# Acceptance Dynamics

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## Abstract

We investigate irreversible acceptance dynamics, leading to phenomena typical for paradigm change not described by widely used reversible and static behavioral models, e.g., multistability, hysteresis, critical parameter values (tipping points), irreversible state changes. Based on a recycling model, we explain these phenomena and develop a simple, generic mathematical model describing the basic traits of acceptance dynamics. Analytical investigations and numerical experiments with this generic model show reproduction of the above mentioned phenomena. In addition, the generic model shows the interplay between internal and external forces. The relation of their time constants is shown to play a crucial role, leading to reversible elasticity dynamics or irreversible acceptance dynamics. Critical parameter values (tipping points) separating elasticity dynamics from acceptance dynamics can be deduced from the generic model. We show that some simplifications applied to the waste recycling model lead to the generic acceptance model. Further, the acceptance model is shown to comprise also the well-known Bass model describing market diffusion of new products. Finally, we discuss benefits of the generic model, its possible extensions to include additional phenomena, and its research implications.

**Keywords:** multistability, hysteresis, tipping points, critical parameter values, irreversibility, choice models, recycling model, generic properties, paradigm change, Bass diffusion model.

## 1. Introduction

Climate change and energy supply issues are triggering global social transformation processes that are based on new technologies, such as energy efficient low carbon vehicle or building technologies. In order to avoid costly, autonomous and radical change processes induced by market forces, key decision makers envision an ecological and effectively managed incremental pathway. Therefore, adequate transition management models are crucial, especially to increase the understanding of processes that influence the acceptance of new technologies.

We define *acceptance* as the act of adoption, with approbation being a function of the attitude, the subjective norm and value system, and the perceived behavioral control (Ajzen and Fishbein 1980, Ajzen 1991). *Acceptance dynamics* describes stabilizing social norm building processes that consolidate observed behavior patterns, and explains adjustment delays of and efforts for behavioral change processes. We all know from personal experience that acceptance, either of new routines or technologies and products, is a complicated process depending on numerous parameters and being subject to dynamics we normally cannot understand, sometimes not even when related to our own decisions (for individual decisions see, e.g., Mathieson 1991). From this point of view, our title *acceptance dynamics* seems to address an intractable topic. However, our intention is to model acceptance dynamics averaged over a large population segment, rather than of single persons. With this simplifying condition, our problem becomes loosely related to widely used decision, choice or marketing models, such as, e.g., logit, probit, or generalizations of them (K.E. Train 2003). These models approximately describe variations of mean choices made by a population segment when attributes of products within a given product assortment change. A simple example would be the choice between different transportation systems (car, railway, bus, bike) characterized by attributes such as price, traveling time, number of bus/train connections per day, distance of next bus/train station, etc. The coefficients needed for these models can be derived from statistical surveys, from the literature, or from educated guesses. A precondition for successful applications of these models is constancy of these coefficients, i.e., the general attitude of people against the transportation systems taken into account does not change, the investigated overall system remains in the same action paradigm (Kuhn 1962, Dosi 1982). In this respect, these models are static and reversible: if, e.g., the price for gasoline (attribute for cars) raises and later-on falls to the previous level, the number of car users temporarily decreases and reaches again the same previous level. The state of the system is a function of the attributes only and never depends on its specific time evolution in the past, in the organizational learning literature known as *single loop learning* (Argyris 1992, 1994, 1999, Argyris & Schön 1996). However, if people adapt to the transient situation of high gasoline prices and learn to value the advantages of public transport, the coefficients change and the system finds a different equilibrium after relaxation of the gasoline prices to the previous level (for the transition to innovative drive technologies see, e.g., Janssen et al. 2006, Struben & Sterman 2006). Such processes are similar to the *double loop learning* concept (Argyris 1992, 1994, 1999, Argyris & Schön 1996) and are a weak form of paradigm shift (Kuhn 1962, Dosi 1982) involving endogenous preference and value change. Due to the gradual change of the preferences of different already existing routines, this process is called *continuous change*. At this point, we leave pure choice models and enter acceptance dynamics of *discontinuous change*, as shown in Fig. 1. In contrast to continuous change, *discontinuous acceptance dynamics* comprises phenomena such as multistability, hysteresis, critical parameter values (tipping points), and irreversibility. It goes beyond already existing (choice-) routines by involving the establishment of new evaluation processes and behavior patterns.

