

# **A First Approach to Model Satisfaction at Work Under Equity Theory Using Fuzzy Set Theory and System Dynamics**

## **ABSTRACT**

This work examines how individuals “determine” their job satisfaction based on changes in situational factors. A simulation model, using Fuzzy Set Theory and System Dynamics, is used. Equity Theory in conjunction with the Equity Sensitivity Index is used as the base models to describe individuals’ behavior. A review of Equity Theory and Equity Sensitivity and their relationship to satisfaction at work is presented, from which an approximation to a formal model is stated. The model presented in this paper is part of a larger ongoing research project that intends to generate a macro-model that is capable of more closely approximating individual behavior in organizations. Data from the ISSP Social Survey is used to construct the model and test the relationships between individual variables.

## **INTRODUCTION**

Job satisfaction has been a popular topic in research for many decades. The interest in this variable has spanned academic fields having been broached by psychologists, management and more recently economists. Economic and industrial globalization, and complexity in organizations and their environments have generated new interest in developing an understanding of how individuals are satisfied or motivated, and the complex relationships that exist between satisfaction and motivation. Lawler (1994) stated three questions that must be answered by any motivation [and satisfaction] theory: (1) What activates behavior? (2) What directs behavior? and (3) What reactions do individuals have to outcomes that result from their behavior? Hereby, it is important to study what is the relationship between satisfaction and motivation at work.

Other questions that can be asked are: (1) What influences individuals’ decisions? (2) Does satisfaction cause an improvement in performance and produce motivation or (3) Does motivation improve performance which consequently generates satisfaction? Lawler (1994)

questioned the causality between satisfaction and performance; postulating that satisfaction is a consequence of the rewards generated by high performance.

In complex systems, it is difficult to determine causality between variables. As Baecker (2001) stated: “Because most, if not all, things (Luhmann, 1990a) happen simultaneously anyway, there cannot be a causal link between them (Whitehead, 1979)”. An approach to overcome this is to model a system as a set of feedback relationships where cause and effect are changing over time. Sterman (2000) stated that, “the most complex behaviors usually arise from the interactions [feedbacks] among the components of the system, not from the components themselves”.

This paper aims to expand the knowledge of the complex interactions that govern individual behaviors within an organizational context. A model using Equity Theory as a basis and Equity Sensitivity to model individual differences is developed. Fuzzy sets are used to approximate individual’s decision-making approaches.

## **THE MODEL**

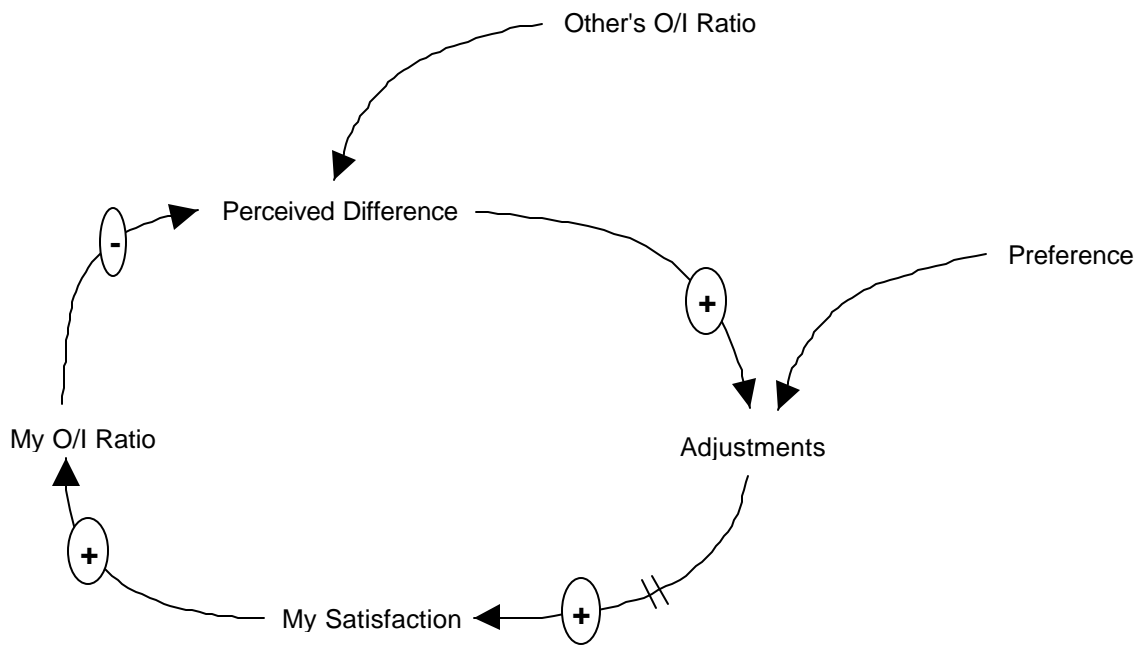
As stated by Senge (1990) there are “systems archetypes” or “generic structures” that allow learning by revealing an elegant simplicity underlying the complex world. One of these generic structures is the balancing feedback, which can be exemplified as follows: “individuals improve themselves for a period of time, then plateau” (Senge 1990, p. 96). The balancing feedback can be used as a generic structure to explain individuals’ behavior.

