

# An Approach to Solve Fuzzy System Dynamics problems

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## **Abstract.**

System Dynamics is an effective method for dealing with time-varying or dynamic interactions among components in the complex system. It is generally used in the domain of social, economic and human activity systems, which deal with imprecise and vague variables or events. In this kind of system casual loop is the main concept for model formulation. Sometimes casual loops can't be explained precisely and have vague and imprecise meanings. In some cases it is better to use system dynamics with the other models. Fuzzy logic is a good interface for solving vague and imprecise problem in the System Dynamics model. This paper develops an approach for investigating fuzzy causal loop to study the behavior of fuzzy relations expressed by linguistic variables and presents an alternative approach for the analysis of problems. This approach has used Expert System as a well known tool in Artificial Intelligence for solving fuzzy system dynamics problems.

**Keyword(s):** System Dynamics; Fuzzy Set Theory; Artificial Intelligence; Expert System; Fuzzy Graph.

## 1. Introduction

Dynamic behavior is an essential feature of most complex systems. By complex we mean that there are different types of variables and causal linkages. The influences of these variables on each other are also complicated. The direction of arrow shows the direction of cause effect relationship. The effect of one variable to the others can be positive or negative or can be represented by linguistic variables. For example Low, High, Very High and so on To use fuzzy set theory to simulate the effects.

### 1.1 System Dynamics

System Dynamics developed by Forrester (1961 -1968) can be used to model and simulate complex systems. System Dynamics have been used for nearly three decades to model economic, social, human activity and other dynamic problems. [2]

System dynamics technique is an effective methodology for dealing with time-varying (dynamic) interactions among components of an analyzed system. A simulation language that is called DYNAMO was designed to simulate system dynamics model. The other application such as ITHINK and STELLA was designed for this purpose.

System dynamics technique identifies two sets of variables, i.e. "level" and "rate". The level variables consist of accumulations of resources within systems. The level equations describe the condition or state of the system at any point in time. The rate variables represent the flow into or out of a level. Dependency between flow rates and levels is represented by rate equations.

The procedure of system dynamics modeling could be broadly broken into six major steps:

- Problem identification and definition
- System conceptualization (Influence diagramming)
- Model formulation (Flow diagramming, Equation)
- Simulation and Validation of the model
- Policy analysis and Improvement
- Policy Implementation

### 1.2 Fuzzy Set Theory

Fuzzy set theory introduced by Zadeh (1965) is an idea to facilitate the use of uncertain concepts in computation processes. Fuzzy number is a convex normalized fuzzy set that shows a number imprecisely.

One of the most important applications of fuzzy sets is computational linguistics. Fuzzy set and linguistic variables can be used to quantify the meaning of natural language, which can be manipulated to solve the real problems. Linguistic variables are assigned values, which are expressed such as words, phrases, or sentences in a natural or artificial language. For example the value of *part quality* could be "high", "very high", "low", "not low"... which are linguistic variables and can be presented by fuzzy sets.

In this example "very" and "not" are modifiers, which used to modify our variables and describe our fuzzy concepts.

A linguistic variable must have a valid syntax and semantic, which can be specified by fuzzy sets and rules.

As already explained, influence or casual loop diagramming is the most important step in system dynamics modeling.

The causal loop diagram explains how the system works. This is created through writing the names of variables and connecting them by an arrow or link. The direction of the arrow shows the direction of causation. This set is a signed directed graph and if a link expresses a fuzzy relation, this graph is named fuzzy signed directed graph (fuzzy-SDG).

A fuzzy graph is a graph that its relations are fuzzy. In figure 1 a fuzzy graph is represented. The relations of this graph are fuzzy numbers between 0 and 1.

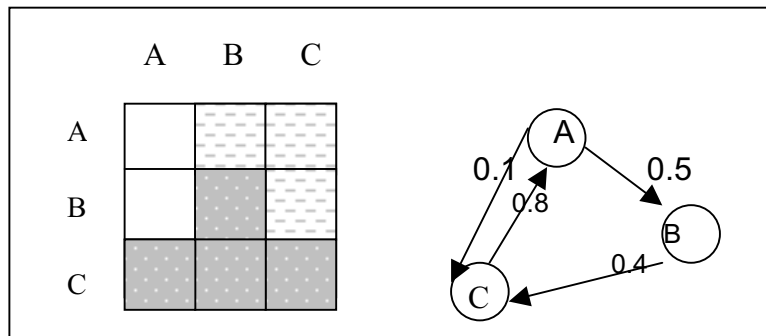


Figure 1. Fuzzy graph

The mathematical notation for a graph is defined as bellow:

$$\tilde{G}(x_i, x_j) = \left\{ \left( (x_i, x_j), \mu_{\tilde{G}}(x_i, x_j) \right) \mid (x_i, x_j) \in E \times E \right\}$$

In this example the notation is represented here:

$$\text{Fuzzy\_G} = \{ ((A,B),0.5), ((B,C),0.4), ((C,A),0.8), ((A,C),0.1) \}$$

(Fuzzy graph is a notation that is used in our new fuzzy system dynamics model.)

