch-vector matrix. $X_{1\times 6}=((R_2,C_{21},L_1),0,0,(R_2,C_{24},L_4),0,(R_2,C_{26},L_6))$ (3)

Then build matrix $A_{6\times 6,}$ which is a salary flow rate fundamental in-tree vector matrix.

$$\overrightarrow{A_{6\times 6}} = \begin{pmatrix} 0 & (R_1,C_{12},L_2) & (R_1,C_{13},L_1) & (R_1,C_{14},L_4) & (R_1,C_{15},L_5) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (R_3,C_{34},L_4) & 0 & (R_3,C_{36},L_6) \\ 0 & (R_4,C_{62},L_2) & 0 & 0 & (R_4,C_{45},L_5) & (R_4,C_{46},L_6) \\ 0 & 0 & 0 & (R_5,C_{54},L_4) & 0 & (R_5,C_{56},L_6) \\ (R_6,C_{61},L_1) & (R_6,C_{62},L_2) & (R_6,C_{63},L_3) & (R_6,C_{64},L_4)+(R_6,C_{24},L_4) & (R_6,C_{65},L_5)+(R_6,C_{625},L_5) & 0 \end{pmatrix} \quad (4)$$

Given $(R_i,C_{ij},L_j)=a_{ij}$

$(R_i,C_{itj},L_j)=a_{itj}$

We can get

$X_{1\times 6}=(a_{21},0,0,a_{24},0,a_{26})$ (5)

$$\vec{A}_{6\times 6}=\begin{pmatrix} 0 & a_{12} & a_{13} & a_{14} & a_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{34} & 0 & a_{36} \\ 0 & a_{42} & 0 & 0 & a_{45} & a_{46} \\ 0 & 0 & 0 & a_{54} & 0 & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64}+a_{624} & a_{65}+a_{625} & 0 \end{pmatrix} \quad (6)$$

3.4.1 The newly gained two-order feedback loops by adopting grade-salary incitements

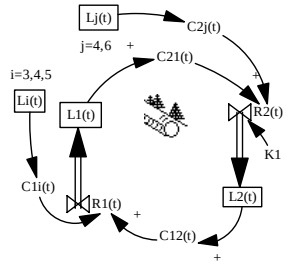
$X_{1\times 6} \bullet \vec{A}_{6\times 6}=(a_{26}a_{61}, \mathbf{a_{21}a_{12}+a_{24}a_{42}+a_{26}a_{62}}, a_{21}a_{13}+a_{26}a_{63}, a_{21}a_{14}+a_{26}a_{64}+a_{26}a_{624}, a_{21}a_{15}+a_{24}a_{46}+a_{24}a_{45}+a_{26}a_{65}, a_{26}a_{625})$ (7)

From the reverse transformation of the second row and $a_{ij}=(R_i(t),C_{ij}(t),L_j(t))$

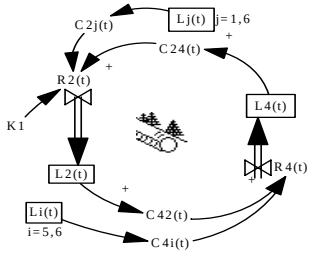
$a_{21}a_{12}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{12}(t),L_2(t))$, we get the two-order positive feedback loop between salary and personnel performance. See Figure 5(a).

$a_{24}a_{42}=(R_2(t),C_{24}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we get the two-order positive feedback loop between salary and the amount of employees. See Figure 5(b).

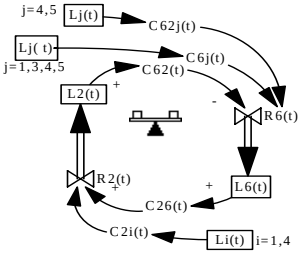
$a_{26}a_{62}=(R_2(t),C_{26}(t),L_6(t),R_6(t),C_{62}(t),L_2(t))$, we get the two-order negative feedback loop of which the salary is restricted by organizational performance. See Figure 5(c).



(a)G12(t) positive feedback loop between salary and personnel performance



（b）G24(t) positive feedback loop between salary and the amount of employees



(c)G26(t) negative feedback loop between salary and organizational performance

Figure 5 newly gained 2-order feedback loops

3.4.2 The newly gained three-order feedback loops by adopting the grade-salary incitement

Given（$X_{1\times 6}$ ● $\vec{A}_{6\times 6}$）● $\vec{A}_{6\times 6}$=($a_{26}a_{61}$, （**$a_{21}a_{12}$+ $a_{24}a_{42}$+ $a_{26}a_{62}$**）,（$a_{21}a_{13}$+ $a_{26}a_{63}$）,（$a_{21}a_{14}$+$a_{26}a_{64}$+$a_{26}a_{624}$）,（$a_{21}a_{15}$+ $a_{24}a_{45}$+ $a_{26}a_{65}$+ $a_{26}a_{625}$）,（$a_{24}a_{46}$）) ●

$$\begin{pmatrix} 0 & a_{12} & a_{13} & a_{14} & a_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{34} & 0 & a_{36} \\ 0 & a_{42} & 0 & 0 & a_{45} & a_{46} \\ 0 & 0 & 0 & a_{54} & 0 & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64}+a_{624} & a_{65}+a_{625} & 0 \end{pmatrix}$$

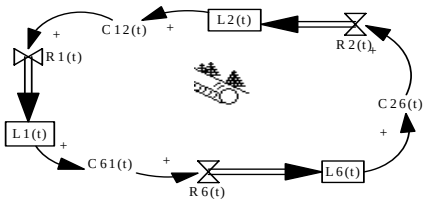
=($a_{24}a_{46}a_{61}$, **($a_{26}a_{61}a_{12}$+$a_{21}a_{14}a_{42}$+$a_{26}a_{64}a_{42}$+$a_{26}a_{624}a_{42}$+$a_{24}a_{46}a_{62}$)**, ($a_{26}a_{61}a_{13}$+$a_{26}a_{46}a_{63}$), ($a_{26}a_{61}a_{14}$+$a_{21}a_{13}a_{34}$+$a_{26}a_{63}a_{34}$+$a_{21}a_{15}a_{54}$ +$a_{26}a_{65}a_{54}$+$a_{26}a_{625}a_{54}$), ($a_{26}a_{61}a_{15}$+$a_{21}a_{14}a_{45}$+$a_{26}a_{64}a_{45}$+$a_{26}a_{624}a_{45}$+$a_{24}a_{46}a_{65}$+$a_{24}a_{46}a_{625}$), ($a_{21}a_{13}a_{36}$+$a_{21}a_{14}a_{46}$+$a_{21}a_{15}a_{56}$+$a_{24}a_{45}a_{56}$)) (8)

By the elements of the second row, we know that there are 5 three-order feedback loops generated by adopting salary incitement. Then Analyze these newly gained three-order feedback loops.

