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# Numerical Simulation of a New Correlation Function for the Climatic Statistical Structure of the Height Dynamic Field

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## ABSTRACT

For the long-term period in the low-latitude region of the earth, the statistical structure of the height dynamic field at 500 mb during the winter seasons has been studied successfully by using a new correlation function  $\{\rho(r) = A(r) * \text{EXP} [ B(r) ]\}$  where  $A(r)$  and  $B(r)$  are two general polynomials. The best selections of the degrees of these two polynomials can be found in the least-squares sense. The results show that this new mixed-type correlation function can yield more accurate fitting than Gandin's formula (1963)  $\{\rho(r) = a * \text{EXP} (- b * r ** 2)\}$ . The height dynamic fields at the regular nest grids are then computed and compared with those obtained from the measured data at irregular observational stations. The troughs of the resulting height dynamic fields can be identified very clearly.

## INTRODUCTION

For the study on macroscale turbulence of meteorology, the univariate and the multivariate scheme are used to analyze the measured data of the upper air observation points. With them, several techniques (Gandin (1963); Kricak (1967); Panchev (1969); Kluge (1970); Steinitz et al. (1970); Schlatter (1975); Schlatter et al. (1976); Thiebaut et al. (1986); Mitchell et al. (1990)) of the optimum interpolation had been developed successfully to study the correlation functions of the height dynamic fields. The computed results based on the above two schemes were also compared with those of the numerical weather forecast (Schlatter et al. (1976)). It was concluded that two univariate schemes (one for height, one for wind) can fit data as well as the multivariate scheme. The present method uses the univariate scheme to analyze the measured data. As an example, for the long-term period in the low latitude region of the earth, the statistical structure of the height dynamic field at 500 mb during the winter seasons has been studied successfully by using a new mixed-type correlation function.

## ANALYSIS

### Correlation Function



$f(r)$ , a meteorological variable, can be defined as the sum of its mean  $\bar{f}(r)$  and the deviation  $f'(r)$  where  $r$  is the distance of two geographic points. Between two meteorological variables  $f$  and  $g$ , the cross-covariance function  $C_{fg}$  can then be defined as

$$C_{fg}(r_1, r_2) = \overline{f'(r_1) \cdot g'(r_2)} \quad (1)$$

where  $r_1$  and  $r_2$  are respectively the distances between one unknown and two known geographic points shown as Fig. 1. Furthermore, the corresponding auto-covariance function  $C_{ff}(r_1, r_2)$  and variance function  $C_{ff}(r_1, r_1)$  can be defined from Eq. (1). Between two meteorological variables  $f$  and  $g$ , the crosscorrelation function can then be defined as

$$\rho_{fg}(r_1, r_2) = \frac{C_{fg}(r_1, r_2)}{[C_{ff}(r_1, r_1) \cdot C_{gg}(r_2, r_2)]^{1/2}} \quad (2)$$

Similarly, the corresponding autocorrelation function  $\rho_{ff}(r_1, r_2)$  can be defined from Eq. (2). Normally, it is homogeneous (position independent) and isotropic (direction independent), i.e.,

$$\rho_{ff}(r_1, r_2) = \rho_{ff}(r_2, r_1) = \rho_{ff}(d_{12}) = \rho_{ff}(d_{21}) \quad (3)$$

The above  $d_{ij}$  can be computed by

$$d_{ij} = \left( r_i^2 + r_j^2 - 2 r_i r_j \cos \theta_{ij} \right)^{1/2} \quad (4)$$

and shown in Fig. 1 with  $i = 1, NT$  and  $j = 1, NT$  where  $NT$  is the total of observational stations. If different and nonzero  $d_{ij}$  are indicated by  $d_k$ , the above  $\rho_{ff}(r_i, r_j)$  can further be represented by  $\rho_{ff}(d_k)$  with  $k = 1, [NT \cdot (NT-1) / 2]$ .

Therefore, the above raw correlation data  $\rho_{ff}(d_k)$  can be computed from the measured values of climatological variables during the long-period of time in a specific region. Then, the corresponding autocorrelation function  $\rho_{ff}(d)$



can be found.

