An Adaptive Expectations Approach to the Mechanisms of Transmission Model of the Central Bank of Colombia

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ABSTRACT

Looking for the potential applications of system dynamics in macroeconomic modeling at the Central Bank of Colombia, the Mechanisms of Transmission Model (MTM) was recast in a system dynamics model. The forward-looking function of the model that, in the case of the MTM is a rational expectations based function, was approached by means of the TREND function. This document describes the system dynamics model and shows comparative impulse-response results between the models, when PULSE and STEP shocks are applied to inflation target, monetary policy, food supply, nominal depreciation rate, and risk premium.

Introduction

This document is the preliminary result of collaborative work made by Pontificia Universidad Javeriana and the Central Bank of Colombia, as part of a research project on potential applications of system dynamics to macroeconomic modeling at the Bank. The Mechanisms of Transmission Model (MTM) of the Bank was chosen to be "recast" as a system dynamics model due to the presence of feedback relationships between variables, and the presence of autoregressive components that can be described by means of stock and flow variables. The MTM has been recently developed, and is intended to be used as a policy design model, specifically for evaluating the effectiveness of changes in the inflation target and the monetary policy by the Bank, and what the Bank should do in the event of external shocks to food supply, nominal depreciation rate and risk premium. The Bank has become interested in system dynamics models because of the transparency of their structure which facilitates public discussion, the possibility to incorporate stocks, and their powerful graphic interface. The main goal of this first part of the research was to find out if, by means of a system dynamics model, it was possible to obtain results similar to those of the MTM. Although the MTM uses a forward-looking function based on rational expectations, an adaptive expectations approach was used for the forward-looking function of the system dynamics model, in order to faster reach preliminary results that could provide ideas on how to proceed further with the research. Several commonly used forward-looking functions, such as moving average, exponential smoothing and Holt's method, were tested. The best results were obtained with the TREND function (Sterman, 1987) and, therefore, this was the one used in the system dynamics model.The first part of the document describes the Bank's model and, in a parallel way, how the system dynamics model was developed, and the second part presents comparative results between the models when disturbed by a shock applied to inflation target, monetary policy, food supply, nominal depreciation rate and risk premium.

The Mechanisms of Transmission Model

Currently, the Central bank of Colombia (Banco de la República) is managing its monetary policy following an operative strategy known as inflation target. This strategy consists, basically, of controlling the interest rate in such a way, that the inflation forecast be aligned with the inflation target. If the inflation forecast is higher than the target, the Central Bank will tend to increase interest rates. This increase in interest rates will activate the diverse channels of transmission of monetary policy, driving inflation to the target. These channels are the aggregate demand channel, the direct and indirect exchange rate channels, and the expectations channel.

Aggregate demand channel

This channel transmits monetary policy to the inflationary pressures arising from aggregate demand. Figure 1 shows an influence diagram of this channel. If the real interest rate increases, GDP decreases, increasing GDP gap, and reducing demand and inflation.

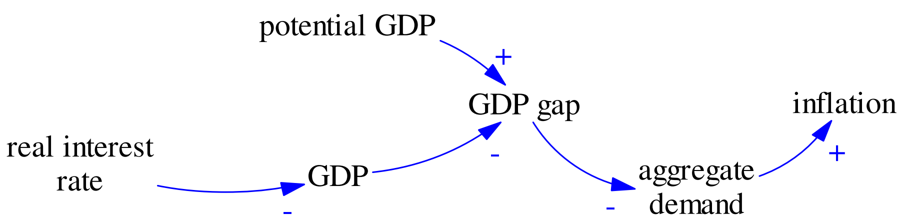


Figure 1. Aggregate demand channel.

Exchange rate direct channel

This channel transmits the pass-through effect of variations in exchange rate on domestic prices. Figure 2 shows an influence diagram of this channel. An increase in domestic interest rates improves the profitability of investment inside the country compared with that of investment outside the country. In a floating exchange rate system, the exchange rate tends to appreciate. For a given level of long term exchange rate, this appreciation produces

an increase in expected devaluation that equals the profitability of investment inside and outside the country.
The exchange rate appreciation reduces inflation of imports. This is the first stage of the direct or pass-through channel. In its turn, the lower inflation of imports reduces inflation of CPI. This is the second stage of pass-through.

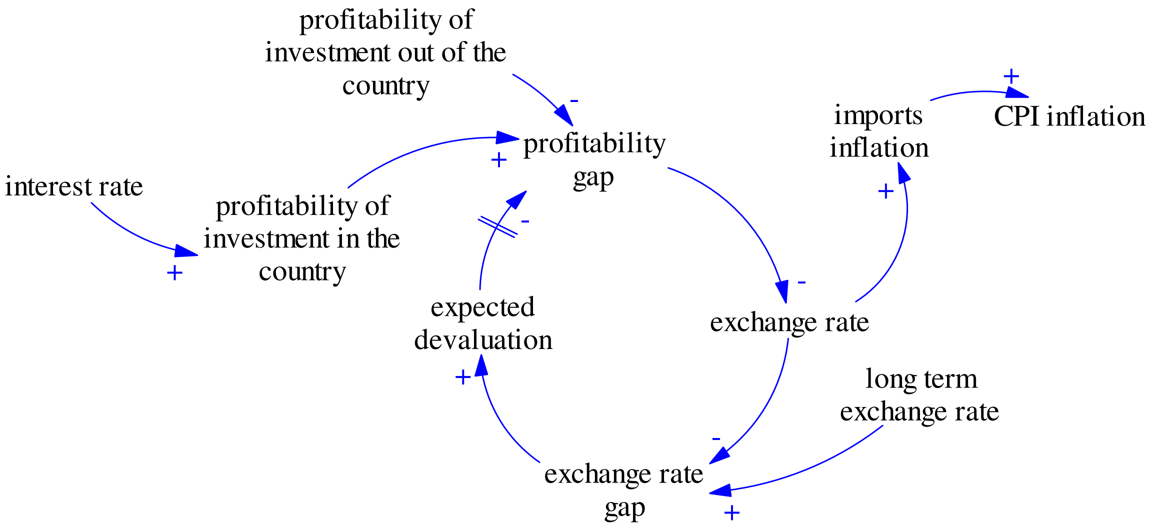


Figure 2. Pass-through channel.