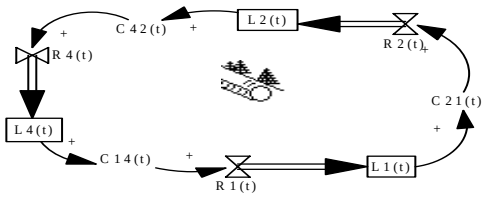
(1) Newly gained three-order feedback loop No.1:

From $a_{26}a_{61}a_{12}$=（R_2(t),C_{26}(t),L_6(t),R_6(t),C_{61}(t),L_1(t),R_1(t),C_{12}(t),L_2(t)）, we can get the three-order positive feedback loop among newly increased salary, personnel performance and organizational performance. See figure 6(a).

This feedback loop indicates the function of three-order growing feedback. That is, the salary-grade incitement system improves personnel performance, personnel performance improves organizational performance, and organizational performance improves the operation of salary-grade incitement system.



(a) Three-order positive feedback loop among salary, personnel performance and organizational performance



(b) Three-order positive feedback loop among salary, personnel performance and the amount of employees

Figure 6 two of the 3-order positive feedback loops

(2) Newly gained three-order feedback loop No.2:

From $a_{21}a_{14}a_{42}$=（R_2(t),C_{21}(t),L_1(t),R_1(t),C_{14}(t),L_4(t),R_4(t),C_{42}(t),L_2(t)）, we can get the three-order positive feedback loop among increased salary, the amount of employees and personnel performance. See figure 6(b).

This feedback loop has illustrated the three-order growing feedback cycling function. This is, after adopting the salary-grade incitement system, this advanced salary system attracts more people to attend the organization, so it increases the amount of employees, thereby increases personnel performance; the improvement of personnel performance can lead to further salary growth.

(3) Newly gained three-order feedback loop No.3:

From $a_{26}a_{64}a_{42}$=（R_2(t),C_{26}(t),L_6(t),R_6(t),C_{64}(t),L_4(t),R_4(t),C_{42}(t),L_2(t)）, we can get the three-order positive feedback loop among salary, the amount of employees and organizational performance. See figure 7.

This feedback loop has presented the three-order growing feedback cycling function. This is, after adopting the salary-grade incitement, salary can increase the amount of employees, therefore accelerate the improvement of organizational performance; better organizational performance can support the execution of the grade-salary incitement.

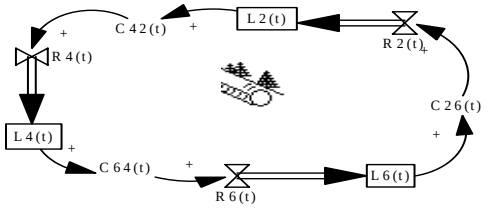


Figure 7 Three-order positive feedback loop among salary, the amount of employees and organizational performance

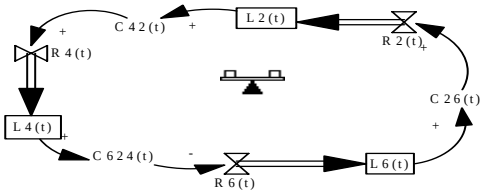


Figure 8 Three-order negative feedback loop among salary, the amount of employees and organizational performance

(4) Newly gained three-order feedback loop No.4:

From $a_{26}a_{624}a_{42}$=（$R_2(t),C_{26}(t),L_6(t),R_6(t),C_{624}(t),L_4(t),R_4(t),C_{42}(t),L_2(t)$）, we can get the three-order negative feedback loop among salary, the amount of employees and organizational performance. See figure 8.

This three-order negative feedback loop showed that, after adopting the salary-grade incitement, salary can increase the amount of employees, but more employees can add cost of the organization, thus it restricts the support of organizational performance on salary.

(5) Newly gained three-order feedback loop No.5:

From $a_{24}a_{46}a_{62}$=（$R_2(t),C_{24}(t),L_4(t),R_4(t),C_{46}(t),L_6(t),R_6(t),C_{62}(t),L_2(t)$）, we can get a newly gained three-order negative feedback loop as showed in figure 9.

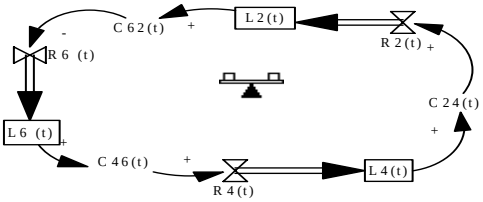


Figure 9 Three-order negative feedback loop among salary, organizational performance and the amount of employees

This three-order negative feedback loop showed that, after adopting the salary-grade incitement, salary raises can increase cost of the organization, decrease the amount of employees, consequently the salary expenditure will be reduced.

By analyzing the above 5 newly gained three-order feedback loops, we can make further conclusions that by adopting the salary-grade incitement to HR management, we can promote the organization to produce growth force continuously.

3.4.3 The newly gained four-order feedback loops by adopting grade-salary incitement

Multiply row matrix （$X_{1\times 6}\bullet\vec{A}_{6\times 6}$）$\bullet\vec{A}_{6\times 6}$ (as formula (8)) by matrix $\vec{A}_{6\times 6}$（(as formula (6)), we get

$(X_{1\times 6}\bullet\vec{A}^{2}{}_{6\times 6})\bullet\vec{A}_{6\times 6}$=($a_{24}a_{46}a_{61}$, $\mathbf{a_{26}a_{61}a_{12}+a_{21}a_{14}a_{42}+a_{26}a_{64}a_{42}+a_{26}a_{624}a_{42}+a_{24}a_{46}a_{62}}$), ($a_{26}a_{61}a_{13}+a_{26}a_{46}a_{63}$), ($a_{26}a_{61}a_{14}+a_{21}a_{13}a_{34}+a_{26}a_{63}a_{34}+a_{21}a_{15}a_{54}+a_{26}a_{65}a_{54}+a_{26}a_{625}a_{54}$), ($a_{26}a_{61}a_{15}+a_{21}a_{14}a_{45}+a_{26}a_{64}a_{45}+a_{26}a_{624}a_{45}+a_{24}a_{46}a_{65}+a_{24}a_{46}a_{625}$),

($a_{21}a_{13}a_{36}+a_{21}a_{14}a_{46}+a_{21}a_{15}a_{56}+a_{24}a_{45}a_{56}$)) $\bullet$

$$\begin{pmatrix} 0 & a_{12} & a_{13} & a_{14} & a_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{34} & 0 & a_{36} \\ 0 & a_{42} & 0 & 0 & a_{45} & a_{46} \\ 0 & 0 & 0 & a_{54} & 0 & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64}+a_{624} & a_{65}+a_{625} & 0 \end{pmatrix}$$

