# A General Stock Control Formulation For Stock Management Problems Involving Delays and Secondary Stocks

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*It is well established that if the stock management formulation ignores the supply line delay, the behavior of the system can be quite oscillatory. There are naturally other types of delays in stock management problems such as information delays in decision processing and delays caused in controlling a primary stock indirectly via a secondary stock. But there exist no general decision rules in system dynamics that explicitly consider these different delays in stock management structures. In this research, we first investigate the implications of ignoring such indirect delays in the decision formulation. We show that the behavioral consequence of ignoring information delays or ignoring the delay implicit via secondary stock control is equivalent to ignoring the supply line delay in the standard case: large oscillations. Next we derive a general stock control heuristic that does take into account these more advanced types of delays and show that the result is a stabilized dynamic behavior. Finally, we implement our decision heuristic on an example involving all three types of delays, demonstrating the "generic" nature of the proposed formulation structure. The combined result is a significant improvement in the stability of the system, when compared against the standard policy that considers the supply line delays only.*

**Keywords:** stock management, secondary stock control, supply line, information delay, virtual supply line.

## INTRODUCTION

In the standard stock management structure, there is typically a material delay (supply line) before control flow actually reaches the stock. (Figure 1.). It is well established that, if the stock control formulation (typically a linear anchor-and-adjust) does not take into account the supply line delay, the behavior of this system can be quite oscillatory. (See for instance Sterman, 1987, 1989, 2000 and Forrester 1961). It is well known that there are other type of delays in stock management problems such as information delays in decision processing, delays caused in trying to control a stock indirectly via a secondary stock and combinations of these. But there are no general decision rules in system dynamics that explicitly consider these different delays in stock management structures. In this research, we first investigate the implications of ignoring such less common and indirect delays in the stock control formulation. We show that the behavioral implication of ignoring information delay in the decision stream or ignoring the delay implicit via secondary stock control is equivalent to ignoring the supply line delay in the standard case: large oscillations. Then we derive a general stock control heuristic that does take into account these more advanced types of delays and show that the result is stabilized dynamic behavior. Finally, we implement the decision heuristic on an example involving all three types of delays, demonstrating the “generic” nature of the proposed formulation structure.

## EQUIVALENCY OF THE THREE STRUCTURES IF DELAYS ARE IGNORED

There can be three types of delay structures in the stock management problem; material supply line, information delay and indirect delay caused by secondary stock control. All these three structures introduce a delay between the control decision and resulting control action.

### Material Supply Line Delay

Material supply line is a material delay structure that shows the actual transportation of quantities from one stock to another. For example, supply line may be *goods on order* in the inventory control problem. (For more examples see for instance Sterman, 2000).

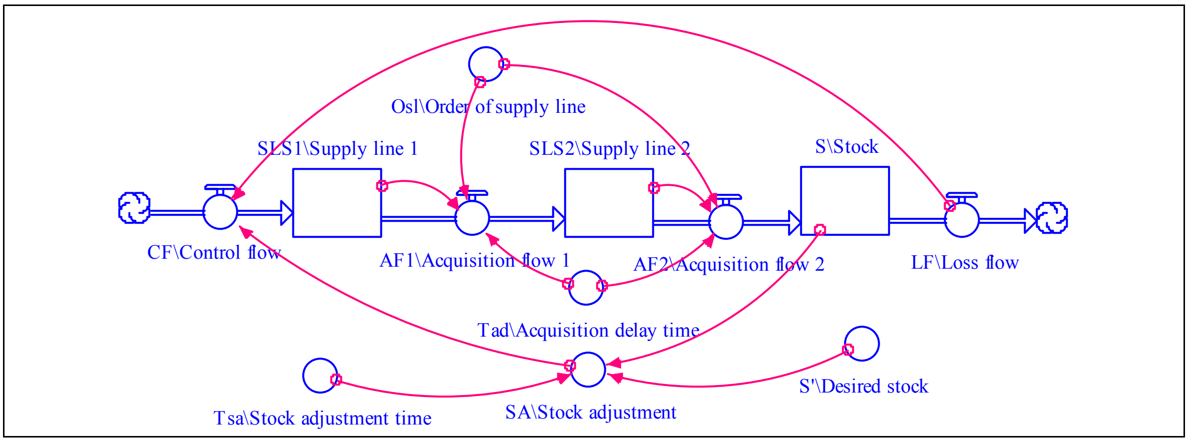


Figure 1. A simple stock management structure involving material supply line

Note that, abbreviated variable names and actual variable names are both shown -separated by a ‘\’ symbol- in the models in Figure 1., Figure 2. and Figure 3. Abbreviated names are used in mathematical equations. For abbreviations we use the following rules:

- All variables are abbreviated in all-capital letters.
- Desired levels and desired values are shown by appending a “ ' ”.
- All stock variables end with “S”.
- All flow variables end with “F”.
- Parameters are abbreviated by an initial capital letter followed by a small letter\letters index. Parameter conventions:
    - C: Coefficient
    - O: Order
    - T: Time
    - W: Weight

$$CF = LF + SA \tag{1}$$

$$AF_1 = \frac{SLS_1}{Tad/Osl} = \frac{SLS_1}{Tad/2} \tag{2}$$

$$AF_2 = \frac{SLS_2}{Tad/Osl} = \frac{SLS_2}{Tad/2} \tag{3}$$

$$SA = \frac{S' - S}{Tsa} \tag{4}$$

In the models in Figure 1., Figure 2. and Figure 3., it is assumed that there are no perception delays in perceiving loss flows or time delays, without loss of generality.

In the first simple model, *Supply line* is ignored in the control flow equation and, *LF* (*Loss flow*), *S'* (*Desired stock*), *Tsa* (*Stock adjustment time*), *Tad* (*Acquisition delay time*), *Osl* (*Order of supply line*) are all assumed to be constants. The stock equations and the reduced stock equations (after all variables and parameters are inserted) can be seen in derivative form in Appendix 1.

A sample behavior of the model can be seen in Figure 4. *Stock1* represents the *Stock* of the model in Figure 1. As we mentioned before, the oscillations can be unstable as a result of ignoring the supply line in the decision formulation.

**Information Delay**

Information delays are involved in the flow of information from one location to another or in information processing. They can be part of some common stock management problems. (i.e. Order-filling delays, ordering decision delays, mail delays, see Forrester, 1961). An example could be the information delay in between two departments of a company.