### 1.3 Expert systems

Expert system is a branch of artificial intelligence that makes extensive use of specialized knowledge to solve problems at the level of human expert. An expert is a person who has expertise in a certain area. The elements of typical expert system are: knowledge base (Rules), inference engine (Agenda), working memory (Facts), explanation facility, and knowledge acquisition facility and user interface.[3]

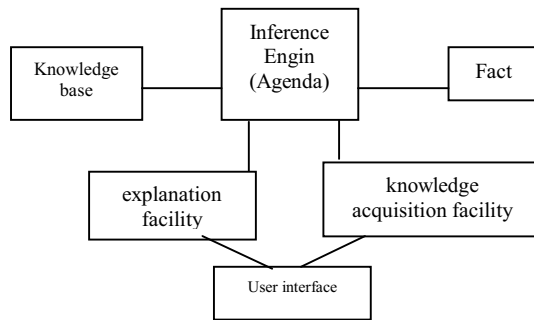


Figure 2. Expert system components

In a rule base expert system the needed knowledge for solving the problems is coded in the form of rules. Other types of expert system use different representations for knowledge. Rules are conditional type statements that comprise of an antecedent and a consequent part. The form of a rule is:

**IF** antecedent **THEN** consequent

For example a fuzzy rule may be expressed as follows:

**IF** quality is low **THEN** cost of repair became very high.

Fuzzy rule is a kind of rule that its element is fuzzy or linguistic variable. Much fuzzy knowledge exists in the real world, i.e. knowledge that is vague, imprecise, uncertain or ambiguous in nature. In crisp system, which based upon classical theory it is very difficult to answer some questions because either they do not have true answer completely, or in many times it is not possible to state the conditions precisely. For example a manager may not be able to state precisely the condition that under it “quality of part” affects on “repair cost of part” but he/she can state that:

*“IF quality of part is low THEN repair cost of part became very high”.*

Thus use of fuzzy expert system helps us to solve this problem and cope with this kind of information.

## 2. System Dynamics with Fuzzy Logic

A system dynamics model can be considered to have fuzzy logic if it has at least one of the characteristics below:

- (a) Some of the level variables are fuzzy.
- (b) The time agent may be vague.
- (c) Some of the rate variables are fuzzy.
- (d) Some of the auxiliary variables and the other variables are fuzzy.
- (e) Some of the relations can be replaced by conditional statements, which include fuzzy variables. These conditional statements generally follow an IF-THEN format. For example in a purchase system the following conditional statement including fuzzy variables may be used to describe the relation between inventory and total cost.

- IF the cost of inventory is very low THEN the total cost becomes very low

- (f) Some of the relations can be presented by fuzzy algorithm, which include fuzzy variables. Fuzzy algorithm is a fuzzy statement, which can be represented by an IF-DO format. [12]
- IF the Cost of Inventory is “very low” Do the Total Cost “very low”
- (g) The degree of uncertainty of variables when the available information regarding them is imprecise or incomplete can be represented by fuzzy probabilities (e.g. likely, unlikely,)
- (h) Some of the operator may be fuzzy

According to these characteristics we proposed an approach for fuzzy system dynamics that will explain it in the next part.

### **3. A New Approach for Fuzzy System Dynamics**

#### **3.1 Steps of Approach**

The proposed Approach has following steps:

- (a) Description of problem
- (b) Collecting related data.
- (c) Definition of imprecise data and linguistic variables
- (d) Definition of the effects of the system component on each other and show their effects in the form of fuzzy graph.
- (e) Translation of the fuzzy graph into fuzzy rules with the IF -THEN form
- (f) Definition of uncertainty of each rule.
- (g) Creating fuzzy facts from the natural behavior of the system that express by experts.
- (h) Implementing a fuzzy expert system via this production rules and facts.
- (i) Analysis of the expert system outputs. (The output can be either fuzzy variables or a defuzzified numbers).

