EXEMPLARY SYSTEM DYNAMICS--INPUT-OUTPUT ANALYSIS MODEL

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ABSTRACT

An exemplary model has been formulated using a methodology which casts a modified version of input-output analysis into system dynamics format. The intent is to utilize the methodology for further study of the concept of a geeignet (appropriate) population for a society. The exemplary model represents a highly aggregated socio-economic system with six sectors. Evaluation of the quality of the society is an important issue in the geeignet population study, and to that end the technique of multiattribute utility measurement (MAUM) has been included in the model. In order to study a mechanism that can minimize the marginal production cost during the time evolution of the system, a Cobb-Douglas production function that permits substitution between two factors has been incorporated into the agricultural sector. Model runs are shown which demonstrate the approach to equilibrium for the society and the time evolution of the society as the agricultural sector changes from a labor intensive to a capital intensive configuration.

I. INTRODUCTION AND PHILOSOPHY

System dynamics, as developed by Jay Forrester [1], and input-output analysis, as developed by Wassily Leontief [2], provide powerful approaches to the study of socio-economic systems. Evidently, a system may be studied with either approach. System dynamics places emphasis on the feedback structure of the system. Input-output analysis places emphasis on the flow of goods between sectors of the system. Concepts of concern in one approach are inherent, albeit perhaps somewhat concealed, in the other approach.

Static input-output analysis is established as a useful planning tool through its ability to relate the amounts of various goods needed in order to fulfill stipulated goals for a society, the "final demands." It is desirable then to not compromise the static input-output analysis in a combined methodology.

The two approaches should represent different analytical formulations of the same problem, which prompted a previous analysis that cast "dynamic input-output analysis" into system dynamics format [3]. An essential modification was necessary in the rule which governed the production rates of the various sectors of the economy. The usual formulation of dynamic input-output analysis yields unstable dynamical behavior, which becomes obvious when the feedback structure is cast into system dynamics form. For simplicity in these tentative studies, a proportional control rule is invoked to govern the regulation of stocks of goods.

Once it is established that more or less equivalent formulations of the dynamical behavior of a societal system are possible, it is a matter of taste as to which to employ. The intent of the present line of work is to continue study of the concept of a geeignet (appropriate) population for a society [4]. The modelling work in support of such a study should describe a society as it might be rather than as it may presently be constituted. The controlling character of the population level is the matter of principal concern, and the model should enable determination of

the best results that are possible, given such considerations as the efficacy of the technology, the resources available, etc. Input-output analysis is well suited to the objectives of this line of investigation. A central concept is the "structural matrix" (comprised of the A_{rs} coefficients), which provides measures of the amounts of each factor needed in the production of each sector output. Changes in the efficacy of technology, changes in returns to scale as production levels change, changes associated with depletion of resources, and impacts of pollution control can be incorporated by adjustments of these coefficients.

Input-output analysis also yields prices which are determined by conservation equations that simply equate monetary inputs and outputs for each sector. There is little arbitrariness in the price determinations. Although the pricing mechanism could be modified to include various effects that cause real prices to vary from those determined strictly on the basis of costs, the idealized pricing determination is appropriate to the objectives of the present work.

Another benefit of idealized modelling is that it can provide a benchmark against which to judge actual society. A conspicuous defect in economic analysis is the absence of standards against which to judge real societal performance. One suspects that much fruitless effort goes into efforts to improve performance by tinkering with economic and political processes when basic limitations, such as population or lack of cheap resources, are present. Societal studies need something akin to the laws of thermodynamics, that provide benchmarks against which to judge practical results in, for example, power generating plants.