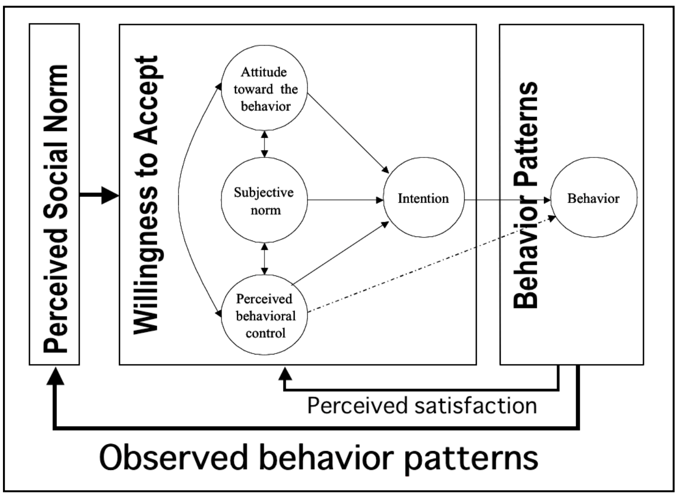


**Fig. 1:** Discontinuous acceptance dynamics of paradigm change arises in situations, where behavior patterns influence the perceived social norm that changes the overall willingness to accept. In contrast to the inner *perceived satisfaction* loop, which reflects a continuous and reversible goal seeking mechanism, the outer *observed behavior patterns* loop explains discontinuous and practically irreversible dynamics, since it can stabilize the system in different states. In order to change a social norm, new behavior patterns must become obvious, often requiring external stimulation and inducing transition costs.

In the present paper, we will investigate the generic dynamics of acceptance, i.e., we will abstract from all specific properties of real systems and keep only their general common structure leading to the above mentioned basic phenomena. A more detailed explanation of the term *generic* is given in the discussion (section 8). After a definition of these phenomena from the viewpoint of nonlinear dynamical systems (section 2), we will illustrate them within the framework of a waste recycling model developed by Silvia Ulli-Beer (2006) on theoretical and experimental grounds (section 3). Based on an abstraction process, we will then deduce the same general phenomena from a simple mathematical model, describing a familiar physical process involving a light ball rolling downhill (section 4). With analytical investigations and numerical experiments, we will discuss the parameters of our generic system interpreted within the framework of acceptance dynamics (section 5). Among these parameters, we will find time constants characterizing the typical (endogenous) system response time (delay effects) and the duration of adjustment processes established by external (policy) interventions (e.g. subsidies, taxes, etc.). To illustrate our concept of a ball rolling downhill, we show that the above mentioned waste recycling model, after some simplifications, can be transformed into the presented generic acceptance model (section 6). To give a link to the widely used Bass diffusion model, we show that our generic acceptance model is also able to reproduce this well-known model (section 7). The understanding of acceptance dynamics in this way established, we will discuss research implications and conclusions towards the development of acceptance models (sections 8 and 9).

## 2. Generic acceptance dynamics as self-organized nonlinear phenomenon

The most obvious and outstanding property of biological and social systems, but also of many abiotic physical and chemical systems, is their capability to produce macroscopic structures seemingly "out of nothing". Famous examples are hexagons in the purely physical Rayleigh-Bénard convection (Lorenz 1963, Assenheimer & Steinberg 1996) that can be observed also in the atmosphere or the Belousov-Zhabotinskii chemical reaction (Zhabotinskii 1967, Field & Noyes 1974) oscillating between two states (e.g. a transparent and a colored state). The first example is self-organization in space and the second in time. In biology, growing, healing, adaptation, and anastasis are typical aspects of self-organization (Mainzer 1997). In the social sciences, self-organization gets visible in fashion trends, the evolution of social norms, value and belief systems, languages, or routines. In the following, we will define properties of self-organized nonlinear systems that we consider being important aspects of acceptance dynamics, namely multistability, hysteresis, structural instability, and abrupt change.