Individuals react in different ways to both perceived equity and inequity. Equity Theory (Adams, 1963) stipulates that individuals, in a cognitive manner, make “decisions” about the state of their satisfaction. According to Equity Theory, individuals compare their own inputs and outputs with those of a reference individual or group. Satisfaction results from a state of “equity”, i.e. when one’s own input-output ratio is equal to that of others. Conceptually, dissatisfaction arises both, when one is under or overcompensated relative to the effort expended, satisfaction decreases when the input-output ratio for oneself is higher *or* lower than that of the reference group.

Equity Theory assumes that all individuals are the same insofar as their perception of equality is concerned. Realistically, however, each individual’s reactions will also depend on individual characteristics. Given the vast variety of variations between individuals, it is pertinent to simplify

the differences. A basic classification can be undertaken using the Equity Sensitivity Index, as described by Huseman et al (1987). The Equity Sensitivity Index classifies individuals into three groups, (1) Egalitarians or Equity Sensitive, (2) Benevolents, and (3) Entitleds.

By using the input-output ratio, the model tries to show how each class of individuals adjusts their preferences. The basic feedback cycle is shown in Figure 1. Here, an adjustment is caused by a perceived difference between an individual's input-output ratio (My O/I Ratio) and the reference group's ratio (Other's O/I ratio). The perceived difference is influenced by the preference (ESI category) of the individual. In turn this adjusted perceived difference affects the individuals satisfaction (My Satisfaction). The change in satisfaction level then affects the perception of the input-output ratio. This feedback cycle tries to explain, in some way, how an individual reacts to changes in his (her) perceptions about the work environment by adjusting his/her I/O ratio according to individual's preferences for different Outcome/Input ratios (Huseman et al 1987).



**Figure 1.** Basic feedback cycle for individual's input-output ratios.

This is a model of a social phenomenon, but if only system dynamics is used, the causal relationships will be represented as mathematical functions. Fuzzy logic prevents the translation

of these hazy concepts (I am satisfy, my coworkers are more satisfied than me, etc.) into exact equations, allowing their formulation in terms of the natural language used by the modeler and/or the system agents. In this way, the model becomes a more realistic representation of the kind of information that is available about the real system (Wenstøp, 1976). This theory allows imprecise evaluation of information that can be defined as a “granule of information” (Zadeh 1979, 1996), and can be represented graphically using functions called “membership functions” (Piegat 2000).

The use of fuzzy logic in system dynamics environments has been previously demonstrated, for example by Bourguet and Soto (2002). Bourguet and Soto (2002) used fuzzy logic to represent qualitative knowledge in dynamics of complex systems. They characterize management policies for a hotel as a fuzzy set (it is important to note that just the abstract of this work has been published). Sanatani (1981) used fuzzy logic to assign numerical values to the parameters of his diffusion model for market penetration of new products in segmented populations.

As Piegat (2000) stated “information obtained from people is usually of less precision (large granularity), while information delivered by measuring devices is of higher precision (small granularity)”. For this model, the information is obtained from people. It measures subjective features of work, consequently making fuzzy set theory a highly applicable technique to evaluate the features. The estimation of the individual’s input-output ratio and the effects of input-output ratio changes on the individual’s satisfaction are evaluated using fuzzy set theory<sup>1</sup>.

The model was constructed following the next steps:

### **Select the information of the ISSP Database**

Data was obtained from the 1997 International Social Survey Program (ISSP) survey. Only individuals currently working for pay in USA were selected (V32 = 1 and V3 = 6). Table 1 shows the variables used to carry out the study and their classification as input (I), output (O), or

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<sup>1</sup> A broader description of -fuzzy set theory can be found in Piegat, A., 2000. Fuzzy Modeling and Control. New York: Physica-Verlag.

as the response variable (A) . These variables were selected in accordance to the study by Sousa-Poza and Sousa-Poza (2000).