II. AN EXEMPLARY MODEL

In order to gain experience in the use of the combined methodology, an exemplary model of a simple socio-economic system has been constructed. In view of the intent to pursue the study of a geeignet population, the model system is chosen to be a helpful basis for further development. Six basic levels or sectors are stipulated: agriculture, industry, service, raw materials, energy, and labor. In further development of the model, the industrial sector should be divided into capital and non-capital categories. Some distinction has been made between skilled and unskilled labor by assignmant of skilled labor to the service sector and unskilled labor to other sectors. A more thorough disaggregation of the labor sector will be needed. No provision is made in the present model for the role of a money supply, and a monetary sector should be incorporated later. In the system dynamics formulation, the levels are the stocks of the various goods held by the sectors. All of the stocks held by one sector are subsumed into a single stock coefficient for that sector, which is in accordance with the formulation of conventional dynamic input-output analysis. This simplification precludes certain kinds of studies, such as the effect of shortage of a particular good, but at least in the exemplary study this

restriction will be retained. The practical complexity of the model is much increased if this restriction is abandoned.

Variables of interest include the following, which may be described as 5-component vectors: stock coefficients (S), sector production rates (Y), and prices (P). The labor sector variables of interest are defined separately and include: the labor supplied (Y9), final demands for each of the goods--a 5-component vector (X), and the wage rate (P9). The wage rate serves as a reference for all prices, hence it is set to some arbitrary constant value. No purpose would be served in this model by variation in the wage rate since other prices would simply be proportional to it. The same wage is used for both skilled and unskilled labor. Upon more complete disaggregation of the labor sector, the relative wages become significant.

Other important quantities in the model include: the 5x5 structural matrix with elements defined as A_{rs} = ratio of input from sector r into sector s to the output of sector s, a 5x5 C matrix that measures the desired stock levels, a 5x5 B matrix that is derived from A, C, and Y, and a 5-component A1 vector which describes the labor needed by each sector. A lagged version of the stock coefficients is used, and a smoothed version of the prices has been incorporated. The "instantaneous" prices, which are calculated as in conventional input-output analysis from a balance of receipts and expenditures for each sector, showed considerable variability in previous studies and are deemed unsuitable as a description of price behavior in the system. Various rate constants are associated with delays, smoothing, and recovery of stocks.

Although the structure of the equations leads to stable behavior of a simple system, incorporation of delays and other complications tends to produce unstable behavior. Derivative feedback has been incorporated into the equations in order to provide one useful stabilizing mechanism. This mechanism can be criticized because it is unclear what real world decision process is supposed to correspond to derivative feedback; it is unlikely that managers consciously use derivative feedback! However, real world behavior sometimes seems to be consistent with the presence of derivative feedback, and such feedback may be viewed as a convenient mathematical artifice to describe a myriad of decision processes that are exercised by prudent managers, often on an intuitive basis. Considerable instability is found to persist in the exemplary model, and further investigation of stabilization of the system will be necessary. Experience with a previous model of the more conventional system dynamics structure [5] suggests that the control equation, which governs the regulation of stocks held by the various sectors, will need modification.

The present form of the control equation is

$$Y = Y3 + BK(R-S4) - \dot{B}\ D\ S4 - B\ D\ \dot{S}4.$$

Y is the vector of sector production rates, Y3 is the equilibrium value of that rate (but Y3 is not constant over time for a

dynamic system), S4 is the lagged stock coefficient vector, R is the desired stock coefficient vector, K is a matrix of parameters that govern the stock recovery rates, and the term ($\dot{B}$ D S4) arises if the matrix B varies with time, as it certainly would in a real system. The last term represents the derivative feedback artifice, with D a matrix of damping coefficients. The proportional control is present in the (R-S4) factor. The equations governing the levels, the stock coefficients, are

$$\dot{S} = B^{-1}[(1-A)(Y-Y3) - \dot{B}\ S] .$$

B^{-1} is the inverse of the B matrix.