$=(a_{24}a_{45}a_{56}a_{61}$, $\mathbf{(a_{24}a_{46}a_{61}a_{12}+a_{26}a_{61}a_{14}a_{42}+a_{21}a_{13}a_{34}a_{42}+a_{26}a_{63}a_{34}a_{42}+a_{21}a_{15}a_{54}a_{42}+a_{26}a_{65}a_{54}a_{42}+a_{26}a_{625}a_{54}a_{42}+a_{21}a_{13}a_{36}a_{62}}$ $\mathbf{+a_{21}a_{14}a_{46}a_{62}+a_{21}a_{15}a_{56}a_{62}+a_{24}a_{45}a_{56}a_{62})}$, $(a_{24}a_{46}a_{61}a_{13}+a_{21}a_{15}a_{56}a_{63}+a_{24}a_{45}a_{56}a_{63}+a_{21}a_{14}a_{46}a_{63})$, $(a_{26}a_{61}a_{13}a_{34}+a_{26}a_{61}a_{15}a_{54}$ $+a_{21}a_{13}a_{36}a_{64}+a_{21}a_{13}a_{36}a_{624}+a_{21}a_{15}a_{56}a_{64}+a_{21}a_{15}a_{56}a_{624})$, $(a_{24}a_{46}a_{61}a_{15}+a_{26}a_{61}a_{14}a_{45}+a_{21}a_{13}a_{34}a_{45}+a_{26}a_{63}a_{34}a_{45}+a_{21}a_{13}a_{36}a_{65}$ $+a_{21}a_{13}a_{36}a_{625}+a_{21}a_{14}a_{46}a_{65}+a_{21}a_{14}a_{46}a_{625})$, $(a_{21}a_{13}a_{34}a_{46}+a_{21}a_{15}a_{54}a_{46}+a_{21}a_{14}a_{45}a_{56}))$ （9）

Perform four-order feedback loops analysis for the 6 elements of the 2nd tier in the above row matrix (formula (9)).

(1) Newly gained four-order feedback loop No. 1

From $a_{24}a_{46}a_{61}a_{12}=(R_2(t),C_{24}(t),L_4(t),R_4(t),C_{46}(t),L_6(t),R_6(t),C_{61}(t),L_1(t),R_1(t),C_{12}(t),L_2(t))$, we can get the 1st four-order new feedback loop which is caused by adopting the salary-grade incitement (positive feedback loop), see figure 10.

(2) Newly gained four-order feedback loop No. 2

From $a_{26}a_{61}a_{14}a_{42}=(R_2(t),C_{26}(t),L_6(t),R_6(t),C_{61}(t),L_1(t),R_1(t),C_{14}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the 2nd four-order new feedback loop which is caused by adopting the salary-grade incitement (positive feedback loop), see figure 11.

(3) Newly gained four-order feedback loop No. 3

From $a_{21}a_{13}a_{34}a_{42}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{13}(t),L_3(t),R_3(t),C_{34}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the 3rd four-order new feedback loop which is caused by adopting the salary-grade incitement mechanism (positive feedback loop), see figure 12.

(4) Newly gained four-order feedback loop No. 4

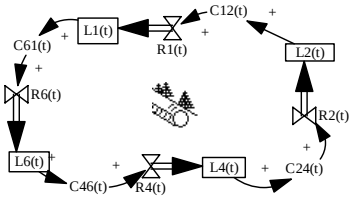


Figure 10 4-order feedback loop No.1

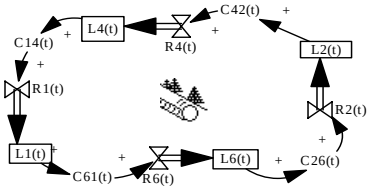


Figure 11 4-order feedback loop No.2

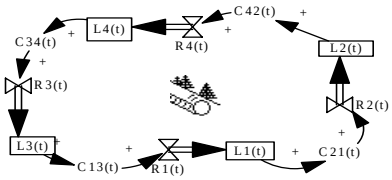


Figure 12 4-order feedback loop No.3

From $a_{21}a_{15}a_{54}a_{42}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{15}(t),L_5(t),R_5(t),C_{54}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the 4th four-order new feedback loop which is caused by adopting the salary-grade incitement (positive feedback loop), see figure 13.

(5) Newly gained four-order feedback loop No. 5

From $a_{26}a_{65}a_{54}a_{42}=(R_2(t),C_{26}(t),L_6(t),R_6(t),C_{65}(t),L_5(t),R_5(t),C_{54}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the 5th four-order new feedback loop which is caused by adopting the salary-grade incitement (positive feedback loop), see figure 14.

(6) Newly gained four-order feedback loop No. 6

From $a_{26}a_{63}a_{34}a_{42}=(R_2(t),C_{26}(t),L_6(t),R_6(t),C_{63}(t),L_3(t),R_3(t),C_{34}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the 6th four-order new feedback loop which is caused by adopting new salary-grade incitement (negative feedback loop), see figure 15.

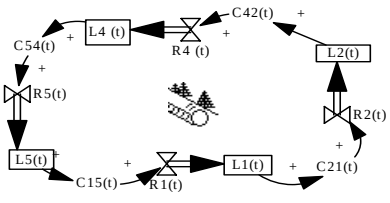


Figure 13 4-order feedback loop No.4

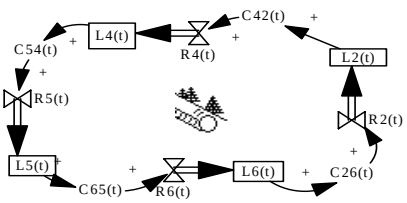


Figure 14 4-order feedback loop No.5

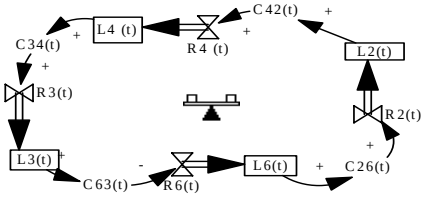


Figure 15 4-order feedback loop No.6

(7) Newly gained four-order feedback loop No. 7

From $a_{26}a_{625}a_{54}a_{42}=(R_2(t),C_{26}(t),L_6(t),R_6(t),C_{625}(t),L_5(t),R_5(t),C_{54}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the 7th four-order new feedback loop which is caused by adopting new salary-grade incitement (negative feedback loop), see figure 16.