The proposed approach is a solution for fuzzy system dynamics. In this model we combined fuzzy graph, fuzzy expert system and many of usual steps of system dynamics models. And at the end of its steps we implemented a fuzzy expert system by means of a Shell (for example fuzzy CLIPS).

#### **3.2 Model validation**

Steps of this new methodology are similar to the usual steps of the system dynamics methods. But in this methodology we use Expert system as a known tool and our method is complex of the system dynamics property and expert system concepts. The below figures compares the new methodology and the System Dynamics Methodology.

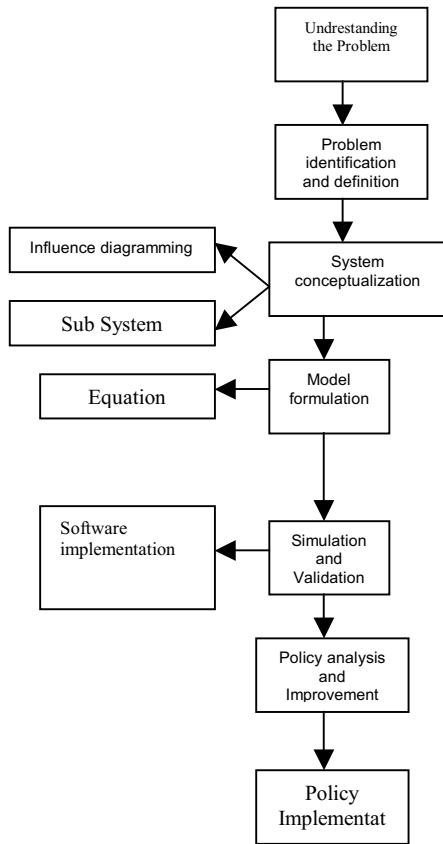


Figure 3.1 Steps of System Dynamics

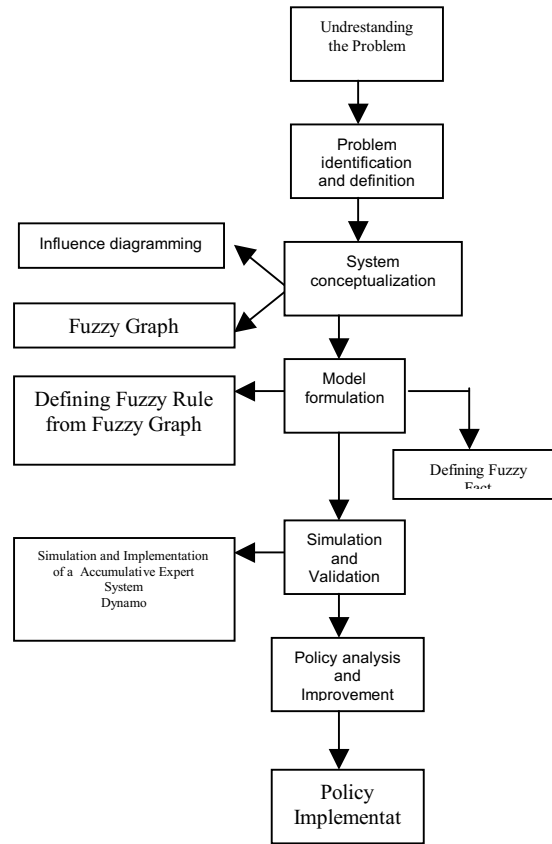


Figure3.2 Steps of New Approach

We add new properties to an expert system in order to implement the system dynamics properties. For example when we want to add accumulation property in our model, it is better to save the last important data and add the last value to the new value for saving information. After firing a rule the old value adds to the new value. This effect helps us to model the accumulation property.

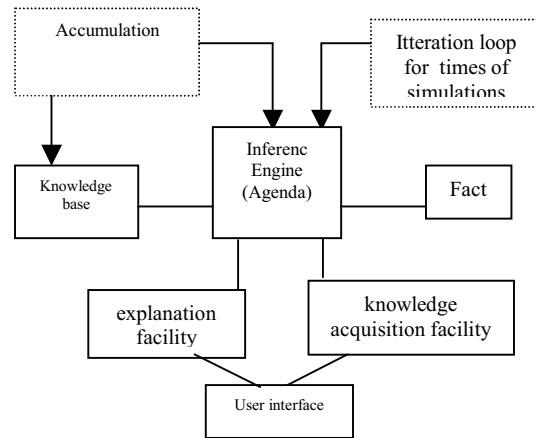


Figure 4. A new changed Expert system components for new model.

The above figure shows these new components that are added to expert system.

