Go Back

"A Rate Variable Fundamental In-tree Modeling and Branch-Vector Approach to Evaluating Feedback Loops"

Jia Renan, Hu Ling, Yang Bo, Ding Rong-hua

Abstract

A new system dynamics rate variable fundamental in-tree modeling and a branch-vector determinant approach to feedback loops analysis are expounded in this paper. These approaches offer an effective means of modeling normally while simultaneously analyzing feedback loops explicitly. A case study of a system of energy and ecology in a village with the principle of sustainable development illustrates these new approaches.

China is a country with an extreme disequilibrium of energy resources and a serious shortage of energy. Especially in vast hilly rural areas, conventional energy resources are very deficient and two-thirds of energy for daily life in these areas is from biomass (mainly including straw, firewood, manure and urine, etc.). As the energy resources from biomass in those areas are not efficiently utilized, it brings about that firewood is seriously felled. This causes that the covering ratio of forests is falling by nearly one-hundredth per annum and that the area of soil erosion is enlarging year by year.

Jiangxi is a province with energy shortage in China. Wangheqiu Village in a county of Jiangxi province is a typical hilly rural area which is lack of energy. Peasants in the village did not attach importance to the re-utilization of energy from biomass. It caused the environment pollution and strongly restricted the development of village economy. For this reason, a research project on the System of Energy and Ecology in Wangheqiu Village was subsidized by the State Science and Technology Commission of China (SSTCC). This project aimed at the exploitation of energy, ecology and economy, while was strongly connected with the population development. We investigated the situation , and designed the interplanting , forests and fruit trees , the methane-generating , the energy-saving and so on. After having obtained a set of data, we made study on simulation, optimization and planning. In this stage, a system dynamics model was used for simulation.

A new approach to modeling

System dynamics modeling is a structural approach, building a feedback model based on causal diagrams and flow diagrams. The change of feedback loops in a model is of great importance to system analyzing, model debugging and simulation output analyzing. Without a normal method, however, it is difficult to obtain all feedback loops in a complex system dynamics flow diagram . Therefore, capturing the essential rate variable in a system, we introduce some concepts such as rate variable fundamental in-tree, embedding operation and so on, then provide a system dynamics rate variable fundamental in-tree modeling. With this new approach, modelers can build models normally while simultaneously analyze feedback loops explicitly. In addition, This approach is advantageous to model building, model debugging and simulation output analyzing.

In order to expound this new approach, some definitions are given in the section.

Definition 1 Let $t \in T$ and a dynamic directed diagram $T(t)=(V(t),X(t))$ be given. There exists a dot $v(t) \in V(t)$,such that for each dot $u(t) \in V(t)$, there is a unique path from u(t) to v(t), then the dynamic directed diagram T(t) is called an in-tree. The dot v(t) is called the tree root. The dot u(t) is called the tree top if $d^-(u(t))=0$. A directed path from the tree top to the tree root is called an in-branch.

Definition 2 Given a SD flow diagram and an in-tree T(t), if a rate variable is the tree root of T(t) and a level variable is the tree top of T(t), T(t) is called a rate variable in-tree.

Definition 3 The number of level variables, which are contained in an in-branch, is called branch-order-length. A rate variable in-tree is said to be a rate variable fundamental in-tree, if each branch-order-length equals one.

On the basis of the concepts above, we give the system dynamics rate variable fundamental in-tree modeling.

•Based on system qualitative analysis, determine level and rate variable pairs of a system dynamics model $\{(Lev_i(t),R_i(t))|i=1,2,\cdots,n\}$, where $\{R_i(t)=R_{i1}(t)-R_{i2}(t)|i=1,2,\cdots,n\}$,$R_{i1}(t)$ is an in-rate variable and $R_{i2}(t)$ is an out-rate variable.

• For each $R_i(t)(i=1,2,\cdots,n)$, build a rate variable fundamental in-tree $T_i(t)$, with $R_i(t)$ being its tree root and level variables, which affect $R_i(t)$ directly or indirectly through auxiliary variables, being its tree tops. Since all in-branches in $T_i(t)$ are ended with level variables, add variables to an in-branch until a level variable.

If a level variable affects $R_i(t)$ through different paths in an in-tree, it will be denoted by a different number and be considered a different variable. For an auxiliary variable, a similar argument is available.

Based on the above, a rate variable fundamental in-tree model is built.

While building an in-tree model, if there are some same auxiliary variables , only remain one of in-subtrees, whose root is the same auxiliary variable. Meanwhile, delete all the other in-subtrees. Then, the in-tree model is called a simplified rate variable fundamental in-tree model.

The relationship between a rate variable fundamental in-tree model and a SD flow diagram

Both a flow diagram model and a rate variable fundamental in-tree model can describe an objective system. They are same in essence, so they can transform from each other. In order to illustrate their relationship, we give two definitions as follows.

Definition 4 A sub-diagram of a flow diagram is called a half flow sub-diagram. A half flow sub-diagram is called a flow sub-diagram, if it contains not only a level variable LEV(t) but also a rate variable R(t), LEV(t) and R(t) being a pair.

Definition 5 Let $t \in T$ and two half flow sub-diagram $G_1(t)=(Q_1(t),E_1(t),F_1(t))$, $G_2(t)=(Q_2(t),E_2(t),F_2(t))$ be given.

1)do $G'(t)=G_1(t) \cup G_2(t)$

2)If a level variable $L_p(t)$ and its corresponding rate variable $R_p(t)$ are contained in $G_i(t)$ (i=1,2), add an arc $R_p(t) \rightarrow L_p(t)$ to $G'(t)$ so as to obtain a half flow sub-diagram G(t).

This operation is defined as embedding operation and noted as . So $G(t)=G_1(t) \quad G_2(t)$

With this embedding operation, a rate variable fundamental in-tree model can be transformed to a flow diagram.

Let a flow diagram G(t) and n rate variable fundamental in-trees $T_1(t)$, $T_2(t)$,$\cdots T_n(t)$ be given. It can be learned that they both describe the same system. Do

- $G_1(t)=T_1(t) \quad T_1(t)$
- $G_i(t)=G_{i-1}(t) \overrightarrow{\cup} T_i(t)(i=2,3,\cdots,n)$

Then $G_n(t)=G(t)$.

The above expounds the means of transforming a rate variable fundamental in-tree model into a flow diagram model. In fact, however, modelers only build an in-tree model of a given system, then construct equations and simulate the system, rather than build a flow diagram.

It should be mentioned that the branch-vector determinant approach would be expounded in the following case study.