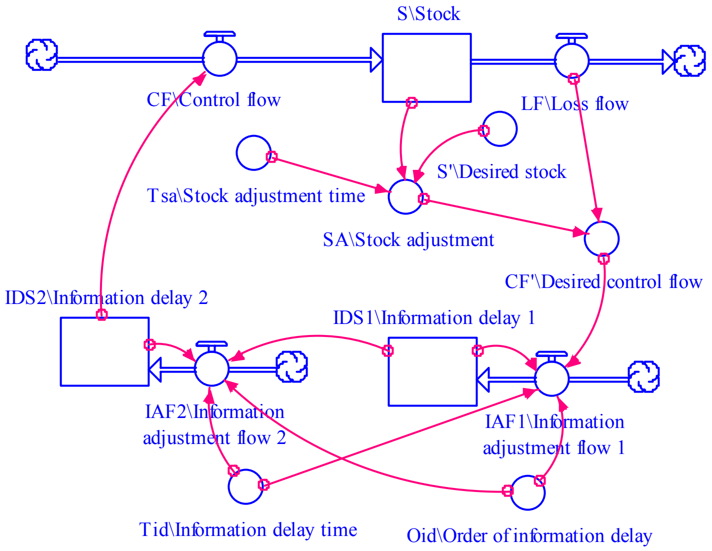


Figure 2. A simple information delay structure in stock control

$$IAF_1 = \frac{CF' - IDS_1}{Tid / Oid} = \frac{CF' - IDS_1}{Tid / 2} \tag{5}$$

$$IAF_2 = \frac{IDS_1 - IDS_2}{Tid / Oid} = \frac{IDS_1 - IDS_2}{Tid / 2} \tag{6}$$

$$CF = IDS_2 \tag{7}$$

$$CF' = LF + SA \tag{8}$$

$$SA = \frac{S' - S}{Tsa} \tag{9}$$

In this simple model, the information delay is ignored in the decision formulation and *LF* (*Loss flow*), *S'* (*Desired stock*), *Tsa* (*Stock adjustment time*), *Tid* (*Information delay time*), *Oid* (*Order of information delay*) are all constants. The stock equations and the reduced stock equations (after all variables and parameters are inserted) can be seen in derivative form in Appendix 1.

A sample behavior of the model can be seen in Figure 4. *Stock2* represents the *Stock* of the model in Figure 2. As we mentioned before, the oscillations can again be unstable as a result of ignoring the information delay in the decision formulation.

## Indirect Control Via a Secondary Stock

Sometimes the stock is controlled via a secondary control. For example, production rate may be controlled indirectly by adjusting the workforce, by hiring\firing workers. (i.e. Customer-Producer-Employment System, Forrester, 1961, Inventory-Workforce Model, A Generic Commodity Market Model, Sterman, 2000) The overall effect of a secondary stock control sub-system is to introduce a delay between the desired production (control decision) and the actual production (resulting control). This is very similar to material supply line delay between orders (control decision) and acquisition rate (resulting control) or, to information delay between decision (control decision) and action (resulting control).

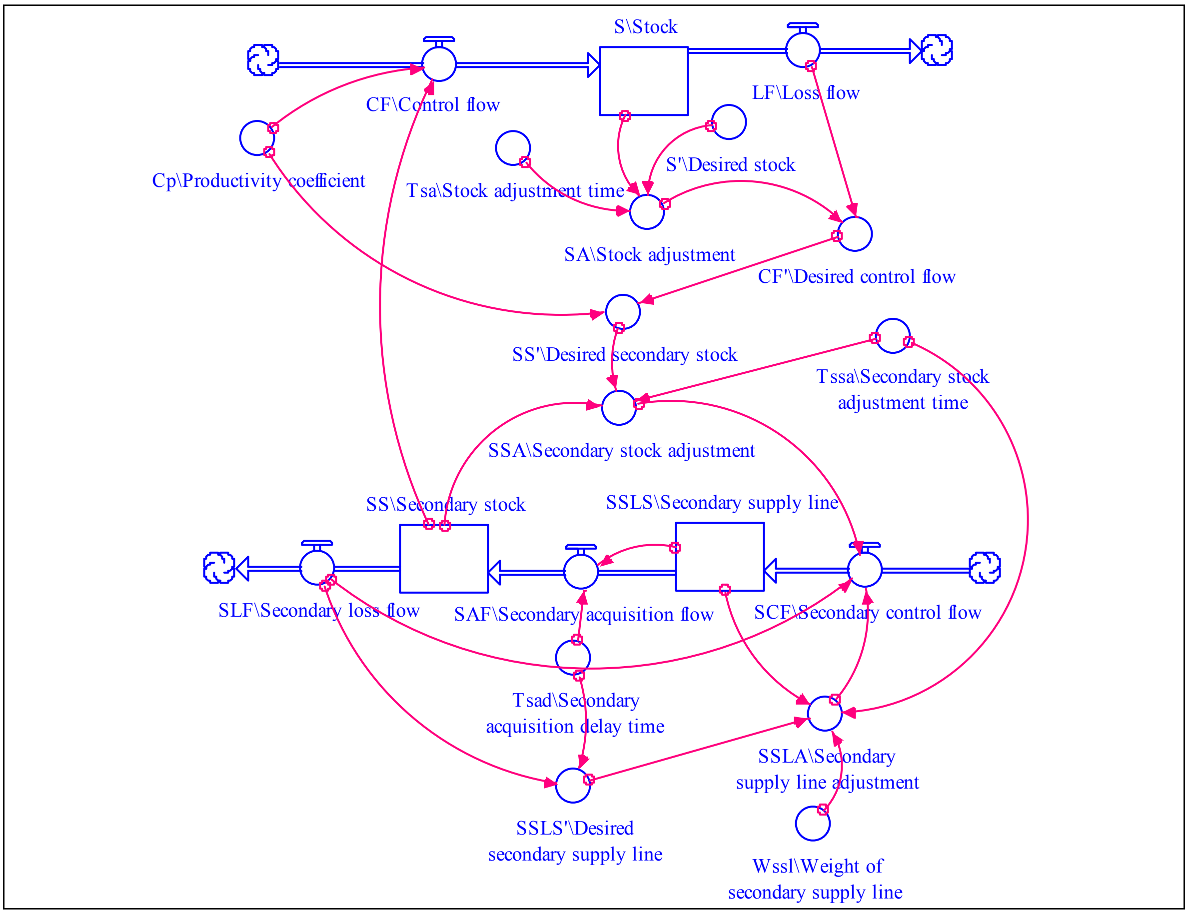


Figure 3. A simple indirect stock control structure via a secondary stock

$$CF = Cp * SS \tag{10}$$

$$SCF = SLF + SSA + SSLA \tag{11}$$

$$SAF = \frac{SSLS}{Tsad} \tag{12}$$

$$SSLA = \frac{Wssl * (SSLS' - SSLS)}{Tssa} \tag{13}$$

$$SSLS' = Tsad * SLF \tag{14}$$

$$SSA = \frac{(SS' - SS)}{Tssa} \tag{15}$$

$$SS' = \frac{CF'}{Cp} \tag{16}$$

$$CF' = LF + SA \tag{17}$$

$$SA = \frac{S'-S}{Tsa} \tag{18}$$

In this simple model, the indirect delay effect of the secondary stock is ignored in the decision formulation and *LF* (*Loss flow*), *SLF* (*Secondary loss flow*), *Cp* (*Production coefficient*), *Tsad* (*Secondary acquisition delay time*), *Wssl* (*Weight of secondary supply line*), *Tssa* (*Secondary stock adjustment time*), *S'* (*Desired stock*), *Tsa* (*Stock adjustment time*) are all constants.

A sample behavior of the model can be seen in Figure 4. *Stock3* represents the *Stock* of the model in Figure 3. Once again, the oscillations can be unstable as a result of ignoring the indirect delay effect via the secondary stock in the decision formulation.