For a specific distance  $D_L$  between two geographic points, the distance-average autocorrelation data with homogeneity and isotropy can be defined as

$$\bar{\rho}_{ff} (D)_L = \frac{\sum_{i=1}^M \rho_{ff} (d_i)}{M_L} \quad (5)$$

where  $(D_L - \Delta D) \leq d_i < (D_L + \Delta D)$  with  $(i = 1, M$  and  $L = 1, N)$

and  $\sum_{L=1}^N M_L = NT \cdot (NT-1)/2$  if  $\Delta D$  is provided and  $N$  is defined as the total of the distance-average autocorrelation data. Also,  $\bar{\rho}_{ff} (D)_L$  with  $L = 1, N$  can be employed to construct the distance-average autocorrelation function  $\bar{\rho}_{ff} (D)$  in the least-squares sense.

#### The Least-Squares Method

The negative squared exponential form (Gandin (1963)),  $\{ \bar{\rho} (D) = a \cdot \text{EXP} (-b \cdot D^{**2}) \}$ , was first proposed to fit the distance-average autocorrelation data  $\bar{\rho}_{ff} (D)_L$  in the least-squares sense where  $L = 1, N$ . The

present method uses the univariate scheme to develop a new correlation function to analyze the distance-average correlation data in the least-squares sense. It is a mixed-type function of the polynomial form and the exponential form. Certainly, it is also general form of the Gandin's (1963) and Buell's (1972) empirical correlation formulas. It can be expressed as

$$\begin{aligned} \bar{\rho} (D) &= [ A (D) ] \cdot [ \text{EXP} ( B (D) ) ] \\ &= [ \sum_{i=0}^{ma} a_i D^i ] \cdot [ \text{EXP} ( \sum_{i=0}^{mb} b_i D^i ) ] \end{aligned} \quad (6)$$

where  $ma$  and  $mb$  may be two arbitrary and different positive integers. The resulting sum of the square errors between the correlation model and the correlation data is

$$\sum_{j=1}^N e_j^2 = \sum_{j=1}^N \{ \ln \bar{\rho}_j - [ ( \sum_{i=0}^{mb} b_i D_j^i ) + \ln ( \sum_{i=0}^{ma} a_i D_j^i ) ] \}^2 \quad (7)$$

where  $\{ (D_j, \bar{\rho}_j) \mid j = 1, N \}$  represents the distance-average autocorrelation data. Differentiate Eq. (7) with respect to  $a_k$  and  $b_k$  where  $k = 0, ma$  for  $a_k$  and  $k = 0, mb$  for  $b_k$  to obtain

$$F_k = \frac{\partial \sum_{j=1}^N \epsilon_j^2}{\partial a_k} = \sum_{j=1}^N \left\{ \left[ \frac{D_j^k}{\sum_{i=0}^{ma} a_i (D_j^i)} \right] \cdot \epsilon_j \right\} \quad (8)$$

and

$$G_k = \frac{\partial \sum_{j=1}^N \epsilon_j^2}{\partial b_k} = \sum_{j=1}^N [D_j^k \epsilon_j] \quad (9)$$

From Eqs. (8) and (9), the  $(ma + mb + 2)$  nonlinear algebraic equations,  $\{ F_k = 0 \mid k = 0, ma \}$  and  $\{ G_k = 0 \mid k = 0, mb \}$ , can be solve simultaneously using the Newton-Raphson method (Chow (1979) and Gerald (1978)) by first guessing arbitrary initial values for  $\{ a_k \mid k = 0, ma \}$  and  $\{ b_k \mid k = 0, mb \}$ . The final numerical solutions of  $a_k$  and  $b_k$  are considered satisfactory when both

$$\text{Max} \{ | a_k^{\text{iter}+1} - a_k^{\text{iter}} | \} < 10^{-5} \quad \text{where } k = 0, ma$$

and

$$\text{Max} \{ | b_k^{\text{iter}+1} - b_k^{\text{iter}} | \} < 10^{-5} \quad \text{where } k = 0, mb$$

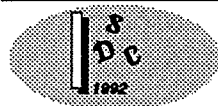
The convergence processes of maximum errors for  $a_k$  and  $b_k$  are indicated in Fig. 2.