There is two alternatives which we can use them to implement this property.

- 1) Using modifiers is the first alternative. “ very” , “little” and so on are sample of this modifiers. We can use these modifiers to calculate a value as an accumulative value.

$$\text{Value ( I+1)} = \text{Value ( I )} + \text{Very High ( Value ( I+1))}$$

- 2) Deffuzifing is the second alternative. This alternative adds the old equivalent defuzzy data to the new equivalent defuzzy data. This adding, will be considered.

$$\text{Deffuzified (Value (I+1))} = \text{Value ( I )} + \text{Deffuzified ( Value ( I+1))}$$

Bellow figure shows these two ways:

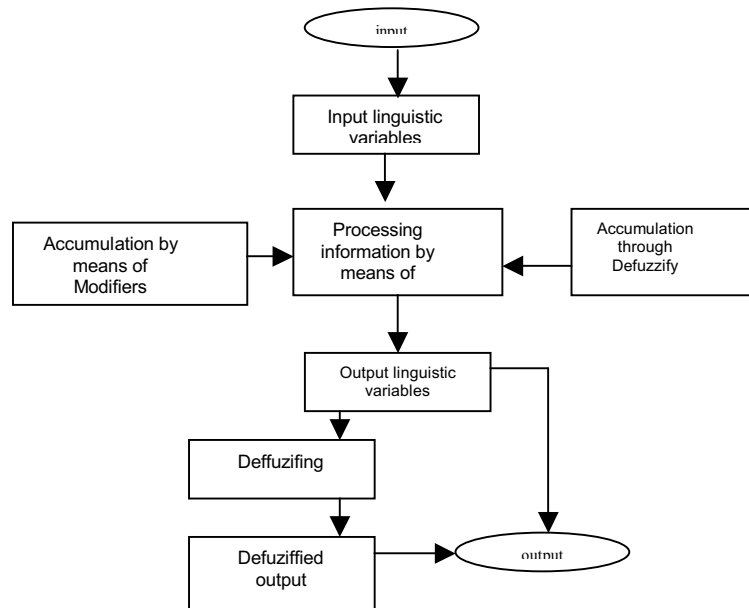


Figure 5 . Two way of the Accumulation

#### 4. Illustration

Example of system dynamics, which include fuzzy variables and linguistic variables, are described below. This proposed method has been illustrated through an example.

The steps of model for this system is here:

- Description of problem
- Collecting related data.

This example is about a purchase system, which consists of the following parameters:

Lead-Time: This is the time, which in that interval purchase must be done.

Supplier number: It is the number of suppliers, which have the acceptable parameters for our purchase. For example their quality must be good and their cost must be low.

Quality: It is quality of parts, which is purchased from the suppliers.

Cost of parts: It is cost of parts, which is purchased from the suppliers.

Cost of repair: It is the repair cost and it is related to quality of parts.

Cost of inventory: It is the cost of parts that exist in the inventory.

Cost of line sleep: It is the cost of sleeping the production line.

Total cost: It is the total cost that influence of inventory cost, cost of line sleep and total cost.

- Definition of imprecise data and linguistic variable
- IF Q equal to low THEN CR with certainty factor 80 percent becomes high  
 IF Supp\_No equal to low THEN Q CR with certainty factor 80 percent becomes low.



IF LT equal to low THEN Supp\_No CR with certainty factor 80 percent becomes low.  
 IF LT equal to low THEN Cinv CR with certainty factor 80 percent becomes low.  
 IF LT equal to low THEN Csleep CR with certainty factor 80 percent becomes high.  
 IF Supp\_No equal to low THEN Cpart CR with certainty factor 80 percent becomes high.  
 IF CR equal to low THEN C\_Total CR with certainty factor 80 percent becomes low.  
 IF Cpart equal to low THEN C\_Total CR with certainty factor 80 percent becomes low.  
 IF Cinv equal to low THEN C\_Total CR with certainty factor 80 percent becomes low.  
 IF Csleep equal to low THEN C\_Total CR with certainty factor 80 percent becomes low.

- Definition of the effects of the system component on each other  
 Figure 2 shows the effects of system component on each other in the form of fuzzy graph. This graph exhibits the relation between components of this purchase system and also shows the fuzzy parameters and linguistic variables.