Exchange rate indirect channel

In the description of the pass-through channel, it was shown how an increase in interest rate produces an appreciation in the nominal exchange rate. This nominal appreciation results in a real appreciation that tends to reduce the competitiveness of exports and increase that of imports and, as a consequence, reduces net exports. This causes a reduction in domestic demand that tends to decrease GDP. Such decrease has an impact on inflation that depends on how much is the gap between current GDP level and its potential level. Figure 3 shows an influence diagram for this channel.

Expectations channel

By this channel, a decrease in inflation expectations tends to reduce the inflation itself. In its turn, inflation expectations may depend on recent results of inflation, the performance of exchange rate, and the monetary policy position.

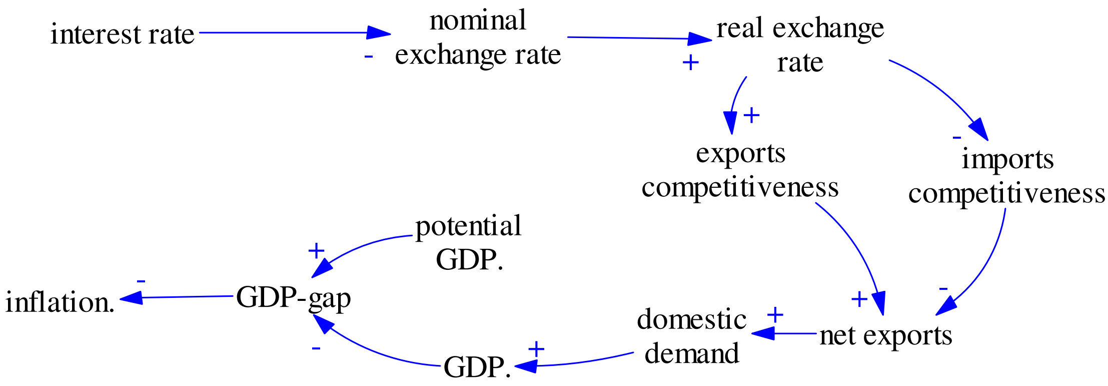


Figure 3. The exchange rate indirect channel.

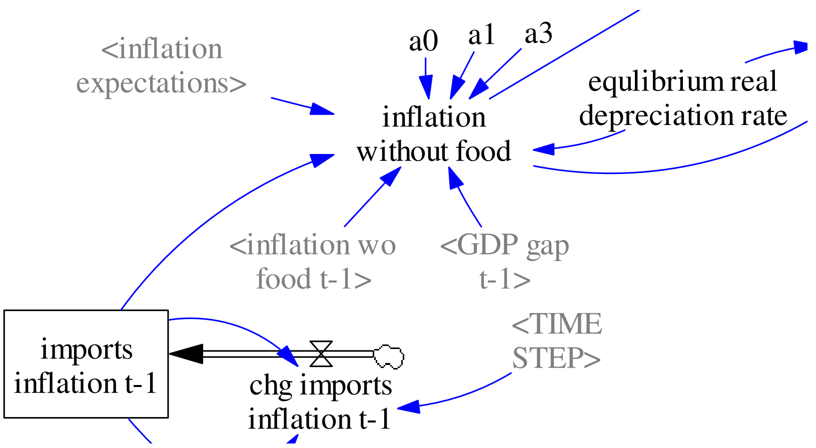
Model equations and system dynamics model

The *inflation without food* is modeled by means of a Phillips curve augmented by expectations:

$$\pi^{sa}_{t} = \alpha_0\pi^{sa}_{t-1} + \alpha_1\pi^{e}_{t} + (1-\alpha_0-\alpha_1)(\pi^{m}_{t-1}-\delta^{z}_{t}) + \alpha_3\hat{y}_{t-1} + \varepsilon^{\pi}_{t} \quad \text{(Eq. 1)}$$

where π^{sa}_{t} denotes the inflation without food, π^{e}_{t} the inflation expectations, π^{m}_{t} the imports inflation, δ^{z}_{t} the equilibrium real depreciation rate, $\hat{y}_t$ the product gap, and ε^{π}_{t} the residual.

Since residuals were not used in the system dynamics model, they will be omitted in the rest of the document. Figure 4 shows the system dynamics structure and equation for inflation without food.



inflation without food= a0"inflation wo food t-1"+a1*("imports inflation t-1"-equlibrium real depreciation rate)+(1-a0-a1)*inflation expectations+a3*"GDP gap t-1"*

Figure 4. Structure and equation for inflation without food

The measurement of expectations used in the model is obtained by the equation:

$\pi^e_t = \alpha_4\pi_{4,t-1} + \alpha_5 E_t\pi_{4,t+1}$ (Eq. 2)

where $\pi_{4,t}$ denotes the annualized quarterly inflation.

As can be seen the equation includes a backward-looking process ($\alpha_4\pi_{4,t-1}$) and a forward-looking process ($\alpha_5 E_t\pi_{4,t+1}$). In the Bank's model, the forward-looking process is approached by using the generalized Schur form to solve a multivariate linear rational expectations model (Klein, 2000). In the system dynamics model, the TREND function (Sterman, 1987) was used as an adaptive expectations approach. The input to the TREND function in the model is the current inflation, and the output (Perceived Trend) is used to calculate the change in expected inflation and, consequently, the expected inflation. The TREND function parameters were set using the Vensim optimization function defining the payoff function as a calibration against the results of inflation obtained with the Bank's model when a shock to the inflation target is applied, and setting one quarter (0.25 year) as the minimum value for the parameters. The result obtained was of 0.25 year for each of the three parameters (Time to Perceive Present Condition, Time Horizon for Reference Condition and Time to Perceive Trend). The structure for the TREND function in the system dynamics model is shown in Figure 5.