In previous work, an important aspect of the time evolution of the system lay in the provision of alternative mechanisms to adjust production together with a decision procedure which would reckon the incremental costs of the various alternatives and adopt the one which minimized the marginal cost. It is desired to incorporate a comparable feature in the present model. One sector has been studied with this in mind. The production rate in the agriculture sector is described by a Cobb-Douglas function of the form

$$Y = r_2^{\ b}\ r_6^{\ (1-b)}/a \qquad (0 \leq b \leq 1).$$

This permits substitution between two production factors, r_2 (capital) and r_6 (labor); "a" and "b" are parameters. When this feature is implemented, the relative mix of capital and labor is continuously adjusted in order to minimize the production cost. The mix depends on the current prices. Furthermore, use of energy by the sector has been linked to the use of capital so that in the determination of the most favorable mix, the price of energy is considered in addition to the labor and goods prices. The determination of the proper r_2 and r_6 values is tantamount to a determination of appropriate coefficients in the A matrix, and adjustment of the elements of the A matrix is the way in which this effect is incorporated into the dynamical equations. The procedure can be extended to other sectors and to include substitution between more than two factors in the production function. Technological change in the society can be reflected by changes in the parameters, the "b" parameter in particular, so as to reflect changing conditions in the relative efficacy of production factors.

III. EQUILIBRIUM

The structure of static input-output analysis has not been compromised in the formulation of the model so that solutions of the static problem are always available, and they may be used, for example, as elements of decision processes. The static equations are

$$Y3 - (1-A)^{-1}X \qquad \text{and } Y4 = A1\ Y3$$

$$P3 = P9(1-A^{*})^{-1}\ A1^{*}\ .$$

The vector of equilibrium production rates in Y3, Y4 is the equilibrium value of labor supplied, and P3 is the vector of equilibrium prices. A^{*} denotes the transpose of the A matrix. A simple dynamic model will smoothly approach the static solutions if started from a non-equilibrium configuration. This is a reasonable expectation and has been a guideline in formulation of the combined methodology. Of course, it is not to be expected that a real system will approach to an equilibrium because of continual changes in societal conditions, as reflected in endogenous or exogenous time evolution of some model parameters.

Results of some approach-to-equilibrium studies for the present model are shown in Figures 1 and 2. Figure 1 depicts the behavior of the stock coefficient for the industrial sector. Initially the coefficient is 0.95, and the equilibrium values for all stock coefficients have been defined as unity. The effect of the damping coefficient, the coefficient for the derivative feedback in the control equation, is evidenced in a comparision of the curve with no added damping, D=0, and the curve with D=3. Some damping is intrinsic to the model, as may be inferred either by perusal of results of model runs or by examination of the model equations in very simple situations where analytic solution is possible. In Figure 2 is depicted the behavior of the "instantaneous" price (P_1) and the smoothed price (PS_1) for agricultural goods.

IV. MAUM

Evaluation of the quality of a society is of more importance in the intended geeignet population studies than in some societal modelling because judgements of relative quality must be made in order to justify the concept of an appropriate population level. An excellent vehicle for the investigation of quality appears to be the "multi-attribute utility measurement" (MAUM) scheme, as

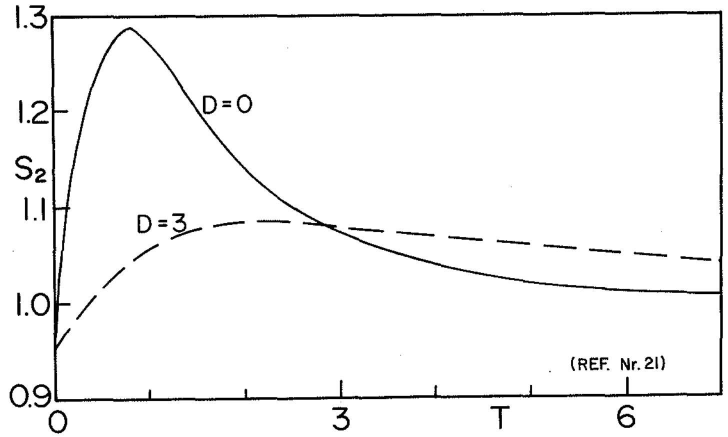

Figure 1. Approach of the stock coefficient for the industrial sector to equilibrium. The D=0 curve represents no damping added to the control equation, while the D=3 curve demonstrates the effect of added damping.