(8) Newly gained four-order feedback loop No. 8

From $a_{21}a_{13}a_{36}a_{62}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{13}(t),L_3(t),R_3(t),C_{36}(t),L_6(t),R_6(t),C_{62}(t),L_2(t))$, we can get the 8th four-order new feedback loop which is caused by adopting the salary-grade incitement (negative feedback loop), see figure 17.

(9) Newly gained four-order feedback loop No. 9

From $a_{21}a_{14}a_{46}a_{62}$=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{14}(t),L_4(t),R_4(t),C_{46}(t),L_6(t),R_6(t),C_{62}(t),L_2(t)), we can get the 9th four-order new feedback loop which is caused by adopting the salary-grade incitement (negative feedback loop), see figure 18.

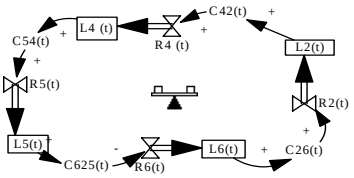


Figure 16 4-order feedback loop No.7

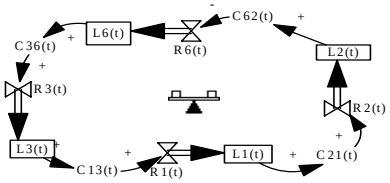


Figure 17 4-order feedback loop No.8

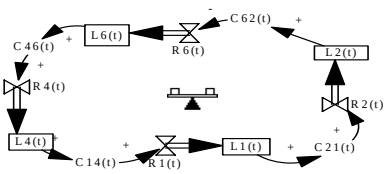


Figure 18 4-order feedback loop No.9

(10) Newly gained four-order feedback loop No. 10

From $a_{21}a_{15}a_{56}a_{62}$=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{15}(t),L_5(t),R_5(t),C_{56}(t),L_6(t),R_6(t),C_{62}(t),L_2(t)), we can get the 10th four-order new feedback loop which is caused by adopting the salary-grade incitement (negative feedback loop), see figure 19.

(11) Newly gained four-order feedback loop No. 11

From $a_{24}a_{45}a_{56}a_{62}$=(R_2(t),C_{24}(t),L_4(t),R_4(t),C_{45}(t),L_5(t),R_5(t),C_{56}(t),L_6(t),R_6(t),C_{62}(t),L_2(t)), we can get the 11th four-order new feedback loop which is caused by adopting the salary-grade incitement (negative feedback loop), see figure 20.

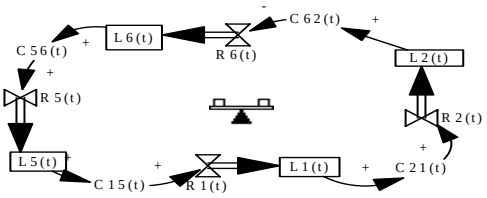


Figure 19 4-order feedback loop No.10

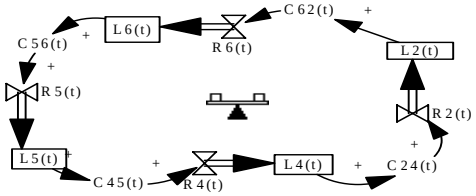


Figure 20 4-order feedback loop No.11

By analyzing these 11 newly added four-order feedback loops, of which 5 are positive feedback loops, 6 are restriction feedback loops, we point out that by adopting the salary-grade incitement on HR management, both personal and organizational performances can be promoted efficiently. But there also exist the restriction effects of the increase in HR management cost and branch management cost. We need an appropriate mechanism to help salary, training, personnel management and branch management to be operated systematically. Only this can we achieve a better function of salary-grade incitement.

3.4.4 The newly gained 5-order and 6-order feedback loops by adopting salary-grade incitement

We can get the 5-order feedback loops by multiplying row matrix （$X_{1\times6}\bullet\vec{A}^{3}_{6\times6}$）(see formula (9)) with matrix $\vec{A}_{6\times6}$ (see formula (6)).

（$X_{1\times6}\bullet\vec{A}^{3}_{6\times6}$）$\bullet\vec{A}_{6\times6}$=($a_{24}a_{45}a_{56}a_{61}$, **$a_{24}a_{46}a_{61}a_{12}$+$a_{26}a_{61}a_{14}a_{42}$+$a_{21}a_{13}a_{34}a_{42}$+$a_{21}a_{13}a_{34}a_{42}$+$a_{26}a_{63}a_{34}a_{42}$+$a_{21}a_{15}a_{54}a_{42}$ +$a_{26}a_{65}a_{54}a_{42}$+$a_{26}a_{625}a_{54}a_{42}$+$a_{21}a_{13}a_{36}a_{62}$+$a_{21}a_{14}a_{46}a_{62}$+$a_{21}a_{15}a_{56}a_{62}$+$a_{24}a_{45}a_{56}a_{62}$)**, ($a_{24}a_{46}a_{61}a_{13}$+$a_{21}a_{15}a_{56}a_{63}$+$a_{24}a_{45}a_{56}a_{63}$ +$a_{21}a_{14}a_{46}a_{63}$)**,** ($a_{26}a_{61}a_{13}a_{34}$+$a_{26}a_{61}a_{15}a_{54}$+$a_{21}a_{13}a_{36}a_{64}$+$a_{21}a_{13}a_{36}a_{624}$+$a_{21}a_{15}a_{56}a_{64}$+$a_{21}a_{15}a_{56}a_{624}$), ($a_{24}a_{46}a_{61}a_{15}$+$a_{26}a_{61}a_{14}a_{45}$ +$a_{21}a_{13}a_{34}a_{45}$+$a_{26}a_{63}a_{34}a_{45}$+$a_{21}a_{13}a_{36}a_{65}$+$a_{21}a_{13}a_{36}a_{625}$+$a_{21}a_{14}a_{46}a_{65}$+$a_{21}a_{14}a_{46}a_{625}$), ($a_{21}a_{13}a_{34}a_{46}$+$a_{21}a_{15}a_{54}a_{46}$+$a_{21}a_{14}a_{45}a_{56}$))

$$\bullet\begin{pmatrix} 0 & a_{12} & a_{13} & a_{14} & a_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{34} & 0 & a_{36} \\ 0 & a_{42} & 0 & 0 & a_{45} & a_{46} \\ 0 & 0 & 0 & a_{54} & 0 & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64}+a_{624} & a_{65}+a_{625} & 0 \end{pmatrix}$$