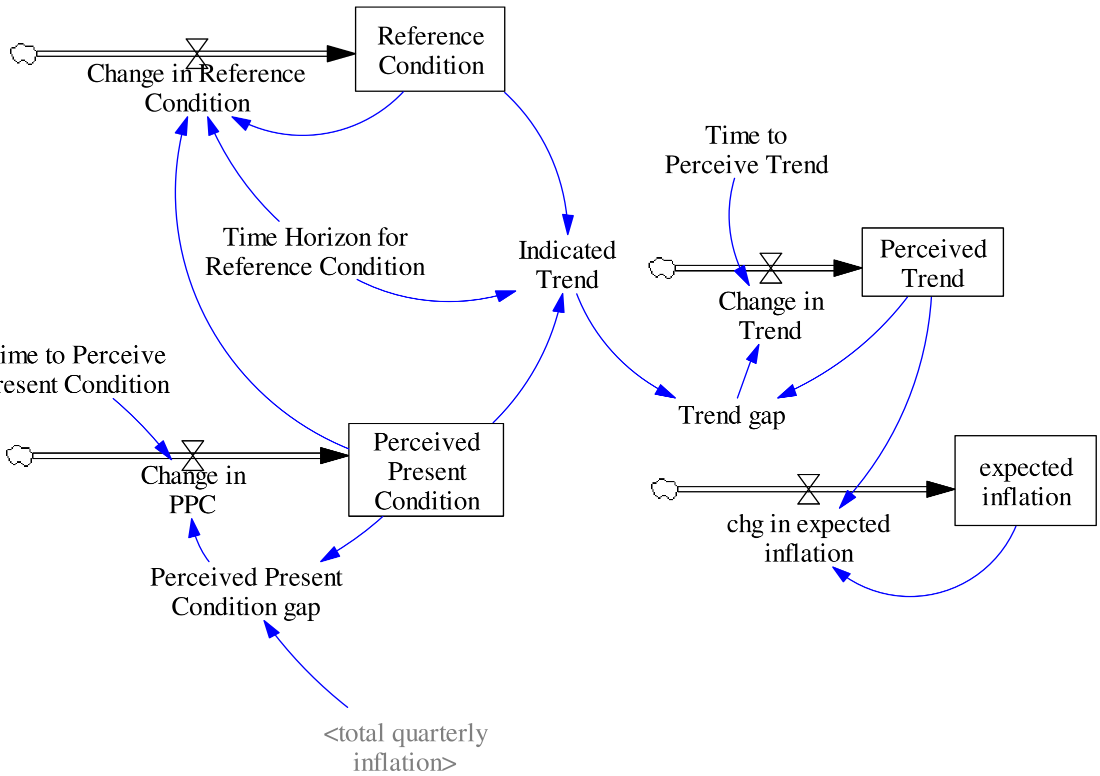


Figure 5. The TREND function.

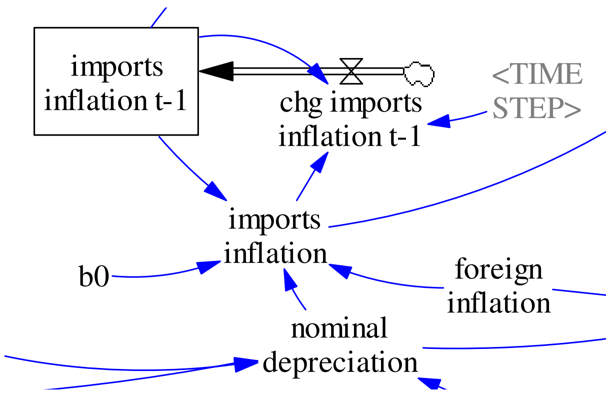
The equations for the TREND function are as follow:

Perceived Present Condition= INTEG (Change in PPC, 0.03)
Perceived Present Condition gap= total quarterly inflation-Perceived Present Condition
Change in PPC= Perceived Present Condition gap/Time to Perceive Present Condition
Time to Perceive Present Condition= 0.25
Reference Condition= INTEG (Change in Reference Condition, 0.03)
Change in Reference Condition= (Perceived Present Condition-Reference Condition)/Time Horizon for Reference Condition
Time Horizon for Reference Condition= 0.25
Indicated Trend= ((Perceived Present Condition-Reference Condition)/Reference Condition)/Time Horizon for Reference Condition
Perceived Trend= INTEG (Change in Trend, 0)
Trend gap= Indicated Trend-Perceived Trend
Change in Trend= Trend gap/Time to Perceive Trend
Time to Perceive Trend= 0.25
chg in expected inflation= expected inflation*Perceived Trend
expected inflation= INTEG (chg in expected inflation, 0.03)

The *imports inflation* is modeled with a partial adjustment equation:

$$\pi^m_t = b_0\pi^m_{t-1} + (1-b_0)(\pi^*_t + \delta_t) \quad \text{(Eq. 3)}$$

where π^*_t denotes the foreign inflation and δ_t the nominal depreciation. Figure 6 shows the structure and equation for foreign inflation.



imports inflation= b0"imports inflation t-1"+(1-b0)*(nominal depreciation + foreign inflation)*

Figure 6. Structure and equation of imports inflation

Food inflation is modeled with a Phillips curve:

$$\pi^{a}_{t} = c_0\pi^{sa}_{t} + c_1\pi^{a}_{t-1} + c_2(\pi^{m}_{t} - \delta^{z}_{t}) + c_3\,\hat{y}_{t} \quad \text{(Eq. 4)}$$

