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Manufacturing Supply Chain Adaptive Management under Changing Demand Conditions

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Abstract

The paper presents a system dynamics approach to analysis of generic manufacturing supply chain model and considers the problem of determination of adaptive effective strategy on production, supply rates and price according to inventory levels and the market situation through a demand curve estimation. The special focus lies on the investigation of the product price influences on demand and its optimal regulation. The simple manufacturing supply chain, which includes manufacturing unit, storage and distribution unit without having a direct customer orders is considered. The model for demand curve adaptive estimation and forecast by ordinary least squares and exponential smoothing methods is proposed. The problem of optimal management for price, distribution rate and production start rate under invariant and shifting demand curve is investigated. The obtained results can be used in research to understand and improve managerial decision making in real complex dynamic systems.

Keywords: system dynamics, adaptive supply chain management, demand curve forecast

Introduction

System dynamics simulation approach to analysis and prediction of complex socio-economic processes became very popular and gained wide dissemination. At the same time, the primary efforts of the scientists are directed to problems of model design, analysis of system dynamics, stability and forecast. Publications investigating the model optimisation problems and optimal decision making are relatively rare. The aim of the paper is to present a system dynamics approach to analysis of generic manufacturing supply chain model and consider the problem of adaptive optimal strategy on production, supply rates and price according to inventory levels and the market situation through a demand curve estimation. While the obtained results can not be directly implemented in practice because of model simplifications, they can be used in researches, directed to understanding and improvement of managerial decision making in real complex dynamic systems.

First, a simple manufacturing supply chain with a feed-back, build on the basis of Sterman's manufacturing firm model [1] is considered. The model includes the process of goods production, transportation to distribution inventory, storage and waste in inventory and distribution to the customers.

On our opinion, the model, considered in the first part of the paper, has a drawback, because it does not pay a sufficient attention to demand curve, the price influences to distribution rate and its regulation. The second part describes the model of distribution unit, which expands the previous model and takes into account the price influences to distribution rate through demand curve. Also, the solution of demand curve estimation problem is provided using the model of linear demand curve estimation and forecast by ordinary least squares and exponential smoothing methods.

Parts 3 and 4 are devoted to determination of optimal price and production rate, maximizing the clear profit rate. The case of invariant market situation, when the optimal firm state is stationary, is considered in the third part. Part 4 discusses the case of variable market situation, when the transitional process from the current state to the optimal one should be optimised. Finally, in the conclusion it is mentioned, that the obtained results can be applied to any linear supply chain models, that can be formulised as a systems of linear differential equations.

1. Manufacturing Supply Chain Model (Model 1)

Consider the simple manufacturing supply chain model (see Fig.1). The excellent investigation of such models is given by Sterman [1]. One may also refer to the Forrester [2], Goodmen [3], Shiryaev[4]. The model includes manufacturing unit, storage and distribution unit without having a direct customer orders. Customers buy products from the store with the distribution rate, depending from the distribution price. Goods at inventory are perishable and waste rate is inversely depended from storage costs. Also, it is assumed, that enough capacity and money is available, and production rate is limited by market requirements only. The adaptation to the changing distribution rate is due to its prediction by exponential smoothing. The aim of a firm is the maximization of the clear profit. Appendix A presents the formulation of the model. Figure 2 shows model dynamics example.