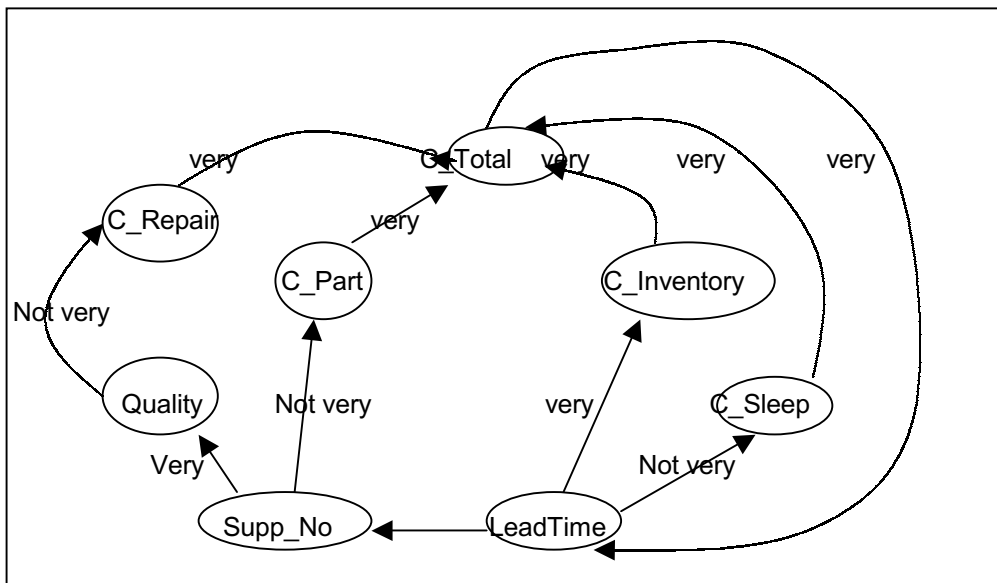


Figure 6. Fuzzy graph for purchase system

Fuzyy\_G = { ( ( Q,CR ) , very ) , ((Supp\_No,Q),very) , ((LT,Supp\_No),very),  
 (( LT, CINV),very) , (( LT, C\_Sleep) ,Not very) , ((Supp\_No,Cpart),Not very),  
 ((CR,C\_Total),very) , ((Cpart,C\_Total),very) , ((Cinv,C\_Total),very) ,  
 ((CSleep,C\_Total),very) }

- Translation of the fuzzy graph into fuzzy rules with the IF -THEN form.
- Definition of uncertainty of each rule.

IF Q==low THEN CR = high (CF=0.8)  
 IF Supp\_No==low THEN Q = low (CF=0.9)  
 IF LT==low THEN Supp\_No = low (CF=0.8)  
 IF LT==low THEN Cinv = low (CF=0.8)  
 IF LT==low THEN Csleep = high (CF=0.8)  
 IF Supp\_No==low THEN Cpart = high (CF=0.8)  
 IF CR==low THEN C\_Total = low (CF=0.8)  
 IF Cpart==low THEN C\_Total = low (CF=0.8)  
 IF Cinv==low THEN C\_Total = low (CF=0.8)  
 IF Csleep==low THEN C\_Total = low (CF=0.8)

- Creating fuzzy facts from the natural behavior of the system that express by experts.
- Implementing a fuzzy expert system via this production rules and facts.

Implementation an expert system from these rules, accomplished by fuzzy CLIPS. By using of this software, we can define linguistic variable and define fuzzy rules.

- Analysis of the expert system outputs. (This outputs can be either fuzzy variables or a defuzzified numbers)

For developing system dynamics accumulative attribute, we use accumulative variables to simulate this attribute. In each loop we add present data of this accumulative variable to the old data. Implementing of this accumulative has done by two ways:

*First way:* By using of modifiers we can accumulate fuzzy variables. For example “high” linguistic variable after accumulation is changed to “very high”, “very very high” and “extremely high”. This method (accumulation of linguistic variable) helps us to change “Static expert system” to “Dynamic expert system” and lets us to simulate this system for the desired steps. For example we can simulate this purchase expert system for 100 times.

*Second way:* In this way we defuzzified the fuzzy numbers and add their values with their old values and during the simulation we transfer their data (each system parameters) to a temporary file. At the end of simulation we transfer this file to MATLAB software and plot the graph of each parameter. In this way we can see the parameter behaviors graphically.

## 5. Conclusion

Social system, economic system and every system that deal with human parameters have imprecise behavior and no one can state those parameters precisely. Therefore fuzzy number and linguistic variable can help us in this type of systems.

In this paper we have discussed the use of fuzzy expert system to represent uncertainty in system dynamics models. The proposed method, can be used for investigating the system dynamics under vague and ambiguity conditions.

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