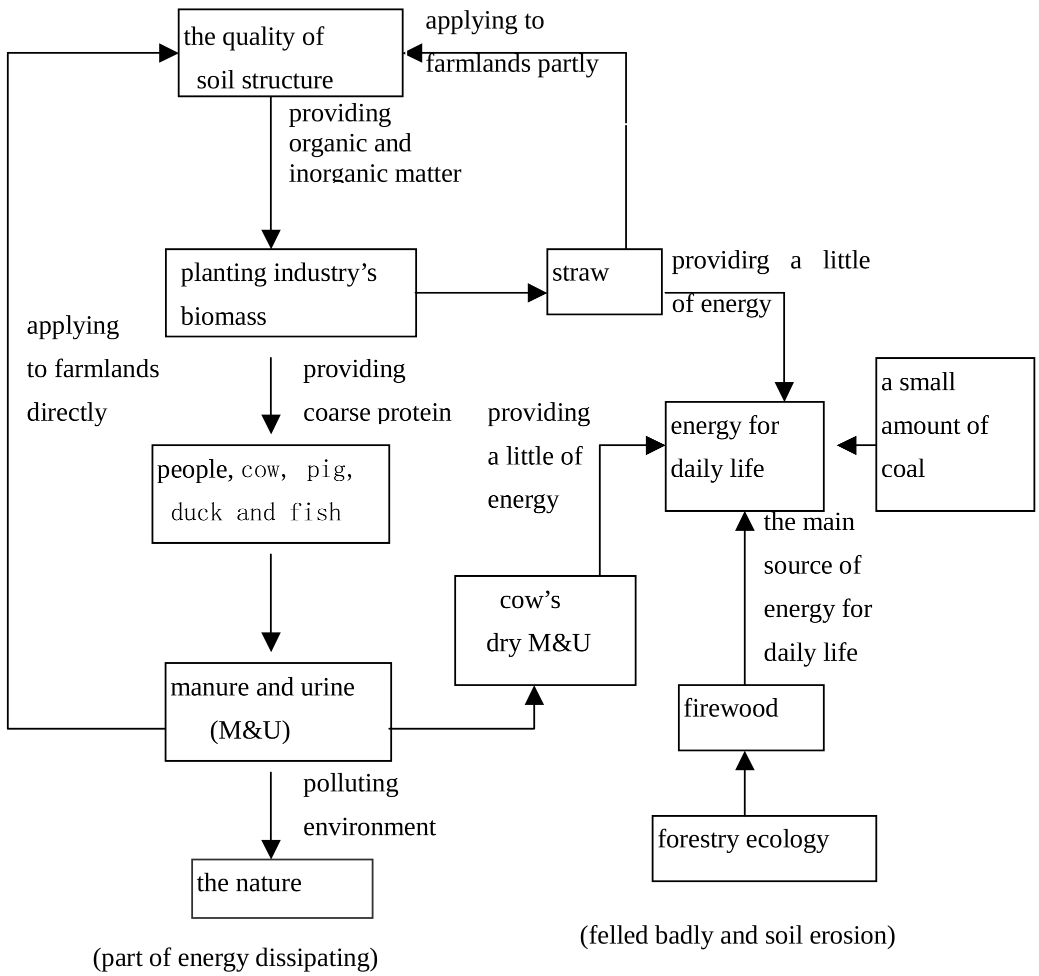
A Case Study

This case is chosen because it got pass of the state-class appraisal and won the second award of the Advancement of Science and Technology granted by JiangXi province’s government in 1998.

The old structure of energy and ecology’s system in Wangheqiu

Based on the situation above ,we obtained a diagram illustrated in figure 1. From the diagram, it could be learned that the main problem of the old system was that manure and urine was not exploited and re-utilized as well as the interplanting was not developed.

Fig 1.
The old structure of energy and ecology in Wangheqiu



The new structure of energy and ecology's system in Wangheqiu

In order to solve the problem above, we designed a new structure of the system. Figure 2 provides a new structure. The most essential distinction between the new and the old structure is re-exploiting the energy resources from biomass by generating methane.