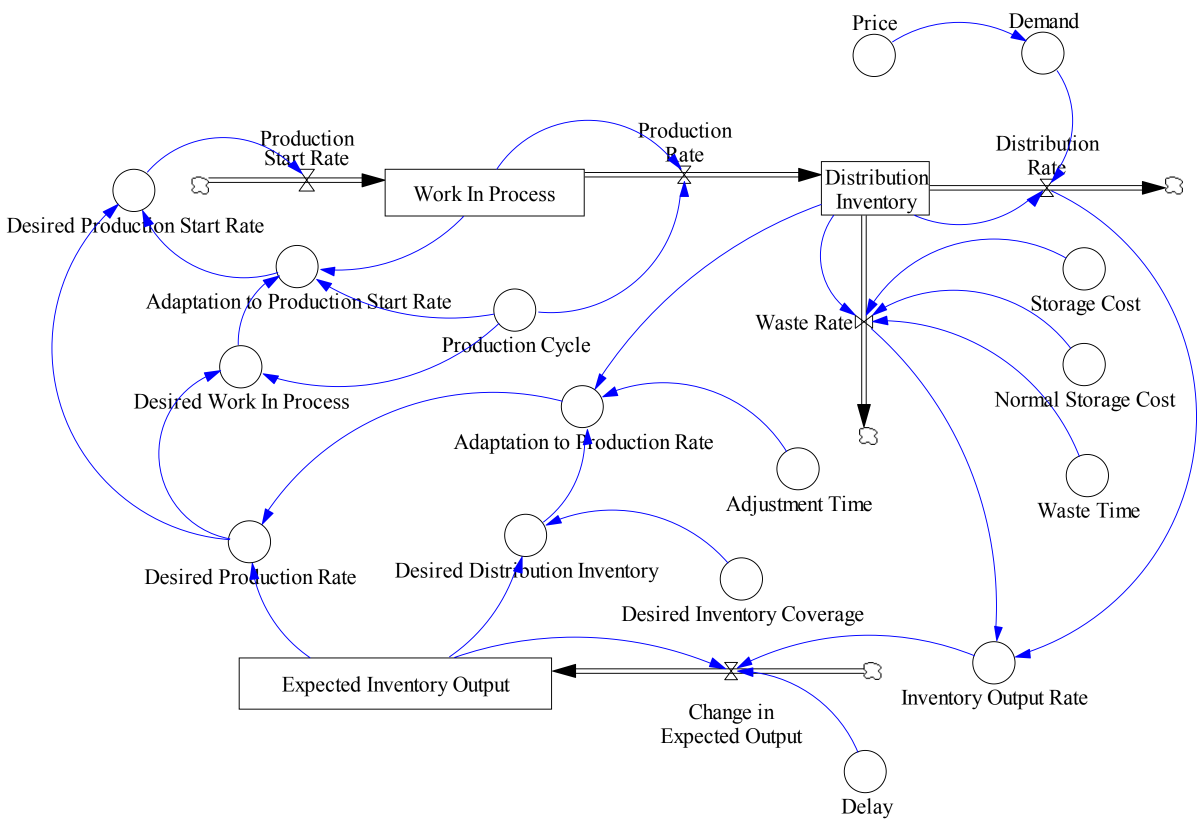

Fig.1. Manufacturing supply chain model with feedback

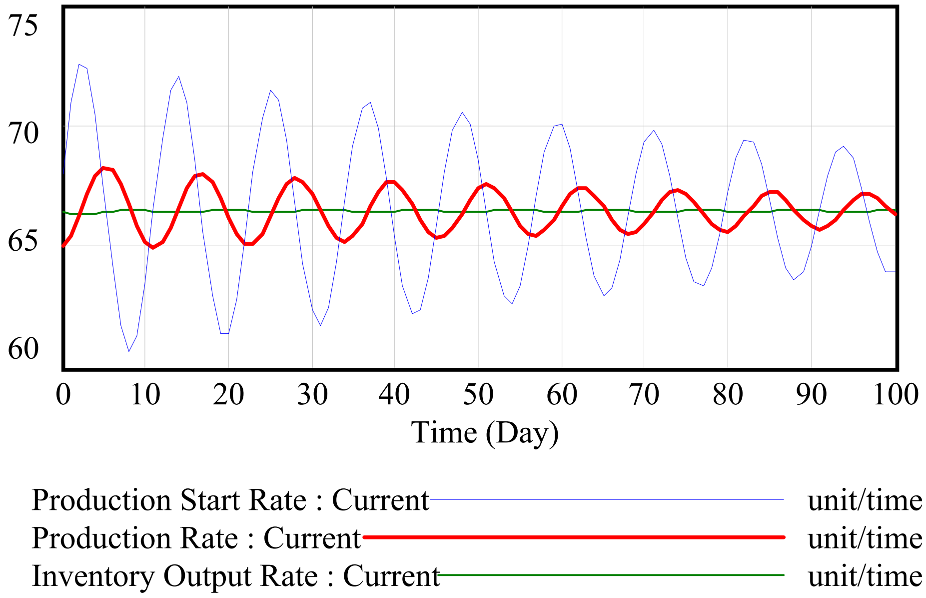

Fig.2. Model 1 dynamics

2. Distribution Unit and Demand Forecasting (Model 2)

On our opinion, the Model 1, considered in the previous part of the paper, has a drawback, because it does not pay a sufficient attention to demand curve, the price influences to distribution rate and its regulation. Consider, that a firm is able to change the distribution price and, consequently, to affect a distribution rate through a demand curve. The model of distribution unit (Fig.3) is added to the Model 1. New model is named as Model 2.

