

**COMPUTING THE OPTIMAL PRODUCTION RATES  
IN A COMPLEX PRODUCTION SYSTEM**

**Authors : Rafael RUIZ USANO**

**Adolfo CRESPO MARQUEZ**

**Dpto. de Organización Industrial y Gestión de Empresas**

**Escuela Superior de Ingenieros Industriales**

**UNIVERSIDAD DE SEVILLA**

**Avda. Reina Mercedes s./n.,**

**41012 - SEVILLA (SPAIN)**

**ABSTRACT.**

It is very well known that the setting of the optimal production rates in a complex production system is one of the most important and difficult decisions to be taken. This decision making process frequently seems to be an artisan job as it is necessary to take into account many influential factors simultaneously. In this paper we discuss the way of setting the optimal production rates, by applying optimization techniques and system dynamics methodologies. We study the effects of this approach on costs in order to minimize them and obtain good overall results, even for non-financial measurements. Simulation results are presented.

**1. INTRODUCTION.**

The use of System Dynamics to build simulation models is widely accepted mainly due (Coyle, 1990) to the speed, which one can model, the transparency of influence diagrams and its adequacy in modelling real situations. A good number of applications have been developed since the appearance of Forrester's models on Industrial Dynamics (Forrester, 1964). At present we can consider the S.D. methodology as a conventional and powerful tool within which a huge scientific and professional community is actively working all over the world in the industrial dynamics field and developing new approaches to improve the results.

In this paper we try to compute the production rates in a production planning system using an optimization model connected with the S.D. model. In section 2, we briefly explain the S.D. model used to reproduce the structure and behaviour of the production planning system. Section 3 shows the optimization model that interacts with the dynamic model. It computes the optimal production rates which are fed into the S.D. model. Section 4 describes the design of the proposed approach and the relationships between both models: S.D. and Optimization. Some simulation results are presented at the end of the paper with a short discussion on the advantages of the proposed mixed approach to modelling the system.



### 3. THE OPTIMIZATION MODEL

The production management tries always to coordinate the strategic and the tactical planning. It therefore has to take into account the real cost structure and constraints of different types: financial, economical, technological, etc.

With this modelling criterion, the production rates can be calculated by computing the optimal solution of a linear programming problem, at regular intervals. The objective function of this problem would incorporate the cost structure of the production system (Ruiz Usano, 1990).

The L.P. model is as follows:

$$\text{MAX} \sum_{i=1}^{i=N} (C_{i1} X_{i1}^k - C_{i2} X_{i2}^k - C_{i3} X_{i3}^k)$$

s. t.:

$$\begin{aligned} X_{i1}^k &\leq DF^k \\ X_{i2}^k &\geq DIN^k \\ X_{i3}^k &\leq MPC^k \\ X_{i1}^k + X_{i2}^k - X_{i3}^k &= INV^k \\ X_{i1}^k, X_{i2}^k, X_{i3}^k &\geq 0 \end{aligned}$$

$$\begin{aligned} i &= 1, 2, \dots, N \quad (\text{Number of products}) \\ k &= 1, 2, \dots, n \quad (\text{Time}) \end{aligned}$$

Subscripts:

- 1 sales for the next period
- 2 inventories at the end of the next period.
- 3 production rates for the next period.

Coefficients of the objective function:

- $C_{i1}$  unit income  
 $C_{i2}$  unit inventory holding cost  
 $C_{i3}$  unit production cost

Right hand side values:

- $DF$  Demand forecasting  
 $MPC$  Maximal production capacity  
 $DIN$  Desired inventory  
 $INV$  Actual inventory of finished products

**4. COMPUTING THE OPTIMAL PRODUCTION RATES.**

In most production planning system the calculation of the production rate is made on a heuristic basis (Forrester, 1964; Coyle, 1977) , as follows:

$$PR = SMOOTHED DEMAND + K * (DESIRED VALUE OF LEVEL VARIABLES - CURRENT VALUE)$$

Where PR is the Production Rate and the variables are usually inventory, backlog, etc.

The Production Output Rate is normally calculated as a delay of the entering Production Rate above (PR).

Is this a good production rate?. On a heuristic basis the answer could be. Why not?

Can we make some progress setting the production rates in a different way? The answer should be: it depends on what objectives you are trying to achieve. In the context of management criterion the setting of the production rates is one of the most important decision to be made.

On the other hand we have powerful optimization techniques which we often use to calculate the optimal decision variables in a rather isolated context, without taking into account the dynamic and interacting environment of the production planning system.