The Vensim equation for food inflation is:

food inflation= c1"food inflation t-1"+c2*(imports inflation - equlibrium real depreciation rate)+c0*inflation without food+c3*GDP gap + food supply shock*food supply shock switch*

The total inflation is calculated as the weighted sum of inflation without food and food inflation. The weighting factors correspond to the participation of each component in the Consumer Price Index:

$$\pi_{t} = \alpha_{sa}\pi^{sa}_{t} + (1 - \alpha_{sa})\,\pi^{a}_{t} \quad \text{(Eq. 5)}$$

The Vensim equation for total inflation is:

*total quarterly inflation= inflation weighting factor*inflation without food+(1-inflation weighting factor)*food inflation*

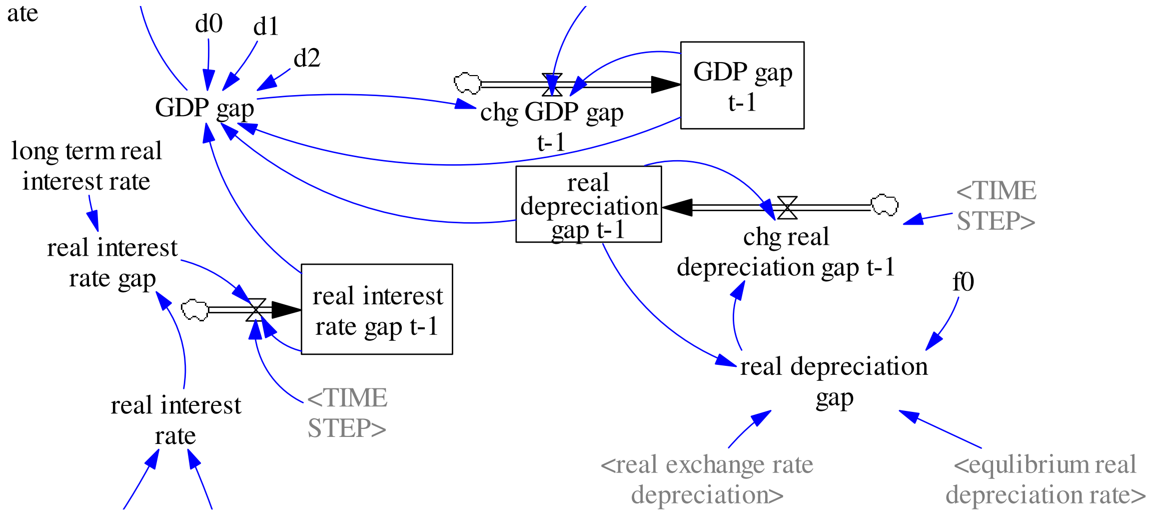
The product gap is modeled with an IS curve:

$$\hat{y}_{t} = d_0\hat{y}_{t-1} - d_1\check{r}_{t-1} + d_2\check{z}_{t-1} \quad \text{(Eq. 6)}$$

$$\check{r}_{t} = r_{t} - r_{l\,t} \quad \text{(Eq. 7)}$$

$$\check{z}_{t} = z_{t} - z_{l\,t} \quad \text{(Eq. 8)}$$

where r_t denotes the real interest rate, $r_{l\,t}$ the long term real interest rate, $\check{r}_t$ the real interest rate gap, z_t the real exchange rate, $z_{l\,t}$ the long term real exchange rate, and $\check{z}_t$ the real exchange rate gap. Figure 7 shows the structure and equation of product gap.



GDP gap= d0"GDP gap t-1"-d1*"real interest rate gap t-1"+d2*"real depreciation gap t-1"*

Figure 7. Structure and equation of GDP gap.

The transmission of interest rates is modeled with the equations:

$$i^p_t = e_0\, i^p_{t-1} + (1- e_0)(i^p_{1\,t} + e_1(\pi^e_{t+k} - \pi^{target}_{t+k}) + e_2\, \hat{y}_t \quad \text{(Eq. 9)}$$

$$\check{i}^p_t = i^p_t - i^p_{1\,t} \quad \text{(Eq. 10)}$$

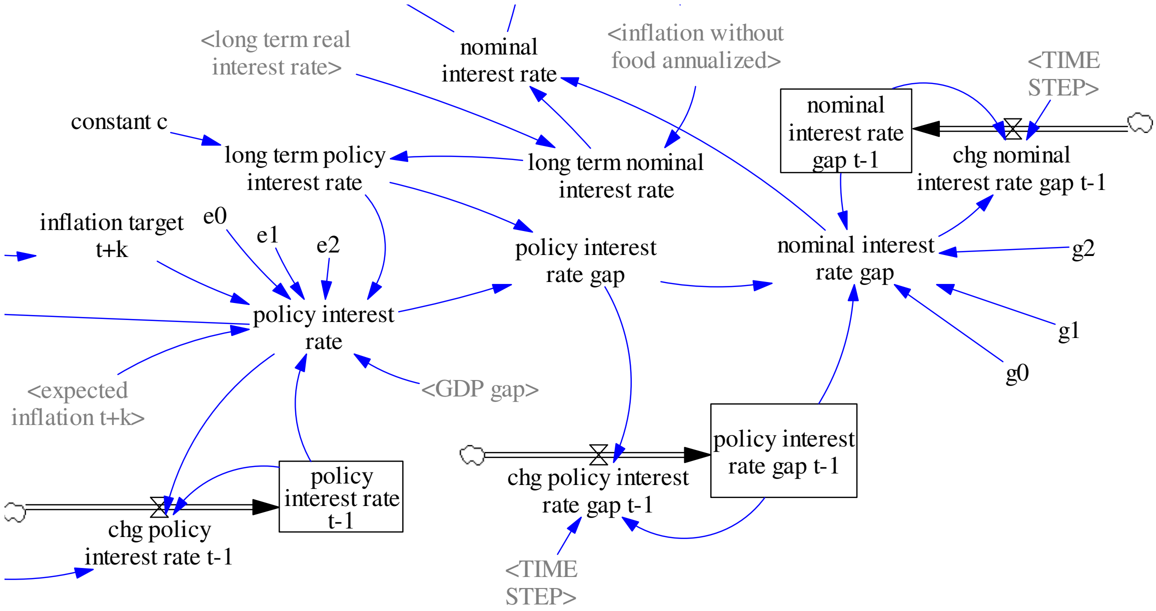
$$\check{i}_t = g_0\, \check{i}_{t-1} + g_1\, \check{i}^p_t + g_2\, \check{i}^p_{t-1} \quad \text{(Eq.11)}$$