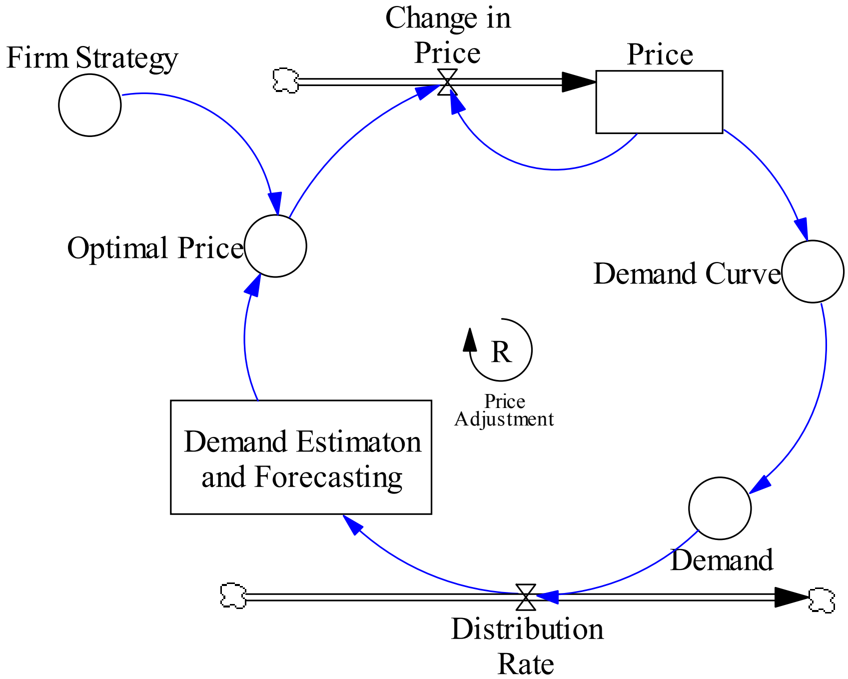

Fig.3. Distribution unit model

Consider the problem of demand curve adaptive forecast using the information, which comes during the process of manufacturing and distribution. The estimation of the demand curve in the Model 2 is performed in the Demand Estimation and Forecasting Block. Fig. 4 shows the model for the forecast of linear demand

$$Demand = b1 - b2 * Price$$

by ordinary least squares and exponential smoothing methods. In the Appendix B the mathematical formulation and solution for the problem of unknown demand estimation using the information on previous sales is considered. The model of Demand Estimation and Forecasting Block, constructed in PowerSim environment, is applied to the paper.

The point of the estimation and forecasting technique is the following. The coefficients b1 and b2 are calculated by ordinary least squares on a sample that includes results of previous sales. The observations are transformed by exponential smoothing to increase the importance of most recent information. This approach allows to follow a dynamics of the demand curve.

The obtained demand estimation should be used for analysis of market situation and determination of price strategy, which is optimal in the sense of firm development criterion. As a criterion, the maximal clear profit rate condition is chose. In the case of linear demand this lead to the problem of quadratic function minimisation (the detailed price control problem is considered below in the parts 3 and 4). Fig. 5-7 show the results for demand curve forecast and price management with the aim of clear profit rate maximisation under instantaneous 10% - decrease of demand from "Initial demand" to "Final demand" at the moment TIME=10.