=(0, **$(a_{24}a_{45}a_{56}a_{61}a_{12}+a_{26}a_{61}a_{13}a_{34}a_{42}+a_{26}a_{61}a_{15}a_{54}a_{42}+a_{21}a_{13}a_{36}a_{64}a_{42}+a_{21}a_{13}a_{36}a_{624}a_{42}+a_{21}a_{15}a_{56}a_{64}a_{42}+a_{21}a_{15}a_{56}a_{624}a_{42}+a_{21}a_{13}a_{34}a_{46}a_{62}+a_{21}a_{15}a_{54}a_{46}a_{62}+a_{21}a_{14}a_{45}a_{56}a_{62})$**, $(a_{24}a_{45}a_{56}a_{61}a_{13}+a_{21}a_{15}a_{54}a_{46}a_{63}+a_{21}a_{14}a_{45}a_{56}a_{63})$, $(a_{21}a_{15}a_{56}a_{63}a_{34}+a_{21}a_{13}a_{36}a_{65}a_{54}+a_{21}a_{13}a_{36}a_{625}a_{54})$, $(a_{26}a_{61}a_{13}a_{34}a_{45}+a_{21}a_{13}a_{36}a_{64}a_{45}+a_{21}a_{13}a_{36}a_{624}a_{45}+a_{21}a_{13}a_{34}a_{46}a_{65}+a_{21}a_{13}a_{34}a_{46}a_{625})$, $a_{21}a_{13}a_{34}a_{45}a_{56}$) (10)

Also we can get the 6-order feedback loops by multiplying row matrix （$X_{1\times 6}\bullet \vec{A}^{4}{}_{6\times 6}$） (see formula (10)) with matrix $\vec{A}_{6\times 6}$ (see formula (6)).

$(X_{1\times 6}\bullet \vec{A}^{4}{}_{6\times 6})\bullet \vec{A}_{6\times 6}$

=(0,**$(a_{21}a_{15}a_{56}a_{63}a_{34}a_{42}+a_{21}a_{13}a_{36}a_{65}a_{54}a_{42}+a_{21}a_{13}a_{36}a_{625}a_{54}a_{42}+a_{21}a_{13}a_{34}a_{45}a_{56}a_{62})$**,0，0，0，0) （11）

(1) Newly gained 5-order feedback loop No. 1

From $a_{24}a_{45}a_{56}a_{61}a_{12}=(R_2(t),C_{24}(t),L_4(t),R_4(t),C_{45}(t),L_5(t),R_5(t),C_{56}(t),L_6(t),R_6(t),C_{61}(t),L_1(t),R_1(t),C_{12}(t),L_2(t))$, we can get the positive 5-order new feedback loop which is caused by adopting the salary-grade incitement, see figure 21.

(2) Newly gained 5-order feedback loop No. 2

By $a_{26}a_{61}a_{13}a_{34}a_{42}=(R_2(t),C_{26}(t),L_6(t),R_6(t),C_{61}(t),L_1(t),R_1(t),C_{13}(t),L_3(t),R_3(t),C_{34}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the positive 5-order new feedback loop which is caused by adopting the salary-grade incitement, see figure 22.

(3) Newly gained 5-order feedback loop No. 3

By $a_{26}a_{61}a_{15}a_{54}a_{42}=(R_2(t),C_{26}(t),L_6(t),R_6(t),C_{61}(t),L_1(t),R_1(t),C_{15}(t),L_5(t),R_5(t),C_{54}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the positive 5-order new feedback loop which is caused by adopting the salary-grade incitement, see figure 23.

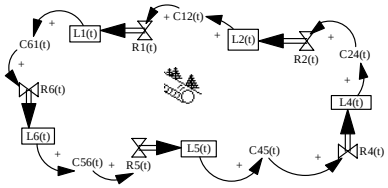


Figure 21 5-order feedback loop No.1

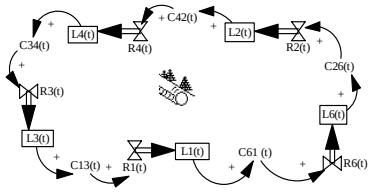


Figure 22 5-order feedback loop No.2

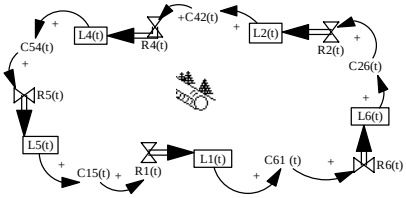


Figure 23 5-order feedback loop No.3

(4) Newly gained 5-order feedback loop No. 4

By $a_{21}a_{13}a_{36}a_{64}a_{42}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{13}(t),L_3(t),R_3(t),C_{36}(t),L_6(t),R_6(t),C_{64}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the positive 5-order new feedback loop which is caused by adopting new salary-grade incitement, see figure 24.

(5) Newly gained 5-order feedback loop No. 5

By $a_{21}a_{15}a_{56}a_{64}a_{42}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{15}(t),L_5(t),R_5(t),C_{56}(t),L_6(t),R_6(t),C_{64}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the positive 5-order new feedback loop which is caused by adopting the salary-grade incitement, see figure 25.

(6) Newly gained 6-order feedback loop No. 1

By

$a_{21}a_{13}a_{36}a_{65}a_{54}a_{42}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{13}(t),L_3(t),R_3(t),C_{36}(t),L_6(t),R_6(t),C_{65}(t),L_5(t),R_5(t),C_{54}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the positive 6-order new feedback loop which is caused by adopting the salary-grade incitement, see figure 26.

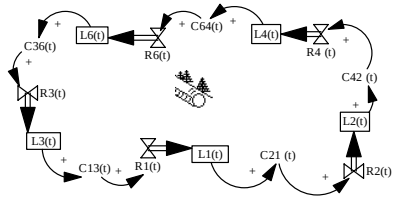


Figure 24 5-order feedback loop No.4

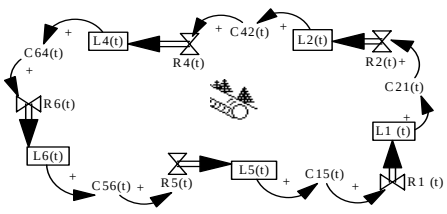


Figure 25 5-order feedback loop No.5

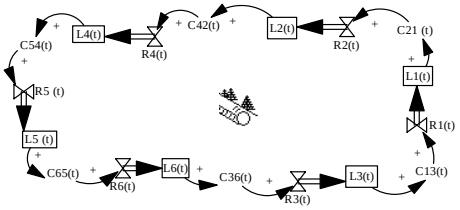


Figure 26 6-order feedback loop No.1

(7) Newly gained 5-order feedback loop No. 6

By $a_{21}a_{13}a_{36}a_{624}a_{42}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{13}(t),L_3(t),R_3(t),C_{36}(t),L_6(t),R_6(t),C_{624}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the 5-order restriction feedback loop by adopting the salary-grade incitement, see figure 27.