**Table 1.** Variables Used.

| Variable | Type | Definition                              |
|----------|------|---|
| D        | I    | Education Categories                    |
| V4       | I    | Time in a paid job                      |
| V43      | I    | To do hard physical work                |
| V44      | I    | Find the work stressful                 |
| V45      | I    | Work in dangerous conditions            |
| V35      | O    | My job is secure                        |
| V36      | O    | My income is high                       |
| V37      | O    | Opportunity for advancement is high     |
| V38      | O    | My job is interesting                   |
| V39      | O    | Can work independently                  |
| V40      | O    | Can help other people                   |
| V41      | O    | Is useful to society                    |
| V52      | O    | Relations: between management/employees |
| V53      | O    | Relations: between workmates/colleagues |
| V54      | A    | How satisfied are you in your job       |

Variable 44 was omitted due to excessive missing cases. Variable D (education categories) ranged from 1 for no education to 7 for university complete. Variable 4 (time in a paid job) corresponds to answering the question “would you like to spend more time in a paid job?”. This variable ranged from 1 (much more time) to 5 (much less time), indicating the preference to spend time in a paid job. The other inputs (V43 and 45) were ranged from 1 (always) to 5 (never). The output variables V35, V36, V37, V38, V39, V40 and V41 ranged from 1 (strongly Agree) to 5 (strongly disagree), while V52 and V53 ranged from 1 (very good) to 5 (very bad). Finally, the response variable (V54) ranged from 1 (completely satisfied) to 7 (completely dissatisfied).

According to Adams (1963), “though inputs and [outputs] may in most cases be measured continuously (ethnicity and sex are obvious exceptions), we have dichotomized them into ‘high’ and ‘low’ for the purpose of simplicity”. Inputs and outcomes can be measured in both continuously or discrete scale, but in this work, fuzzy set theory is used to convert discrete data from the survey into continuous data to construct the model.

## Statistical support of some assumptions in the model

The comparison between individual's and other's outcome/input ratio determines the satisfaction level, (Huseman et al 1987). For simplicity, one input and one outcome were chosen from correlation indexes (shown in Table 2).

**Table 2.** Correlation indexes between satisfaction and inputs and outcomes.

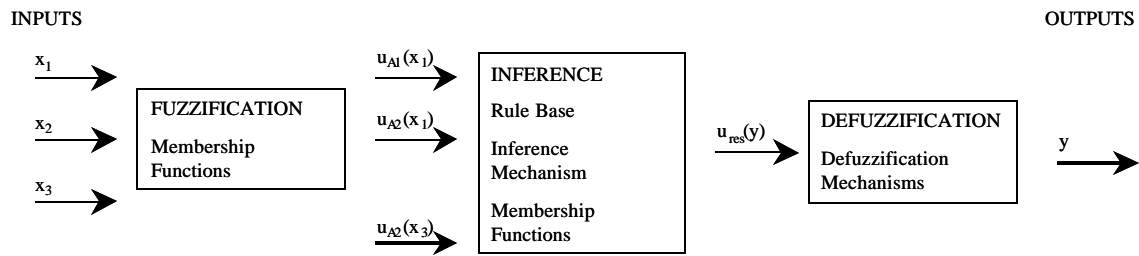
|            | D            | V4          | V43         | V45         | V35         | V36         | V37         | V38         | V39         | V40         | V41         | V52         | V53         | <b>V54</b> |
|------------|--------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|------------|
| D          | 1            |             |             |             |             |             |             |             |             |             |             |             |             |            |
| V4         | 0.07         | 1           |             |             |             |             |             |             |             |             |             |             |             |            |
| V43        | 0.18         | 0.11        | 1           |             |             |             |             |             |             |             |             |             |             |            |
| V45        | 0.05         | 0.07        | 0.72        | 1           |             |             |             |             |             |             |             |             |             |            |
| V35        | -0.06        | 0.06        | 0.42        | 0.43        | 1           |             |             |             |             |             |             |             |             |            |
| V36        | -0.15        | 0.03        | 0.40        | 0.47        | 0.72        | 1           |             |             |             |             |             |             |             |            |
| V37        | -0.09        | 0.05        | 0.40        | 0.43        | 0.66        | 0.70        | 1           |             |             |             |             |             |             |            |
| V38        | -0.15        | 0.04        | 0.40        | 0.45        | 0.73        | 0.71        | 0.73        | 1           |             |             |             |             |             |            |
| V39        | -0.05        | 0.01        | 0.46        | 0.45        | 0.65        | 0.66        | 0.61        | 0.74        | 1           |             |             |             |             |            |
| V40        | -0.08        | 0.02        | 0.47        | 0.48        | 0.63        | 0.60        | 0.62        | 0.75        | 0.70        | 1           |             |             |             |            |
| V41        | -0.12        | 0.05        | 0.46        | 0.53        | 0.65        | 0.61        | 0.60        | 0.76        | 0.66        | 0.81        | 1           |             |             |            |
| V52        | 0.02         | 0.09        | 0.23        | 0.17        | 0.37        | 0.32        | 0.35        | 0.34        | 0.34        | 0.32        | 0.32        | 1           |             |            |
| V53        | -0.03        | 0.08        | 0.21        | 0.21        | 0.36        | 0.31        | 0.32        | 0.35        | 0.28        | 0.33        | 0.33        | 0.67        | 1           |            |
| <b>V54</b> | <b>-0.02</b> | <b>0.09</b> | <b>0.18</b> | <b>0.13</b> | <b>0.43</b> | <b>0.40</b> | <b>0.42</b> | <b>0.52</b> | <b>0.45</b> | <b>0.45</b> | <b>0.41</b> | <b>0.57</b> | <b>0.47</b> | <b>1</b>   |