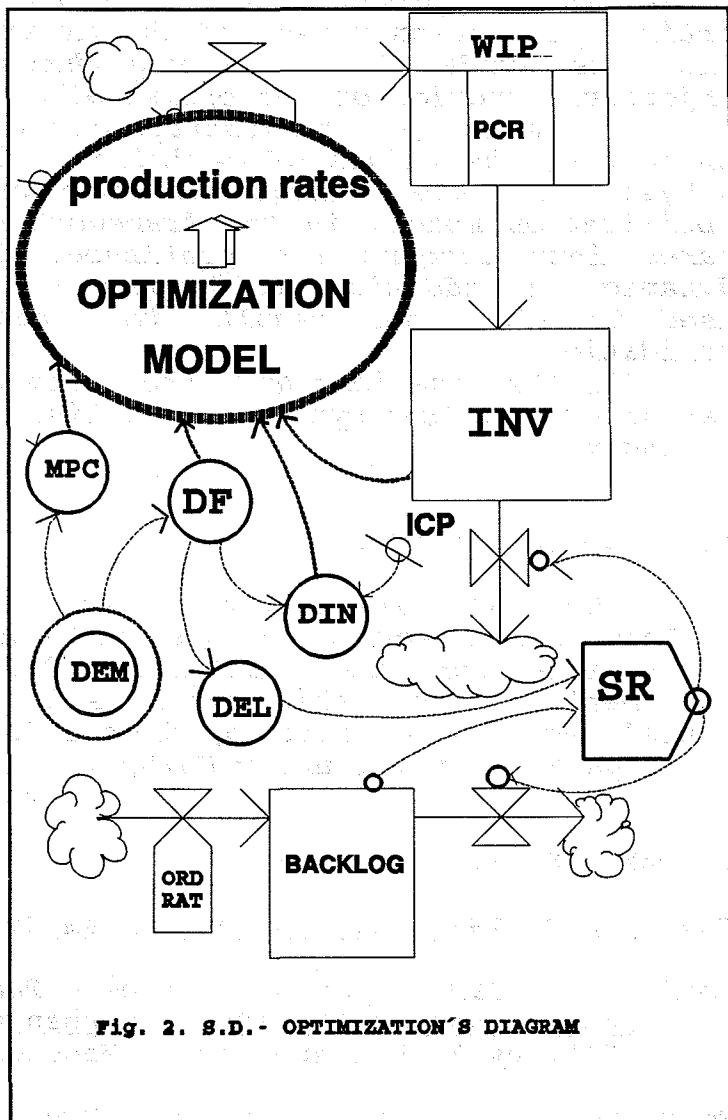


Fig. 2. S.D.- OPTIMIZATION'S DIAGRAM

Figure 2. S.D. - Optimization's Diagram.

Therefore, the basic idea is to design an optimization model that computes the optimal production rates , with an objective function given by the system's manager and these are fed into the S.D. model.

The S.D.-Optimization's diagram is shown in Figure 2. As it can be seen a part of the system has been replaced by the optimization model. The arrows entering the optimization model are the values calculated by the S.D. model and fed into the optimization model in order to update the parameters it needs.

## 5. SIMULATION RESULTS.

We show the results obtained by simulation of S.D.-Optimization approach. Figure 3 and 4 represent respectively the optimal production rates and the inventories for both products in the case study. Figure 5 reproduces the value taken by the objective function of the optimization model

An important advantage of using this approach is the way of setting the values of decision variables. These variables are subject to the management criterion translated into the optimization model, in the framework of the whole system, which takes into account the continuous interaction of the system dynamics. In addition, this approach (Wolstenholme, 1990) can be used to fit model results to actual observations for model validation.

All the equations and the parameter's numerical values can be found in the Dynamo source list of the model provided in the Appendix.

## 6. ACKNOWLEDGEMENTS.

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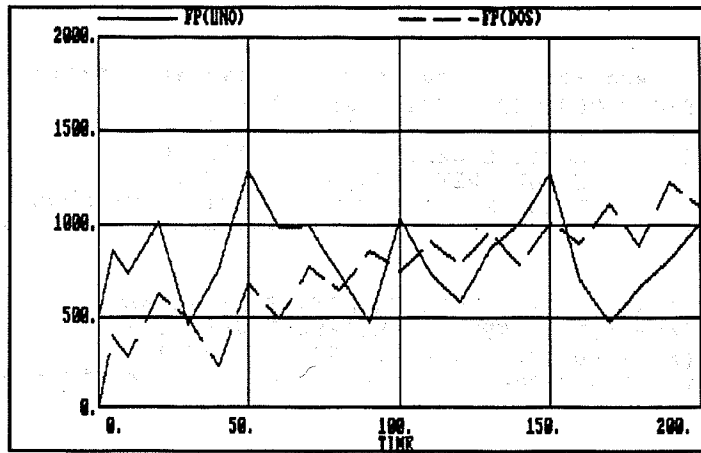


Figure 3. Optimal Production Rates.