(8) Newly gained 5-order feedback loop No. 7

By $a_{21}a_{15}a_{56}a_{624}a_{42}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{15}(t),L_5(t),R_5(t),C_{56}(t),L_6(t),R_6(t),C_{624}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the 5-order restriction feedback loop after adopting the salary-grade incitement, see figure 28.

(9) Newly gained 5-order feedback loop No. 8

By $a_{21}a_{13}a_{34}a_{46}a_{62}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{13}(t),L_3(t),R_3(t),C_{34}(t),L_4(t),R_4(t),C_{46}(t),L_6(t),R_6(t),C_{62}(t),L_2(t))$, we can get the 5-order restriction feedback loop after adopting the salary-grade incitement, see figure 29.

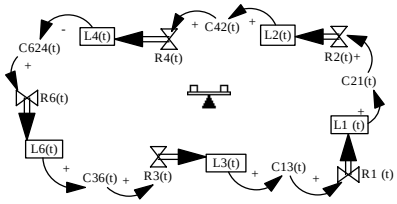


Figure 27 5-order feedback loop No.6

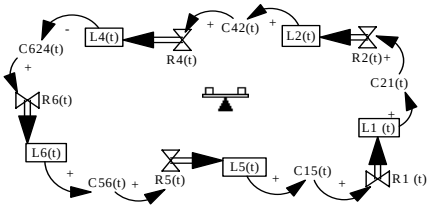


Figure 28 5-order feedback loop No.7

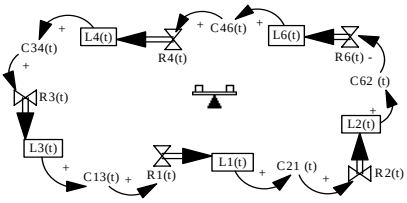


Figure 29 5-order feedback loop No.8

(10) Newly gained 5-order feedback loop No. 9

By $a_{21}a_{15}a_{54}a_{46}a_{62}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{15}(t),L_5(t),R_5(t),C_{54}(t),L_4(t),R_4(t),C_{46}(t),L_6(t),R_6(t),C_{62}(t),L_2(t))$, we can get the 5-order restriction feedback loop after adopting the salary-grade incitement, see figure 30.

(11) Newly gained 5-order feedback loop No. 10

By $a_{21}a_{14}a_{45}a_{56}a_{62}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{14}(t),L_4(t),R_4(t),C_{45}(t),L_5(t),R_5(t),C_{56}(t),L_6(t),R_6(t),C_{62}(t),L_2(t))$, we can get the 5-order restriction feedback loop after adopting the salary-grade incitement, see figure 31.

(12) Newly gained 6-order feedback loop No. 2

From

$a_{21}a_{15}a_{56}a_{63}a_{34}a_{42}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{15}(t),L_5(t),R_5(t),C_{56}(t),L_6(t),R_6(t),C_{63}(t),L_3(t),R_3(t),C_{34}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the 6-order restriction feedback loop after adopting the salary-grade incitement, see figure 32.

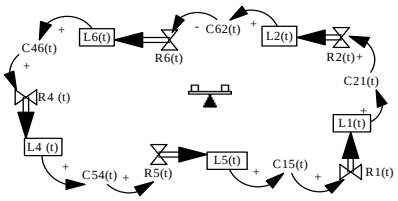


Figure 30 5-order feedback loop No.9

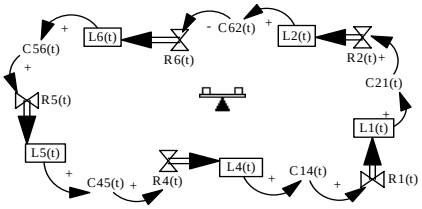


Figure 31 5-order feedback loop No.10

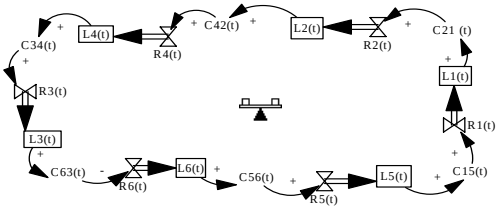


Figure 32 6-order feedback loop No.2

(13) Newly gained 6-order feedback loop No. 3

From

$a_{21}a_{13}a_{36}a_{625}a_{54}a_{42}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{13}(t),L_3(t),R_3(t),C_{36}(t),L_6(t),R_6(t),C_{625}(t),L_5(t),R_5(t),C_{54}(t),L_4(t),R_4(t),C_{42}(t),L_2(t))$, we can get the 6-order restriction feedback loop after adopting the salary-grade incitement, see figure 33.

(14) Newly gained 6-order feedback loop No. 4

From

$a_{21}a_{13}a_{34}a_{45}a_{56}a_{62}=(R_2(t),C_{21}(t),L_1(t),R_1(t),C_{13}(t),L_3(t),R_3(t),C_{34}(t),L_4(t),R_4(t),C_{45}(t),L_5(t),R_5(t),C_{56}(t),L_6(t),R_6(t),C_{62}(t),L_2(t))$, we can get the 6-order restriction feedback loop after adopting the salary-grade incitement, see figure 34.

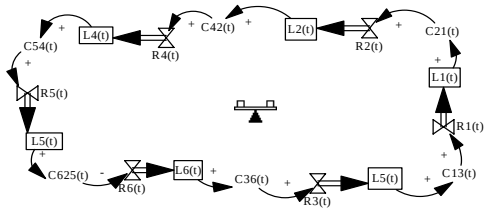


Figure 33 6-order feedback loop No.3

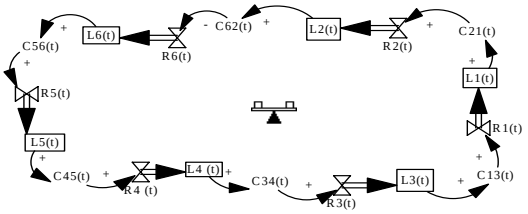


Figure 34 6-order feedback loop No.4

3.4.5 The comprehensive analysis of the Newly gained feedback loops by adopting salary-grade incitement

By using the formula of branch-vector matrix feedback loops of Theorem 1, we get 33 newly gained feedback loops(three 2-order, five 3-order, eleven 4-order, ten 5-order, six 4-order). There are 16 positive feedback loops and 17 negative feedback loops. We do comprehensive analysis for the effect of them.