The above table shows the impact of inputs and outcomes to explain satisfaction at work. The chosen output was V52 (Relations: between management/employees) that can explain a 57% of change in satisfaction. All inputs have low values of simple correlation (below 50%). V43 (To do hard physical work) can explain an 18% of change in satisfaction and was chosen since it had the biggest value of all outputs. To model the equity sensitive index (ESI) it is necessary to have one or more inputs and one or more outcomes, in spite of the low correlations of the inputs with job satisfaction.

## Fuzzy Set Theory for modeling

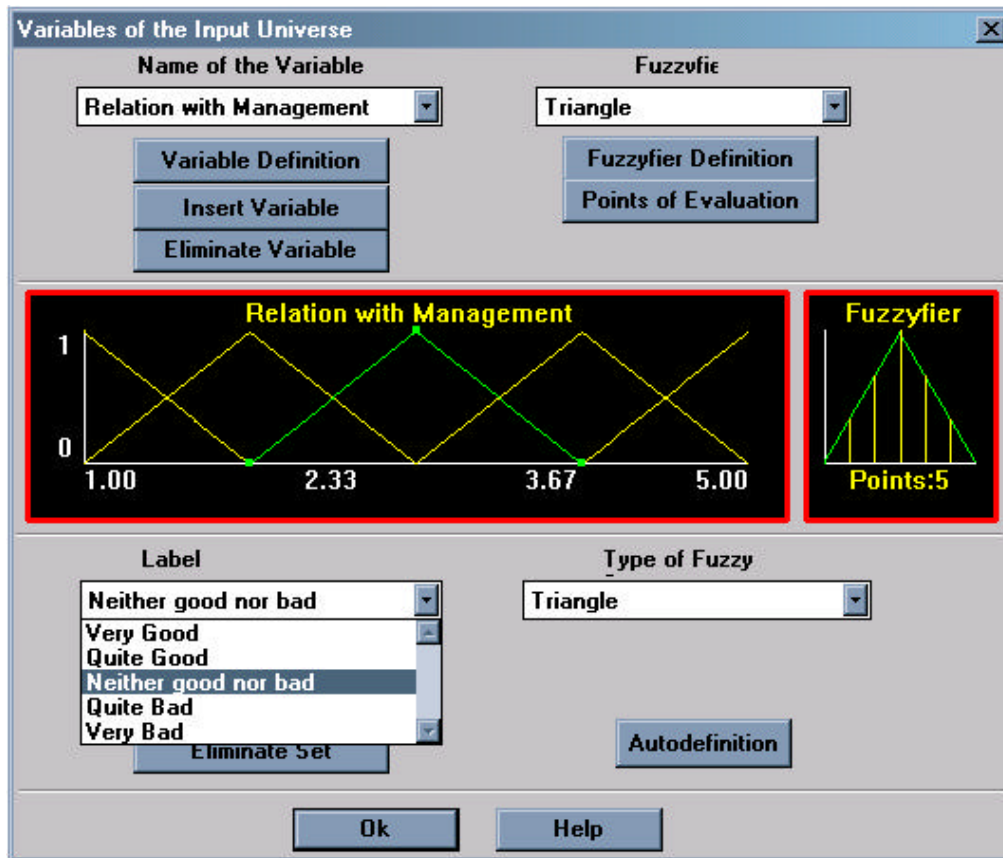
As stated by Piegat (2000), "fuzzy sets are generally applied by people for qualitative evaluation of physical quantities, states of plants and systems and for comparison to each other". Thereby this theory can be useful to model how individuals at work make comparisons between each other (Adams 1963).

The structure of a typical fuzzy model is depicted in Figure 2.



**Figure 2.** Structure of a fuzzy model.

In the fuzzy model inputs, some crisp, numeric value can be introduced, (two in the model to calculate the perceived ESI). The block *Fuzzification* calculates their membership grades in particular sets. Five sets (very good, quite good, neither good nor bad, quite bad and very bad) are used for the evaluation of output (Relations: between management/employees). This block must be provided with precisely defined membership functions of the inputs as shown in Figure 3. The membership grades inform how high membership is of the numerical value of an output in particular sets, i.e. how very good, quite good, neither good nor bad, quite bad or very bad relations between management and employees are.



