

## The Dynamics of Transportation Systems

Dr. J. A. Charlesworth  
University of Bradford Management Centre

### ABSTRACT

The techniques currently used for the management of urban road transportation systems are briefly reviewed, and the extent to which they take account of the dynamics of the system examined. Recent work on the development of mathematical models of urban traffic systems is described and the applicability of the models to real-life traffic systems explored. In particular, the ability of these models to reflect temporal as opposed to spacial properties of the system is examined, as well as their ability to assist in the formulation of strategies for system control. The role that system dynamics might play in overcoming some of the problems encountered is then discussed.

### INTRODUCTION

The increase in the number of vehicles on the road in the last few decades has necessarily led to problems of lack of capacity both on roads connecting towns and in particular on urban streets. The effects of this lack of capacity have been poor access to the outlying regions of some countries with consequent depressing effects on their economy, and in the urban context a deterioration in the environment. In urban areas, these environmental problems include, for example, safety both of drivers and pedestrians, pollution of the atmosphere, damage to the structure of buildings alongside busy roads, low levels of service in public transport, and lack of access for emergency services such as fire and ambulance services.

Many of the problems associated with large volumes of through-traffic have been solved by the construction of high-speed roads connecting and bypassing major conurbations, firstly in the United States and later in European countries and Japan. However, increasing affluence has generated a large amount of intra-urban traffic in the form of personal transport by car at the expense of public transport. In the U.K., this trend led to the formulation of grandiose schemes of urban renewal involving the investment of large sums of money for the provision of urban motorways or expressways to provide easy access from residential areas to the business districts. Such schemes envisaged a further increase in the use of personal transport with steady growth in the urban economy. The downturn in the world economy, particularly as a result of the rapid increase in fuel prices, has meant that where such schemes have been started, the pace at which additions to the road system have been made has been considerably slowed, and long-term plans have had to be revised. In addition, the upheaval caused to suburban communities by the development of such road systems has led to a shift of opinion about their desirability. The result of these experiences has been to bring about a situation in which increasingly the emphasis has been on making the best use of the resources available. To this end, the traffic engineer has been greatly aided by the increasing availability of relatively cheap and increasingly sophisticated tools for control of urban traffic.

Transportation system management (TSM) plays an important role in the formulation of long-term regional and urban strategic plans. In the urban content, in particular, TSM lays stress on the control of demand by spreading demand over space, time, mode (e.g. bus vs. car) and/or reducing the overall demand level.

System Dynamics has played a role in the formulation of such long-term strategies. (e.g. Ali et al 1980).

As far as the short-term situation is concerned, we are more concerned with what is usually labelled traffic system management, or even simply traffic management (TM). TM comprises operational techniques formulated to cope with a much higher degree of both spacial and temporal detail than TSM. Detailed TM studies can interact with long-term TSM studies with regard to such factors as capacity or demand. For instance, public transport priority measures can effect the network flow pattern and demand levels in the network. System dynamics, has not to the author's knowledge, been applied in this area. The purpose of this paper, therefore, is to examine some of the techniques which are either being used or in the course of development, and the extent to which they might benefit from a system dynamics treatment.

#### TRAFFIC MANAGEMENT TECHNIQUES

The techniques used for traffic management can be broadly split into those used on urban streets and those used on motorways and expressways. The interaction of those techniques in the case of urban motorways and expressways represents a special problem.

##### Traffic Management on Streets

Perhaps the most obvious form of traffic management to the road user is the traffic signal encountered at road junctions. For isolated junctions, the timing of such signals can be set to optimise some objective function, such as overall delay or overall capacity (e.g. Allsop 1971, 1974). Problems naturally occur in the setting of such signals when the junction as a whole is overloaded (Mayne 1979). The signal timings are usually set on an hourly basis to reflect the expected traffic flow on the approaches and are controlled by a control unit sited nearby. Such timings need to be updated occasionally, by on-site measurements of traffic flow. At relatively low levels of traffic flow, additional benefits can be gained by the use of vehicle-actuated signals, whereby the approach of vehicles to the signal is detected by, for instance, inductive loops set in the roadway. Maintenance costs are however higher for the latter - a fact that should be borne in mind when appraising the benefits of such signals.