(1) Added positive feedback loops produced by adopting the grade-salary incitement

After an organization adopting the grade-salary incitement in its HR management, and combined other HR management measures such as training, internal branches setup, etc., the newly gained positive feedback loops are as following:

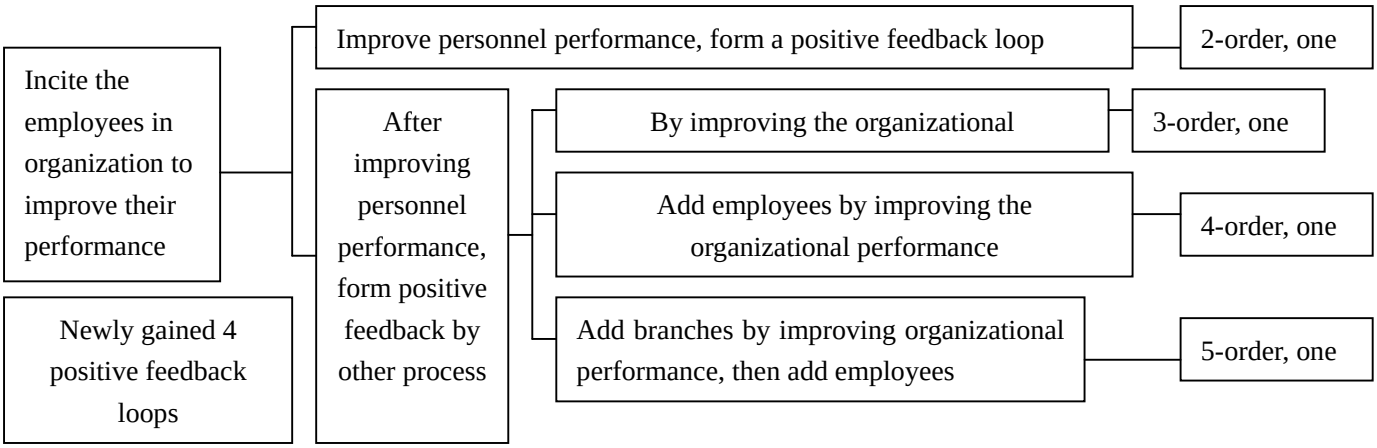


Figure 35 newly gained positive feedback of inciting the motivity of personnel to improve their performance

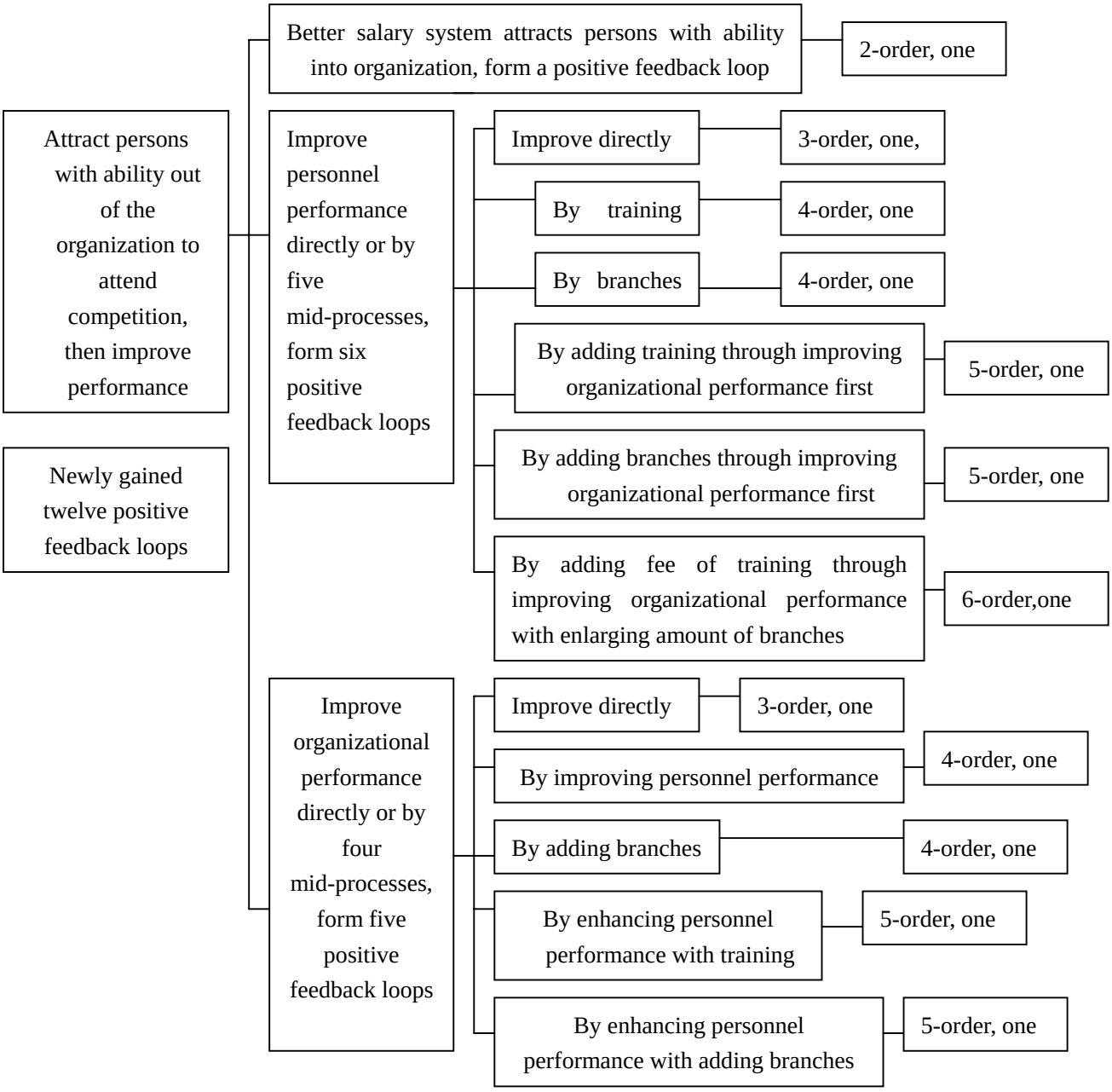


Figure 36 newly gained positive feedback to attract persons with ability

There are two two-order added positive feedback loops, one of these is produced by the increased amounts of employees, which exacerbates the competitions within an organization, and thus promotes the performance of an organization; the other is directly produced by the directly prompting employees to improve their performance.

There are four three-order added positive feedback loops. Three of them are produced by hiring more employees to enhance the personnel performance and organizational performance, the fourth one is produced by motivating employees.

There are five four-order added positive feedback loops. Four of them are from hiring more employees. The other one comes from directly motivating employees to promote their performance.

There are five five-order added positive feedback loops. Four of them come from hiring more employees. The fifth one roots in directly motivating employees to promote their performance.

There is only one six-order added positive feedback loop, which is produced by directly motivating employees to promote their performance.