## Behavior of the Three Models

The above three models cause their stocks to exhibit exactly the same behaviors when parameter values are selected appropriately; (i.e. $Tssa = Tsad = \frac{Tid}{2} = \frac{Tad}{2}$) For example:

- *LF* (*Loss flow*) = 4
- *SLF* (*Secondary loss flow*) = 0.2
- *Cp* (*Production coefficient*) = 12
- *Wssl* (*Weight of secondary supply line*) = 1
- *Tsa* (*Stock adjustment time*) = 4
- *Tad* (*Acquisition delay time*) = 21
- *Tid* (*Information delay time*) = Tad = 21
- *Tsad* (*Secondary acquisition delay time*) = Tad/2 = 10.5
- *Tssa* (*Secondary stock adjustment time*) = Tad/2 = 10.5

All stocks are initialized to their equilibrium levels.
At time four, *S'* (*Desired stock*) is decreased from ten to nine.

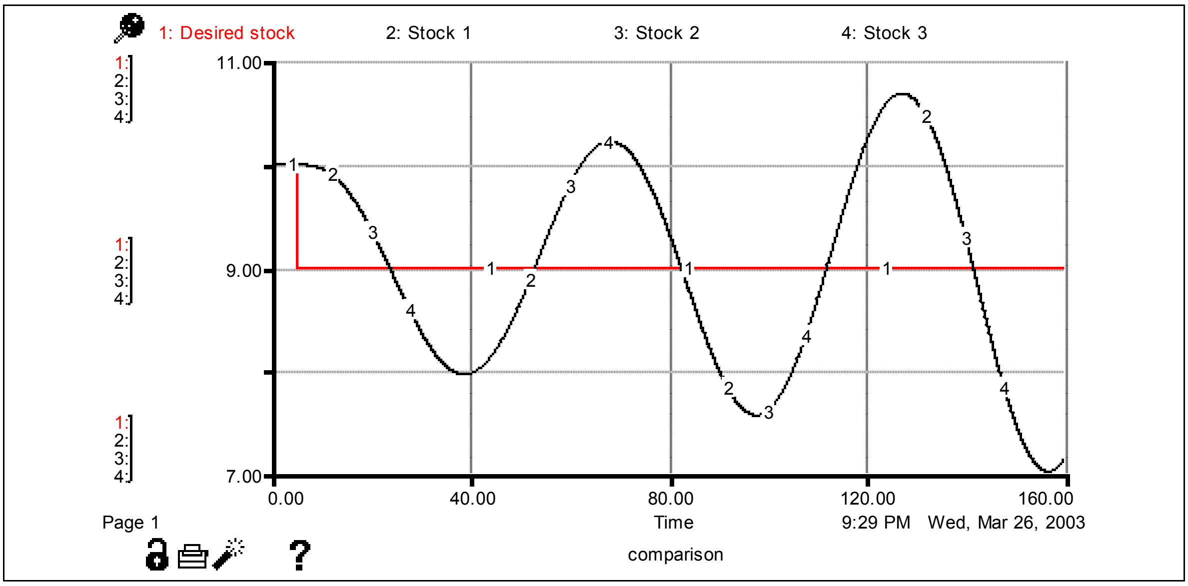


Figure 4. Output behaviors of three models without considering the different delays

In the above figure, *Stock1*, *Stock2* and *Stock3* are the primary control stocks of the models in Figure 1., Figure 2. and Figure 3. respectively. (It is a well-known fact that when supply line is ignored in the decision rule, oscillatory behavior may be obtained. We selected parameter values so as to demonstrate an extreme case of unstable oscillations).
It can be seen above that the three structures generate exactly the same behavior patterns. The equivalence can also be mathematically proven (see Appendix 2).

## THE VIRTUAL SUPPLY LINE CONCEPT

It is well known that to obtain stable behavior, considering the supply line in stock control decisions is critical (Sterman, 1987, 1989, 2000 and Forrester 1961). In the same way, we claim that considering other type of delays like information delay and indirect secondary stock delays can also be very important. Oscillations can be dampened by incorporating such delays in the stock control decisions.

Supply line is considered by simply including a supply line adjustment term in control flow. It is not possible to include secondary stock and information delay in the same way, since their role and even their units are different from the primary supply line. But there must be a way to handle these delays since they are behaviorally and mathematically equivalent to supply line delay when they are seen as input/output systems. To account for these two types of delays we propose the ***virtual supply line*** concept that has unit consistency with the primary stock.

### The Supply Line Adjustment in the Model with Supply Line Delay (Figure 1.)

In the improved model, the *Adjusted control flow* (*ACF*) is used as input to supply line, instead of *Control flow* (*CF*). Equation for *Adjusted control flow* can be given as

$$ACF = CF + SLA = LF + SA + SLA \tag{19}$$

where

$$SLA = Wsl * \frac{SLS' - SLS}{Tsa} \tag{20}$$

$$SLS' = Tad * LF \tag{21}$$

*SLA* is *Supply line adjustment*, *Wsl* is *Weight of supply line* and, *SLS'* is *Desired supply line*.

A sample behavior of the model can be seen in Figure 5. *Stock1* represents the *Stock* of the model in Figure 1. Note that model is modified with the above equations to consider supply line delay in decisions and *Wsl* is taken to be one. It is known that for *Wsl* = 1, supply line and stock system reduces effectively to a first order system that is perfectly stable (Sterman, 1989 and Yasarcan 2003).

**Introducing the Virtual Supply Line in the Model with Information Delay (Figure 2.)**

Again, the *Adjusted desired control flow* (*ACF'*) can be used as input to information delay instead of *Desired control flow* (*CF*). Equation for *Adjusted desired control flow* can be given as

$$ACF' = CF' + VSLA = LF + SA + VSLA \tag{22}$$

where

$$VSLA = Wvsl * \frac{VSLS' - VSLS}{Tsa} \tag{23}$$

$$VSLS' = Tid * LF \tag{24}$$

$$VSLS = (Tid / 2) * IDS_1 + (Tid / 2) * IDS_2 \tag{25}$$

*VSLA* is *Virtual supply line adjustment*, *VSLS* is *Virtual supply line*, *VSLS'* is *Desired virtual supply line* and *Wvsl* is *Weight of virtual supply line*. Note that the *Virtual supply line* stock *VSLS* is a hypothetical (virtual) stock that has the same dynamic (and mathematical) role as having a supply line delay in the stock control structure. This concept is thus used in order to account for the information delay in the decision formulation.

A sample behavior of the model can be seen in Figure 5. *Stock2* represents the *Stock* of the model in Figure 2. Note that the model is modified with the above equations to consider the information delay in decisions and *Wvsl* is taken to be one. The structure with information delay reduces effectively to a first order system and a perfectly stable behavior results.