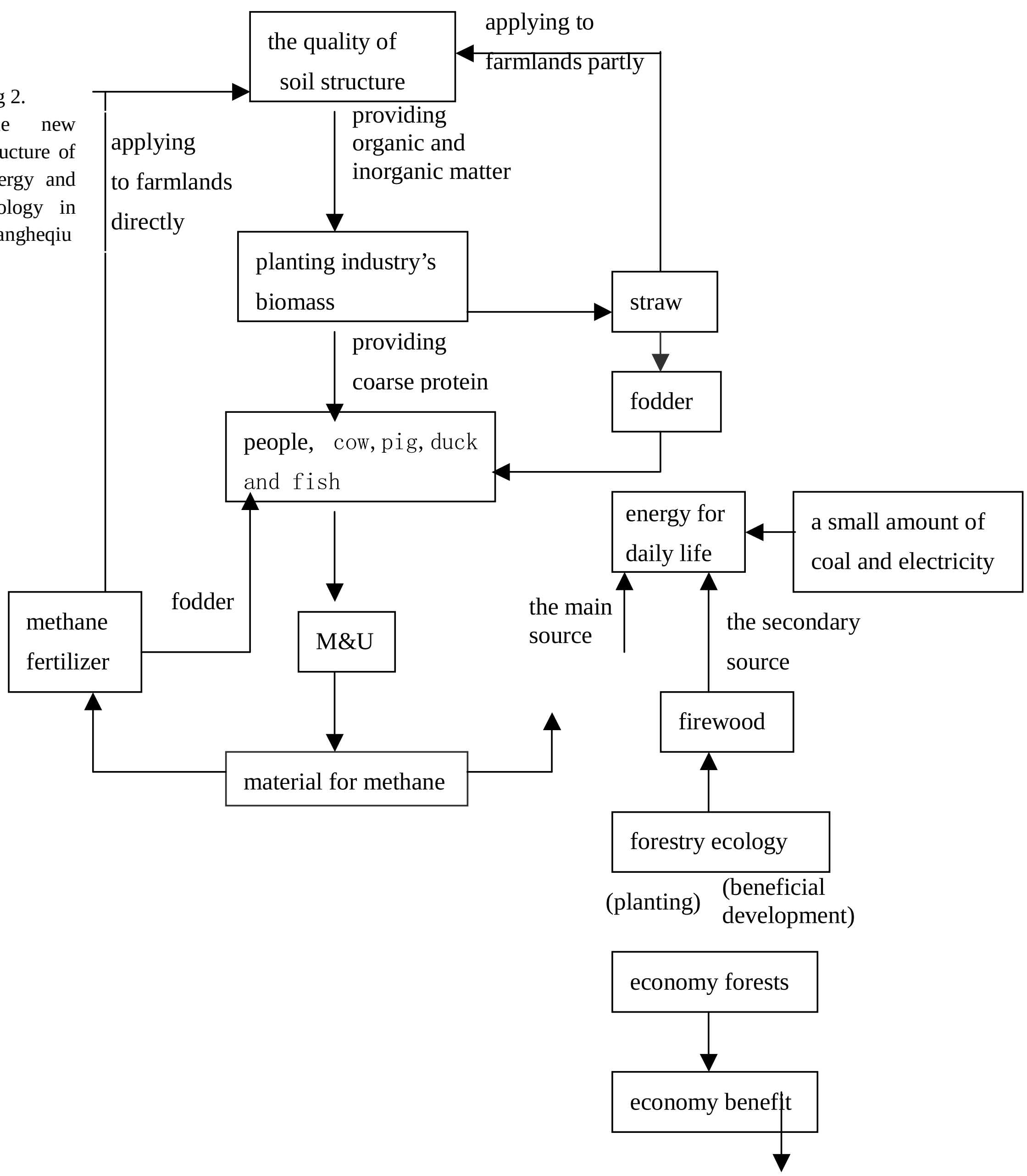


Fig 2. The new structure of energy and ecology in Wangheqiu

Build simplified and strongly simplified rate variable fundamental in-tree model

In the light of system qualitative analysis, we divided the system into Energy. Population, Interplanting, Forests, Livestock and Economy subsystems. Then we built all level and rate variable pairs as follows.

1) Energy substructure (Methane, Methane Rate)

2) Population substructure (Population, Population Rate)

3) Interplanting substructure (Rice, Rice Rate)

4) Forests substructure (Firewood Area, Firewood Area Rate)

5) Livestock substructure (Pig, Pig Rate)

6) Economy substructure (Total Output Value. Total Output Value Rate)

On the basis of practical background, we build a leading simplified rate variable fundamental in-tree model (SI) (see Fig.3). The sixth substructure will be discussed in the later.

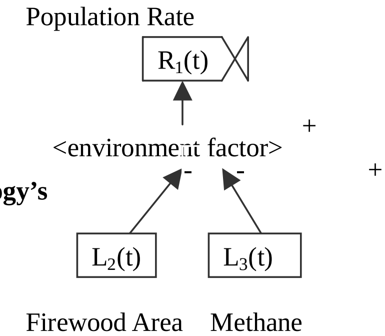
In order to analyze feedback loops conveniently in the following branch-vector determinant approach, we provide a concept of strongly simplified rate variable fundamental in-tree model on the basis of simplified rate variable fundamental in-tree model.

Definition 6 Given a simplified rate variable fundamental in-tree model (SI). Handle every in-tree as follows: delete all non-repeated auxiliary variable of every branch, reconnect the remained variable in the previous direction. After that, maybe there are P same branches, only remain one and change its top variable $V_k(t)$ to $P \times V_k(t)$ (where $V_k(t)$ is repeated variable or level variable). By this handling, we call the in-tree model strongly simplified rate variable fundamental in-tree model(SSI).

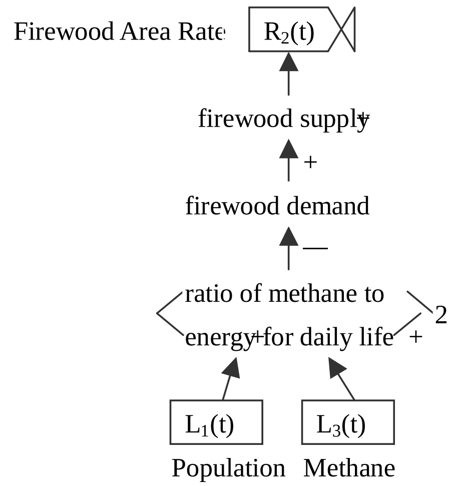
Also it can be proved that the number of feedback loops obtained from SI equals the number that is obtained from its corresponding SSI with embedding operation SSI. The feedback loop obtained from SSI is said to be the core of feedback loops. It can be learned that the core of feedback loops is corresponding to the feedback loop. So,only we obtain the core of feedback loop, we obtain the feedback loop.

Based on the concept above, we obtain the SSI of Wangheqiu energy and ecology's system. (see Fig 4).

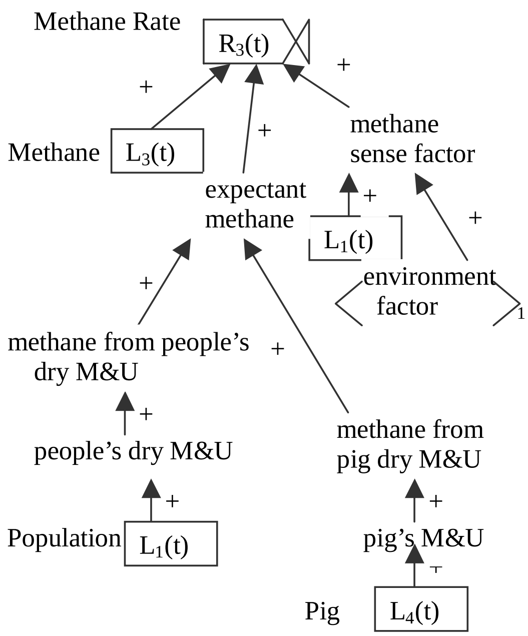
Fig 3. The SI of Wangheqiu energy and ecology's system