When signalised junctions are close together, it is important that the timings of the signals are coordinated, as the output profile of traffic from one signalised junction will necessarily determine the timings of the adjacent signalised junction, if for instance overall delay is to be minimised. A technique has been developed for setting the timings of signals to minimise overall network delay for a system of fixed-time signals, (i.e. not vehicle-actuated) namely the TRANSYT programme (Robertson 1969). The programme has been considerably developed over the years and has been used worldwide. Modifications involve the ability to take account of bus-only lanes, priority junctions and fuel consumption. A requirement of this method of coordination is that a set of signals should operate on a common cycle time determined by a particular critical junction. An urban area is split into subareas with different cycle times, the delineation of such areas requiring a certain amount of care (Ferguson 1976). One of the drawbacks of the method is that traffic volumes and turning movements need to be continuously monitored in order that the timings of the signals (which usually change from hour to hour) reflect the pattern of traffic in the network.

One way of overcoming the problem of constant monitoring of traffic flows is to go to a system of vehicle-actuated signals, in which traffic flows are measured by for instance inductive loops in the road. These local variations in traffic flow can adjust local timings but this adjustment must be constrained by the effect on adjacent signals. Several methods have been used in the past but the most recent methods are SCOOT developed at TRRL (Hunt et al 1982) and SCAT in Australia (Sims 1979). With all such systems, the cost of maintenance of vehicle detection equipment must be borne in mind, in weighing up the benefits of such systems.

Apart from controls by signals, there are of course other methods of traffic management in towns which should not be forgotten. Such measures include one-way systems, bus-only lanes, parking controls and "cordon" controls. The latter two measures constitute measures of deterrance whereby drivers are discouraged from using particular parts of the urban traffic system.

#### Traffic Management on Motorways

The traffic management measures usually encountered by motorists on motorways are speed restrictions, displayed on overhead gantries. Such controls may have been initiated as a result of abnormal congestion ahead, perhaps resulting from an incident on the motorway. The way in which such congestion is detected varies from system to system. Detection can consist of visual CCTV surveillance or can be semi-automatic by means of inductive loops set in the roadway. In the latter case, occupancy algorithms are used to decide whether congestion is present, with varying degrees of success (e.g. Collins et al 1979).

In addition to speed controls, there exists the possibility of control of traffic wishing to enter the motorway system. In the U.S., the tendency has been to meter the flow of traffic at on-ramps by means of signals if the section of motorway ahead is congested. Such a system requires sufficient queueing space for vehicles and/or the availability of reasonably uncongested lateral routes which is usually the case in U.S. urban traffic systems. A freeway model FREQ3 (Stock et al 1973) has been developed to calculate expected loadings on a freeway and appropriate controls for the on-ramps.

The above-mentioned freeway model is essentially deterministic in nature: much work, especially in the U.S. has centred on the problems associated with the development of controls which take into account the stochastic and dynamic features of freeway traffic, (see for instance the useful review by Papageorgiou et al 1983).

In extreme circumstances, traffic may be diverted totally from the motorway at off-ramps. Where sufficient capacity exists for such diversions, for example on to a parallel motorway, few problems may accrue. Where the diversions must involve the use of urban streets, as for instance in the case of some urban motorways, then considerable thought should be given to deciding where such diversions should be instituted if at all.

#### Corridor traffic management

Control of on-ramps and off-ramps usually requires the control of a corridor which should embrace all streets likely to be effected by motorway controls. For instance, the corridor control model CORQ1C (Ortleb and May 1974) combines the FREQ3 freeway model with TRANSYT and a linear programming decision model. The model is essentially deterministic and a simulation is necessary to check whether the predicted controls are feasible or desirable. Its applicability to

the more congested cities of Europe must be questioned in that sufficient ramp queueing space or diversion capacity may not be available. In such circumstances, for instance, complete on-ramp closures may be preferred instituted by means of secret signs at on-ramp entries along with relevant diversion route signing. Such a system is envisaged for the Centrally Integrated Traffic Control (CITRAC) system for the City of Glasgow (Mowatt 1984).

Naturally, diversions of traffic from urban motorways on to urban streets requires that the traffic control system on the urban streets should be able to react in an essentially dynamic way to the extra flow of traffic on diversion routes. Such a reaction can be centrally controlled or can be affected by the provision of vehicle-activated signals on the prescribed routes.