$$i^p_{lt} = i_{lt} - c \quad \text{(Eq. 12)}$$

$$i_t = \check{i}_t + i_{lt} \quad \text{(Eq.13)}$$

$$i_{lt} = r_{lt} + \pi^{sa}_{4,t} \quad \text{(Eq. 14)}$$

where i^p_t denotes the policy interest rate, i^p_{lt} the long term policy interest rate, π^e_{t+k} the expected inflation in the period t+k, π^{target}_{t+k} the inflation target in the period t+k (k=1 for the system dynamics model), $\check{i}^p_t$ the policy interest rate gap, $\check{i}_t$ the nominal interest rate gap, i_{lt} the long term nominal interest rate, c a constant, i_t the nominal interest rate, and $\pi^{sa}_{4,t}$ the inflation without food annualized. Figure 8 shows the structure of interest rates transmission and Vensim equations for policy interest rate and nominal interest rate gap.



policy interest rate= e0*"policy interest rate t-1"+(1-e0)*(long term policy interest rate+e1*("expected inflation t+k" -"inflation target t+k")+e2*GDP gap)

Figure 8. Structure of interest rates transmission.

Depreciation and risk premium are determined in the following equations:

$$\delta^e_t = i_t - i^*_t - i^{prem}_t \qquad \text{(Eq. 15)}$$

$$i^{prem}_t = \gamma_0\, i^{prem}_t + (1 - \gamma_0)\, i^{prem}_{lt} \qquad \text{(Eq. 16)}$$

$$\delta^z_t = \delta_{t\,+}\, i^*_t - \pi^{sa}_t \qquad \text{(Eq. 17)}$$

$$\check{z}_t = \check{z}_{t-1} + f_0(\delta^z_t - \delta^z_{lt}) \qquad \text{(Eq. 18)}$$

where δ^e_t denotes the expected nominal depreciation, i^*_t the foreign interest rate, i^{prem}_t the risk premium, i^{prem}_{lt} the long term risk premium, δ^z_t the real exchange rate depreciation, and δ^z_{lt} the long term real exchange rate depreciation. Figure 9 shows the structure of depreciation and risk premium.

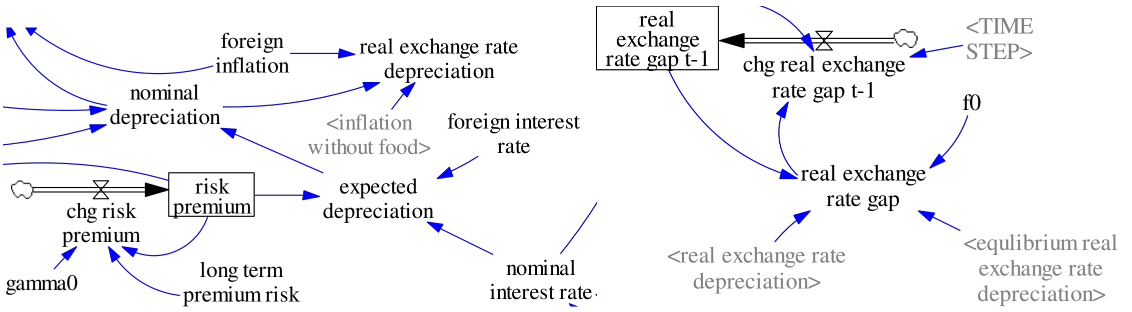


Figure 9. Structure of depreciation and risk premium.

Impulse-response results

Diverse shocks were applied to the system dynamics model and the results compared with those of the MTM. The comparison is made between Vensim custom graphs for the system dynamics model, and graphs of the MTM results, scanned from a document of the Central Bank (Banco de la República, 2003).

Inflation target

A reduction from 3% to 2% in the inflation target was set by means of a STEP function in year 1. Figure 10 shows the comparative results for inflation (total, without food, and food), Figure 11 the results for imports inflation and depreciation (nominal and real), Figure 12 the results for GDP gap and real interest rate gap, and Figure 13 the results for policy and real interest rates.

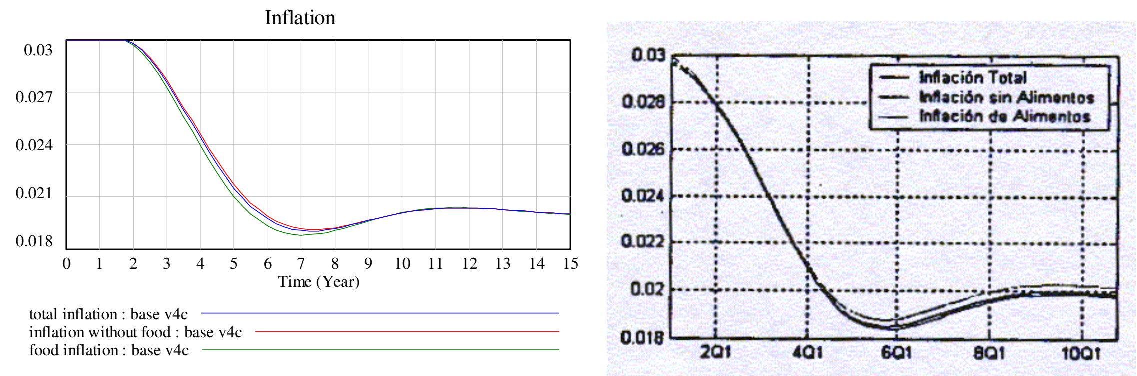


Figure 10. Results for inflation after a shock to inflation target, for the SD model (left) and the MTM (right). Notation of the right-hand graph must be read as year-quarter (for example, 2Q1 must be read as second year-first quarter).