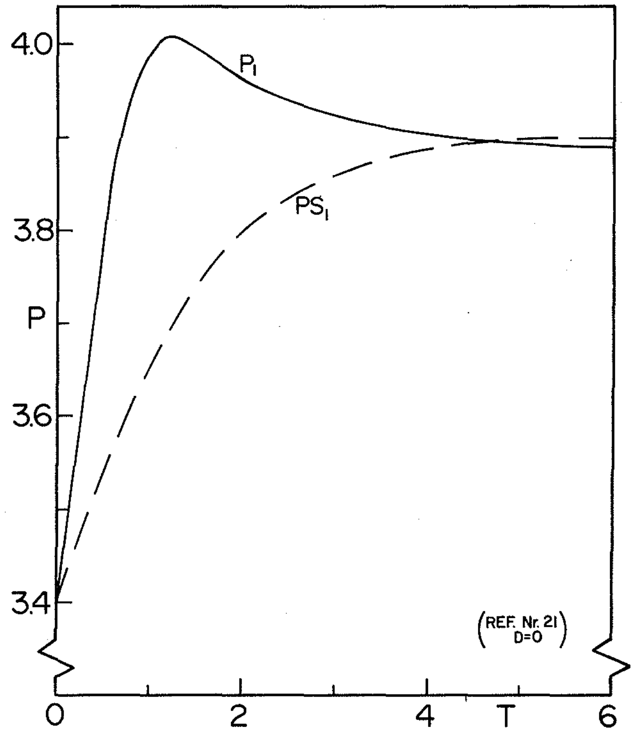

Figure 2. Approach of prices for the agricultural sector to equilibrium. P_1 represents the "instantaneous" price, as determined in the usual manner in input-output analysis, while PS_1 represents a smoothed version of the price.

applied by Gardner and Ford [6] in a system dynamics study. A review of the steps involved in the MAUM procedure will be given in order to explain what this part of the model does. The quality of the society is evaluated by various "constituencies." For purposes of the exemplary model, the constituencies are denoted unskilled workers, skilled workers, industrial leaders, political leaders, and planners. The latter constituency is supposed to be a group that does not have selfish interests at stake and that is concerned with long term effects; it would be difficult to identify an influential constituency with these characteristics in a real society.

A list of attributes of the society is drawn up which includes those measurable characteristics of the society that are of interest to any constituency. Indices are defined that provide measures of the attributes. Fourteen indices have been defined including, as examples, unskilled employment, services (provided by the service sector of the system) to the skilled labor sector, environmental loading, and overall societal productivity. For each attribute of interest to a particular constituency, a utility function is needed. The utility functions are likely to be very non-linear. For example, the utility of added food is great when food is deficient, but the added utility of more food after there is adequate food becomes almost zero.

For each constituency, relative weighting factors are assigned which weigh the utility of the various goods or services.

The utility of the society, or quality of the society, as judged by constituency α, can then be written as

$$U_\alpha = \sum_i w_{i,\alpha} \, u_{i,\alpha}(v_i) \; .$$

Here the α index denotes the constituency, the i index denotes the attribute, the $w_{i,\alpha}$ are the weighting factors, and the $u_{i,\alpha}(v_i)$ are the utility functions, which are functions of the indices v_i. It is possible to adduce arguments which would make it improper to represent the utility judgement in this relatively simple form. For example, one might argue that the weights should depend on the v_i indices. Such complications could be incorporated into the formulation. Although it is sometimes argued that a model should have sufficient robustness to represent a society under extreme conditions, that may entail excessive complications that are irrelevant to more reasonable conditions, which are the only ones of interest.

The MAUM part of the model presently is used only for evaluation and does not provide feedback to the equations which govern the operation of the model. There is no reason why such feedback should not be incorporated into a model, and contemplation of that possibility raises some interesting opportunities. There are certain obvious feedbacks that might be incorporated. For example, a labor sector that is unhappy about its employment index could provide feedback to the structural matrix A in such a way as to move the production function towards utilization of more labor. This would simulate the political and economic clout that the labor sector was capable of exerting.