#### Summary

The control systems used on city streets can only satisfactorily take account of the dynamics of traffic flow if there is some method of vehicle detection, e.g. by inductive loops, in operation. Such systems can take account of the stochastic nature of traffic flow at levels of flow below congestion, but usually revert to fixed-time operation as congestion increases. Thus even vehicle-actuated signals need to operate to a predetermined fixed-time plan. However, if the induction loop detectors are used in conjunction with a data collection facility, then nominal fixed-time plans can be continuously updated, avoiding the need for repeated on-site surveys of traffic volumes. Such systems are particularly useful however in coping with abnormal loadings such as are produced by diversions of traffic from motorways. Alternatively, if the installation costs and maintenance costs of such systems are considered excessive, then fixed-time operation of signals can be employed in conjunction with a limited number of detectors sited at critical points in the network.

On motorways, a great deal of control can be affected visually by CCTV but clearly inductive loop detectors are of great assistance in automatically detecting congestion, caused perhaps by an incident. Whichever system is used, algorithms need to be developed for the particular area to decide what response should be initiated. For instance, emergency services usually like traffic to be cleared from an incident site, but diversion of this traffic on to ordinary streets may produce excessive congestion and subsequent lack of access for such services. As mentioned above, diversions on to streets can be handled in a dynamic way if suitable detection equipment is available on the street system.

#### URBAN TRAFFIC MODELS

The trend away from the provision of large systems of high-speed roads in urban areas to a situation in which emphasis is placed on making the best use of available resources, has led to the development of relatively sophisticated models of urban traffic networks. Only by the use of such models, can strategies for control of traffic systems be tested cheaply. The complexity of urban traffic systems has necessitated the development of different types of model capable of producing answers to specific problems. For instance, some models lay emphasis on special detail, whilst others attempt to model temporal effects. Most models however tend to regard traffic signals as operating under fixed time control for simplicity. From the foregoing discussion, this is not too unreasonable in that vehicle-actuated signals operate to an underlying fixed-time plan, and it is these underlying fixed-time plans which will determine the broad pattern of network control.

### "Steady-state" Models

It was recognised at an early stage in the testing of systems of coordinated signals (Almond and Lott 1968) that users of the network reacted to particular signal timings. In other words, the traffic flow pattern used as a basis for calculating, for instance, overall delay-minimising settings, is no longer valid some months later. Essentially, the control imposed by the signals initiates feedback in which users of the network alter their routes in an effort to minimise their perceived journey cost (Wardrop 1952), Beckman et al (1956) showed that this equilibrium pattern of flow on a network was equivalent to the minimisation of an objective function  $Z$  given by

$$Z = \sum_{\substack{\text{links} \\ i}} \int_0^{q_i} c_i(q) dq$$

where  $q_i$  is the traffic flow on link  $i$  of the network, and  $c_i(q)$  is the perceived cost to an average user of using link  $i$  at flow level  $q$ . The function  $c_i(q)$  is required to be monotonic increasing in order that  $Z$  be convex. If one interprets  $c_i(q)$  as the time to traverse link  $i$  (as is often the case) and link  $i$  terminates in a traffic signal, then clearly  $c_i(q)$  will be a function of the settings of signals in the immediate area of link  $i$ . Thus signal settings instituted to minimise delay, will encourage a reassignment of flows which may or may not result in a reduction in delay. The important point however is that the signal settings can be used to influence the traffic patterns in the network (Allsop and Charlesworth 1977). The programme TRANSIGN developed by Charlesworth (1979) combines the simulation and control programme TRANSYT with the equilibrium assignment programme TRAFIC developed by Sang Nguyen (1975), and was used in a feasibility study for the CITRA<sup>C</sup> System for Glasgow (TORG, 1980).

The technique has not been subsequently developed due to lack of funding by the author's then employers. A very similar programme, namely the Saturn programme, has been subsequently developed and tested at various locations (Hall et al 1980). SATURN uses an identical simulator to TRANSYT and identical assignment technique, but has no signal control optimiser - a major failing in the author's opinion.

One great advantage of both these methods lies in the use of an analytic traffic assignment method which can be extended to deal with elastic demand, modal split and system optimisation, providing useful feedback for TSM.