### *Multistability*

A system able to spontaneously organize itself, normally finds different quasi-stable states for a given set of fixed parameters. This property is illustrated with an animated simulation of a waterwheel with 12 leaking buckets, a magnetic brake and a water-supply tub located exactly over the rotation axis (Gassmann 1996). With the same parameter values (water supply rate, leaking rate, brake strength, moment of inertia), this waterwheel shows five different quasi-stable states in certain regions of parameter space: a stationary rotating state clockwise or counter-clockwise, a non-stationary clockwise or counter-clockwise rotating state with oscillating velocity, and a non-rotating oscillating state. The term *quasi-stable* points to the fact that external forces can induce phase transitions into another state. In systems with non-moving quasi-stable states (e.g. a ferromagnet: an iron bar that can be magnetized in one or the opposite direction by an external magnetic field), these are normally called *fixed points* in state space, *locally stable equilibrium points* or simply *local equilibria*. Using the more general term of *attractor*, we can distinguish three different types of attractors: point attractors (local equilibria of the ferromagnet), periodic attractors (waterwheel) and chaotic or strange attractors (Lorenz-attractor). For an illustrative example in the framework of system dynamics, reference is made to stable and unstable equilibria in the market penetration process of hydrogen vehicles (Struben 2004, p.27ff.). The entity of all starting points in *state space* for trajectories leading to a specific attractor are called *basin of attraction* for this specific attractor. In simple cases, the different basins of attraction have smooth *boundaries* (with a dimension one unit lower than the dimension of state space). For one variable (i.e. a one-dimensional state space), these boundaries are isolated points. For a two-dimensional state space, basins of attraction can be patches reminiscent of different districts on a town map. In *parameter space*, *stability regions* for different quasistable states can overlap. For one parameter and two possible quasistable states, the overlapping multistable region consists of an interval limited by two critical parameter values (also called *tipping points* according to Sterman 2000, p. 305 ff.). For two parameters, stability regions for different states are analogous to different layers in a geographical information system (GIS) concerning, e.g., soil type, district, land use type. Fig. 2 gives an example for a two-dimensional parameter space. If one parameter is held constant, a one-dimensional parameter space with tipping-points results.

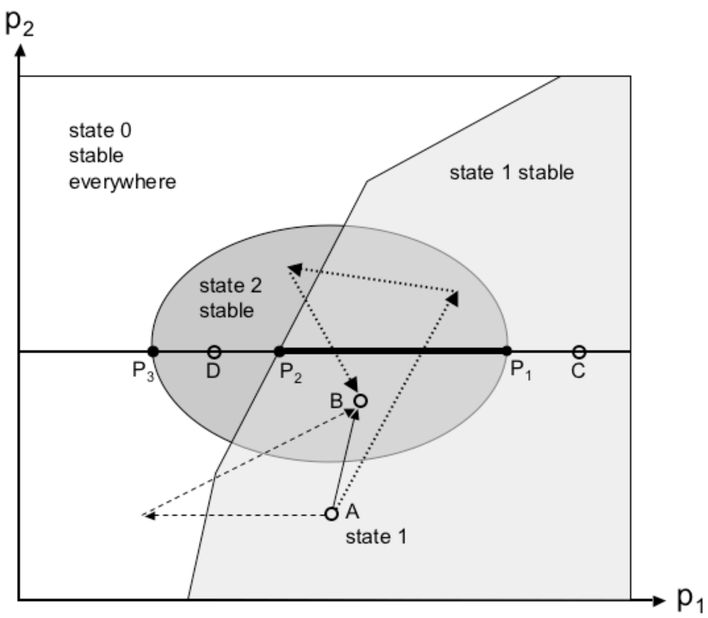


**Fig. 2:** Two-dimensional parameter space with overlapping stability regions for three different quasistable states. Starting at A in state 1, the system will arrive at B in state 1, 0 and 2 or 0 when paths along the continuous, dashed and dotted lines are chosen, respectively. This shows that history plays a decisive role. If parameter $p_2$ is fixed, the resulting one-dimensional parameter space $p_1$ shows three tipping points $P_1$, $P_2$, $P_3$. The interval between $P_1$ and $P_2$ (thick line) is the hysteresis region for variation of parameter $p_1$ between C and D (see text).

