EMPLOYMENT POLICY AND EXPECTATIONS

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ABSTRACT

The paper studies with the help of a model the employment decisions of a firm through a business cycle which maximizes its discounted income and assumes that the forecasts are perfectly corred.

The firm produces a output function of number of workers employed by the firm at time t. The firm's labor force increases over time as a result of layoffs and quits. The firm can recalls workers at time t only if it has an inventory of previonsly laid off workers. The firm's output is supposed superior or equal to demand (function of price and time) at any time. The solution to the maximization problem will then yield an optimal output and employment plan which the firm proceeds to implement until its expectations about demand at time t.

1. INTRODUCTION

Recent papers have examined the decisions of layoffs and hires of the firm when the expectatives on the demand are cyclical. For example, Nickell (1978), Leban and Lesourne (1980, 1983), these papers assume that all layoffs are permanent.

A paper by Barron, Loewenstein and Black, considers workers employed by the firm that can be lays off today, are available for employment in the future. This paper show that layoffs can be optimal when the potencial to recall is introduced, this result is established in a model in which the firm can vary their labor input, not only by changing the number of workers that are employed,but also by changing the number of hours that employees work.

In this paper, we examine the policies of layoff, recall and hire of the firm when the demand expectatives are cyclical.We assume in the same way Barron, Loewenstein and Black did, that there is an inventory of previous laid off workers and as distinquished from what they did, we assume that the number of hours worked by each employee, can not be modifiered.

The paper is structured as follows. In section 2 we are presenting the model. The output, is assumed to be a function only of the number of workers employed by the firm and superior or equal to demand. Section 3 analyce the validity of the different policies abaut the output price, hires, laidoffs and recalls, through a bussines cycle. In section 4, we determine the firm's strategy if at the initial moment the firm is hiring, differing if at the initial moment the inventory of previous laid off workers is positive or null. Section 5 contains some concluding remarks.

2. THE MODEL

Consider a firm that produces an output y(t), give 'y

$$y(t) = f(N(t)) \qquad (1)$$

where N(t) denote the number of workers employed by the firm at time t. The function f(.) is assumed to be a concave given by

$$f[N(t)] = [N(t)]^{\alpha} \quad , \; 0 < \alpha < 1 \qquad (2)$$

For simplicity, we allow the firm to vary neither its capital stock nor its rate of capital utilization.

The firm's labor force increases over time as a result of new hires and recalls and it decreases as a result of layoffs and quits. Letting A(t) denote the number of workers hired at time t, L(t) the number of workers laid off at time t, R(t) the number of previous laid off workers recalled at time t, and q the rate at which employed workers quit, the rate of change in the firm's labor force at time t is given by

$$\dot{N}(t) = A(t) + R(t) - L(t) - qN(t) \qquad (3)$$

The firm can recall workers at time t only if it has an inventory of previous laid off workers. Denote this inventory of laid off individuals by S(t). The rate of change in this stock is given by

$$\dot{S}(t) = L(t) - R(t) - \theta S(t) \qquad (4)$$

where θ is the rate at which laid off workers find employment else where.

Let, a, the cost to the firm of hiring and training new workers at time t, let l and r the costs to the firm of laying off and recalling workers at time t, respectively and finally, let ω the wage rate. To simplify the analysis a, l, r, ω, will be assumed positive and constant through time.

We assume that the demand level at time t, x[p(t),t], where p(t) denote the price that the firm receives for its output at time t, is inferior or equal to output at any time

$$x[p(t),t] \leqslant y(t) \qquad (5)$$

Moreover, the demand is assumed perfectly known by the firm throughout the horizon.

The firm is assumed to maximize net present value, i.e., is maximizes

$$\int_0^{\infty} [p(t)x(p,t) - aA - rR - lL - \omega N] \, e^{-\rho t} \, dt$$

subject to the constraint (3), (4), (5) and non-negativity constraints on the control and state variables, where ρ is the rate disconunt.

The Hamiltonian function in our problem is

$$H(p,A,R,L,N,S,\psi_1,\psi_2) = p(t).x[p(t),t] - a.A(t) - r.R(t) - l.L(t) - \omega.N(t) + \psi_1[A+R-L-q.N] + \psi_2[L-R-\theta S] + \alpha_1.A + \alpha_2.R + \alpha_3.L + \alpha_4[y(t)-x(p,t)] + \Phi[L-R-\theta S] \quad (6)$$

where ψ_1 and ψ_2 are the costate variables associated with the state variables N and S; α_1, α_2 and α_3 are the Kuhn-Tucker multipliers associated with the non-negativity constraints on the control variables A, R and L, and Φ is the multiplier corresponding to the non-negativity constraint on the state variable S (that is not in the objective function). The solution to the optimal control problem must satisfy the conditions presented in the appendix.