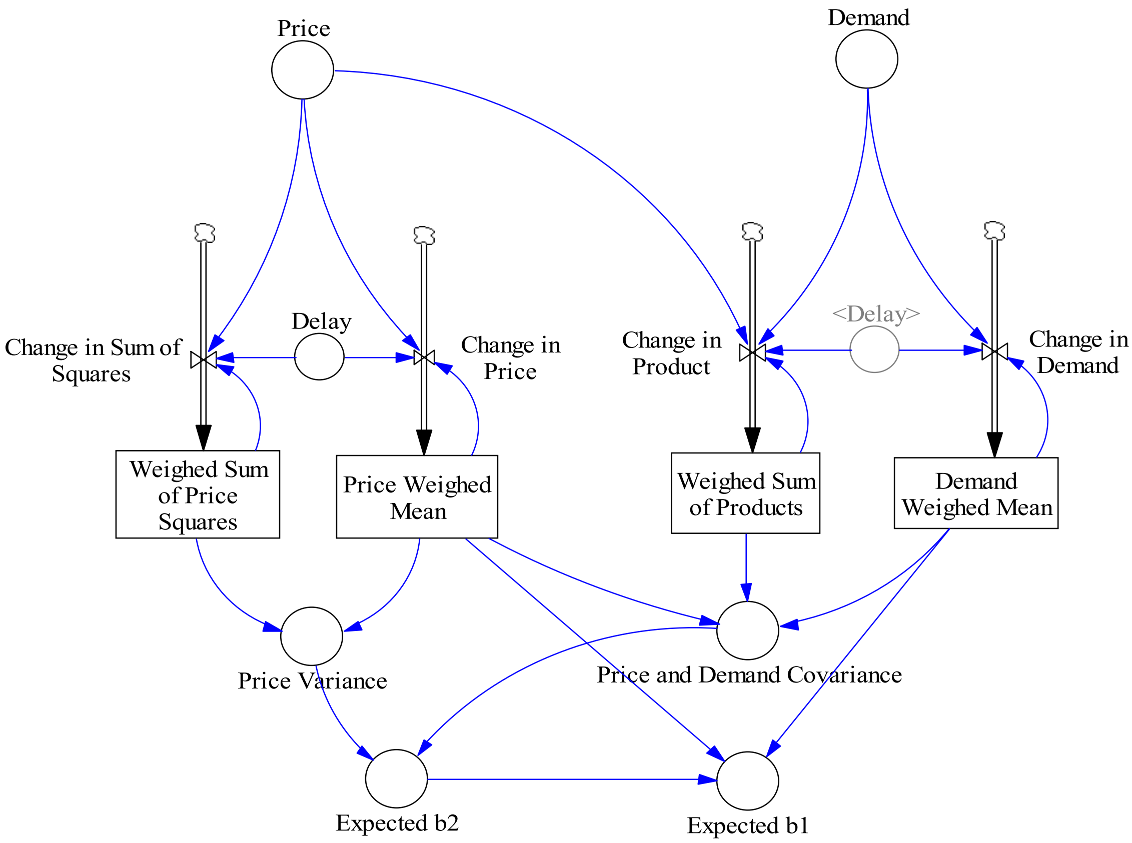

Fig.4. Demand estimation and forecasting block

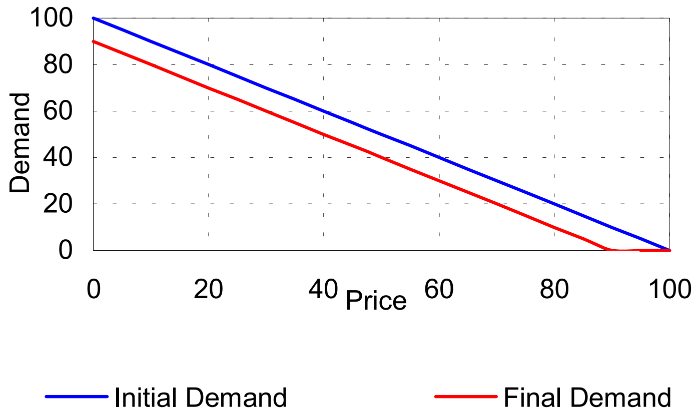

Fig.5. Initial and final demand curve

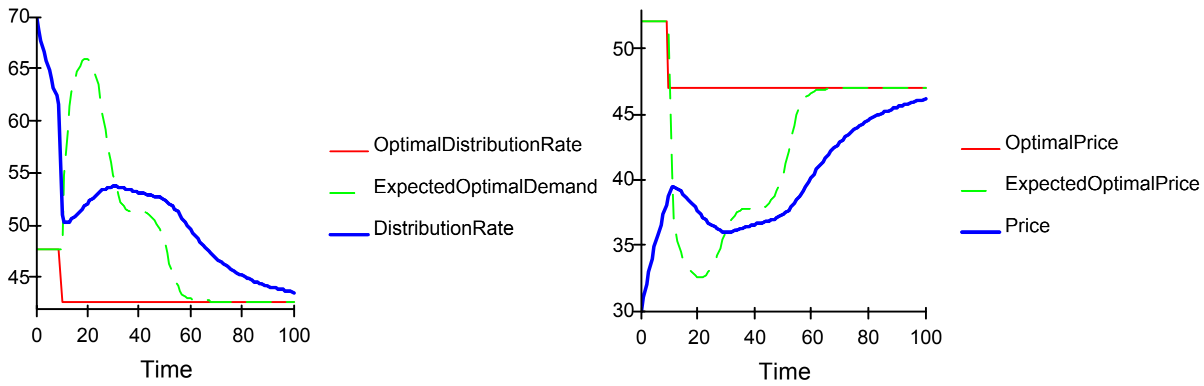

Fig.6,7. Results of demand forecast and price management
after instantaneous 10% - decrease of demand

The described approach may be used for forecasting of nonlinear demand curve with minimal modifications.

3. Management under Invariant Demand

The next step after demand curve estimation is the problem of management for price and material flows between manufacturing supply chain units. In the Model 2 material flows are defined with the help of expected flow values. In turn, these expected values are defined by other model components. Fig. 2 shows, that under stable market situation, in spite of oscillation, rates of material flows converge to fixed values. One can obtain these values through the analysis of firm dynamics. On the other hand, under invariant market situation it is possible to calculate directly the optimal firm policy, including flows and levels, maximising clear profit flow.

First, let the market situation, including demand curve and resource price, is invariant and determined. It is obvious, that the optimal firm state is equilibrium and manufacturing chain units act invariantly. So flows and levels do not change in time in this case. Under the clear profit flow maximisation criterion one can find optimal price and production rates providing the most effective firm activity in the current market situation (Appendix C provides the mathematical definition and solution of this problem). These optimal price and rates in case of invariant market ensure the firm activity that maximises the selected criterion.