**Introducing the Virtual Supply Line in the Model with Secondary Stock (Figure 3.)**

Once again, the *Adjusted desired control flow* (*ACF'*) can be used as input to secondary stock sub-structure instead of *Desired control flow* (*CF*). Equation for *Adjusted desired control flow* can be given as

$$ACF' = CF' + VSLA = LF + SA + VSLA \tag{26}$$

where

$$VSLA = Wvsl * \frac{VSLS' - VSLS}{Tsa} \tag{27}$$

$$VSLS' = \left(Tssa + Tsad\right) * LF \tag{28}$$

$$VSLS = Cp * \left(\left(Tssa + Tsad\right) * SS + Tsad * \left(SSLS - SSLS'\right)\right) \tag{29}$$

*VSLA* is *Virtual supply line adjustment*, *VSLS* is *Virtual supply line*, *VSLS'* is *Desired virtual supply line* and *Wvsl* is *Weight of virtual supply line*. Note that the *Virtual supply line* stock *VSLS* is a hypothetical (virtual) stock that has the same dynamic (and mathematical) role as having a supply line delay in the stock control structure. This concept is thus used in order to account for the implicit delay caused by the indirect secondary stock control, in the decision formulation.

A sample behavior of the model can be seen in Figure 5. *Stock3* represents the *Stock* of the model in Figure 3. Note that the model is modified with the above equations to consider secondary stock sub-structure in decisions and *Wvsl* is taken to be one. The secondary stock and primary stock system reduces effectively to a first order system and a perfectly stable behavior results.

**Behavior of the Three Models (Figure 1., Figure 2. and Figure 3.) with Supply Line Adjustment and/or with Virtual Supply Line Adjustment**

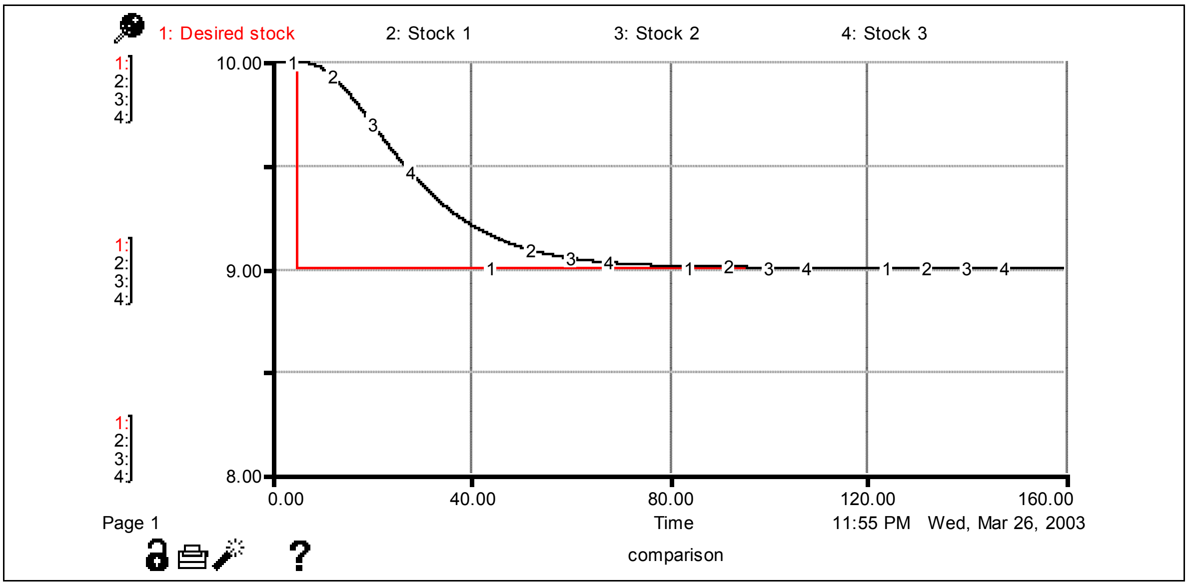


Figure 5. Output behaviors of three structures after considering the different delays

In the above figure, *Stock1*, *Stock2* and *Stock3* are the primary stocks of the models in Figure 1., Figure 2. and Figure 3. respectively. Note that the models are modified to take delays into consideration and all supply line weights are set to one. As it can be seen above, the primary stocks in the three structures generate exactly the same behaviors. This proves that the *Virtual supply line* concept is structurally equivalent to *Supply line*. The equivalence can also be mathematically proven (see Appendix 3).

Note that parameter values are the same as for the run in Figure 4. It can be seen that output behaviors in Figure 5. are stable. Behavior is stabilized by incorporating the effects of delays in control decisions.

## EXAMPLE MODEL WITH ALL THREE DELAY STRUCTURES

In this example, we incorporate supply line, information delay and secondary stock sub-structures simultaneously. Information delay is in between inventory control department and human resources department. Production start rate is controlled by changing the numbers of workers (labor). There is a supply line delay representing the manufacturing process. For simplicity, we assume that there are no delays in perceiving the quit rates or delay times and, no layoffs are allowed, without loss of generality.