We have 32 regimes a priori possible corresponding to the various combinations of positive or zero α_i's (i = 1,2,..,4) and Φ, but it is easy to prove that some are not liable to be part of an optimal strategy on the basis of the following propositions.

(P.1): The multipliers α_2 and α_3 can not be null simultaneously

Proof. From the conditions of the appendix, we have:

$$H'_R + H'_L = 0 = -(r + l) + (\alpha_2 + \alpha_3)$$

It is easy to see that Proposition (P.1) excludes 8 regimes.

(P.2): If the firm hire (A(t) > 0 over an interval), it has not excess capacity ($\alpha_4 > 0$).

Proof. If A(t) > 0 the costate variable $\psi_1 = a$. If the firm has excess capacity, the costate variable ψ_1 verifies

$$\dot{\psi}_1 = (\rho + q).\psi_1 + \omega$$

therefore $(\rho + q).a + \omega = 0$, but this is a contradiction since the rates are positives.

(P.3): If the multiplier Φ is positive ($\Phi > 0$), the firm neither lays off nor recalls.

Proof. If $\Phi > 0$, S = 0 therefore R = 0. If S = 0, L-R = θS = 0, therefore L = 0.

3. BUSSINES CYCLE

Assume now that the firm's expectations are:

$$x[p(t),t] = \begin{cases} M[p(t)]^{-\varepsilon}.e^{\delta t} & \text{for all } t \in [0,t_o) \\ M[p(t)]^{-\varepsilon}.e^{\delta t-\mu(t-t_o)} & \text{for all } t \in [t_o,t_1] \\ M[p(t)]^{-\varepsilon}.e^{\delta t-\mu(t_1-t_o)} & \text{for all } t \in (t_1,+\infty) \end{cases}$$

with $\mu - \delta > 0$, $\mu > 0$, $\delta > 0$, $M > 0$.

The demand curve is thus of constant elasticity with an upward trend expept in a slump between t_o and t_1 in which demande declines.

Furthermore, the following assumption is made: the function of revenue marginal

$$R[p(t)] = \frac{x[p(t),t]}{x'_p[p(t),t]} + p(t) = \frac{\varepsilon - 1}{\varepsilon} p(t)$$

is always positive: $\varepsilon > 1$, that means the Hamiltonian is a concave function.

As the multiplier $\alpha_4 = R\,[p(t)]$ it is easy to establish the following proposition:

(P.4): The firm has no excess capacity.

From the propositions (P.1), (P.2), (P.3) and (P.4), we have only six policies to be part of an optimal strategy. These six policies are:

POLICY 1. $S > 0$, $R = 0$, $L > 0$, $A = 0$

POLICY 2. $S > 0$, $R > 0$, $L = 0$, $A = 0$

POLICY 3. $S > 0$, $R = 0$, $L = 0$, $A > 0$

POLICY 4. $S > 0$, $R = 0$, $L = 0$, $A = 0$

POLICY 5. $S = 0$, $R = 0$, $L = 0$, $A > 0$

POLICY 6. $S = 0$, $R = 0$, $L = 0$, $A = 0$

Before discussing the firm's optimal strategies, we sketch the steps that will allow us characterize these policies.

Remark that the rate θ can be superior, inferior o equal to q. We analyse the case $\theta > q$, if $\theta \leq q$ an analoque study can be realized.

a) As the demand function $x[p(t),t]$ is equal to $y(t)$ along any optimal sequence, we have the price and the number of employed workers by the firm:

$$\delta = \varepsilon \frac{\dot{p}}{p} + \alpha \frac{\dot{N}}{N} \qquad (7)$$

for all $t \in [0,t_o) \cup (t_1, +\infty)$ and

$$\delta - \mu = \varepsilon \frac{\dot{p}}{p} + \alpha \frac{\dot{N}}{N} \qquad (8)$$

for all $t \in (t_o,t_1)$.

b) If $H(t) = \alpha.R[p(t)].[N(t)]^{\alpha-1}$ then from the conditions that are presented in the appendix and (1), (2), we have the following results in the six policies:

In the policy 1, as $\psi_1 \leq a$, we have $\omega-(\rho+\theta)l-(\theta-q)a \leq H(t)$, we assumed that $\omega-(\rho+\theta)l - (\theta-q)a \geq 0$, for characterize this policy. (Notice of that this policy is not feasible in expansion phase if $\theta = q$). In this policy the function H(t) is increasing, since $\dot{\psi}_1 = \dot{\psi}_2 = (\rho+\theta)\psi_2 = \dot{H}$. The number of workers employed by the firm is decreasing, and the inventory of laidoffs is increasing, decreasing or being constant if L(t) is superior, inferior or equal to $\theta.S$. As $0 < L/N < 1$, the price is increasing in expansion phase and in recession phase:

$$\alpha q + \delta - \mu < \varepsilon \frac{\dot{p}}{p} < \alpha(q+1) + \delta - \mu$$

then, the price can be increasing, decreasing or constant depending of the relations between the rates.

In the policy 2, also $\psi_1 \leq a$, we have $\omega + r(\rho+\theta) - (\theta-q)a \leq H$ (Notice that $\omega + r(\rho+\theta) - (\theta-q)a \geq \omega - (\rho+\theta)l - (\theta-q)a$). The behavior of the functions $\dot{\psi}_1$ and $\dot{\psi}_2$ is equal in the policy 1 and 2, therefore the function H(t) is increasing in both policies. Now, the inventory of laidoffs is always decreasing. The number of workers employed by the firm increases, decreases or is constant if R is equal, superior or inferior to qN.

In the policies 3 and 5, the function H(t) is constant, therefore:

$$\frac{\dot{p}}{p} = (1-\alpha) . \frac{\dot{N}}{N} \qquad (9)$$

In the expansion phase the price and the number of workers employed are increasing, while in recession phase both decreas. As in this policy $A > 0$, this policies are feasible in recession phase if $q > b$, where $b = \alpha + \varepsilon(1-\alpha)$. The inventory of laidoffs is null in the policy 5 and in the policy 3 is decreasing.

In the policies 4 and 6, the number of workers employed is decreasing. The price in the expansion phase is increas, while in the recession phase is constant, increasing or decreasing if $\delta - \mu$ is equal to $\alpha.q$, superior or inferior, then the function H(t) is decreasing in the expansion phase and increasing, decreasing or constant if q.b is equal, superior or inferior to $\mu - \delta$ in recession phase. In the policy 4, the inventory of laidoffs is decreasing and the policy 6 is null.

c) Since that, the coestate variable ψ_1 is continuous and the coestate variable ψ_2 is, in general, only piecewise continous with jumps on the right, only the following links have generality to be considered from the conditions presented in the appendix:

$1 \longleftrightarrow 4$; $1 \longrightarrow 5$; $2 \longrightarrow 3$; $2 \longleftrightarrow 4$

$2 \longrightarrow 5$; $2 \longrightarrow 6$; $3 \longrightarrow 4$; $4 \longrightarrow 3$

$5 \longleftrightarrow 6$; $6 \longrightarrow 1$

In the following section, we study how the various are linked during the demande cycle. Assumed that at the initial moment

$t = 0$, the function $H(0) = (\rho + q).a + \omega$ with $S(0) = 0$ or $S(0) \neq 0$, therefore we start with the policy 3 or 5.

4. THE DIFFERENT STRATEGIES

Assumed that at the initial moment the function H is equal to $(\rho + q)a + \omega$. The following propositions present and discuss the different strategies that the firm adapts.

(P.5): If the rate at which employed workers quit is superior to $(\mu - \delta)/b$, then the firm always follows a policy of hire positive.

Proof. If the inventory at the initial moment is positive, then the conditions of the policy 3 are verifying and if - $S(0) = 0$, we have the conditions of the policy 5. In this case, these policies are possible in expansion and recession phase. The behavior of hire is:

$$A(t) = N(t)[\frac{\delta}{b} + q]$$

from (3), (7) and (8) in the expansion phase; and

$$A(t) = N(t)\ [\frac{\delta - \mu}{b} + q]$$

from (3), (7) and (9) in the recession phase.

(P.6): If the rate at which employed workers is equal to $(\mu - \delta)/b$ then the firm does not hire in the recession phase.

Proof. If $S(0)$ is positive and $q = (\mu - \delta)/b$, the policy 3 is not possible in recession phase and since only the policy 4 ($A = 0$) can follow to policy 3, this will be adapted in the recession phase. In the policy 4, the number of workers decreas and the function $H(t)$ is constant, therefore in the second expansion phase the firm adapts a hire positive.