4. Management under Changing Demand

Consider the situation, when demand curve has changed under the influence of unknown factors. Suppose that new demand curve parameters are determined either by demand estimation and forecasting model considered below or by other formal or informal methods. As noted, with the help of observation on previous sales one can find firm parameters, optimal in the sense of chosen criterion, for example, parameters maximising clear profit flow. The new problem – is the optimisation of transitional process from the current state to the optimal one by price and production starts management.

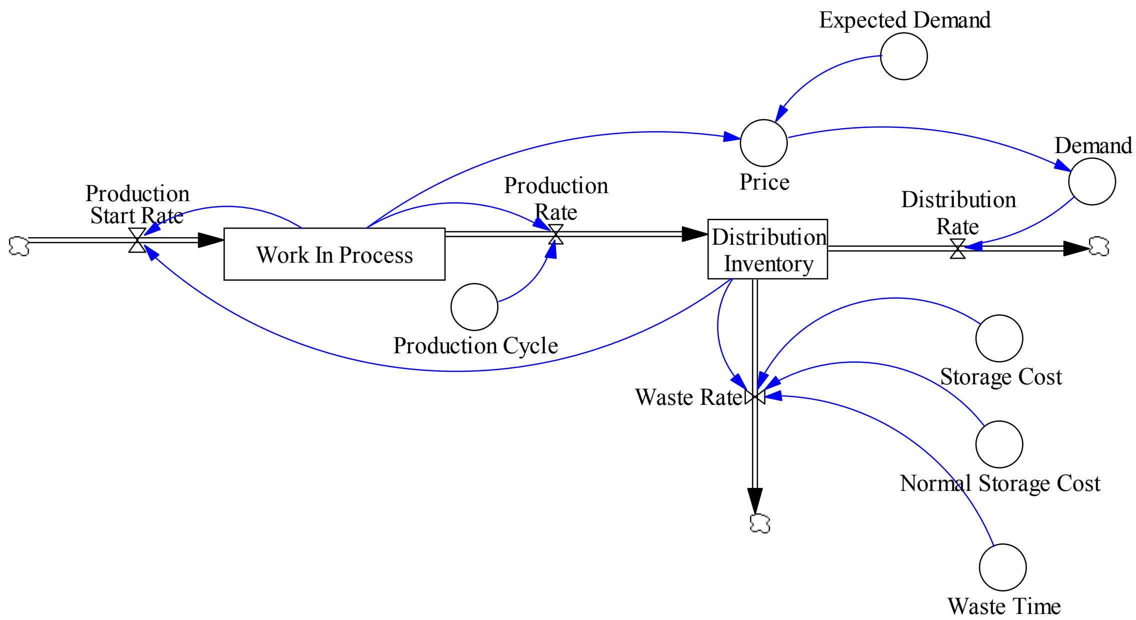

Fig.8. Manufacturing supply chain optimal control model

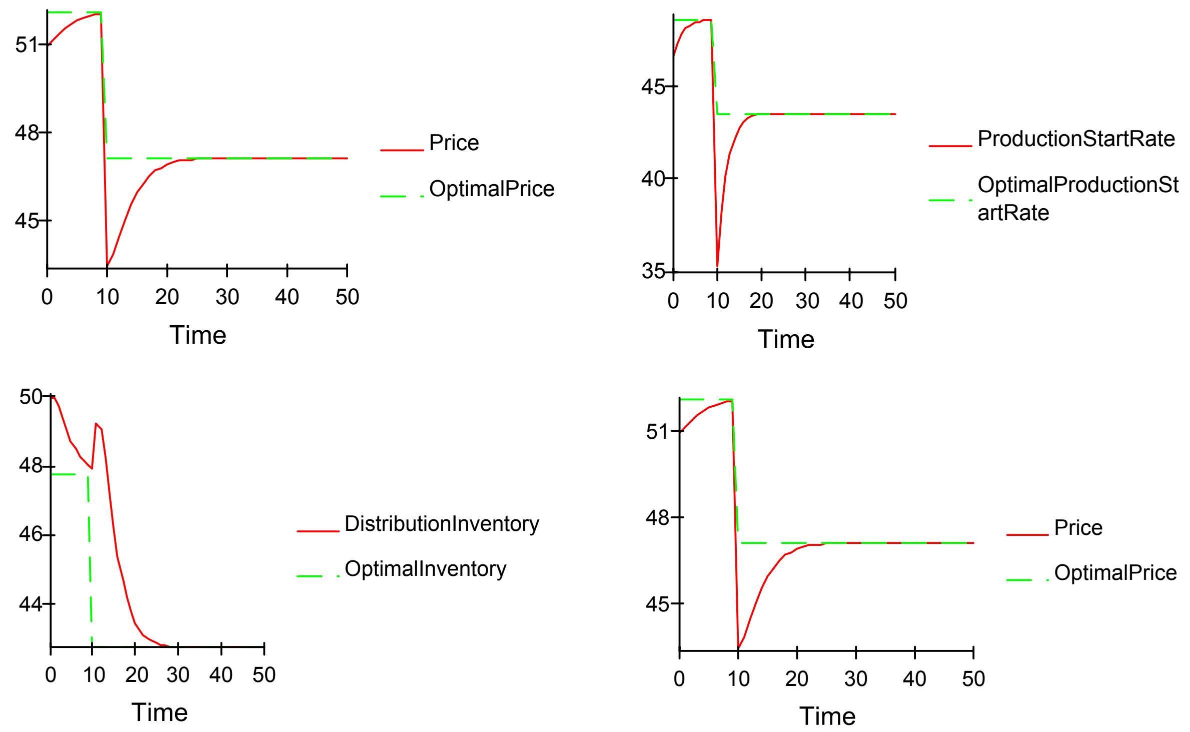

Fig.9. Results of price and production start rate optimal management after instantaneous 10% - decrease of demand

It is found, that optimal control theory allows to determine the dependence between the current firm state (inventories and production levels and demand curve) and optimal controllable rates (production start rate and distribution rate, determined by distribution price) (Appendix D provides the mathematical formulation and solution of this problem). As a result, the amount of units in the model has decreased because of simplifying and optimisation of algorithm for model adaptation to the external market changes (see Fig.8). Fig.9 shows the results of model management under instantaneous 10% - decrease of demand.

Conclusion

The paper considers the simple generic model of manufacturing supply chain and tries to present the approach, which can improve managerial decision making through market estimation and the optimal price and production strategy determination. While the obtained results can not be directly implemented in practice because of model simplification, they can be used in research to understand and improve managerial decision making in real complex dynamic systems.

The problem of adaptive estimation of the demand curve using previous sales information by econometric methods is discussed and the model of distribution unit with demand estimation block is presented. Results of numerical experiment are shown.