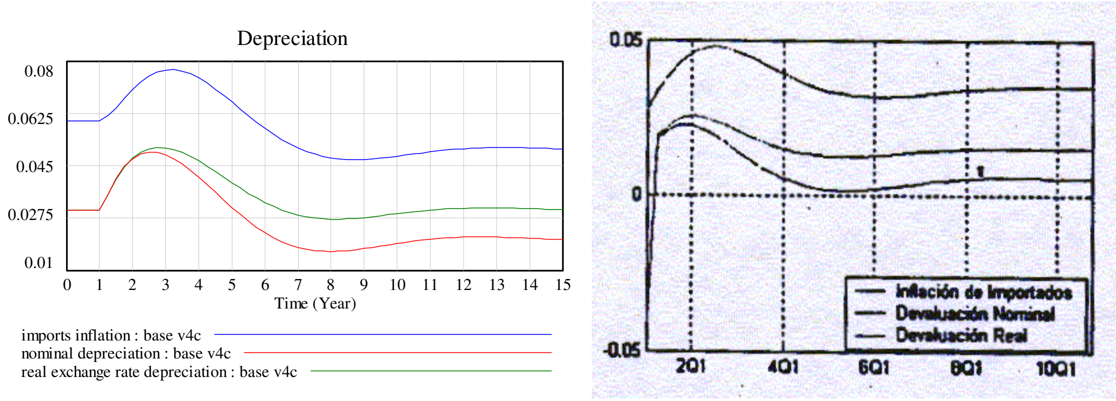


Figure 11. Results for imports inflation and depreciation (nominal and real) after a shock to inflation target, for the SD model (left) and the MTM (right). Notation of the right-hand graph must be read as year-quarter (for example, 2Q1 must be read as second year-first quarter).

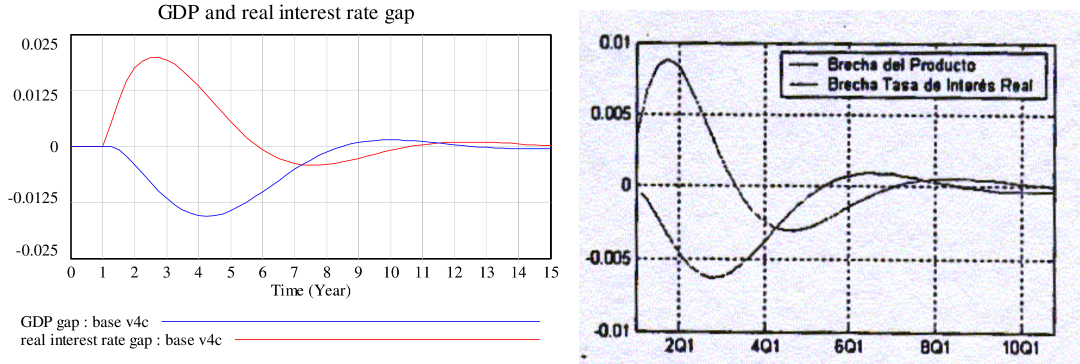


Figure 12. Results for GDP and real interest rate gap after a shock to inflation target, for the SD model (left) and the MTM (right). Notation of the right-hand graph must be read as year-quarter (for example, 2Q1 must be read as second year-first quarter).

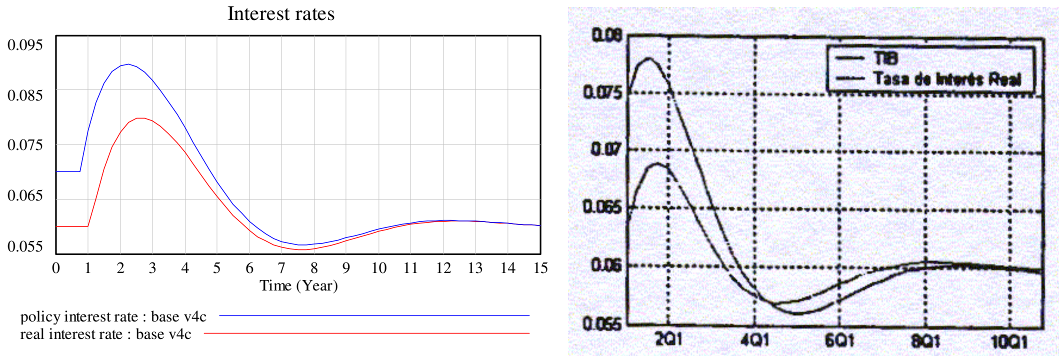


Figure 13. Results for policy and real interest rates after a shock to inflation target, for the SD model (left) and the MTM (right). Notation of the right-hand graph must be read as year-quarter (for example, 2Q1 must be read as second year-first quarter).

Monetary Policy

The policy interest rate was increased 100 basic points during 4 quarters, beginning in year 1, by means of the Vensim RC STEP function. Figure 14 shows the comparative results for inflation (total, without food, and food), Figure 15 the results for imports inflation and depreciation (nominal and real), Figure 16 the results for GDP gap and real interest rate gap, and Figure 17 the results for policy and real interest rates.