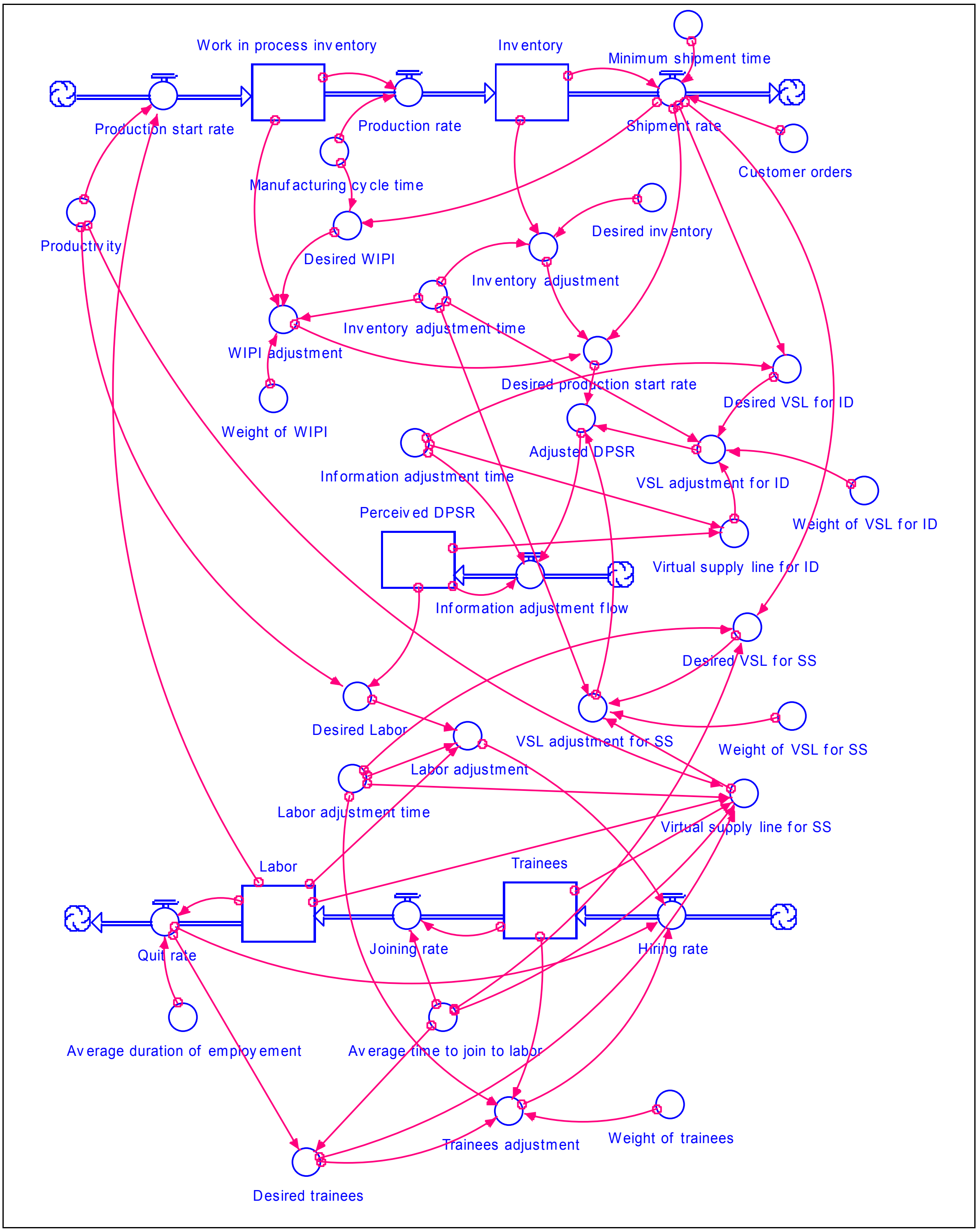


Figure 6. Example model using all three delay structures

## Equations of The Example Model

$$Adjusted\_DPSR = \begin{pmatrix} Desired\_production\_start\_rate \\ +VSL\_adjustment\_for\_SS+VSL\_adjustment\_for\_ID \end{pmatrix} \tag{30}$$

$$Average\_duration\_of\_employement = 40 \tag{31}$$

$$Average\_time\_to\_join\_to\_labor = 9 \tag{32}$$

$$Customer\_orders = 4000 + \mathrm{STEP}(500,4) \tag{33}$$

$$Desired\_inventory = 60000 \tag{34}$$

$$Desired\_labor = \frac{Perceived\_DPSR}{Productivity} \tag{35}$$

$$Desired\_production\_start\_rate = \begin{pmatrix} Shipment\_rate + Inventory\_adjustment \\ + WIPI\_adjustment \end{pmatrix} \tag{36}$$

$$Desired\_trainees = Average\_time\_to\_join\_to\_labor * Quit\_rate \tag{37}$$

$$Desired\_VSL\_for\_ID = Information\_delay\_time * Shipment\_rate \tag{38}$$

$$Desired\_VSL\_for\_SS = \begin{pmatrix} Labor\_adjustment\_time \\ + Average\_time\_to\_join\_to\_labor \end{pmatrix} * Shipment\_rate \tag{39}$$

$$Desired\_WIPI = Manufacturing\_cycle\_time * Shipment\_rate \tag{40}$$

$$Hiring\_rate = Quit\_rate + Labor\_adjustment + Trainees\_adjustment \tag{41}$$

$$Inventory\_adjustment = \frac{Desired\_inventory - Inventory}{Inventory\_adjustment\_time} \tag{42}$$

$$Information\_adjustment\_flow = \frac{Adjusted\_DPSR - Perceived\_DPSR}{Information\_delay\_time} \tag{43}$$

$$Inventory\_adjustment\_time = 7 \tag{44}$$

$$Joining\_rate = \frac{Trainees}{Average\_time\_to\_join\_to\_labor} \tag{45}$$

$$Labor\_adjustment = \frac{Desired\_Labor - Labor}{Labor\_adjustment\_time} \quad (46)$$

$$Labor\_adjustment\_time = 16 \quad (47)$$

$$Manufactoring\_cycle\_time = 9 \quad (48)$$

$$Minimum\_shipment\_time = 1.6 \quad (49)$$

$$Production\_rate = \frac{Work\_in\_process\_inventory}{Manufacturing\_cycle\_time} \quad (50)$$

$$Production\_start\_rate = Productivity * Labor \quad (51)$$

$$Productivity = 10 \quad (52)$$

$$Quit\_rate = \frac{Labor}{Average\_duartion\_of\_employement} \quad (53)$$

$$Shipment\_rate = \mathrm{MIN}(Customer\_orders, Inventory/Minimum\_shipment\_time) \quad (54)$$

$$Trainees\_adjustment = Weight\_of\_trainees * \frac{Desired\_trainees - Trainees}{Labor\_adjustment\_time} \quad (55)$$

$$\begin{aligned} &VSL\_adjustment\_for\_ID = \\ &\quad Weight\_of\_VSL\_for\_ID * \frac{Desired\_VSL\_for\_ID - Virtual\_supply\_line\_for\_ID}{Inventory\_adjustment\_time} \end{aligned} \quad (56)$$

$$\begin{aligned} &VSL\_adjustment\_for\_SS = \\ &\quad Weight\_of\_VSL\_for\_SS * \frac{Desired\_VSL\_for\_SS - Virtual\_supply\_line\_for\_SS}{Inventory\_adjustment\_time} \end{aligned} \quad (57)$$

$$Virtual\_supply\_line\_for\_ID = Information\_delay\_time * Perceived\_DPSR \quad (58)$$

$$\begin{aligned} &Virtual\_supply\_line\_for\_SS = \\ &\quad Productivity * \begin{pmatrix} (Labor\_adjustment\_time + Average\_time\_to\_join\_to\_labor) * Labor \\ + Average\_time\_to\_join\_to\_labor * (Trainees - Desired\_trainees) \end{pmatrix} \end{aligned} \quad (59)$$

$$WIPI\_adjustment = Weight\_of\_WIPI * \frac{Desired\_WIPI - Work\_in\_process\_inventory}{Inventory\_adjustment\_time} \quad (60)$$

All stocks are at their equilibrium levels initially. At time four *Customer_orders* is increased from 4000 to 4500 to disturb the system from its equilibrium level.

**Runs of The Model**

First run:

- *Weight_of_trainees* = 0
- *Weight_of_WIPI* = 0
- *Weight_of_VSL_for_ID* = 0
- *Weight_of_VSL_for_SS* = 0

Second run:

- *Weight_of_trainees* = 1
- *Weight_of_WIPI* = 0
- *Weight_of_VSL_for_ID* = 0
- *Weight_of_VSL_for_SS* = 0

Third run:

- *Weight_of_trainees* = 1
- *Weight_of_WIPI* = 1
- *Weight_of_VSL_for_ID* = 0
- *Weight_of_VSL_for_SS* = 0

Fourth run:

- *Weight_of_trainees* = 1
- *Weight_of_WIPI* = 1
- *Weight_of_VSL_for_ID* = 1
- *Weight_of_VSL_for_SS* = 0

Fifth run:

- *Weight_of_trainees* = 1
- *Weight_of_WIPI* = 1
- *Weight_of_VSL_for_ID* = 1
- *Weight_of_VSL_for_SS* = 1