#### Optimum Interpolation Based On Observational Stations

For an observational point  $k$ , the deviation  $f'(r)$  can be found from

$$f'(r)_k = f(r)_k - \bar{f}(r)_k \quad (10)$$

$f(r)_k$  and  $\bar{f}(r)_k$  are respectively the measured meteorological and the cli-



mathematical mean values. Therefore, the analyzed deviation of the meteorological parameter  $f$  at an arbitrary geographic point  $G$  can be evaluated by

$$\hat{f}'(r)_G = \sum_{k=1}^{NT} \alpha_k f'(r)_k \quad (11)$$

if weighting coefficients,  $\{\alpha_k | k = 1, NT\}$ , are provided. Actually, the weighting coefficients are not given up to now. However, they can be found by using the method of the optimum interpolation in the least-squares sense. The resulting simultaneous equations for the weighting coefficients  $\{\alpha_k | k = 1, NT\}$  are

$$\sum_{k=1}^{NT} \rho_{ff}(r, r)_m \alpha_k = \rho_{ff}(r, r)_m G \quad (12)$$

for  $m = 1, NT$ .

#### Reliability of Distance-Average Autocorrelation Models

For the observational raw autocorrelation data as shown in Fig. 3, the distance-average autocorrelation data computed from Eq. (5) are then employed to generate the present new mixed-type autocorrelation model in the least-squares sense. This model is compared with the distance-average autocorrelation data and the Gandin's model. The results are shown in Fig. 4.

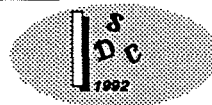
From Fig. 4, it can be concluded that the present new mixed-type distance-average autocorrelation model is much more accurate and reliable than Gandin's model for representing the distance-average autocorrelation data in the low-latitude region.

#### SAMPLE PROBLEM

In this study, the basic measured data are provided by 78 observational stations as shown in Fig. 5. Their height field on 500 mb and at 1200 GMT during 3 winter months (December, January, and February) of each year from 1977 to 1983 is then employed to generate a new mixed-type distance-average autocorrelation model. This model is finally used to compute height fields on the  $18 \times 16$  grid for two prescribed days. For these days, the height fields based on the measured data of 78 observational stations are contoured as Figs. 6 and 7 for the following comparing work. Based on the above results of objective analysis, the meteorological parameter at the regular grid can then be employed to serve as the initial guess of the numerical weather prediction (NWP).

#### NUMBER OF OBSERVATIONAL STATIONS USED FOR OBJECTIVE ANALYSES

Between computed and measured deviations of the meteorological parameter at a target point  $G$ , the error ( $E'$ ) in the least-squares sense depends on the num-



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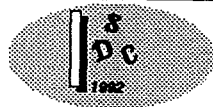
ber ( NO ) of observational stations used for objective analyses. ( Gandin ( 1963 ) ; Wang ( 1986 ) ) The relation between E'and NO is shown in Fig. 8. It was concluded that the measured data of the nearest seven observational stations around a target point are enough to compute accurate meteorological parameters. ( Wang ( 1986 ) ) In this study, the number of observational stations used for objective analyses is eight.

#### RESULTS AND CONCLUSIONS

1. Fig. 4 compares this new mixed-type model with the distance-average autocorrelation data and the Gandin's model. As can be seen, the mixed-type model is much more accurate and reliable than Gandin model for representing the distance-average autocorrelation raw data in the low-latitude region. Specially, the above phenomena are more obvious when the separation distances between two geographic points are less than 370 KM and greater than 2250 KM. Since the large areas in the low-latitude are the sea. The observational stations on the sea are sparse, the separation distances between the target points on the sea and the nearest 8 observational stations may be greater than 2000 KM. Therefore, the present autocorrelation model is much more reliable for computing the meteorological parameters in the low-latitude region than the Gandin's model.
2. Figs. 6 and 7 show the height fields based on the measured data of 78 observational stations. Several obvious troughs of height fields are found in these contours and marked by arrows. For each figure, the high and low of the height field are indicated and the gradient of the height field across Japan in the S-N direction is also found to be higher than those of the other regions in the figure.
3. As mentioned above, Fig. 8 shows that the measured data of the nearest seven observational stations around a target point are enough to compute the accurate meteorological parameters. This result can also be found by comparing Fig. 9 with Fig. 10. They are the interpolation-error contours of height fields computed respectively from the nearest 8 and 20 observational stations. As can be seen, these two contours are very similar.
4. Figs. 11 and 12 are the height fields computed respectively on January 7 and January 10 of 1983 from this new mixed-type autocorrelation model by optimum interpolation method of objective analyses. They are respectively compared with Figs. 6 and 7. The same analyzed results can be found.