**Figure 3.** Membership functions to a fuzzy variable (relation with management).

The *Inference* block has as inputs the membership degrees  $u_{\text{very good}}(x_1)$ ,  $u_{\text{good}}(x_1)$ , and so on and calculates as an output a so-called resulting membership function  $u_{\text{equity}}(\text{ESI})$  of the model output. This function is determined by means of inference, which has to include the following, strictly defined elements (Piegat 2000):

- Rule Base
- Inference mechanism
- Membership functions of the model-output

The rule base consists of logical rules determining causal relationships existing in the system between fuzzy sets of inputs and outputs. The inference mechanism determines the resulting membership function. The defuzzification block calculates from the resulting membership function, a crisp, numeric value  $y$ .



As mentioned above, fuzzy set theory was used to estimate the individual's input-output ratio and the effects of the input-output ratio changes on the individual's satisfaction. As shown in Table 2, the estimation of the individual's ratio used one input (to do hard physical work) and one output (relations between management/employees). The two variables are used as inputs to the fuzzy logic model. Five fuzzy sets with their respective membership functions are given to each fuzzy variable. The input (to do hard physical work) has the following fuzzy sets:

- Always
- Often
- Sometimes
- Hardly ever
- Never

The output (relations between management/employees) has the following fuzzy sets:

- Very good
- Quite good
- Neither good nor bad
- Quite bad
- Very bad

Their membership functions are shown in Figure 3.

Piegat (2000) stated, "To calculate membership grades in particular fuzzy sets, their membership functions must be precisely defined with respect to quality (function type) and quantity (function parameters). Both, the parameters and the shape of membership functions strongly influence the model accuracy (Baglio, 1994)". In this work, function parameters are defined in accordance to the given scales from the ISSP survey, i.e. from 1 to 5 for relations between management/employees. Functions types are defined consisting of straight segments, which are commonly used in practice for their simplicity (Piegat 2000). The forms of the most often applied functions are polygons like triangles and trapezes.

Rule Base and Inference mechanism are defined by rules that can be stated as follows:

IF

Relationship with Management IS Very Good AND

Hard Physical Work IS Always

THEN

Index\_Position IS High (Input-Output Ratio)

A complete set of rules must be defined in order to determine, from the input membership grades, the resulting membership function of the model output by using the inference mechanism (Piegat 2000).

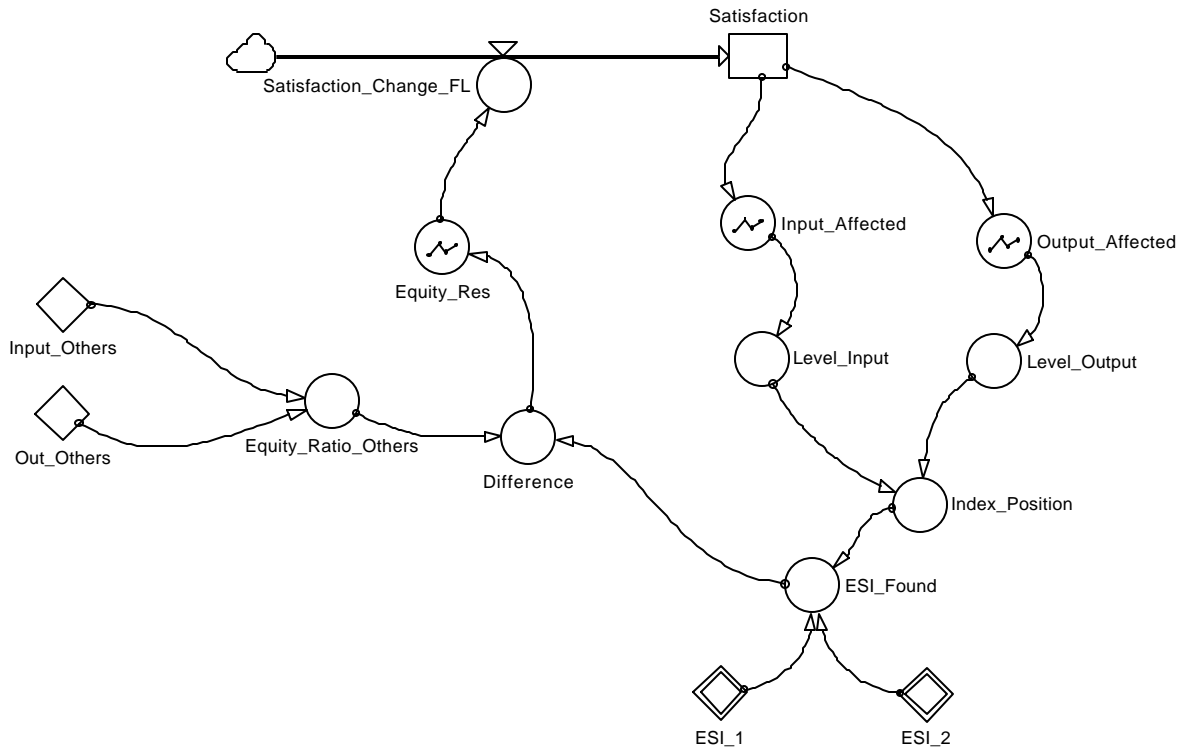
Finally, through defuzzification of a fuzzy set, a crisp value of the model output can be determined. Several defuzzification methods exist. In this work, due to its simplicity and facility to program (Piegat 2000), the middle of maxima method was used.