If $S(0) = 0$, the firm starts with the policy 5. Now then, this policy is not possible in recession phase. Only the policy 6 can follow to policy 5. In the policy 6, the function $H(t)$ is constant, we have $H(t_1) = H(t_o) = \omega + (\rho + \theta).a$, therefore, the firm can continue in the second recession phase with the policy 5.

In the following propositions, we assume that the rate at which employed workers quit is inferior to $(\mu - \delta)/b$.

Assumed that $S(0)$ is positive, then like in the cases preceding the firm starts with the policy 3 that is not possible in recession phase, therefore, in this phase to adapts a policy of null hire and positive inventory. In this policy, the function $H(t)$ is decreasing and

$$H(t_1) = H(t_o).e^{\frac{\delta - \mu + q.b}{\varepsilon}(t_1 - t_o)} \qquad (10)$$

with $H(t_o) = \omega + (\rho + \theta).a$.

(P.7): If $H(t_1)$ is superior or equal to $\omega + r(\rho + \theta) - (\theta - q).a$ then the following strategies are possible: 3 - 4 - 2 - 3, 3 - 4 - 2 - 4 - 3, 3 - 4 - 2 - 6 - 5.

Proof. Now, the conditions of policy 2 verify. Then, the firm recalls, but R can be inferior, superior or equal to -- q.N(t).

If $S(t_1) > q.N(t_1)$, the firm can considerer a $R > q.N$. Now, $S(t_1) > q.N(t_1)$ if

$$q.\frac{N(0)}{S(0)} < e^{-(\theta - q)t_1 - (\frac{\delta}{b} + q).t_o}$$

If the firm adapts R = q.N, then N is constant and S is decreasing, therefore, the relation $S(t) > q.N(t)$ is not always valid. Let us, denote t_2 at moment where $S(t_2) = q.N(t_2)$, then since the function H(t) is increasing in the policy 2, we have three possibilities: $H(t_2) = H(t_o)$ $H(t_2) < H(t_o)$ or $H(t) = \omega + (\rho + \theta).a$ with $t < t_2$. If $H(t_2) = H(t_o)$, the firm can continue with policy 3. If $H(t_2) < H(t_o)$, the firm continues with policy 4, where the function H(t) is increasing, until the function H(t) attains $H(t_o)$ to continue with policy 3. Finally , if $H(t) = \omega + a.(\rho + \theta)$ with $t < t_2$, the firm will adapt policy 3.

If the firm adapts R > q.N, then the number of workers increases and the inventory decreases. Also now, the condition S > q.N is not always valid and we are in the case preceding.

If $S(t_1) \leq q.N(t_1)$, the firm adapts a recall R < q.N . Now, the inventory and the number of workers decreases and the function H(t) increases. This policy can be maintened until the inventory is null. If t_2 is the moment where $S(t_2) = 0$, we have:

a) If $H(t_2) = \omega + (\rho + \theta).a$, the firm continues with policy 3.

b) If $H(t_2) < \omega + (\rho + \theta).a$, the firm continues with the policy 6, until the function H attains $\omega + (\rho + \theta).a$

c) If $H(t) = \omega + (\rho + \theta).a$, with $t < t_2$, the firm adapts at t the policy 3.

(P.8): If $H(t_1) \geq \omega - l(\rho + \theta) - (\theta - q).a$, where $H(t_1)$ is given by (10), then the following strategies are possibles for the firm: 3 - 4 - 1 - 3 , 3 - 4 - 1 - 4 - 3, 3 - 4 - 1 - 5, 3 - 4 - 1 - 6 - 5.

Proof. Now, we have the conditions of policy 1, therefore the firm laids off. If $N(t_1) > \theta . S(t_1)$, then $L \geq \theta.S$. The condition $N(t_1) > \theta . S(t_1)$ is true if

$$\theta \frac{N(0)}{S(0)} < e^{-(\theta - q)t_1 - (\frac{\delta}{b} + q)t_0}$$

If the firm adapts $L \geqslant \theta . S$, the relation $N(t) > \theta .S(t)$ is not always possible, since, the inventory $S(t)$ is constant or is increasing and $N(t)$ is decreasing. Since in policy 1 the function $H(t)$ is increasing, we have different sttuations. If at the t_2 moment: $N(t_2) \geqslant \theta .S(t_2)$ and $H(t_2) = \omega + a(\rho + \theta)$, the firm continues with policy 3 and if at the t_2 moment $N(t_2) = \theta .S(t_2)$ and $H(t_2) < \omega + a(\rho + \theta)$ the firm continues with policy 4 until the function attains $\omega + a(\rho + \theta)$ to continue with policy 3 afterwards.