From the above results and experiences, the following conclusions can be drawn :

1. For the low-latitude region, since the large areas are the sea, the accuracy of the autocorrelation model is very important for the full range of the separation distances between the target points and the observational stations.
2. Because of the similarity of correlation data distributions for the mid and high latitudes, it can be anticipated that the algorithm of the numer-



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ical simulation for this new mixed-type correlation function will also be valid for different latitudes regions of the earth provided the measured correlation raw data are given.

#### APPENDIX

In Fig. 4, the resulting coefficients of Eq. (6) for the  $m_a = 5$  and  $m_b = 5$  case are

a = 0.99998D-01	a = 0.23054D-02	a = -0.33989D-03
0	1	2
a = -0.11815D-02	a = -0.10476D-04	a = 0.46725D-04
3	4	5
b = 0.22635D+01	b = -0.35200D+00	b = -0.68385D-01
0	1	2
b = -0.71690D-02	b = 0.12955D-02	b = 0.91964D-03
3	4	5

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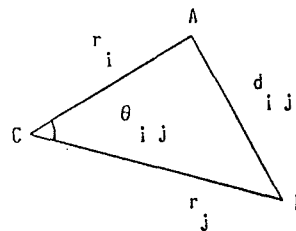


Fig. 1. Definitions of distances among three geographic points (A, B, C).

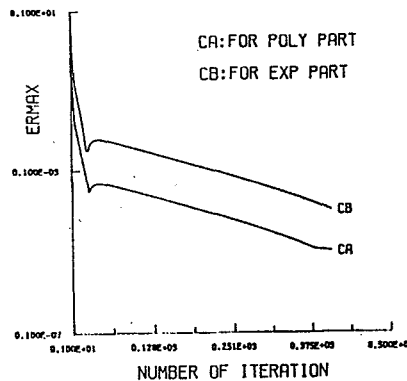
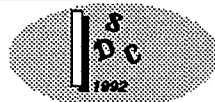


Fig. 2. The convergence processes of maximum errors for coefficients ( a | k = 0, 5 ) and ( b | k = 0, 5 ).





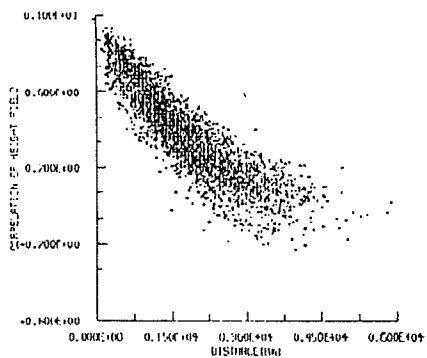


Fig. 3. The observational raw autocorrelation data for the height field as a function of the separation distance between every pair of points for 78 radio-sonde stations of the low latitude region

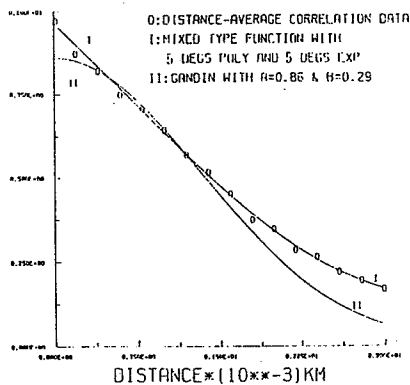


Fig. 4. The new mixed-type autocorrelation model and its distance-average raw autocorrelation data. ( Gandin's model for the low latitude is presented together as a reference. )

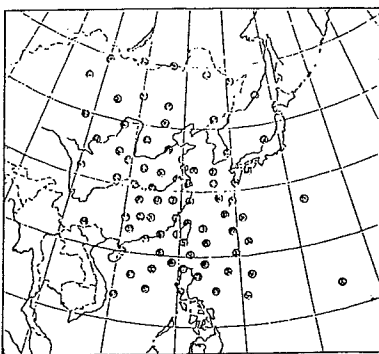


Fig. 5. Geographic Locations of 78 observational stations.