DeGreene [7] has suggested interesting ideas for incorporation into system dynamics models, which might be implemented with the aid of the MAUM procedure. MAUM can serve as a "field," defined in loose analogy with that concept as developed by Lewin [8]. This field could monitor the society with the idea that a certain minimal utility level, or a certain pattern of utility evaluations, might trigger a severe and sudden change in the structure of the society. If employed in this sense, the MAUM part of the model might lie dormant until some "critical point" was reached, at which juncture it would trigger changes in the model, and then resume its dormant state.

V. EXEMPLARY BEHAVIOR OF MODEL

In order to study the behavior of the model in this preliminary stage of development, essentially arbitrary values were assigned to the many model parameters, including the structural matrix (A) and the exogenous final demands of the labor sector for the products of the other five sectors (the elements of the vector X). It was desired to simulate a scenario in which societal conditions evolved in such a way that the most advantageous mix between labor and capital in the agricultural sector gradually shifted from labor towards capital. This is simulated in the model by exogenous adjustment of the "a" and "b" parameters in the Cobb-Douglas production function for the agricultural sector. The model then adjusts the mix to minimize the marginal cost. The variation in the Cobb-Douglas parameters simulates changes in technological development or societal changes of a

political or sociological nature.

Figure 3 depicts the time evolution of the outputs of the agricultural, industrial, and raw materials sectors, A, G, and R, respectively. Final consumption of all goods and services, the useful net output of the society, is held fixed so that changes in sector outputs reflect changes in demands for various goods by the sectors in order to meet the stipulated final consumption. Over the time span depicted, the agricultural output rises slightly (some agricultural output is used as an input to industry) while the outputs of the industrial and raw materials sectors rise markedly. The raw materials sector supplies major inputs to both industry and to the energy sector, whose output is not plotted but it is linked to the industrial sector.

The curve denoted RE depicts the dependence of the equilibrium output of the raw materials sector on time. Comparison of R and RE shows the extent of the discrepancy at any moment of the actual output of the sector and the production rate that would obtain if the society were allowed to come to equilibrium, with all parameters frozen at whatever values they had at that stage of societal evolution. Equilibrium values for system variables are available at all times through solution of the static input-output analysis equations. There is no guarantee that the system would, indeed, come to an equilibrium. It might oscillate for a long time and eventually approach equilibrium, it might smoothly approach equilibrium but with a long time constant, or

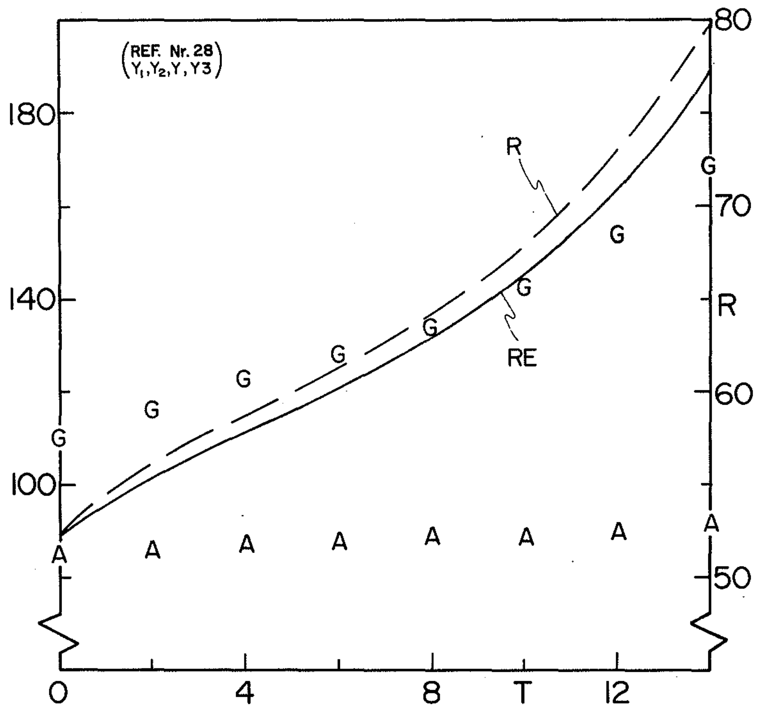

Figure 3. Time evolution of the system as the agricultural sector shifts from labor intensive to capital intensive. Points "A" represent output of the agricultural sector, points "G" output of the industrial sector, and curve R the output of the raw materials sector. The curve RE depicts the output of the raw materials sector if the system were to come to equilibrium, but the existence of a stable equilibrium requires separate investigation. (Right hand scale applies to R and RE.)