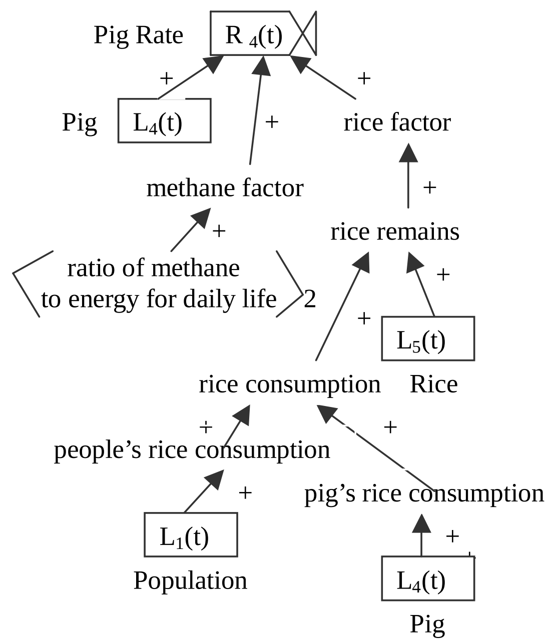
(a) Population SI $T_1(t)$



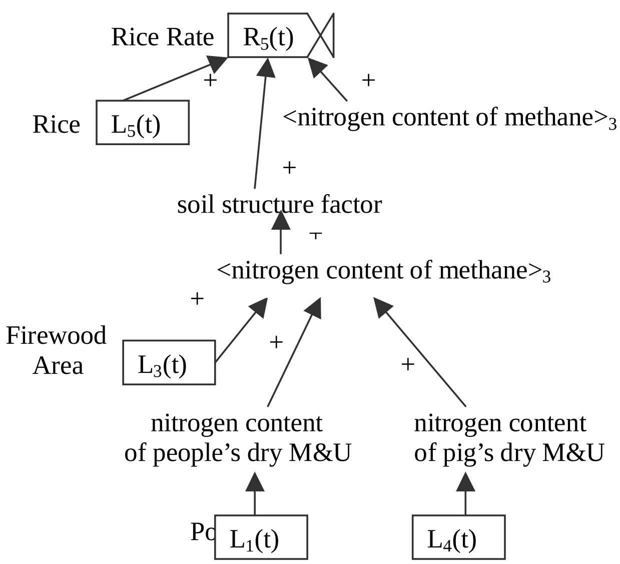
(b) Firewood Area SI $T_2(t)$



(c) Methane SI $T_3(t)$

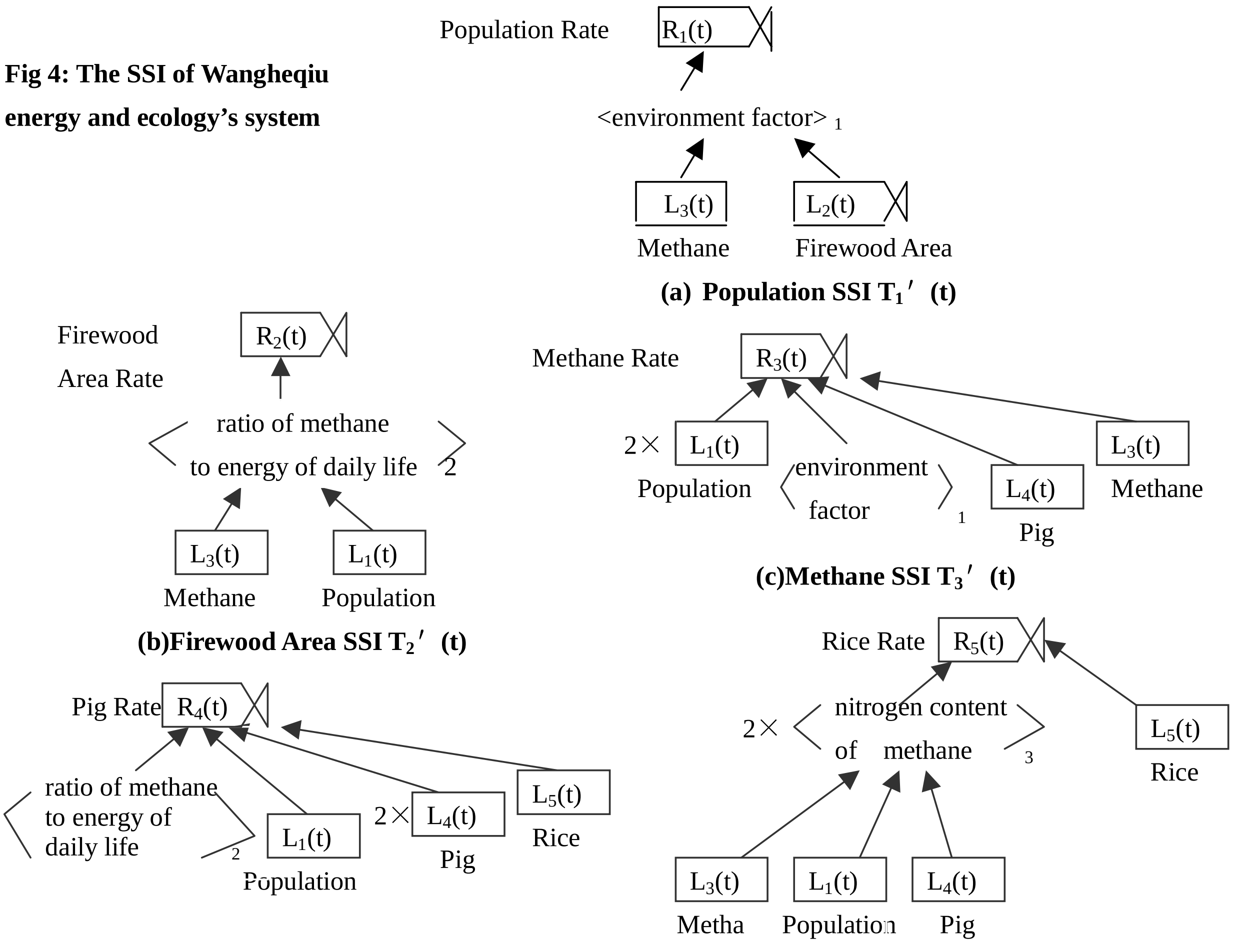


(d) Pig SI $T_4(t)$



(e) Rice SI $T_5(t)$

Fig 4: The SSI of Wangheqiu energy and ecology's system



(a) Population SSI $T_1'(t)$

(b)Firewood Area SSI $T_2'(t)$

(c)Methane SSI $T_3'(t)$

(d)Pig SSI $T_4'(t)$

(e)Rice SSI $T_5'(t)$

A branch—vector determinant approach to feedback loops analysis

In the above, using SD rate variable fundamental in-tree modeling, we have already constructed an in-tree model of a given complex system . Then we can directly build equations and compile programs with Vensim sofeware to simulate , Furthermore, in order to well consult with correspondents and meet the debugging needs , the following important work that we need to do is to analyze feedback loops. As the above mentioned, conventional analyzing method is difficult to obtain all the feedback loops of a complex SD flow diagram. So, in the sections below , we provide a new algebraic means to analyzing feedback loops on the basis of an in-tree model and embedding operation. First, some relevant concepts are proposed (in order to depict simply and conveniently, above-mentioned kinds of in-tree are generally designated as in-tree).

Definition 7 Given an in-tree, if there exists a directed path $R_i(t) \leftarrow q_1 \times m_1(t) \leftarrow q_2 \times m_2(t) \leftarrow \cdots \leftarrow q_r \times m_r(t) \leftarrow L_j(t)$ (the polarity is neglected), where $R_i(t)$ is a rate variable, $L_j(t)$ is a level variable, $m_i(t)(i=1,2, \cdots, r)$ is an auxiliary variable and $m_i(t)$ is different from $m_j(t)$ $(i \neq j)$, then this directed path is called an in-branch, written as $p_1(R_i, m_1, m_2, \cdots, m_r, L_j)$, where $p_1 = q_1 \times q_2 \times \cdots \times q_r$. And $p_1(R_i, m_1, \cdots, m_r, L_j)$ is called a branch-vector, $R_i(t)$ being the root of branch-vector and $L_j(t)$

being the top. If i equals j, this branch-vector is said to be a root-top-connected branch-vector. The sum of branch-vectors only indicates all the branches in the in-trees, which correspond to each branch-vector of the sum.