Both of these models have however one serious drawback. Neither can deal satisfactorily with time of day variations in network flow except by time slicing as is used in the calculation of fixed-time signal plans. This problem is not too serious when levels of flow are relatively light but as soon as congestion occurs, then various shortcomings come to light, namely:

- i) shortest routes for a journey are calculated on the basis of link costs and flows which are independent of time.
- ii) journeys may cross time slice boundaries
- iii) shortest routes may proceed by temporarily overloaded junctions.

In an effort to overcome some of these difficulties, the cost function  $c_i(q)$  can be amended to take account of oversaturation (Catling 1977) with queues at the end of one time slice being passed on to subsequent time slices by again amending the  $c_i(q)$ , but this process is unsatisfactory in the author's opinion.

### Temporal Models

The idea of equilibrium can be extended to deal with situations in which for certain periods, junctions are overloaded and queues are retained at the end of signal cycles. This situation can be regarded as a sort of equilibrium in that users still endeavour to minimise their journey cost, i.e. we have a quasi-steady-state. The above mentioned models are not able to cope satisfactorily with this phenomenon. One programme that attempts to model this situation, which occurs particularly in peak periods, is the CONTRAM model developed at TRRL (Leonard et al 1978). Here, a time-dependent input flow profile is split into "packets" of vehicles which are assigned to a network - time slice matrix on the basis of shortest routes. Thus, packets of vehicles only contribute to the link flow during appropriate time slices. The assignment technique is somewhat heuristic and the treatment of link travel times does not take into account the effects of linking between signals. Results are however encouraging. The model has certain other applications, for instance, packets can be categorised allowing discrimination between certain types of user of the network.

Another model that is useful for examination of small networks is the discrete entity simulation programme TRAFFICQ developed by Logie (1979) and approved by the U.K. Department of Transport. The programme is, however, purely a simulation programme, initial assignments and control parameters being input from an accompanying programme LP-Plan. No allowance is made for user feedback in the way of reassignment of traffic.

Recently, there have been theoretical or semi-theoretical studies directed at obtaining equilibrium flows for time-dependent flow patterns (Hurdle 1981, Ben-Akiva et al 1984, Smith 1984a, 1984b, Mahmassani and Herman 1984). Naturally, these studies are concerned with very simple networks, and it remains to be seen whether this work can be extended to deal with networks of a reasonable size.

### INTERRUPTED FLOW CONDITIONS

The problems associated with the modelling of steady-state or quasi-steady-state conditions are as nothing compared with the problems associated with the modelling of interrupted flow conditions, brought about, for instance, by roadworks or by incidents, especially on motorways. Roadworks are less of a problem in that traffic patterns should settle down to an equilibrium over a period of a few days, whereas incidents are essentially short-term in character and thus require the use of temporal models. Ideally, a discrete entity simulation with provision for rerouting should be used, but this may necessitate very long runs if the network is of a reasonable size. The model that comes nearest to satisfying the requirements for this type of situation is CONTRAM. Of particular value, is the ability of the user of the programme to distinguish between different types of user by labelling "packets" in different ways, and thus perhaps calculate different preferred diversion routes for these users of the network.

### A ROLE FOR SYSTEM DYNAMICS

Clearly by its nature system dynamics will have a role to play in attempting to model traffic patterns which have a time component, but it is perhaps pertinent to explore its possible advantages and shortcomings in this context. The first requirement is that we are able to represent the network of roads in some suitable way.

### Network Representation

The simplest form of representation is to represent the queue at a stop-line as a state or level. The rate of inflow to this queue will be determined by the rate of inflow of vehicles to the upstream end of the link. The input pattern to the link may alter on progressing down the link as in the case of signalised links (a feature known as dispersion of platoons and modelled for instance in the TRANSYT programme). The rate of outflow from this queue will depend on the aspect of the signal (red vs. green) and also on whether a queue is present at any particular time. This output flow will be split into contributions to each of the downstream links.

It would perhaps be convenient to code each link by means of a specially written macro inserted into whatever system dynamics programme is to hand, e.g. DYSMAP (Cavana and Coyle, 1982). The macro would clearly require the link parameters, the downstream connections, and possibly the average flows to be specified.

### "Steady-state" Modelling

There is little to be gained from using system dynamics in the classical steady-state situation, as programmes like TRANSIGN or SATURN can deal quite adequately with such situations, and to a fair level of special detail. However, the transition from one signal plan to another (which usually occurs every hour or half-hour) does require a temporal model, as does the transition from one subarea to another, where there is a lack of synchronisation between adjacent signals.