it might diverge from the indicated equilibrium. To establish the pattern of an approach to equilibrium, investigations of the sort depicted in Figures 1 and 2 are necessary at selected points along the time path, with appropriate parameters inserted into the system equations and with the exogenous time evolution of parameters in the Cobb-Douglas function suppressed. The Cobb-Douglas production function will still show a time dependence during the approach to equilibrium as the function adjusts itself to minimize marginal production costs. Very limited study suggests that this freedom of the production function to adjust during the approach to equilibrium may tend to destabilize the system.

For the conditions under which the exemplary runs were obtained, the system showed good stability when the agricultural sector was mostly labor intensive, with decreasing stability as the sector became more capital intensive. It is interesting to notice that the dynamic behavior of the system, as depicted in Figure 3, may appear quite smooth while an approach-to-equilibrium run may show no stable equilibrium. The oscillatory and divergent tendencies of the system may be masked by the continued time evolution of the system. Of course, a real system is always on a dynamic path of the type depicted in Figure 3, and real experiments of the type depicted in Figures 1 and 2 are not possible. Potentially grave implications may be associated with such a state of affairs. If the dynamic development of the system which has effectively masked an intrinsic instability ceases or changes significantly, the underlying instability may assert itself, which will almost certainly lead to undesirable societal conditions. The insidious aspect of the situation lies in the lack of obvious signs of instability from an inspection of time series associated with the history of the system. Hence, one may not recognize a need to embark on preventive stabilization measures until the time has passed when such actions might have been feasible.

Figure 4 depicts some employment information as the system travels along the time path already shown in the previous figure. Y9 is the employment of workers from both unskilled and skilled labor sectors. $U_{1,2}$ is the utility function, from the MAUM part of the model, as perceived by the unskilled labor constituency in its evaluation of unskilled labor employment. $U_{2,3}$ represents the utility function of the skilled labor constituency in its evaluation of skilled labor employment. The reactions of the two sectors as history unfolds would evidently be rather different. The demand for skilled workers improves monitonically, through an increasing demand for output from the service sector, while the demand for unskilled workers shows a decline followed by improvement.

Conditions in the unskilled category are probably more volatile than suggested by the behavior of $U_{1,2}$. Employment in the agricultural sector drops rapidly as the sector becomes more capital intensive. Employment in other sectors picks up,

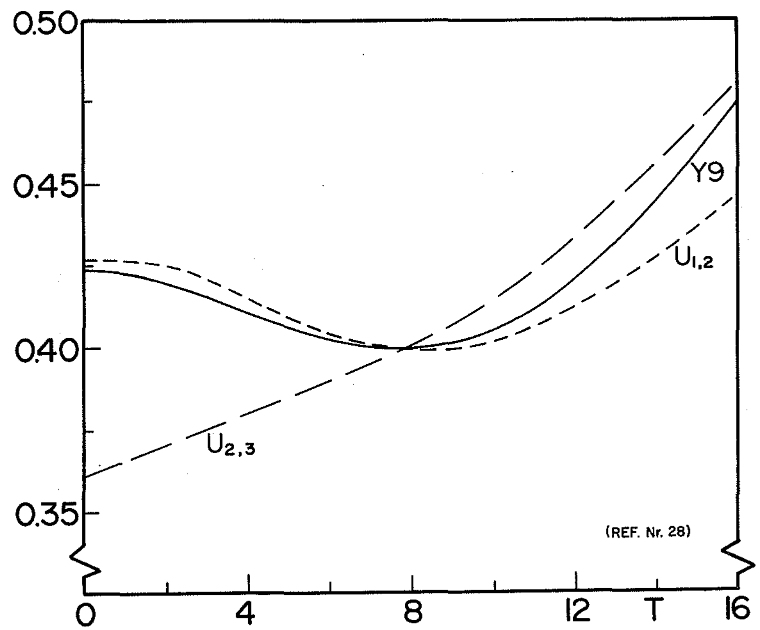

Figure 4. Behavior of overall employment, Y9, and utility functions indicative of employment conditions as viewed by the unskilled and skilled labor constituencies, $U_{1,2}$ and $U_{2,3}$, respectively.