From all of above, 16 positive feedback loops have been added. All of these 16 increased positive feedback loops reflect that the human resource management system, which its core is the grade -salary incitement based on personnel performance and assisted by training and department management, not only can promote personnel performance within an organization and cause well performed employees to gain higher grade salary, and thus they have incitement to achieve more income; at the same time it results the badly performed employees manage to promote their salary grades under the spur of grade-salary system; but also can attract multi-levels external human resource, especially the high-level human resource, by the competent diversified grade-salary system. Thus, the performance of an organization can be promoted in the round.

(2) The added negative feedback loops caused by adopting grade-salary incitement

When an organization adopts the human resource management system, which its core is the grade-salary incitement and assisted by training and department management, the 17 added negative feedback loops are as follows:

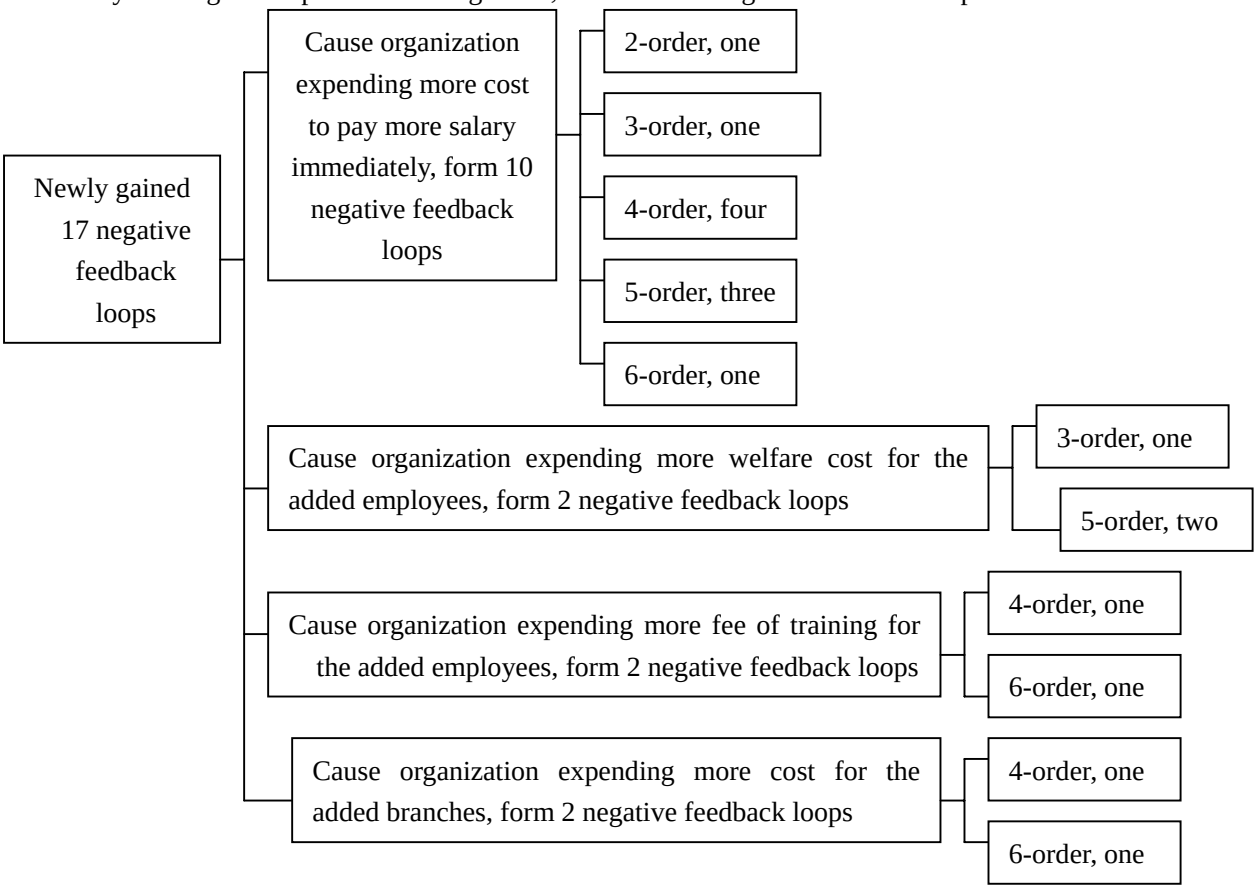


Figure 37 newly gained negative feedback loops to expend more cost

There is one added two-order negative feedback loop: the compensation cost results in the more expansion on salary, and

therefore it gives rise to a negative feedback loops.

There are two added three-order negative feedback loops: one comes from the compensation cost results in the expansion of the amount of employees, which aggrandize the welfare cost in an organization, and therefore it gives rise to a negative feedback loops. The other comes from more expansion on salary.

There are six added four-order negative feedback loops: the training fee of the increased employees caused by the compensation gives rise to expansion in the cost of an organization, and thus causes a negative feedback loop. One negative feedback loop mechanism is that the compensation causes an organization to hire more employees, which results in the organization setting up more departments, and thus the operation fees of increased departments raise the cost. Four of them are that compensation causes more salary expansion.

There are three added five-order negative feedback loops: one of them comes directly from the more salary decline of the performance of an organization; the rest two negative feedback loops are indirectly induced by the compensation, the mechanism is that the compensation induces the increase of employees, which aggrandizes the labor cost, and thus the performance of an organization declines.

There are three added sixth-order negative feedback loops: one of them comes directly from the more salary decline of the performance of an organization; the rest two negative feedback loops are indirectly induced by the compensation, the mechanism is that the compensation induces the increase of employees, which aggrandizes the training cost, and thus the performance of an organization declines.

From all of above, 17 negative feedback loops have been added. The implement of the human resource management system, which its core is the grade-salary incitement and assisted by training and department management requires enormous cost, especially the compensation cost, and the compensation cost of an organization is the crucial restricted factor and the parameters in the model。

4. Conclusion

By introducing and applying the diagonal-0 branch-vector matrix algorithm, building the HR management SD model focused on adopting the grade-salary incitement. This method is operational by following the principal from simple to complex, combination of reduction and integration. Also it is easy to discover the essence of practical issues and beneficial for adjustment analysis by computer emulation. By analyzing the cases, we concluded that the HR management system of an organization focused on adopting the grade-salary incitement is a growth limited flow chart. Using this policy can motivate employees to increase their performance and absorb ability labors out of the system. But in order to realize it, we should adjust the restricted elements. Using this theory, we assist some enterprise to improve on their HR management and have got good result. Now, we are doing some quantity research on it, and a lot of meaningful problems are waiting us to study in this field.

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