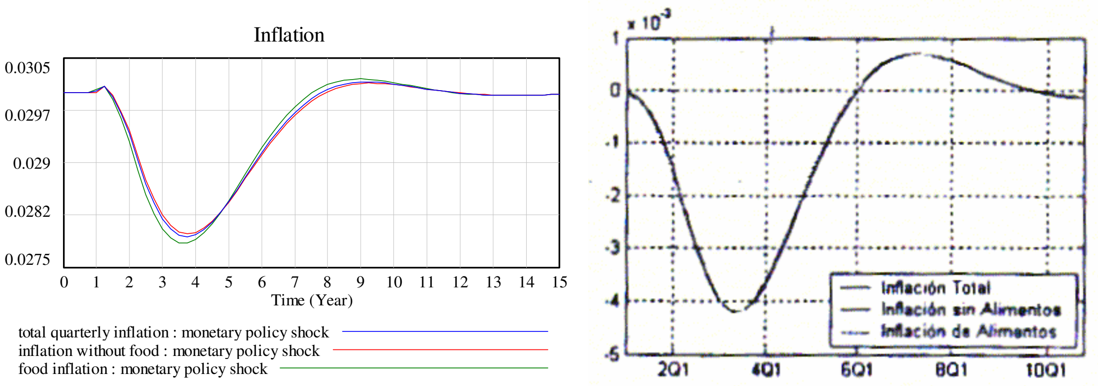


Figure 14. Results for inflation after a shock to monetary policy, for the SD model (left) and the MTM (right). Notation of the right-hand graph must be read as year-quarter (for example, 2Q1 must be read as second year-first quarter).

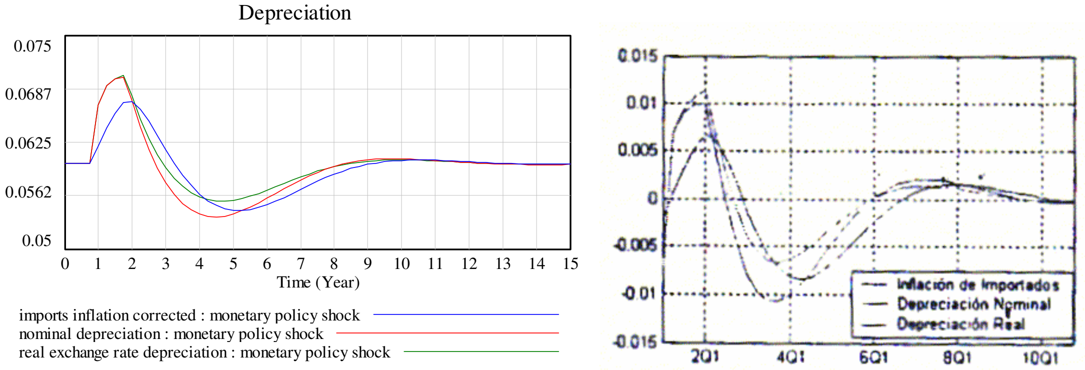


Figure 15. Results for imports inflation and depreciation after a shock to monetary policy, for the SD model (left) and the MTM (right). Notation of the right-hand graph must be read as year-quarter (for example, 2Q1 must be read as second year-first quarter).

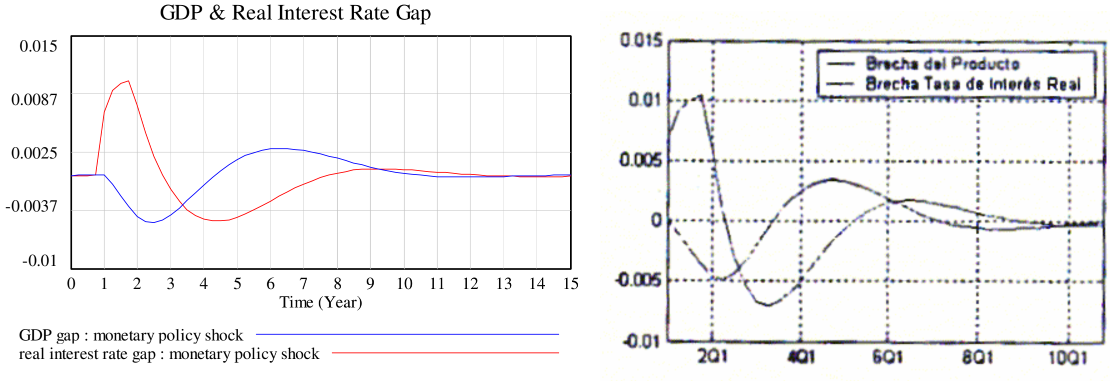


Figure 16. Results for GDP and real interest rate gaps after a shock to monetary policy, for the SD model (left) and the MTM (right). Notation of the right-hand graph must be read as year-quarter (for example, 2Q1 must be read as second year-first quarter).

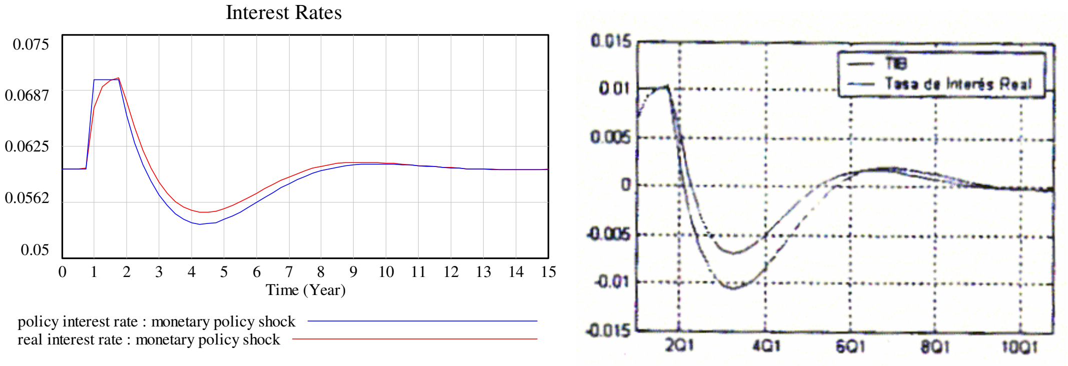


Figure 17. Results for policy and real interest rates after a shock to monetary policy, for the SD model (left) and the MTM (right). Notation of the right-hand graph must be read as year-quarter (for example, 2Q1 must be read as second year-first quarter).