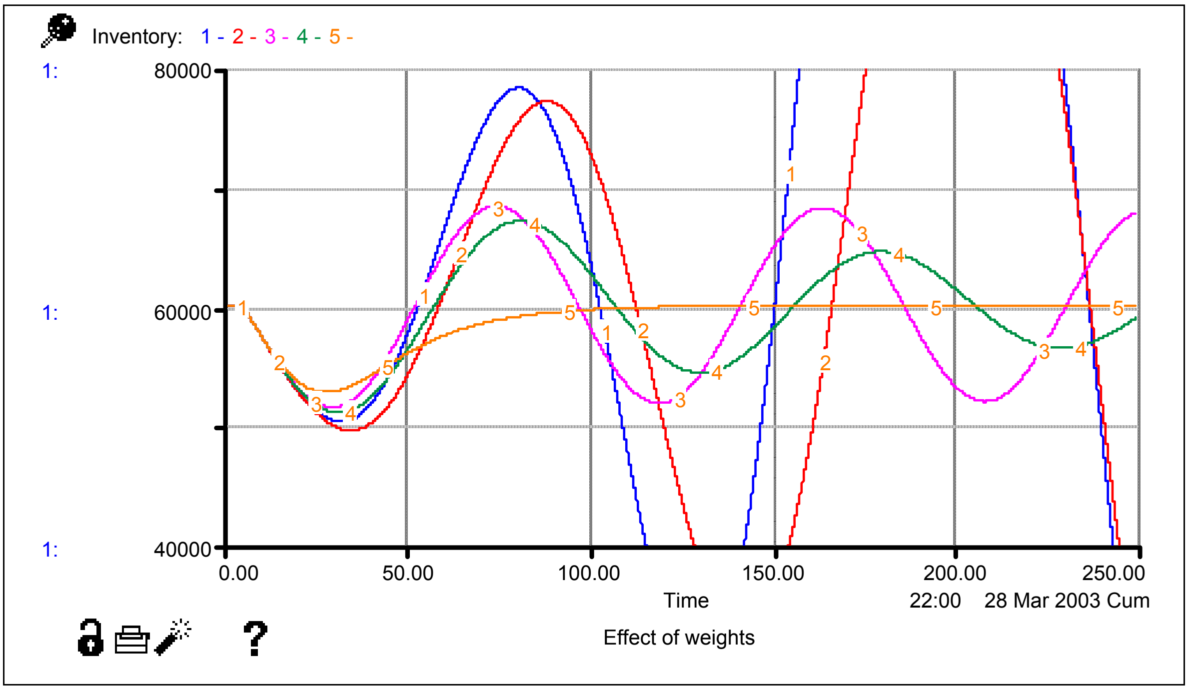


Figure 7. Output behaviors of the example model with different supply line and virtual supply line weight values

A typical system dynamics decision formulation run is the third run, consisting of oscillations. In this run, supply line of the primary stock and supply line of the secondary stock are both considered in the decisions, but the information delay and indirect secondary stock delay effects are ignored. When the information delay and indirect secondary stock delays are also considered in decisions using our formulations, the behavior is improved significantly (fifth run). When all delays are considered optimally, oscillations are completely eliminated.

## IMPLEMENTATION ISSUES IN VIRTUAL SUPPLY LINE ADJUSTMENT

Virtual Supply Line adjustment necessitates delay durations, orders and the stock values in the delay structure to be known or estimated.

In the secondary stock control structure, the stocks of the structure is already monitored and known, so the only unknown may be the delay durations. In this case we propose to use the estimates of the delay durations for calculation of Virtual Supply Line. Ignoring delays are a far bigger mistake than using estimates. Furthermore, the Virtual Supply Line adjustment is quite robust, so it works quite well with estimates of delay durations. (See Yaşarcan 2003).

Information delay structure may be a harder case. For some cases decision maker can only perceive the last stock of the delay structure. In this case, not only the delay durations but also the stock values in the delay structure must be estimated, which may not be a simple task. Furthermore, in some cases even delay order may not be available to the decision maker. For these more complicated cases, if we know the initial value of the delay stock, we can continuously update the value of the stock by using a “stock type virtual supply line” assuming that we have access to the outflow of the delay. (This structure is skipped in this article; see Yaşarcan, 2003).

## CONCLUSION

It is well known that, if the stock management formulation ignores the supply line delay, the behavior of the system can be quite oscillatory. There are naturally other types of delays in stock management problems such as information delays in decision processing and delays caused in controlling a primary stock indirectly via a secondary stock. But there are no general decision rules in system dynamics that explicitly consider these different delays in stock management structures. In this research, we first investigate the implications of ignoring such indirect delays in the stock control formulation. We show that the behavioral consequence of ignoring information delay in the decision stream or ignoring the delay implicit via secondary stock control is equivalent to ignoring the supply line delay in the standard case: large oscillations. Next we define the ‘virtual supply line’ concept and derive a general stock control heuristic that does take into account these more advanced delays and show that the result is stabilization of the dynamic behavior. We prove again that the improvement obtained by incorporating these less common delays is equivalent to the improvement obtained by incorporating the supply line delay in the standard case. Finally, we implement our decision heuristic on an example involving all three types of delays, demonstrating the “generic” nature of the proposed formulation structure. The combined effect is a significant improvement in the stability of the system, when compared against the standard policy that considers the supply line delays only. Future work will involve testing our “generalized” stock adjustment formulation in actual models that involve further loops and non-linearities.

## APPENDIX 1. EQUATIONS OF SIMPLE DELAY STRUCTURES

### Stock Equations of Model with Supply Line Delay (Figure 1.)

$$\dot{S} = AF_2 - LF \tag{61}$$

$$\dot{SLS_1} = CF - AF_1 \tag{62}$$

$$\dot{SLS_2} = AF_1 - AF_2 \tag{63}$$

### Reduced Stock Equations of Model with Supply Line Delay (Figure 1.)

$$\dot{S} = \frac{SLS_2}{Tad / 2} - LF \tag{64}$$

$$\dot{SLS}_1 = LF + \frac{S' - S}{Tsa} - \frac{SLS_1}{Tad/2} \tag{65}$$

$$\dot{SLS}_2 = \frac{SLS_1}{Tad/2} - \frac{SLS_2}{Tad/2} \tag{66}$$

**Stock Equations of Model with Information Delay (Figure 2.)**

$$\dot{S} = CF - LF \tag{67}$$

$$\dot{IDS}_1 = IAF_1 \tag{68}$$

$$\dot{IDS}_2 = IAF_2 \tag{69}$$

**Reduced Stock Equations of Model with Information Delay (Figure 2.)**

$$\dot{S} = IDS_1 - LF \tag{70}$$

$$\dot{IDS}_1 = \frac{LF + \frac{S' - S}{Tsa} - IDS_1}{Tid/2} \tag{71}$$

$$\dot{IDS}_2 = \frac{IDS_1 - IDS_2}{Tid/2} \tag{72}$$

**Stock Equations of Model with Secondary Stock (Figure 3.)**

$$\dot{S} = CF - LF \tag{73}$$

$$\dot{SS} = SAF - SLF \tag{74}$$

$$\dot{SSLS} = SCF - SAF \tag{75}$$

**Reduced Stock Equations of Model with Secondary Stock (Figure 3.)**

$$\dot{S} = Cp * SS - LF \tag{76}$$

$$\dot{SS} = \frac{SSLS}{Tsad} - SLF \tag{77}$$

$$\dot{SSLS} = SLF + \frac{\left(\frac{LF + (S'-S)/Tsa}{Cp} - SS\right)}{Tssa} + \frac{Wssl*(Tsad*SLF - SSLS)}{Tssa} - \frac{SSLS}{Tsad} \tag{78}$$

## APPENDIX 2. MATHEMATICAL EQUIVALENCY OF THE THREE STRUCTURES WHEN DELAYS ARE IGNORED

It can be shown mathematically that the three structures can be made equivalent. We will consider material supply line, information delay and secondary stock structures as separate input\output systems and ignore primary stocks for simplicity.