but not rapidly enough to absorb the displaced workers until some time has elapsed. Finally, the unskilled labor sector will benefit from the change in orientation of the agricultural sector. However, the inadequacy of the oft quoted adage to not worry about displacement of labor is well illustrated by the model outputs. In the first place, there is a time lapse, and Lord Keynes has been quoted to the effect, "In the long run we won't be here." Moreover, structural unemployment will be worse than depicted in the plot because individuals cannot be interchanged willy-nilly between sectors. Again there is an insidious aspect to the implications of the system behavior. In the face of a declining overall picture for unskilled labor, there may be shortages for labor of the same genre in some sectors (or geographical locations) which can be interpreted as no labor problem at all or as a reflection upon the work ethic of the labor force. An obvious implication is the need for an effective program which moves displaced labor into more appropriate categories as a concomitant of the changing character of the society.

In Figure 5 several other model outputs are depicted during the time evolution of the system. P_1 is the smoothed price of agricultural sector output; the model has a fixed price base, viz. the price of labor. Vl_{10} is a measure of environmental loading, and it monitonically worsens as would be expected. The full implications of this are not presently incorporated into the model; no provision is made for extra efforts that might be needed in some sectors to control pollution. The index is used

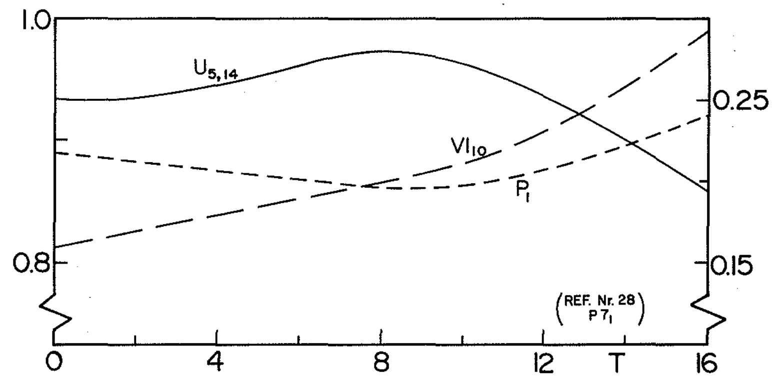

Figure 5. Behavior of the price of agricultural goods, P_1, environmental loading index, Vl_{10}, and the utility function $U_{5,14}$, which is a measure of the net output of the society to final consumption, normalized to the total labor force. (Right hand scale applies to P_1 and Vl_{10}.)

by MAUM as an attribute which figures in the evaluation of the society by some constituencies.

$U_{5,14}$ is the ratio of final outputs to the labor, or consumer, sector normalized to the total labor absorbed by the society. The outputs represent a weighted sum of components of the X vector. Here the weights are arbitrarily assigned, but a realistic assignment depends on an evaluation of relative utilities of diverse goods and services. This index is a type of efficiency index, an efficiency for employment of labor. Note that the weighted outputs used here are not related to a "gross national product." Outputs used elsewhere as inputs do not count. If there were outputs to control pollution, they would not count. Much of the "overhead" associated with output of the service sector does not count. This utility function has been included to direct attention to the need for indices which might not normally arise through the evaluation of the society by recognized constituencies. Here, the agricultural sector will experience greater labor productivity, and both labor sectors will finally realize more employment. However, in the end the society as a whole devotes more labor to the same useful result. In the model, this utility is considered only by the "planners," a probably non-existent constituency.

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