### "Quasi-steady-state" Modelling

If the traffic pattern (i.e. the input flow profiles, the routings from each origin to each destination) and the system parameters (capacities, undelayed link times, link connections, signal settings) are specified, then system dynamics can be used to simulate the operation of the system over whatever time period is required. The technique would then be playing a similar role to CONTRAM except

- i) it ought to be able to produce a better simulation of link behaviour
- ii) it would not be calculating equilibrium routings.

It would be interesting to explore the possibility of using a system dynamics package to produce these equilibrium routings. A macro would have to be written to calculate shortest routes through a network, which would then be combined in an iterative scheme with the standard simulation. The possibility also arises of using an optimisation facility (Luostarinen 1982) to simultaneously calculate the appropriate signal settings.

### Interrupted flow conditions

Perhaps the greatest benefit to be achieved from the use of system dynamics in traffic modelling, is in situations where flow is interrupted in some way, and the progression of traffic through the network, perhaps on diversion routes could be readily modelled. Thus alternative strategies for control of such situations could be compared with respect to any particular specified objective function. Again, it would be interesting to explore the possibility of using system dynamics to actually elucidate optimum control policies by means of an optimisation facility.

### Example Calculations

To demonstrate the feasibility of using system dynamics for traffic modelling, calculations were carried out by a MBA student at the University of Bradford Management Centre (Gunawan 1984). The examples were naturally concerned with simple networks and in fact consisted of a single link on which either an incident had taken place or on which there were roadworks, together with an alternative diversionary link. The results for the roadwork example were particularly interesting. The analysis followed that of Davies et al (1981) who were interested in improving the U.K. Department of Transport's cost-benefit programme (Department of Transport 1978) in its assessment of delays caused by roadworks. Gunawan found in his short study that delays were highly dependent on the time taken to achieve equilibrium. The process of adjustment of route choice, resulting in a flip-flop of flows between the roadworks route and the alternative route was a significant factor, which the analysis of Davies et al did not allow for.

### CONCLUSIONS

The purpose of this paper has been to review the techniques used for the management of traffic systems and the models being developed to aid this process, and to explore the extent to which they take account of dynamic behaviour. The paper has pointed to various areas in which system dynamics might play a useful role in overcoming some of the problems encountered in the modelling of traffic systems.

This paper is primarily intended as a discussion paper. The author would be grateful for useful comments and suggestions.



REFERENCES

- Ali, S., A.J. Bencosme and J.S. Dajani, "A Dynamic Simulation Approach to Transportation Planning", Proceedings of the International Conference on Cybernetics and Society, IEEE, New York, 1980.
- Allsop, R.E. "SIGSET: A computer program for calculating traffic signal settings", Traffic Engineering Control, Volume 13, Number 13, Number 2, pp. 58-60, February, 1971.
- Allsop, R.E., "Computer program SIGCAP for assessing the traffic capacity of signal-controlled road junctions - description and manual for users", TORG Working Paper No. 11, University of Newcastle upon Tyne, 1974.
- Allsop, R.E. and J.A. Charlesworth, "Traffic in a signal-controlled road network - an example of different signal settings inducing different routings", Proceedings of the Moscow Automobile and Road Construction Institute, 1976, Traffic Engineering Control, volume 18, number 5, pp. 262-264, May 1977.
- Almond J. and R.S. Lott, "The Glasgow experiment: implementation and assessment", TRRL Report LR 142, Transport and Road Research Laboratory, Crowthorne, Berks, U.K., 1968.
- Beckman M., C.B. McGuire and C.B. Winston, "Studies in the Economics of Transportation", Yale University Press, New Haven, 1956.
- Ben-Akiva, M., M. Cyna and A. de Palma, "Dynamic model of peak period congestion", Transportation Research, volume 16B, number 4/5, pp. 339-355, 1984.
- Catling, I., "Development of Junction Delay Formulae", Department of Transport Working Paper no. 2, 1977.
- Cavana, R.Y. and R.G. Coyle, "DYSMAP User Manual", System Dynamics Research Group, University of Bradford, 1982.
- Charlesworth, J.A., "Control and routing of traffic in a road network", Traffic Engineering Control, volume 21, number 10, pp. 460-466, October 1979.
- Collins, J.R., C.M. Hopkins and J.A. Martin, "Automatic incident detection - TRRL algorithms HIOCC and PATREG", TRRL Supplementary Report, SR 526, Transport and Road Research Laboratory, Crowthorne, Berks, U.K., 1979.
- Davies, R.F., J.F. Vincent and R.G. Jacoby, "A method of assessment of the delays caused by roadworks", Traffic Engineering Control, volume 22, number 1, pp. 9-13, January 1981.
- Department of Transport, "COBA - A method of economic appraisal of highway schemes", Economic Highway Division, London, 1978.
- Ferguson, J. A., "Computer Control of Traffic: Combining Subareas", Proceedings of the 3rd IFAC/IFIP/IFORS International Symposium on Control in Transportation Systems, Columbus, Ohio, pp. 363-374, August, 1976.
- Gunawan, E., "Management of traffic on an incident road", MBA Dissertation, University of Bradford, 1984.
- Hall, M.D., D. van Vliet and L.G. Willomsen, "SATURN - a simulation-assignment model for the evaluation of traffic management schemes", Traffic Engineering Control, volume 21, number 4, pp. 168-177, April 1980.
- Hunt, P.B., D.I. Robertson and R.D. Bretherton, "The SCOOT on-line traffic signal optimisation technique", Traffic Engineering Control, volume 23, number 4, pp. 190-199, April 1982.