Also, fuzzy set theory was used to determine how the difference between the individual's and other's ratio affects the individual's satisfaction. In the same way, one input variable and one output variable were defined. "Difference" was defined as input and "Delta Satisfaction" was defined as output.

### **Simulation Model**

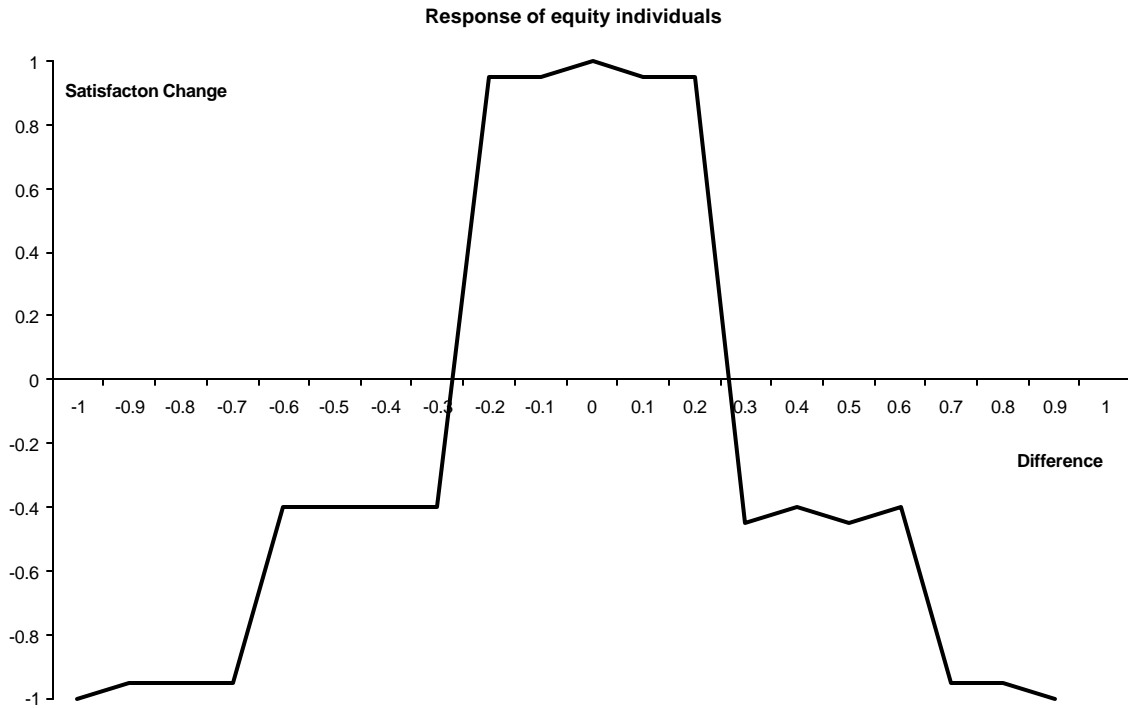
By using a System Dynamics approach, a model to simulate satisfaction changes was constructed in order to model the feedback process, which is present in many social and physical complex systems (Sterman 2000). Time delays and nonlinearities in systems can be also be modeled. As shown in Figure 1, a simple feedback-balancing loop is defined. According to Sterman (2000), "this kind of loop acts to bring the state of the system in line with a goal or desired state". Here, "equity" is sought for individuals depending of their preferences (Huseman et al 1987).

Fuzzy set theory is used to determine some nonlinear relationships found in the modeling of satisfaction. Powersim<sup>®</sup>, a computer program is used to developed the system dynamics model. It permits the user to define graphically a set of differential equations and solve them using numeric methods. In Figure 4, a graphical representation of the model is shown.



**Figure 4.** Graphical representation of the model.

The egalitarian individuals are modeled obtaining both *ESI\_Found* and *Equity\_Res* using fuzzy logic. Figure 5 shows the shape of the response of equity individuals to changes in the perceived difference of the input-output ratio. This shape looks similar to the shape defined for equitably rewarded individuals (Huseman et al 1987). The difference value is defined as the reference group equity ratio (*Equity\_Ratio\_Others*) minus the individual's equity ratio (*ESI\_Found*). It is used to show under-rewarded ( $\text{Difference} < 0$ ) and over-rewarded ( $\text{Difference} > 0$ ) states and their effect on satisfaction.