Branch-vector multiplication formula

Given two branch-vectors, $p_1(R_i, m_1, m_2, \cdots, m_r, L_j)$ and $p_2(R_t, s_1, s_2, \cdots, s_k, L_n)$, then $p_1(R_i, m_1, m_2, \cdots, m_r, L_j) \times p_2(R_t, s_1, s_2, \cdots, s_k, L_n)$

$$= \begin{cases} (p_1 \times p_2)(R_i, m_1, \cdots, m_r, L_j, R_t, s_1, \cdots, s_k, L_n) & \text{if } R_t \text{ and } L_j \text{ is a pair and all the variables are different} \\ (p_1 \times p_2)(R_t, s_1, \cdots, s_k, L_n, R_i, m_1, \cdots, m_r, L_j) & \text{if } R_i \text{ and } L_n \text{ is a pair and all the variables are different} \\ 0 & \text{otherwise} \end{cases}$$

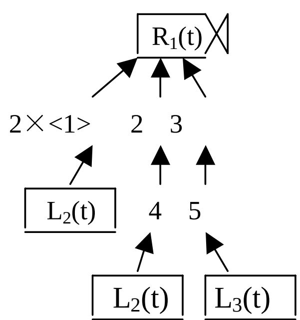
This formula is based on the practical background that feedback loops are formed by root-top-connected branches without same variables. From it, we can also learn that the commutative law and distributive law of the branch-vector multiplication are truth.

Definition 8 Let $T_i(t)$ and $T_j(t)$ be two in-trees, with $R_i(t)$ and $R_j(t)$ being respectively the roots of $T_i(t)$ and $T_j(t)$, $(R_i(t), L_i(t))$ and $(R_j(t), L_j(t))$ being pairs, then in $T_i(t)$ the branches whose tops are $L_j(t)$ are called root-connected branches of $T_j(t)$ to $T_i(t)$, and the sum of them is called root-connected branch-vector of $T_j(t)$ to $T_i(t)$, written as a_{ij}.

Example An in-tree $T_1(t)$ is illustrated in Fig 5 (the polarity is neglected), numbers 1,2,3,4,5 stand for auxiliary variables and the root of another in-tree $T_2(t)$ is $R_2(t)$,($R_2(t)$, $L_2(t)$) being a pair. Then the root-connected branch-vector of $T_2(t)$ to $T_1(t)$ is $2(R_1, 1, L_2) + (R_1, 2, 4, L_2)$, written as a_{12}.

Fig.5

An in-tree $T_1(t)$



Definition 9

Definition 10 The complement minor of a_{ij} in *n*th order branch-vector determinant A is (*n*-1)th order determinant obtained by omitting the *i*th row and *j*th column from A.

By definition 9 and 10, it also can be learned that *n*th order branch-vector determinant equals a sum of multiplying all the elements of its one free row(colum) with their corresponding complement minors.

Definition 11 A determinant, which is obtained by keeping the values of a_{ij} for $i \neq j$ and letting all the elements a_{ii} （i=1, ⋯,n）be one in the *n*th branch-vector determinant of the in-tree model, is said to be a 1-diagonal determinant of this in-tree model.

Definition 12 A determinant, which is obtained by keeping the values of a_{ij} except letting a_{nn} be zero in the *n*th 1-diagonal determinant of the in-tree model,is said to be a *n*th 1-0-diagonal determinant of this in-tree model.

In the sections below, two approaches to evaluating feedback loops with branch-vector determinants are provided. The method I is used to evaluate the number of new-added feedback loops generated in each embedding operations, and with method Ⅱ, we can obtain all the feedback loops of the in-tree model at one time.

Approach Ⅰ to evaluating feedback loops with branch-vector determinants

Let an in-tree model (Ⅰ): $T_1(t)$, ⋯,$T_n(t)$,its corresponding SI(Ⅱ) : $T'_1(t)$, ⋯,$T'_n(t)$,SSI(Ⅲ): $T''_1(t)$, ⋯,$T''_n(t)$and embedding operations $G_1(t)=T_1(t)\vec{\cup}T_1(t)$, $G_i(t)=G_{i-1}(t)\vec{\cup}T_i(t)$(i=2,3, ⋯,n) be given.

Step 1 According to the root-top-connected branches of each in-tree $T_i(t)$(i=1,2, ⋯,n), directly obtain all the one-order feedback loops embedded by the model (Ⅰ).

Step 2 On the basis of the model (Ⅲ), for i=2,3, ⋯,n, respectively evaluate the corresponding 1-0-diagonal determinant

$$A_i=\begin{vmatrix} 1 & a_{12} & \cdots & a_{1,i-1} & a_{1i} \\ a_{21} & 1 & \cdots & a_{2,i-1} & a_{2i} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i-1,1} & a_{i-1,2} & \cdots & 1 & a_{i-1,i} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{i,i-1} & 0 \end{vmatrix} \qquad (i=2,3,\cdots,n)$$

If $A_i=0$, it shows that there is no any new added feedback loops (more than one-order) generated in embedding $T_i(t)$ into $G_{i-1}(t)$. Conversely, if A_i equals a sum of Non-zero Root-top-connected Branch-vectors (NRB), then what is linked with every NRB is the core of the new-added feedback loop.

Step 3 For all the cores of feedback loops, apply the inverse transformation from SI to SSI, i.e. supplementing the non-repeated auxiliary vertices that are cancelled in the transformation from SI to SSI. After this inverse transformation , the core is namely the required feedback loop.

According to step 2 and 3, after i runs over 2,3, ⋯,n ,we can obtain all the feedback loops.

Approach Ⅱ to evaluating feedback loops with branch-vector determinants

As the above mentioned, the approach Ⅰ can evaluate the number of new-added feedback loops in every embedding operation, which is very useful in assessing the effects of every in-tree to the whole system. In addition, we have another approach Ⅱ that can evaluate all the feedback loops at one time. In order to get it, we only need to change step 2 of the method Ⅰ to evaluate the 1-diagonal determinant of the model (Ⅲ)

$$A = \begin{vmatrix} 1 & a_{12} & \cdots & a_{1n} \\ a_{21} & 1 & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{vmatrix} .$$

If A=0, it shows that there is no any feedback loops (more than one-order) generated in the process of embedding operations for the in-tree model(Ⅰ).Conversely, if A equals a sum of NRB, then what is linked with every NRB is the core of the new-added feedback loop.

By the definition of branch-vector determinant, the form of feedback loops and the branch-vector multiplication formula, we can prove the following theorem to verify the correctness of two approaches above.