On the basis of demand curve estimation and forecast the problem of optimal price and production strategy is considered. It is shown, that manufacturing supply chain model can be mathematically defined as linear differential optimal control problem with quadratic criterion. Further, the optimal price and production strategy, which depends only from the current state of the chain, is found.

Finally, it is should be mentioned, that the obtained results can be applied to any linear supply chain models, that can be formulised as a systems of linear differential equations. Appendix E provides the mathematical definition and solution of generic N-dimensional model.

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Appendix A

Manufacturing Supply Chain Model Formulation

(01) Adaptation to Production Rate=
(Desired Distribution Inventory-Distribution Inventory)/Adjustment Time
Units: unit/time

(02) Adaptation to Production Start Rate=
(Desired Work In Process-Work In Process)/Production Cycle
Units: unit/time

(03) Adjustment Time=1
Units: time

(04) Change in Expected Output=
-(Expected Inventory Output-Inventory Output Rate)/Delay
Units: (unit/time)/time

(05) Delay=10
Units: time

(06) Demand=100-1*Price
Units: unit/time

(07) Desired Distribution Inventory=
Expected Inventory Output*Desired Inventory Coverage
Units: unit

(08) Desired Inventory Coverage=1
Units: time

(09) Desired Production Rate=Adaptation to Production Rate+Expected Inventory Output
Units: unit/time

(10) Desired Production Start Rate=
Desired Production Rate+Adaptation to Production Start Rate
Units: unit/time

(11) Desired Work In Process=Desired Production Rate*Production Cycle
Units: unit

(12) Distribution Inventory= INTEG (Production Rate-Distribution Rate-Waste Rate,70)
Units: unit

(13) Distribution Rate=MAX(Demand,Distribution Inventory)
Units: unit/time

(14) Expected Inventory Output= INTEG(Change in Expected Output,65)
Units: unit/time

(15) FINAL TIME = 100
Units: Day
The final time for the simulation.

(16) INITIAL TIME = 0
Units: Day
The initial time for the simulation.

(17) Inventory Output Rate=Distribution Rate+Waste Rate
Units: unit/time

(18) Normal Storage Cost=0.1
Units: $/(unit*time)

(19) Price=35
Units: $/unit

(20) Production Cycle=7
Units: time

(21) Production Rate=Work In Process/Production Cycle
Units: unit/time

(22) Production Start Rate=Desired Production Start Rate
Units: unit/time

(23) SAVEPER = 1
Units: Day
The frequency with which output is stored.