For material supply line structure input is *CF* and output is *AF2* and, for secondary stock and information delay structures input is *CF'* and output is *CF*.

We will re-write reduced stock equations ignoring primary stock.

**Reduced Supply Line Equations (Figure 1.)**

$$\dot{SLS}_1 = CF - \frac{SLS_1}{Tad/2} \tag{79}$$

$$\dot{SLS}_2 = \frac{SLS_1}{Tad/2} - \frac{SLS_2}{Tad/2} \tag{80}$$

From Equation (80) the following two equations are obtained:

$$SLS_1 = (Tad/2)*\dot{SLS}_2 + SLS_2 \tag{81}$$

$$\dot{SLS}1 = (Tad/2)*\ddot{SLS}_2 + \dot{SLS}_2 \tag{82}$$

Equation (81) and Equation (82) can be inserted in Equation (79) to obtain the following equation:

$$(Tad/2)*\ddot{SLS}_2 + \dot{SLS}_2 = CF - \frac{(Tad/2)*\dot{SLS}_2 + SLS_2}{Tad/2} \tag{83}$$

Equation (83) can be simplified to the following:

$$(Tad/2)^2 * \ddot{SLS}_2 + Tad * \dot{SLS}_2 + SLS_2 = (Tad/2) * CF \tag{84}$$

Equation (84) can be re-written for *AF2* with using the relationship given for *SLS2* and *AF2* in Equation (3)

$$(Tad/2)^2 * \ddot{AF}_2 + Tad * \dot{AF}_2 + AF_2 = CF \tag{85}$$

**Reduced Information Delay Equations (Figure 2.)**

$$\dot{IDS}_1 = \frac{CF' - IDS_1}{Tid/2} \tag{86}$$

$$\dot{IDS}_2 = \frac{IDS_1 - IDS_2}{Tid/2} \tag{87}$$

From Equation (87) the following two equations are obtained:

$$IDS_1 = (Tid/2) * \dot{IDS}_2 + IDS_2 \tag{88}$$

$$\dot{IDS}_1 = (Tid/2) * \ddot{IDS}_2 + \dot{IDS}_2 \tag{89}$$

Equation (88) and Equation (89) can be inserted in Equation (86) to obtain the following equation:

$$(Tid/2) * \ddot{IDS}_2 + \dot{IDS}2 = \frac{CF' - (Tid/2) * \dot{IDS}_2 + IDS_2}{Tid/2} \tag{90}$$

Equation (90) can be simplified to the following:

$$(Tid/2)^2 * \ddot{IDS}_2 + Tid * \dot{IDS}_2 + IDS_2 = CF' \tag{91}$$

Equation (84) can be re-written for *CF* with using the relationship given for *IDS2* and *CF* in Equation (7).

$$(Tid/2)^2 * \ddot{CF} + Tid * \dot{CF} + CF = CF' \tag{92}$$

Letting $Tid = Tad$ the following is obtained:

$$(Tad/2)^2 * \ddot{CF} + Tad * \dot{CF} + CF = CF' \tag{93}$$

If Equation (93) is compared with Equation (85) it can be seen that the two differential equations are the same except for the variable names. This proves that as input-output systems, supply line and information delay structures can be identical with appropriate selection of parameter values.

**Reduced Secondary Stock Equations (Figure 3.)**

$$\dot{SS} = \frac{SSLS}{Tsad} - SLF \tag{94}$$

$$\dot{SSLS} = SLF + \frac{(SS' - SS)}{Tssa} + \frac{Wssl * (Tsad * SLF - SSLS)}{Tssa} - \frac{SSLS}{Tsad} \tag{95}$$

From Equation (94) the following two equations are obtained:

$$SSLS = Tsad * \dot{SS} + Tsad * SLF \tag{96}$$

$$\dot{SSLS} = Tsad * \ddot{SS} \tag{97}$$

Equation (96) and Equation (97) can be inserted in Equation (95) to obtain the following equation:

$$Tsad * \ddot{SS} = SLF + \frac{(SS' - SS)}{Tssa} + \frac{Wssl * \left( Tsad * SLF - Tsad * \dot{SS} + Tsad * SLF \right)}{Tssa} - \frac{Tsad * \dot{SS} + Tsad * SLF}{Tsad} \tag{98}$$

Equation (98) can be simplified to the following:

$$Tsad * Tssa * \ddot{SS} + Tssa * \dot{SS} + Wssl * Tsad * \dot{SS} + SS = SS' \tag{99}$$

Letting $Wssl = 1$ and $Tsad = Tssa = \frac{Tad}{2}$ the following is obtained:

$$(Tad/2)^2 * \ddot{SS} + Tad * \dot{SS} + SS = SS' \tag{100}$$

Equation (100) can be re-written for *CF* with using the relationship given for *SS* and *CF* in Equation (10).

$$\left(Tad/2\right)^2 * \ddot{CF} + Tad * \dot{CF} + CF = CF' \qquad (101)$$

If Equation (101) is compared with Equation (85) and Equation (93) it can be seen that it is same with these equations. This proves that as input-output systems, supply line, information delay and secondary stock structures can be identical with appropriate selection of parameter values.

## APPENDIX 3. MATHEMATICAL EQUIVALENCY OF THE THREE STRUCTURES WHEN DELAYS ARE CONSIDERED

It can be shown mathematically that the three structures can made equivalent. We will consider material supply line, information delay and secondary stock structures as separate input\output systems and ignore primary stocks for simplicity.

For material supply line structure input is *CF* and output is *AF2* and, for secondary stock and information delay structures input is *CF'* and output is *CF*.