Theorem Given an in-tree model $T_1(t)$, $T_2(t)$, $\cdots T_n(t)$ and embedding operations $G_1(t)=T_1(t) \vec{\cup} T_1(t)$, $G_k(t)=G_{k-1}(t) \vec{\cup} T_k(t)(k=2,3\cdots,n)$,then all the feedback loops (more than one-order) contained in the diagram $G_n(t)$ correspond, one by one, to the value of 1-diagonal determinant formed by $T_1(t)$, $\cdots$,$T_n(t)$. Moreover, the new-added feedback loops from two-order to (k+1)-order generated by embedding $T_{k+1}(t)$ into $G_k(t)$ correspond, one by one, to the value of 1-0-diagonal determinant A_{k+1} formed by $T_1(t)$, $\cdots$, $T_{k+1}(t)$,where

$$A_{k+1} = \begin{vmatrix} 1 & a_{12} & \cdots & a_{1k} & a_{1,k+1} \\ a_{21} & 1 & \cdots & a_{2k} & a_{2,k+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{k1} & a_{k2} & \cdots & 1 & a_{k,k+1} \\ a_{k+1,1} & a_{k+1,2} & \cdots & a_{k+1,k} & 0 \end{vmatrix}$$

In this section we take the above-mentioned SD model of Wangheqiu energy and ecology system for example to expound the new evaluating method of feedback loops.

Application of the method Ⅰ

According to the step one, we can learn that there are five one-order feedback loops in this system (see Fig.3).

• **Feedback loop 1 (one-order):** $R_3(t)$ Methane Rate (m^3 per year) $\overset{-}{\leftarrow}$ $L_3(t)$ Methane (m^3) $\overset{+}{\leftarrow}$ $R_3(t)$

This negative feedback loop depicts a cycle law, that is, on account of restriction of natural resources, the increase in output of methane leads to the diminution of relative increase, thus results in the relative decrease of methane.

• **Feedback loop 2 (one-order):** $R_3(t)$ Methane Rate (m^3 per year) $\overset{+}{\leftarrow}$ Methane Sense Factor $\overset{+}{\leftarrow}$ Environment Factor $\overset{+}{\leftarrow}$ $L_3(t)$ Methane (m^3) $\overset{+}{\leftarrow}$ $R_3(t)$.

This positive feedback loop depicts an increasing cycle law, that is, the increase in Methane leads to the improvement of Enrironment Factor and Methane Sense Factor, thus results in the re-growth of Methane .

- **Feedback loop 3-5 are omitted.**

By the step two and three, all the new-added feedback loops generated in every embedding operation can be evaluated as follows.

$$1)A_2 = \begin{vmatrix} 1 & (R_1,1,L_2) \\ (R_2,2,L_1) & 0 \end{vmatrix} = (R_2,2,L_1,R_1,1,L_2)$$

It shows that a piece of two-order feedback loop's core is generated in embedding $T_2(t)$ to $G_1(t)$,which is $R_2(t)\leftarrow$ <Ratio of Methane to Energy of Daily Life>$_2 \leftarrow L_1(t) \leftarrow R_1(t) \leftarrow$ <Environment Factor>$_1$ $\leftarrow L_2(t)$.Because that $R_2(t)$ $\leftarrow$< Ratio of Methane to Energy of Daily Life >$_2\leftarrow L_1(t)$ (in SSI) is formed by canceling non-repeated auxiliary variable vertexes, Firewood Supply and Firewood Demand, from the branch $R_2(t)$ $\leftarrow$Firewood Supply$\leftarrow$Firewood Demand$\leftarrow$< Ratio of Methane to Energy of Daily Life >$_2\leftarrow L_1(t)$(in SI) and according to the inverse transformation from SI to SSI, the obtained feedback loop core can be added with cancelled vertexes to form a piece of new-added feedback loop of two-order embedded by $T'_2(t)$ and $T'_1(t)$.

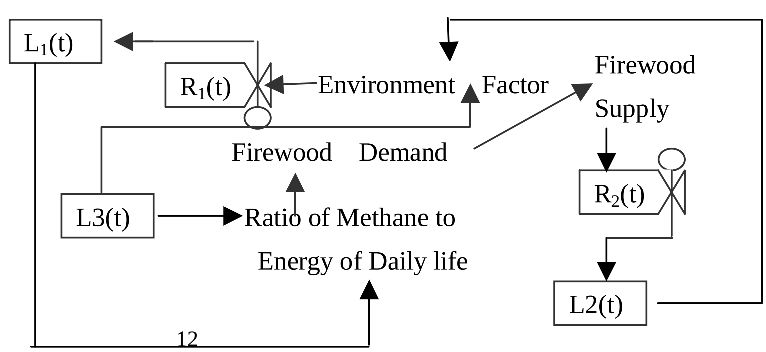
- **Feedback loop 6 (two-order):**$R_2(t)$ Firewood Area Rate (hectare per year) $\xleftarrow{+}$Firewood Supply $\xleftarrow{+}$ Firewood Demand $\xleftarrow{-}$Ratio of Methane to Energy of Daily Life $\xleftarrow{-}$ $L_1(t)$ Population (person) $\xleftarrow{+}$ $R_1(t)$ Population Rate (person per year) $\xleftarrow{+}$Environment Fator $\xleftarrow{-}$ $L_2(t)$Firewood Area (hectare) $\xleftarrow{+}R_2(t)$.

This negative feedback loop depicts a restricting cycle law. In the condition of generating methane, the integration of stereo-forestry and population development leads to the increase in amount of fruit trees, the diminution of Firewood Area, the better environment, the increase in population and the decrease in Ratio's, thus results in the growth of Firewood Demand and, finally, the increase in Fireword Area .

In addition, in order to further show that flow diagrams and in-tree models are interlinked, we can draw the Fig 6 which depicts the relationship between two fundamental variables, Population and Firewood Area. But it should be mentioned that in the practical application it is not necessary to draw this flow diagram.

Fig.6

$G_2t)=G_1(t)\overleftarrow{\cup}T'_2(t)$



$$A_3 = \begin{vmatrix} 1 & (R_1,1,L_2) & (R_1,1,L_3) \\ (R_2,2,L_1) & 1 & (R_2,2,L_3) \\ 2(R_3,L_1) & (R_3,1,L_2) & 0 \end{vmatrix}$$

$$= 2(R_3,L_1,R_1,1,L_3) + (R_3,1,L_2,R_2,2,L_3) + 2(R_3,L_1,R_1,1,L_2,R_2,2,L_3)$$

Corresponding with $2(R_3,L_1,R_1,1,L_3)$, the feedback loop's core embedded by $T''_3(t)$ and $T''_1(t)$ is $R_3(t)\leftarrow 2L_1(t)\leftarrow R_1(t)\leftarrow$<Environment Factor>$_1\leftarrow L_3(t)\leftarrow R_3(t)$(see Fig 4), where $R_3(t)\leftarrow 2L_1(t)$ is formed by canceling non-repeated variable vertexes from two branches of $T'_3(t)$. Concretely, one piece of $R_3(t)\leftarrow L_1(t)$ is formed by deleting Expectant Methane, Methane from People's Dry M&U and People's Dry M&U from $R_3(t)$ ←Expectant Methane←Methane from People's Dry M&U← People's Dry M&U←$L_1(t)$, the other is formed by canceling Methane Sense Factor from $R_3(t)$← Methane Sense Factor←$L_1(t)$. So, by the inverse transformation, two new-added feedback loops of two order can be obtained , as analyzed below.