Food Supply

A pulse increment of 0.25 was made to food inflation, beginning in year 1. Figure 18 shows the comparative results for inflation.

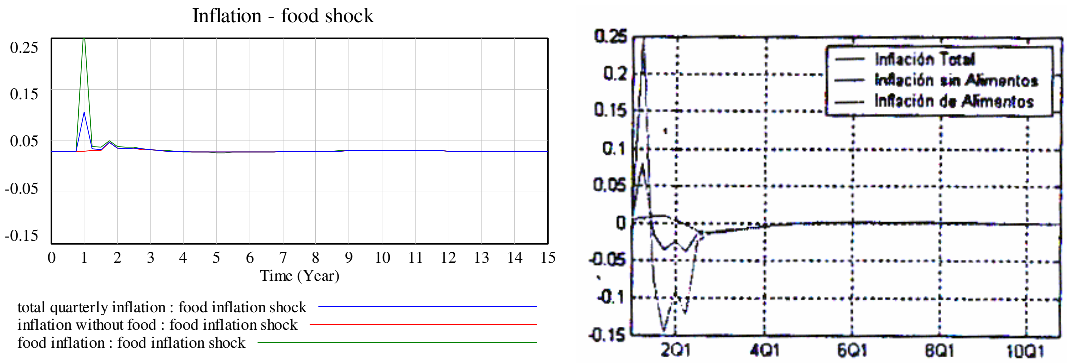


Figure 18. Results for inflation after a shock to food supply, for the SD model (left) and the MTM (right). Notation of the right-hand graph must be read as year-quarter (for example, 2Q1 must be read as second year-first quarter).

Nominal Depreciation

The risk premium was increased 100 basic points during 4 quarters, beginning in year 1, by means of the Vensim RC STEP function. Figure 19 shows the comparative results for inflation.

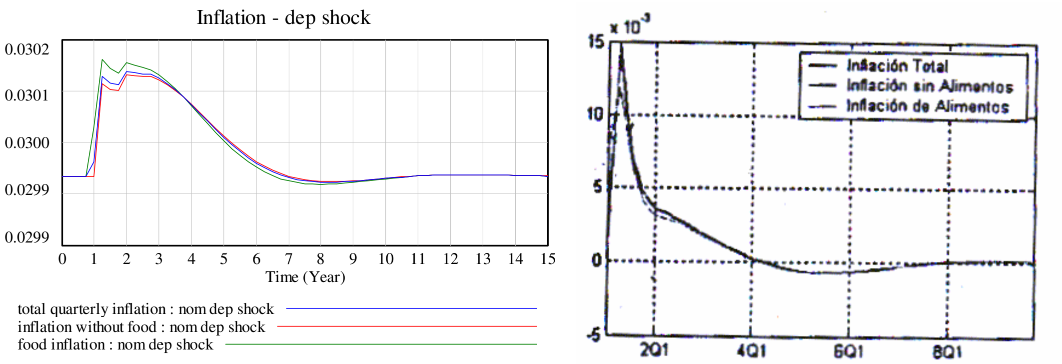


Figure 19. Results for inflation after a shock to nominal depreciation, for the SD model (left) and the MTM (right). Notation of the right-hand graph must be read as year-quarter (for example, 2Q1 must be read as second year-first quarter).

Risk Premium

A pulse increase of 0.01 was made to the risk premium, beginning in year 1. Figure 20 shows the comparative results for inflation.

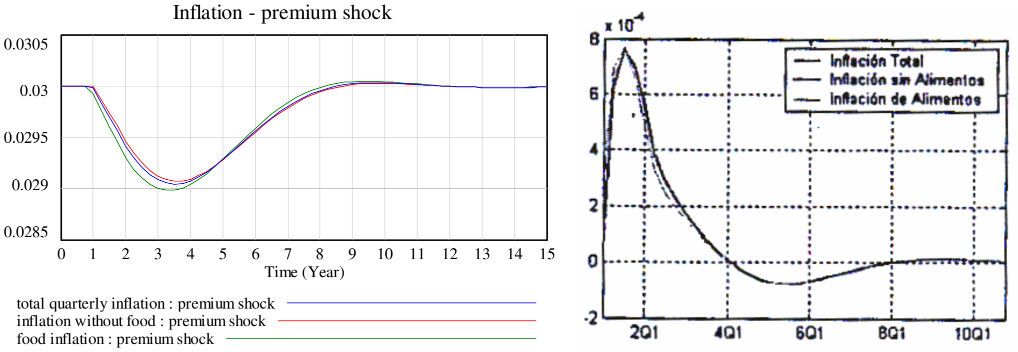


Figure 20. Results for inflation after a shock to risk premium, for the SD model (left) and the MTM (right). Notation of the right-hand graph must be read as year-quarter (for example, 2Q1 must be read as second year-first quarter).

Conclusions

The pattern of performance the variables in the SD model, for shocks to the inflation target and monetary policy, is quite similar to that of the MTM, but, looking carefully at the MTM results in Figures 11 and 15, a downward jump of depreciation in year 1 can be noticed. This jump is a consequence of the solution of the rational expectations component of the MTM. This performance makes a noticeable difference between the SD and MTM results when shocks are applied to food supply, nominal depreciation, and risk premium. In spite of the differences, the TREND function seems to be an appropriate adaptive expectations approach to this model.

Based on the results obtained, the Bank considers that system dynamics models represent a potentially useful tool for policy design and will continue supporting the research that has originated this document.

In the specific case of the MTM, the Bank considers the adaptive expectations approach as interesting but too deterministic, and will support a rational expectations approach to the model, from the system dynamics perspective.

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