*Hysteresis*

A property immediately following from the existence of several simultaneously stable attractors is *hysteresis*, clearly displayed in *parameter space* (see Fig. 2). Let our system be in state 1 at point C and consider a slow variation of $p_1$ along the indicated line towards point D. As soon as $p_1$ meets the tipping point $P_2$, the system undergoes a phase transition into state 2. If the same path is followed backwards, the system will change back to the original state 1 at the tipping point $P_1$, i.e., the state of the system within the multistable region between $P_1$ and $P_2$ is history dependent, rather than being a function of $p_1$. The physical example coining the term *hysteresis* is the ferromagnet (at constant temperature $p_2$), where the parameter $p_1$ is the strength of an external magnetic field, and the system variable is the magnetization of the ferromagnet bar. For switching the bar-magnetization from negative to positive, a positive external magnetic field $>P_+$ is needed. To reverse the magnetization to negative, a negative field $<P_-$ is needed (for symmetry reasons $P_- = -P_+$). As long as the external field is between $P_-$ and $P_+$, magnetization of the bar resists the external magnetic force without changing direction (i.e. without changing state). In other words, both directions (states) of the bar magnetization are stable for external fields between $P_-$ and $P_+$.

*Structural instability*

The above explained critical parameter values (tipping points) lead to another property called *structural instability*. When a system is operated near stability boundaries in parameter space, small variations of one or several parameters might cause the system to change state. For systems with one parameter only, its critical values can be determined experimentally or theoretically and stable operation of the system (i.e. robust policies) can be guaranteed by avoiding parameter excursions near to the critical values. If there are several parameters, more-dimensional boundaries exist in parameter space, as depicted for two parameters in Fig. 2. In general, a slow and gradual parameter variation can guide the system near to a boundary, where the actual state looses stability. In this situation, a small variation of a system parameter can cause an abrupt phase transition into another state. If we are not aware of the respective stability boundary, as is normally the case for systems with many parameters, such a state transition comes as a surprise.

*Abrupt change*

The different quasi-stable states in most nonlinear systems exclude each other and therefore, a mixture of states is impossible. For the nonlinear waterwheel, no combination of stationary rotation and chaotic movement is possible. In contrast, for the linear system of oscillating strings, superposition of different oscillation modes is possible, giving a music instrument its characteristic sound. As different states in nonlinear systems cannot be combined, simple and short or long and complicated transitions between two such states arise. The short transitions are called *abrupt transitions* to indicate that a state (e.g. chaos) suddenly ceases and is replaced by another state (e.g. stationary rotation). The long transitions are called *transient chaos*, because their trajectories and lengths are highly unpredictable. The waterwheel shows transient chaos in several regions of parameter space. Interestingly, in specific regions, these chaotic transients can be shortened by several decades with the application of irregular (noisy) disturbance of a variable or parameter (Gassmann 1997a). In our context of acceptance dynamics, abrupt transitions seem to be the rule, rather than transient chaos. For chaotic systems see also System Dynamics Review 1988 and Rasmussen et al. 1985.

*The impossibility of causal analytic investigations*

The basic structure of self-organizing systems includes at least one feedback-loop. In most systems, many interaction-chains form closed loops and therefore, we find the notion *circular causality* to be more adequate than feedback (see also Mainzer 1997). Especially in biological systems with myriads of chemical substances intricately linked to each other by innumerable chemical reactions, speaking of causes for certain effects (e.g. illness, mental disorder) is a radical simplification in many cases. Which parameter combination (e.g. stress, nutrition, bacteria, etc.) is pushing a system over which unknown boundary in parameter space is subject of speculation in all but a few clear situations (e.g. attack of an aggressive virus). In social systems, there might be a fewer number of interactions when compared to biological systems, but still enough to lead to the same conclusions. Inadmissible simplifications can lead to mental models suggesting counter-productive actions and unintended consequences. To avoid such mistakes, models can help to better understand systems with circularly causal interaction chains (Dörner 1980, 1993, Gassmann 1997b).