Hurdle, V.F., "Equilibrium Flows on Urban Freeways", Trans. Sci., volume 15, pp. 255-293, 1981.

Leonard, D.R., J.B. Tough and P.C. Baguley, "CONTRAM: A traffic assignment model for predicting flows and queues during peak periods", TRRL Report LR841, Transport and Road Research Laboratory, Crowthorne, Berkshire, U.K. 1978.

Logie, D.M.W., "Computer-aided design and evaluation of traffic management schemes: programs LP-Plan and Trafficq", Traffic Engineering Control, volume 18, number 7/8, pp. 347-353, July/August 1977.

Luostarinen, A., "DYSMOD Users Manual", Helsinki School of Economics, edited by R.G. Coyle, System Dynamics Research Group, University of Bradford, 1982.

Mayne, A., "Computer program SIGSAT for traffic signal calculations at temporarily overloaded junctions - description and manual for users", Transport Studies Group, University College, London, 1979.

Mowatt, A., "CITRAC - the first five years", Traffic Engineering Control, volume 25, number 5, pp. 251-259, May 1984.

Nguyen, S. and L. James, "TRAFFIC: an equilibrium assignment program", Publication number 17, Centre de Recherche sur les Transports, University of Montreal, 1975.

Ortlieb, M. and A.D. May, "Freeway Corridor Control Strategies", Traffic Equilibrium Symposium Proceedings, University of Montreal, November 1974.

Papageorgiou, M., B. Posch and G. Schmidt, "Comparison of macroscopic models for control of freeway traffic", Transportation Research, volume 17B, number 2, pp. 107-116, 1983.

Robertson, D.I., "TRANSYT: A Traffic Network Study Tool", TRRL Report, LR253, Transport and Road Research Laboratory, Crowthorne, Berks, U.K., 1969.

Sims, A.G., "The Sydney Coordinated Adaptive Traffic System", Proceedings Engineering Foundation Conference on Research Priorities in Computer Control of Urban Traffic Systems, ASCE, New York, pp. 12-27, 1978.

Smith, M.J., "The Stability of a Dynamic Model of Traffic Assignment - An Application of a method of Lyapunov", Trans. Sci. volume 18, number 3, pp. 245-252, 1984.

Smith, M.J., "The Existence of a Time-Dependent Equilibrium Distribution of Arrivals at a Single Bottleneck", Trans. Sci. volume 18, number 4, pp. 385-394, 1984.

Stock, W.A., R.C. Blankenhorn, and A.D. May, "The FREQ3 Freeway Model", Freeway Operations Study - Phase III, Report No. 73-1, Institute of Transportation and Traffic Engineering, University of California, June 1973.

TORG., "Strathclyde CITRAC Feasibility Study", Final Summary Report, Technical Note No. 19 (TORG Technical Report No. 47), Transport Operations Research Group, University of Newcastle upon Tyne, November 1980.

Wardrop, J.G., "Some theoretical aspects of road traffic research", Proceedings Institute of Civil Engineers (Part II), volume 1, number 2, pp. 325-362, 1952.