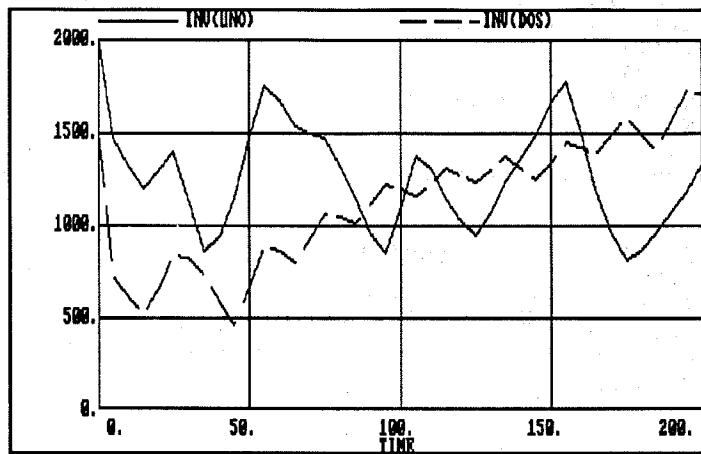
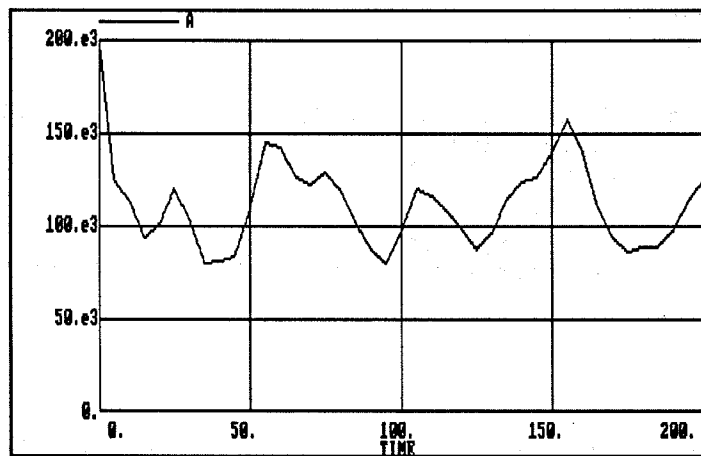


Figure 4. Levels of Inventory.



Ilustr. 5. Objective Function.

## 8. APPENDIX.

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* *****
*          BASIC PRODUCTION-INVENTORY MODEL          *
* *****
FNCTN MAXIMIZ(7,1,9)      ROUTINE CALL FOR OPTIMIZATION
INSERT MODFOR             MODULE WITH FOR' DEFINITIONS
DMNSN VRES(VAR)           RESULTS VECTOR FROM OPTIMIZATION MODEL
* *****
* LEVEL EQUATIONS
*
L CART.K(ART)=CART.J(ART)+(DT)*(FCON.JK(ART)-SUMI.JK(ART))      BACKLOG
L INV.K(ART)=INV.J(ART)+(DT)*(FPT.JK(ART)-SUMI.JK(ART))          INVENTORY
L WIP.K(ART)=WIP.J(ART)+(DT)*(FP.JK(ART)-FPT.JK(ART))           WIP
* *****
* AUXILIARY EQUATIONS
*
A D.K(ART)=TABLE(TDEM(*,ART),TIME.K,0,200,10)                   DEMAND
A PRD.K(ART)=SMOOTH(D.K(ART),TAD)                                 FORECAST
A IDES.K(ART)=PRD.K(ART)*GCI
A RETRASO.K(ART)=TABLE(TETRAS(*,ART),INV.K(ART)/PRD.K(ART),0,10,1)
A MCP.K(ART)=SMOOTH(1.8*D.K(ART),TAC)                            MAX.CAPACITY
A INVAGR.K=SUM(INV.K(*))
A WIPAGR.K=SUM(WIP.K(*))
A DINSAT.K(ART)=CART.K(ART)-SUMI.KL(ART)
A TINV.K(ART)=WIP.K(ART)+INV.K(ART)
A TINVAGR.K=WIPAGR.K+INVAGR.K
* *****
* FLOW EQUATIONS
*
R FPT.KL(ART)=DELAY3(FP.KL(ART),LT(ART))
R FCON.KL(ART)=D.K(ART)
R SUMI.KL(ART)=CART.K(ART)/RETRASO.K(ART)
INSERT MODOPT             MODULE FOR THE MATRIX RIGHTHAND SIDE UPDATE
* *****
* CONSTANTS AND PARAMETERS
*
P TAC=8                   SEMANAS
P TAD=3                   SEMANAS
P GCI=3                   SEMANAS
C LT(*)=0.4,0.4           SEMANAS
P POND=0.8                80 %
P TAI=1.5                 SEMANAS
* *****
* TABLES
*
T TDEM(*,DOS)=500,300,600,500,250,650,500,750,650,850,750,900,800,950^
,800,1000,900,1100,900,1200,1100
T TDEM(*,UNO)=1000,750,1000,500,750,1250,1000,1000,750,500,1000,750,600,850^
,1000,1250,750,500,650,800,1000
T TETRAS(*,UNO)=6,1,1,1,1,1,1,1,1,1,1,1
T TETRAS(*,DOS)=6,1,1,1,1,1,1,1,1,1,1,1
* *****
* INITIAL CONDITIONS
*
N CART(ART)=FCON(ART)
N INV(ART)=IINV(ART)
I IINV(*)=2000,1500
N WIP(ART)=D(ART)*LT(ART)
* *****
* SIMULATION SPEC
*
SPEC LENGTH=200/DT=0.1/PRTPER=5/SAVPER=5/TINIC=0/TCAMB=200
SAVE INV,WIP,CART
SAVE D,PRD,IDES,RETRASO,MCP,INVAGR,WIPAGR,DINSAT,TINV,TINVAGR
SAVE FPT,FCON,FP,SUMI