•**Feedback loop 7(two-order):** $R_3(t)$ Methane Rate (m^3 per year) $\xleftarrow{+}$Expectant Methane (m^3) $\xleftarrow{+}$ Methane from People's Dry M&U (m^3) $\xleftarrow{+}$People's Dry M&U (kg) $\xleftarrow{+}$$L_1(t)$Population (person) $\xleftarrow{+}$$R_1(t)$Population Rate (person per year) $\xleftarrow{+}$Environment Factor$\xleftarrow{+}$$L_3(t)$Methane($m^3$) $\xleftarrow{+}$$R_3(t)$

This positive feedback loop depicts an auto-increasing cycle law, that is, the development of generating methane leads to the increase in Methane, the improvement of environment and the growth of population, thus brings about the increase in amount of M&U and, finally, the re-growth of Methane.

• **Feedback loop 8(two-order):** $R_3(t)$ Methane Rate (m^3 per year) $\xleftarrow{+}$Methane Sense Factor$\xleftarrow{+}$ $L_1(t)$Population (person) $\xleftarrow{+}$$R_1(t)$Population Rate (person per year) $\xleftarrow{+}$Environment Factor$\xleftarrow{+}$$L_3(t)$ Methane (m^3) $\xleftarrow{+}$$R_3(t)$.

This positive feedback loops depicts an auto-increasing cycle law, that is, the development of generating methane leads to the increase in Methane, the improvement of environment and the growth of population whose Methane Sense Factor is strengthened correspondingly, thus results in the re-increase in Methane.

These two feedback loops above realize the harmonious development of methane's output, environment and population.

Similarly, we can obtain three feedback loops that respectively correspond to $(R_3,1,L_2,R_2,2,L_3)$

and $2(R_3, L_1, R_1, 1, L_2, R_2, 2, L_3)$. Here they are omitted for the limited space.

3)

$$A_4 = \begin{vmatrix} 1 & R_1,1,L_2) & (R_1,1,L_3) & 0 \\ (R_2,2,L_1) & 1 & (R_2,2,L_3) & 0 \\ 2(R_3,L_1) & (R_3,1,L_2) & 1 & (R_3,L_4) \\ (R_4,2,L_1)+(R_4,L_1) & 0 & (R_4,2,L_3) & 0 \end{vmatrix}$$

$=(R_4,2,L_3, R_3,L_4)+ (R_4, 2,L_1, R_1,1,L_3\ R_3,L_4)+ (R_4, L_1, R_1,1,L_3\ R_3,L_4)+ (R_4, L_1, R_1,1,L_2\ R_2,2, L_3, R_3, L_4)$

By the inverse transformation, we can obtain all the new-added feedback loops embedded by $T''_4(t)$ and $G_3(t)$, one of them is analyzed below.

• **Feedback loop 12(two-order):** $R_4(t)$ Pig Rate (head per year) $\xleftarrow{+}$ Methane Factor $\xleftarrow{+}$ Ratio of Methane to Energy of Daily Life $\xleftarrow{+}$ $L_3(t)$Methane (m^3) $\xleftarrow{+}$ $R_3(t)$Methane Rate (m^3 per year) $\xleftarrow{+}$ Expectant Methane $\xleftarrow{+}$ Methane from Pig's Dry M&U (m^3) $\xleftarrow{+}$ Pig's Dry M&U (kg) $\xleftarrow{+}$ $L_4(t)$Pig((head) $\xleftarrow{+}$ $R_4(t)$

This positive feedback loop depicts an auto-increasing cycle law, that is, the close integration of raising pigs and generating methane leads to the increase in amount of pig and M&U, thus brings out the more output of methane and, because that methane liquid can be used to feed pigs, strengthens Methane Factor, finally results in the re-growth of amount of pig. This feedback loop realizes the sustainable development of pig's amount and methane's output.

$$A_5 = \begin{vmatrix} 1 & (R_1,1,L_2) & (R_1,1,L_3) & 0 & 0 \\ (R_2,2,L_1) & 1 & (R_2,2,L_3) & 0 & 0 \\ 2(R_3,L_3) & (R_3,1,L_2) & 1 & (R_3,L_4) & 0 \\ (R_4,2,L_1)+(R_4,L_1) & 0 & (R_4,2,L_3) & 1 & (R_4,L_5) \\ 2(R_5,3,L_1) & 0 & 2(R_5,3,L_3) & 2(R_5,3,L_4) & 0 \end{vmatrix}$$

$=2(R_5,3,L_4, R_4,L_5)+2(R_5, 3,L_3, R_3,L_4, R_4,L_5)+2(R_5, 3,L_1, R_1,1,L_3\ R_3,L_4,R_4,L_5)+ 2(R_5, 3,L_1, R_1,1,L_2\ R_2,2, L_3, R_3, L_4, R_4, L_5)$.

We only analyze two new-added feedback loops.

• **Feedback loop 16(two-order):** $R_5(t)$ Rice Rate (ton per year) $\xleftarrow{+}$ Nitrogen Content of Methane (kg) $\xleftarrow{+}$ Nitrogen Content of Pig's M&U (kg) $\xleftarrow{+}$ $L_4(t)$Pig (head) $\xleftarrow{+}$ $R_4(t)$Pig Rate (head/ per year) $\xleftarrow{+}$ Rice Factor $\xleftarrow{+}$ Rice Remains (ton) $\xleftarrow{+}$ $L_5(t)$Rice(t)(ton) $\xleftarrow{+}$ $R_5(t)$

This positive feedback loop depicts an auto-increasing cycle law, that is, in the condition of developing the production of rice, the integration of generating methane and raising pigs leads to the increase in amount of rice and pig and the improvement of the nitrogen content of methane from pigs, thus results in the re-increase in amount of rice.

- **Feedback loop 17-22 are omitted.**
- **Feedback loop 23(five-order):**

$R_5(t)$ Rice Rate (ton per year) $\xleftarrow{+}$ Soil structure Factor $\xleftarrow{+}$ Nitrogen Content of Methane (kg) $\xleftarrow{+}$ Nitrogen Content of People's M&U (kg) $\xleftarrow{+}$ $L_1(t)$Population (person) $\xleftarrow{+}$ $R_1(t)$Population Rate (person per year) $\xleftarrow{+}$ Environment Factor $\xleftarrow{-}$ $L_2(t)$Firewood Area(hectare) $\xleftarrow{+}$ $R_2(t)$Firewood Area Rate (hectare per year) $\xleftarrow{+}$ Firewood Supply $\xleftarrow{+}$ Firewood Demand $\xleftarrow{-}$ Ratio of Methane to Energy of Daily life $\xleftarrow{+}$ $L_3(t)$Methane(m^3) $\xleftarrow{+}$ $R_3(t)$Methane Rate(m^3 /per year) $\xleftarrow{+}$ Expectant Methane(m^3) $\xleftarrow{+}$ Methane from Pig's Dry M&U(m^3) $\xleftarrow{+}$ Pig's Dry M&U(kg) $\xleftarrow{+}$ $L_4(t)$Pig(head) $\xleftarrow{+}$ $R_4(t)$Pig Rate(head per year) $\xleftarrow{+}$ Rice Factor $\xleftarrow{+}$ Rice Remains(ton) $\xleftarrow{+}$ $L_5(t)$ Rice(ton) $\xleftarrow{+}$ $R_5(t)$.