**Reduced Supply Line Equations when Supply Line is Considered (Figure 1.)**

$$\dot{SLS}_1 = CF + \frac{Wsl * \left(Tad * LF - SLS\right)}{Tsa} - \frac{SLS_1}{Tad/2} \qquad (102)$$

$$\dot{SLS}_2 = \frac{SLS_1}{Tad/2} - \frac{SLS_2}{Tad/2} \qquad (103)$$

From Equation (103) the following two equations are obtained:

$$SLS_1 = \left(Tad/2\right) * \dot{SLS}_2 + SLS_2 \qquad (104)$$

$$\dot{SLS}_1 = \left(Tad/2\right) * \ddot{SLS}_2 + \dot{SLS}_2 \qquad (105)$$

Equation (104) and Equation (105) can be inserted in Equation (102) to obtain the following equation:

$$\begin{aligned}&\left(Tad/2\right) * \ddot{SLS}_2 + \dot{SLS}_2 = \\ &CF + \frac{Wsl * \left(Tad * LF - \left(Tad/2\right) * \dot{SLS}_2 - SLS_2\right)}{Tsa} - \frac{\left(Tad/2\right) * \dot{SLS}_2 + SLS_2}{Tad/2}\end{aligned} \qquad (106)$$

Equation (106) can be simplified to the following:

$$(Tad/2)^2 * Tsa * \ddot{SLS}_2 + \left(Tad * Tsa + Wsl * (Tad/2)^2\right) * \dot{SLS}_2 + (Tsa + Wsl * Tad) * SLS_2 = Tsa * (Tad/2) * CF + Wsl * \left(Tad^2/2\right) * LF \quad (107)$$

Equation (84) can be re-written for *AF2* with using the relationship given for *SLS2* and *AF2* in Equation (3)

$$(Tad/2)^2 * Tsa * \ddot{AF}_2 + \left(Tad * Tsa + Wsl * (Tad/2)^2\right) * \dot{AF}_2 + (Tsa + Wsl * Tad) * AF_2 = Tsa * CF + Wsl * Tad * LF \quad (108)$$

**Reduced Information Delay Equations when Virtual Supply Line is Considered (Figure 2.)**

$$\dot{IDS}_1 = \frac{CF' + Wvsl * \dfrac{Tid * LF - (Tid/2) * IDS_1 - (Tid/2) * IDS_2}{Tsa} - IDS_1}{Tid/2} \quad (109)$$

$$\dot{IDS}_2 = \frac{IDS_1 - IDS_2}{Tid/2} \quad (110)$$

From Equation (110) the following two equations are obtained:

$$IDS_1 = (Tid/2) * \dot{IDS}_2 + IDS_2 \quad (111)$$

$$\dot{IDS}_1 = (Tid/2) * \ddot{IDS}_2 + \dot{IDS}_2 \quad (112)$$

Equation (111) and Equation (112) can be inserted in Equation (109) to obtain the following equation:

$$(Tid/2) * \ddot{IDS}_2 + \dot{IDS}_2 = \frac{\begin{pmatrix} CF' \\ + \dfrac{Wvsl}{Tsa} * \begin{pmatrix} Tid * LF - (Tid/2)^2 * \dot{IDS}_2 \\ - (Tid/2) * IDS_2 - (Tid/2) * IDS_2 \end{pmatrix} \\ - (Tid/2) * \dot{IDS}_2 \\ - IDS_2 \end{pmatrix}}{Tid/2} \quad (113)$$

Equation (113) can be simplified to the following:

$$(Tid/2)^2 * Tsa * \ddot{IDS}_2 + \left(Tid * Tsa + Wvsl * (Tid/2)^2\right) * \dot{IDS}_2 + (Tsa + Wvsl * Tid) * IDS_2 \\ = Tsa * CF' + Wvsl * Tid * LF \tag{114}$$

Letting *Tid* = *Tad* and using the relationship given for *IDS2* and *CF* in Equation (7), Equation (84) can be re-written for *CF*:

$$(Tad/2)^2 * Tsa * \ddot{CF} + \left(Tad * Tsa + Wvsl * (Tad/2)^2\right) * \dot{CF} + (Tsa + Wvsl * Tad) * CF \\ = Tsa * CF' + Wvsl * Tad * LF \tag{115}$$

If Equation (115) is compared with Equation (108) it can be seen that the two differential equations are the same except for the variable names. This proves that, with appropriate selection of parameter values, *Virtual supply line* for information delay is mathematically equivalent to *Supply line*.

**Reduced Secondary Stock Equations when Virtual Supply Line is Considered (Figure 3.)**

$$\dot{SS} = \frac{SSLS}{Tsad} - SLF \tag{116}$$

$$\dot{SSLS} = \left( \begin{array}{l} SLF + \\ \dfrac{\left( \dfrac{CF' + \dfrac{Wvsl}{Tsa} * \left( \begin{array}{l} (Tssa + Tsad) * LF \\ - Cp * \left( \begin{array}{l} (Tssa + Tsad) * SS \\ + Tsad * (SSLS - Tsad * SLF) \end{array} \right) \end{array} \right)}{Cp} - SS \right)}{Tssa} \\ + \dfrac{Wssl * (Tsad * SLF - SSLS)}{Tssa} - \dfrac{SSLS}{Tsad} \end{array} \right) \tag{117}$$

From Equation (116) the following two equations are obtained:

$$SSLS = Tsad * \dot{SS} + Tsad * SLF \tag{118}$$

$$\dot{SSLS} = Tsad * \ddot{SS} \tag{119}$$

Equation (118) and Equation (119) can be inserted in Equation (117) to obtain the following equation:

$$Tsad*\ddot{SS}=\left(\begin{array}{l} SLF+ \\ \dfrac{\left(\dfrac{CF'+\dfrac{Wvsl}{Tsa}*\left(\begin{array}{l}(Tssa+Tsad)*LF \\ -Cp*\left(\begin{array}{l}(Tssa+Tsad)*SS \\ +Tsad*\left(Tsad*\dot{SS}+Tsad*SLF-Tsad*SLF\right)\end{array}\right)\end{array}\right)}{Cp}-SS\right)}{Tssa} \\ +\dfrac{Wssl*\left(Tsad*SLF-Tsad*\dot{SS}-Tsad*SLF\right)}{Tssa}-\dfrac{Tsad*\dot{SS}+Tsad*SLF}{Tsad}\end{array}\right) \quad (120)$$

Equation (120) can be simplified to the following:

$$\begin{aligned} &Cp*Tsad*Tssa*Tsa*\ddot{SS}+Cp*\left(Wssl*Tsad*Tsa+Tssa*Tsa+Wvsl*Tsad^{2}\right)*\dot{SS} \\ &+Cp*\left(Tsa+Wvsl*\left(Tssa+Tsad\right)\right)*SS=Tsa*CF'+Wvsl*\left(Tssa+Tsad\right)*LF \end{aligned} \quad (121)$$

Letting $Wssl = 1$ and $Tsad = Tssa = \dfrac{Tad}{2}$ the following is obtained:

$$\begin{aligned} &Cp*\left(Tad/2\right)^{2}*Tsa*\ddot{SS}+Cp*\left(Tad*Tsa+Wvsl*\left(Tad/2\right)^{2}\right)*\dot{SS} \\ &+Cp*\left(Tsa+Wvsl*Tad\right)*SS=Tsa*CF'+Wvsl*Tad*LF \end{aligned} \quad (122)$$

Equation (122) can be re-written for *CF* with using the relationship given for *SS* and *CF* in Equation (10).

$$\begin{aligned} &\left(Tad/2\right)^{2}*Tsa*\ddot{CF}+\left(Tad*Tsa+Wvsl*\left(Tad/2\right)^{2}\right)*\dot{CF}+\left(Tsa+Wvsl*Tad\right)*CF \\ &=Tsa*CF'+Wvsl*Tad*LF \end{aligned} \quad (123)$$

If Equation (123) is compared with Equation (108) and Equation (115) it can be seen that it is same with these equations. This proves that, with appropriate selection of parameter values, *Virtual supply line* for secondary stock is mathematically equivalent to *Supply line* and *Virtual supply line* for information delay.

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