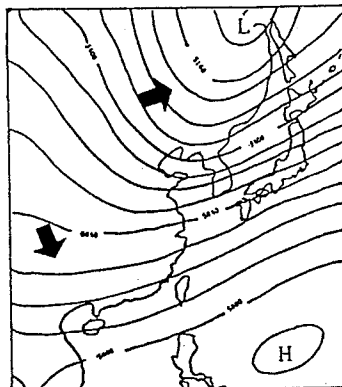


Fig. 6. The height field based on the measured data of January 7, 1983 at 78 observational stations. ( From Central Weather Bureau, R.O.C. )

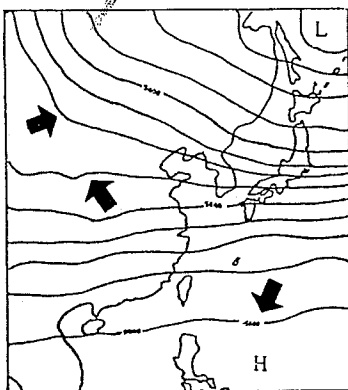


Fig. 7. The height field based on the measured data of January 10, 1983 at 78 observational stations. ( From Central Weather Bureau, R.O.C. )

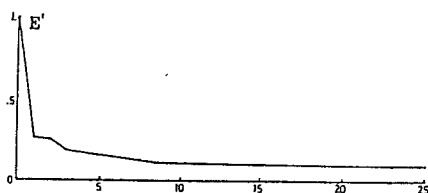


Fig. 8. The relation between the interpolation error ( E' ) and the number ( N ) of observational stations used.

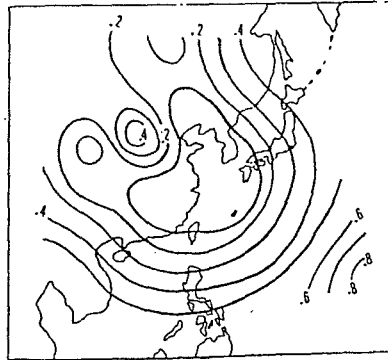


Fig. 9. The interpolation-error contour of the height field computed from the nearest 8 observational stations.

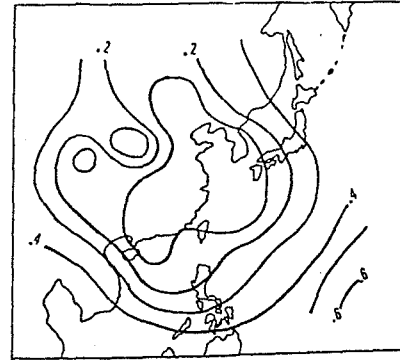


Fig. 10. The interpolation-error contour of the height field computed from the nearest 20 observational stations.

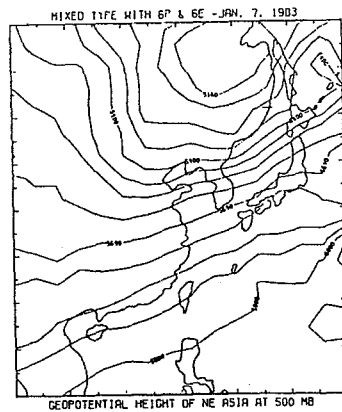


Fig. 11. The height field computed by optimum interpolation method of objective analyses from the new mixed-type autocorrelation model. ( January 7, 1983 )

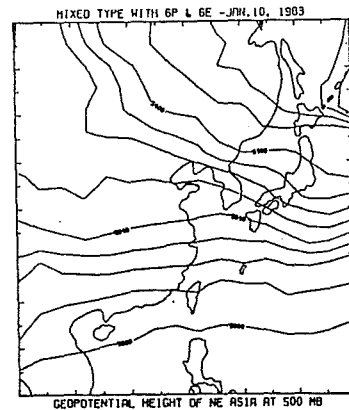


Fig. 12. The height field computed by optimum interpolation method of objective analyses from the new mixed-type autocorrelation model. ( January 10, 1983 )