(24) Storage Cost=0.5
Units: $/(unit*time)

(25) TIME STEP = 1
Units: Day
The time step for the simulation.

(26) Waste Rate=
Distribution Inventory/Waste Time*(Normal Storage Cost/Storage Cost)
Units: unit/time

(27) Waste Time=10
Units: time

(28) Work In Process= INTEG (+Production Start Rate-Production Rate,65*7)
Units: unit

Appendix B

The Problem for Estimation of Unknown Demand Using Previous Distribution Information. Mathematical Definition and Solution

Suppose that demand curve is

$$D = b_1 + b_2 P, \tag{B.1}$$

where D –Distribution Rate; P –Price; b_1, b_2 – undefined parameters, $b_1>0$, $b_2<0$.

Consider the problem of parameters b_1, b_2 estimation using the previous sale information. Let this information include n pares $(P_i, D_i), i = 1,..n$, where P_i - the price and D_i - corresponding distribution rate at the moment i. According to the ordinary least squares one has estimates of b_1, b_2

$$\hat{b}_2 = \frac{Cov(P,D)}{Var(P)}, \tag{B.2}$$

$$\hat{b}_1 = \overline{D} - \hat{b}_2 \overline{P}, \tag{B.3}$$

where $Cov(P,D) = \frac{1}{n}\sum_{i=1}^{n}(P_i - \overline{P})(D_i - \overline{D})$ – covariance P and D; $Var(P,D) = \frac{1}{n}\sum_{i=1}^{n}(P_i - \overline{P})^2$ – variance P; $\overline{P}, \overline{D}$ - sample means.

In the Model 2, observations are transformed by exponential smoothing in order to increase the importance of most recent information and then estimations $\hat{b}_1, \hat{b}_2$ of parameters b_1, b_2 are found through (B.2, B.3). The model of Demand Estimation and Forecasting Block is enclosed to the paper.

Appendix C

The Problem of the Manufacturing Supply Chain Optimization in Case of Invariant Demand Curve. The Mathematical Definition and Solution

Mathematical model of the manufacturing supply chain is defined by the system of differential equations:

Distribution Inventory Derivative

$$\dot{I} = -D(P) - \frac{k_1}{k_2} I + \frac{1}{k_3} WIP, \tag{C.1}$$

where I - goods amount in distribution inventory; $D(P)$ - demand curve (dependence of distribution rate from price); $\frac{k_1}{k_2} I$ - goods waste rate, $k_1 = \frac{NormalStorageCost}{WasteTime}$, k_2 - Storage Cost; WIP - work in progress; k_3 - production cycle. Work in progress derivative

$$\dot{WIP} = PSR - \frac{1}{k_3} WIP, \tag{C.2}$$

where PSR - Production Start Rate.

Receipts:

$$\dot{R} = P \cdot D(P). \tag{C.3}$$

Costs:

$$\dot{C} = k_4 D(P) + k_2 I + k_5 PSR, \tag{C.4}$$

where k_4 - distribution cost; k_5 - production cost.

Optimality criterion is the maximum of clear profit

$$Q = \alpha R(T) + \beta C(T) \to \max_{P, PSR}, \tag{C.5}$$

where α, β include VAT and profit tax; T - time period.

In case of invariant market state, i.e. demand curve $D=D(P)$ does not change through the time. It is obvious, that the optimal state of the system (C.1)-(C.5) is stationary and, consequently, derivatives should be equal to zero. So one has

$$\alpha P \cdot D(P) + \beta k_4 D(P) + k_5 PSR \to \max_{P}. \tag{C.6}$$

It is clear, that the optimal policy in the sense of criterion (C.6) is the Just In Time technology and, hence, I should be equal to zero. However, in practice it is not always possible to work without any stocks. So let the optimal I is

$$I^* = k_6 D^*(P^*), \tag{C.7}$$

where k_6 - Desired Inventory Coverage , e.g. $k_6=1$.

Then one has the optimal Work In Progress level

$$WIP^* = k_3\left(k_6 D^*\left(P^*\right) + \frac{k_1}{k_2} I^*\right) \tag{C.8}$$

and optimal Production Start Rate is

$$PSR^* = \frac{1}{k_3} WIP^* . \tag{C.9}$$

Supposing the demand curve to be linear

$$D(P) = b_1 + b_2 P , \tag{C.10}$$

substituting (C.7) in (C.6) and differentiating by *P* one obtains optimal price value

$$P^* = -\frac{b_1 + \beta k_4 b_2 + \beta k_2\left(b2 + \frac{k_1}{k_2} k_6\right)}{2\alpha b_2} . \tag{C.11}$$

Substituting (C.11) into (C.10) one has the expected optimal distribution rate

$$D^* = b_1 - \frac{\alpha b_1 + \beta k_4 b_2 + \beta k_2\left(b2 + \frac{k_1}{k_2} k_6\right)}{2\alpha} P^* . \tag{C.12}$$

So, in case of estimated and invariant demand, the optimal manufacturing supply chain state is stationary and its parameters are P^*, I^*, WIP^*, PSR^*.