This positive feedback loop depicts a sustainable development cycle law, that is, the integration of improving the quantity of rice, further raising pigs, generating methane , developing population and comprehensively managing mountain forests leads to the increase in amount of rice, pig and methane resources, the Methane's growth, the Firewood Demand's relative diminution, the forestry and fruitage development, the mountain greenbelt's extension, the more beautiful life environment and the increase in population, which, because that people's M&U can ferment to become methane that is used in the farmland, brings out the less loss of nitrogen, the improvement of the nitrogen content and the soil quality's structure, thus results in the increase in amount of rice.

These two feedback loops above realize the sustainable increase of amount of rice.

So far, according to five rate variable fundamental in-trees established with the SD rate variable fundamental in-tree modeling, twenty-three feedback loops of this SD model are obtained with branch-vector determinant without any flow diagram and some of them are analyzed. From this case we can learn that this modeling has not necessity of drawing a flow diagram and it has two functions---normally modeling while simultaneously obtaining all feedback loops. The latter provides convenience of searching and analyzing dominant loops.

Moreover, according to the method Ⅱ. We can also obtain all the feedback loops of the whole system only with the evaluation of the following 1-diagonal branch-vector determinant.

$$
A = \begin{vmatrix}
1 & (R_1,1,L_2) & (R_1,1,L_3) & 0 & 0 \\
(R_2,2,L_1) & 1 & (R_2,2,L_3) & 0 & 0 \\
2(R_3,L_3) & (R_3,1,L_2) & 1 & (R_3,L_4) & 0 \\
(R_4,2,L_1)+(R_4,L_1) & 0 & (R_4,2,L_3) & 1 & (R_4,L_5) \\
2(R_5,3,L_1) & 0 & 2(R_5,3,L_3) & 2(R_5,3,L_4) & 1
\end{vmatrix}
$$

Further Work

In the preceding sections, we have constructed the five rate variable fundamental in-tree model of Wangheqiu energy and ecology's system (see Fig 3).In further work, considering that country's output value and income with energy and ecology are interdependent and improve each other, as well as thinking of the development of raising cows, we further add four branches, which respectively depict the influence of cow's M&U to methane and nitrogen as well as the influence of population and pigs to economy, and one economy rate variable fundamental in –tree .

Outcome

The above-mentioned rate variable fundamental in-tree modeling and the branch vector determinant approach to evaluating feedback loops have been compiled to a software (named as RFITM). With this software, we obtain all the 163 feedback loops contained in the Wangheqiu villages in-tree model. As the feedback loops analysis is very explicit and understandable, we can successfully consult with the peasants in Wangheqiu Village. On the basis of the six rate variable fundamental in-tree model above, we make further quantitative analysis and then obtain more than 200 equations of the in-tree model. By debugging the parameters with Venisim software, we achieve three forecasting schemes: the expectant scheme, the comprehensive scheme and the recursire scheme. According to the practical situation, the fundamental scheme was carried out. Generating methane resources solves the problem of energy shortage , improves the environment and develops the forests and fruit trees system, which makes Wangheqiu village progressively realize the harmonious and sustainable development of population, resources, environment and economy.

Summary

Based on the preceding and other applications of the rate variable fundamental in-tree modeling and the branch-vector determinant approach to feedback loops analysis, we make the following conclusion, that is, these approaches offer an effective tool for building models normally and analyzing feedback loops explicitly. With these approaches, modelers only need to capture the key rate variables in a system, and to build an in-tree model by adding in-branches whose tops are all level variables , rather than build a flow diagram model . And modelers can build equations on the basis of these in-trees, simulate the system, then achieve the simulation results. Moreover, with the branch-vector determinant approach, all the feedback loops in a given SD model can be obtained. This is advantageous to feedback analysis . Take the World Model Ⅱ built by Jay. W.Forrester and his cooperators for example, on the basis of these above-mentioned approaches, all the 81 concrete feedback loops can be obtained only with a 5th branch-vector determinant.

References

Mashijun. 1983. Biology Engineering-Applications on Biology System Principles. *Journal of Biology* 5 (1):1~6.

MaShijun and WangRusong. 1987. *Agricultural Biology Engineering of China*. The Science and Technology Publishing House of China, Beijing.

The Planning Commission of JiangXi Province. 1991. *Report of Research on the Rural Energy Region Program of JiangXi Province*. The Science and Technology Publishing House of JiangXi, Nanchang.

Wang, ZhaoJian. 1988. Optimization of Rural Biology System Structure *Journal of Biology.* 7(3):1~6.

J.A.Bondy and U.S.R.Murty. 1976. *Graph Theory with Applications*. The Macmillan Press LID.

Jia, Renan. 1992. *A Course on System Dynamics*. The People's Publishing House of Jiangxi, Nanchang

Wang, Qifan. 1995. *Advanced System Dynamics*. The Tsinghua University Press. Beijing.

Forrester.Jay.W. 1968. *Principles of systems*. Cambridge, Mass. The MIT Press.

Forrester.Jay.W. 1961. *Industrial Dynamics*. Cambridge. Mass. The MIT Press.

Jia, Renan. 1995. "A Mathematical Definition of System Dynamics", *SYSTEM DYNAMICS*'95 ISBN:994-8256-33-2, Tokyo, Japan.

Jia, Renan. 1994. "A study of Output Intersection in the case of Delay: (I=1,2)", *SYSTEM DYNAMICS'*94, Methodology & Technology, ISBN:91-9129-004-1,Scotland.

Jia, Renan. 1991. "the Graph Theory Analysis of Flow Diagram of System", *SYSTEM DYNAMICS*'91. ISBN:974-8201-961, Thailand.

Xu, Nansun and Jia, Renan.1998 *Research on the System Engineering of Energy and Ecology in Wangheqiu Village*, The Science and Technology Publishing House of JiangXi, Nanchang.

David N. Ford. 1998. A behavioral approach to feedback loop dominance analysis. *System Dynamics Review.* 15(1):3~36.R

Jia Renan is a professor of Nanchang University in China. He is the head of Institute of Systems Engineering, where he conducts research on the theory and applications of system dynamics.

E-mail:dingrh@public.nc.jx.cn