**Figure 5.** Response of egalitarian individuals.

Generally, a system dynamics model can be understood as a system of differential equations, in which one or more state variables (stocks) changes as a result of a change in their flows over the time. Table 3 shows the system of differential equations corresponding to the system shows in the Figure 4.

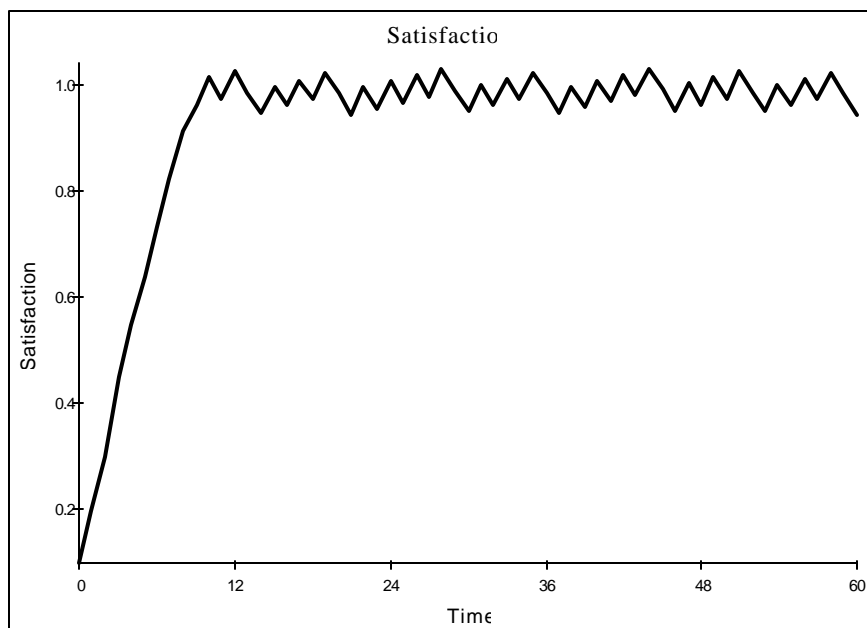
**Table 3.** Model's Equation.

| Variable | Equation  |
|----------|---|
| Init     | Satisfaction = .5   |
| Flow     | Satisfaction = +dt*Satisfaction_Change_FL   |
| Aux      | Satisfaction_Change_FL = Equity_Res   |
| Aux      | Difference = (Equity_Ratio_Others-ESI_Found)/100  |
| Aux      | Equity_Ratio_Others = (Out_Others/Input_Others)*100   |
| Aux      | Equity_Res = GRAPH(Difference,-1,0.1,[-0.73, -0.61, -0.53, -0.34, -0.22, -0.11,0.06,0.22,0.39,0.39,0.75,0.56, 0.46, 0.32, 0.31, 0.08, -0.15, -0.33, -0.45, -0.646, -0.743 "Min:-1;Max:1;Zoom"]) |
| Aux      | ESI_Found = IF(Index_Position>250,LOOKUP(ESI_2, Index_Position-250),LOOKUP(ESI_1, Index_Position))  |
| Aux      | Index_Position = Level_Input+Level_Output-1   |

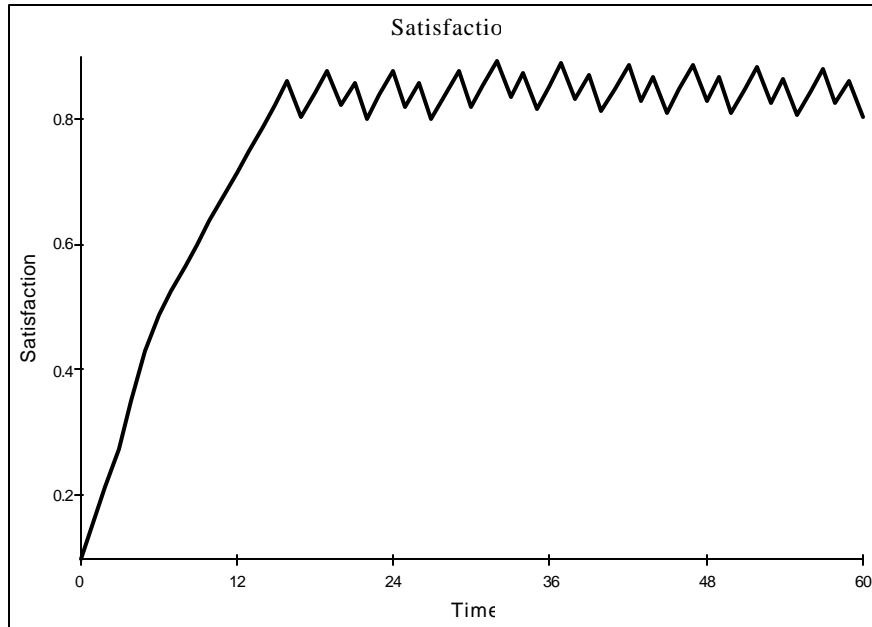
| Variable | Equation  |
|----------|---|
| Aux      | Input_Affected = GRAPH(Satisfaction,0,0.1,[5,4.39,3.82,3.58,3.37,3.07,2.89,2.72,2.4,2.16,2.19"Min:1;Max:5;Zoom"]) |
| Aux      | Level_Input = (INT((Input_Affected-1)/0.2))+1   |
| Aux      | Level_Output = (INT((Output_Affected-1)/0.2)*21)+1  |
| Aux      | Output_Affected = GRAPH(Satisfaction,0,0.1,[4.67,4.4,4.07,3.54,3,2.6,2.39,2.26,2.23,2.12,2"Min:1;Max:5;Zoom"])    |
| Dim      | ESI_1 = (1..250)  |
| const    | ESI_1 = VECTOR  |
| Dim      | ESI_2 = (1..191)  |
| const    | ESI_2 VECTOR  |
| const    | Input_Others = 5  |
| const    | Out_Others = 5  |