Appendix D

The Problem of Manufacturing Supply Chain Optimisation in Case of Changed Demand Curve. The Mathematical Definition and Solution

Suppose, that dynamics of manufacturing supply chain is defined by the system of differential equations (C.1) and (C.2) (see Appendix C). The problem of optimal firm state (P^*, I^*, PSR^*, WIP^*) determination under known demand curve is considered in Appendix C. Now consider the problem of optimal firm policy starting from the current state (P^0, I^0, PSR^0, WIP^0) to the effective one (P^*, I^*, PSR^*, WIP^*) through the management by price P and Production Start Rate PSR under linear demand curve

$$D = b_1 + b_2 P. \tag{D.1}$$

As a criterion of transitional process effectiveness consider

$$Q = \int_0^T \left[c_1 \left(I - I^* \right)^2 + c_2 \left(WIP - WIP^* \right)^2 + c_3 \left(P - P^* \right)^2 + c_4 \left(PSR - PSR^* \right)^2 \right] dt \to \min, \tag{D.2}$$

where c_i, $i=1,..,4$ - coefficients to be chosen by the firm.

The problem (C.1)-(C.2),(D.1)-(D.2) is linear problem with quadratic criterion and may be solved by optimal control theory methods according to Pontryagin's maximum principle:

Euler equations

$$\dot{p}_1 = p_1 \frac{k_1}{k_2} + 2c_1 \left(I - I^* \right); \tag{D.3}$$

$$\dot{p}_2 = -p_1 \frac{1}{k_3} + p_2 \frac{1}{k_3} + 2c_2 \left(WIP - WIP^* \right), \tag{D.4}$$

where p_1, p_2 - Lagrange multipliers,

control stationarity conditions

$$p_1 b_2 + 2c_3 \left(P - P^* \right) = 0; \tag{D.5}$$

$$p_3 + 2c_4 \left(PSR - PSR^* \right) = 0. \tag{D.6}$$

One can find the solution of the problem (C.1)-(C.2),(D.1)-(D.6) searching the Lagrange multipliers p_1, p_2 in the form

$$p_1 = f_1^1 I + f_2^1 PSR + f_3^1; \tag{D.7}$$

$$p_2 = f_1^2 I + f_2^2 PSR + f_3^2, \tag{D.8}$$

where unknown f_i^j shoul be defined. Unfortunately, analytical solution of the problem (C.1)-(C.2),(D.1)-(D.8) is quite intricate. In particular, under firm parameters from Table 1 the optimal control is

$$P = -0.977 \cdot \mathrm{I} - 0.0693 \cdot \mathrm{WIP} + 114.069; \tag{D.9}$$

$$PSR = -0.0693 \cdot \mathrm{I} - .874 * \mathrm{WIP} + 373.490 . \qquad \text{(D.10)}$$

Table 1. Testing Firm Parameters

b_1	b_2	k_1	k_2	k_3	c_1	c_2	c_3	c_4
90	*-1*	*0.01*	*0.5*	*7*	*1*	*1*	*1*	*1*

Appendix E

The Problem of Large Scale Model Optimization in Case of Changed Demand Curve. The Mathematical Definition

Consider the problem of optimal management for large scale manufacturing supply chain in case of changed demand curve. One can see, that mathematical definition of the model is the system of linear differential equations

$$\dot{X} = AX + BU + C, \qquad \text{(E.1)}$$

where X - vector , that consists of levels of chain units; U - controllable rates of material flows between chain units; A, B - known matrixes, C - known vector.

In fact, for the Model 2 one has

$$X' = [I \;\; WIP];\; U' = [P \;\; PSR];\; A = \begin{bmatrix} -\frac{k_1}{k_2} & \frac{1}{k_3} \\ 0 & -\frac{1}{k_3} \end{bmatrix};\; B = \begin{bmatrix} -b_2 & 0 \\ 0 & 1 \end{bmatrix};\; C' = [-b_1 \;\; 0].$$

Matrix form of quadratic criterion (D.2) is

$$Q = \int_0^T (X' R_1 X + U' R_2 U) dt \to \min_U . \qquad \text{(E.2)}$$

The increase in chain units quantity without changing units structure implies the extension of the problem (E.1)-(E.2) dimension only. At the same time, methods of problem solution remain the same: the problem (E.1)-(E.2) has the linear structure with quadratic criterion and in general form can be solved by optimal control theory methods.