If $N(t_1) \leqslant \theta . S(t_1)$ the firm adapts a laidoff $L < \theta .S$ the firm can follow this policy until the function $H(t)$ (that is increasing) attains $\omega + (\rho + \theta).a$ continuing with policy 3 or until the inventory is null, then the firm adapts policy 6 and then policy 5.

Remark that the function $H(t)$ at the t_1 moment has not to verify the condition of the propositions (P.7) or (P.8), then the firm can support the policy 4 in the second expansion phase until can be applied policy 1 and then we are in a case preceding.

The propositions (P.7) and (P.8) assume a positive inventory at the moment initial. Analyse now, the case $S(0) = 0$, the firm adapts policy 5 in the expansion phase, that in not possible in recession phase. The policy 6 can only follow to policy 5, therefore this policy is adapt in recession phase. Policy 1 and policy 5 are the anly policies that can follow policy 6. Now then, both require determined conditions about the $H(t)$ function. Policy 5 cannot be considered at the end of the recession phase because of the values that is reaches there, then we can adapt policy 1 if its conditions are verified, being in a situation already analized previously, in another case, we will adapt policy 6 until we can adapt policy 1.

5. CONCLUSION

This paper has analyzed the policies of hire, recall , layoff and prices of a firm in a bussines cycle. The paper can be considered as a Barron, Loewenstein and Black's work, accounting the bussines cycle. The rates of model, that have been assumed constant, as soon as the values at the initial moment of the state variables and output price have shown that play a crucial role to determine the policy that the firm adapts at the initial moment.

Assuming that the firm starts hiring, we have analyzed the different behaviors that the firm adapts as much in the expansion phase as in the recession phase. It is necessary to bring out the different behaviors of the firm after the recession phase, since the firm can adapt policies of recall,layoff, hire or policies of modification only of the price, until the firm reaches the situation of departure. These strategies depend on the rates and the duration on the recession phase.

Obviouly, these considerations have been realized in the particular case that the model analyses.

APPENDIX

From (6), a solution to the optimal control problem must satisfy the following conditions:

$$\frac{\partial H}{\partial p} = x(p,t) + p(t).x'_p(p,t) - \alpha_4.x'_p(p,t) = 0$$

$$\frac{\partial H}{\partial A} = -a + \psi_1 + \alpha_1 = 0$$

$$\frac{\partial H}{\partial R} = - r + \psi_1 - \psi_2 + \alpha_2 - \Phi = 0$$

$$\frac{\partial H}{\partial L} = - 1 - \psi_1 + \psi_2 + \alpha_3 + \Phi = 0$$

$$-(\dot{\psi}_1 - \rho\psi_1) = - \omega - q\,\psi_1 + \alpha_4.f'_N(N)$$

$$-(\dot{\psi}_2 - \rho\psi_2) = - \theta\psi_2 - \theta\Phi$$

$$\alpha_4\,[f[N(t)] - x(p,t)] = 0 \quad ; \quad \alpha_4 \geqslant 0$$

$$\alpha_1.A = 0 \quad ; \quad \alpha_1 \geqslant 0$$

$$\alpha_2.R = 0 \quad ; \quad \alpha_2 \geqslant 0$$

$$\alpha_3.L = 0 \quad ; \quad \alpha_3 \geqslant 0$$

$$\Phi.S = 0 \; ; \; \Phi[L - R - \theta S] = 0 \quad ; \quad \Phi \geqslant 0$$

The transversality conditions corresponding to the control problem are

$$\lim_{t \to \infty} \psi_1 \geqslant 0 \quad ; \quad \lim_{t \to \infty} \psi_1.N = 0$$

$$\lim_{t \to \infty} \psi_2 \geqslant 0 \quad ; \quad \lim_{t \to \infty} \psi_2.S = 0$$

The costate variable ψ_2 is, in general, only piecewise continuous. At each jump point t_i

$$\psi_2^{+}(t_i) = \psi_2^{-}(t_i) + K_i$$

where the non-negative values of K_i are chosen so as to insure the conditions above.

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