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*   COMPUTING THE OPTIMAL PRODUCTION RATES
*   *****
OBJECTIVE FUNCTION COEFFICIENTS
I   IMAT(*,FOB)=0,250,120,-25,-25,-100,-60
MAX. CAPACITY CONSTRAINTS
I   IMAT(*,R1)=4400,0,0,0,0,-1,0
I   IMAT(*,R2)=2200,0,0,0,0,0,-1
SALES CONSTRAINTS
I   IMAT(*,R3)=1000,-1,0,0,0,0,0
I   IMAT(*,R4)=500,0,-1,0,0,0,0
INVENTORY CONSTRAINTS
I   IMAT(*,R5)=2000,0,0,-1,0,0,0
I   IMAT(*,R6)=1500,0,0,0,-1,0,0
INVENTORY BALANCE EQUATIONS
I   IMAT(*,R7)=2000,-1,0,-1,0,1,0
I   IMAT(*,R8)=1500,0,-1,0,-1,0,1
N   MAT(J,I)=IMAT(J,I)
*   *****
NUMBER OF VARIABLES SENT TO SIMPLEX L.P. MODEL
P   N=6
NUMBER OF CONSTRAINTS <=
P   M1=4
NUMBER OF CONSTRAINTS >=
P   M2=2
NUMBER OF CONSTRAINTS =
P   M3=2
P   ER=10E-3
*   *****
*   SIMPLEX MATRIX RIGHT HAND SIDE UPDATE
A   MAT.K(OBJ,R1)=MCP.K(UNO)
A   MAT.K(OBJ,R2)=MCP.K(DOS)
A   MAT.K(OBJ,R3)=PRD.K(UNO)
A   MAT.K(OBJ,R4)=PRD.K(DOS)
A   MAT.K(OBJ,R5)=0.5*IDES.K(UNO)
A   MAT.K(OBJ,R6)=0.5*IDES.K(DOS)
A   MAT.K(OBJ,R7)=INV.K(UNO)
A   MAT.K(OBJ,R8)=INV.K(DOS)
*   *****
CALL TO MAXIMIZE ROUTINE ( A = OBJECTIVE FUNCTION )
A   A.K=MAXIMIZ(VRES.K,MAT.K,N,M1,M2,M3,ER)
OPTIMAL PRODUCTION RATES
R   FP.KL(UNO)=VRES.K(VA5)
R   FP.KL(DOS)=VRES.K(VA6)

```

```
SAVE VRES,MAT,FP,A
```

```
MODFOR.DYN
```

```
FOR ART=UNO,DOS
FOR I=FOB,R1,R2,R3,R4,R5,R6,R7,R8
FOR J=OBJ,V1,V2,I1,I2,P1,P2
FOR VAR=VA1,VA2,VA3,VA4,VA5,VA6
```

```
PRODUCTS
ROWS
COLUMNS
SIMPLEX OUTPUT VARIABLES
```