## Discussion

As shown in Figures 6 and 7, the satisfaction level of the egalitarian individual tries to reach the reference group satisfaction (other's satisfaction). As Huseman et al (1987) stated: "equity sensitives are most content when their outcome/input ratio equal those of the comparison other". Satisfaction levels oscillate in both cases due to the delay between when an individual feels distress and when their actions to reestablish the equilibrium take effect. Also, more time is required to reestablish the equilibrium when an over rewarded state takes place, and the recurrence cycle is shorter.



**Figure 6.** Satisfaction when other's ratio = 1 (Equity).



**Figure 7.** Satisfaction when other's ratio = 0.8 (Over-rewarded).

## CONCLUSIONS

The aim of this work was to construct a model, using fuzzy set theory and system dynamics, to show how individuals react to changes in their input-output ratio. Important things were observed developing this work. First, fuzzy set theory can be a useful tool to understand, and model, qualitative variables related to work satisfaction.

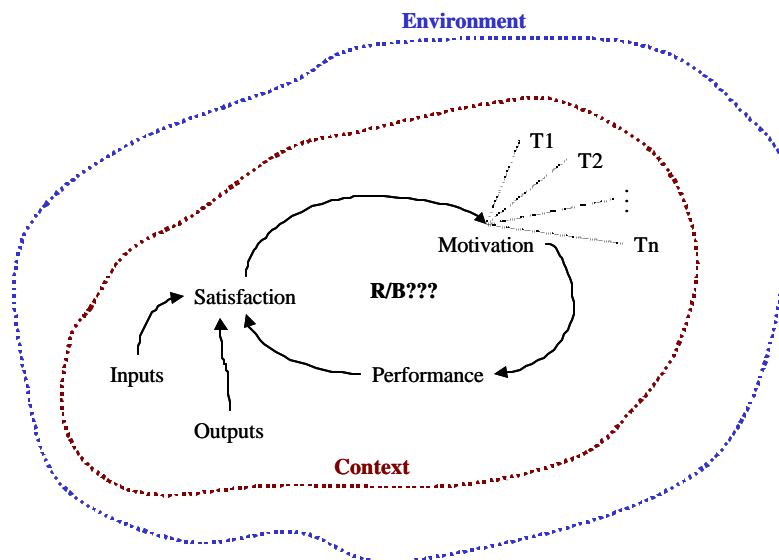
Second, system dynamics shows its versatility as a tool to model complex dynamics in systems including a feedback process, time delays and nonlinearities. In this case, this approach was useful to show how the balancing feedback process of work satisfaction operates in equity theory.

Finally, by using combinations of tools, such as fuzzy set theory and system dynamics, individuals and organizations can model real systems. As Sterman stated (2000) modeling is learning, and some meaningful understanding of the real world can be acquired when modeling is carried out.

## FURTHER RESEARCH

Future areas of research, some of which is already planned, are identified.

- There are multiple theories about job satisfaction. This work presents one: equity theory. The presented methodology could be used to integrate other existing theories to come up with a more robust model of the dynamics of individuals' job satisfaction. The researchers must be aware of the ontological and epistemological constraints of integrating multiple theories. Also, the conceptual congruency of the resulting model must be tested.
- Analyzing the feedback process shown in the Figure 8 could be a remarkable research topic. Here System Dynamics becomes a useful tool for carrying out the modeling process including feedbacks, delays, and nonlinearities.
- Equity theory could be generalized not only for satisfaction but also motivation. Here, retaining the idea of how individuals compare themselves against a reference group in organizations, a new way to measure how individuals react to changes in performance and motivation could be developed.
- Fuzzy set theory could be used to evaluate the effects of changes in motivation and satisfaction on performance at work. Besides, more variables can be included in order to measure performance and motivation, including satisfaction.



**Figure 8.** Feedback process including motivation and performance.

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