## 3. The example of waste recycling

To illustrate the above mentioned properties of nonlinear systems, we present a model developed by one of the authors (Ulli-Beer 2006) to simulate waste separation and recycling by citizens with a garbage bag charge imposed. A special weight was put on the formulation of the decision process guiding citizens' behavior to separate by addressing interactions between contextual and personal factors (Kaufmann-Hayoz & Bättig et al. 2001). It was assumed that people decide once to separate or not and so initiate a new routine (see e.g. Dahlstrand & Biel 1997). This implies that people can be divided into two main groups: a group x willing to separate and a group 1-x not willing to separate. In each group, subgroups were distinguished mediating the transition of individuals between the main groups. Fig. 3 shows the main structure of the model resembling an electronic flip-flop, i.e., the basic bistable device of computers. If the majority of people are willing to separate (x near to unity), the perceived social norm exerts a pressure on the remaining fraction 1-x of non-recyclers and motivate them (lower processes in Fig. 3). These processes stabilize the system on the recycling side ($x \approx 1$). Analogously, if the majority of people are not willing to separate (x near to zero), the perceived norm will drive remaining recyclers to loose their motivation to continue and so stabilizes the system on the non-recycling side ($x \approx 0$). The garbage bag charge helps to stabilize the recycling state of the system by compensating time investments in separating activities and other inconveniences (e.g. unattractive collection points because of crowding) imposed by recycling.

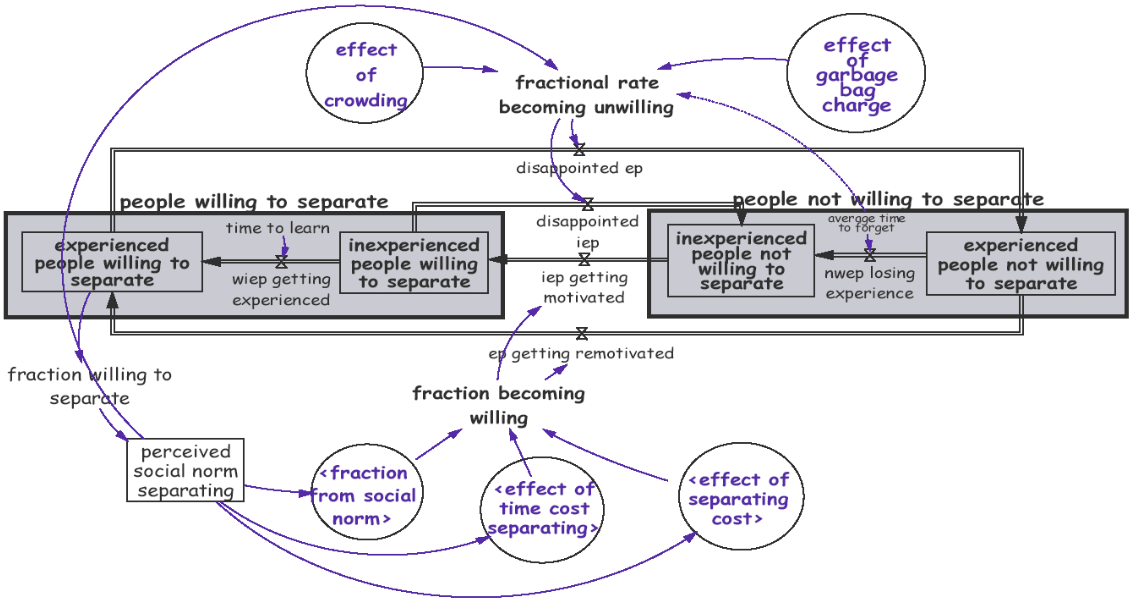


**Fig. 3:** Structure of the model to simulate waste separation and recycling. Changes in citizen's willingness to separate lead to stabilization of the system either on the recycling side or the non-recycling side (ep: experienced people, iep: inexperienced people, wiep: willing inexperienced people, nwep: not willing experienced people). From S. Ulli-Beer (2006, p. 96).

The above described generic properties of nonlinear systems can be found in this waste separation and recycling model in the following way:

*Multistability*: The model has two locally stable states: Nearly nobody recycles ($x \approx 0$) or almost everybody does it ($x \approx 1$).

*Hysteresis*: Investigating transients from state $x \approx 0$ to state $x \approx 1$, we find a critical parameter value $P_1$ (tipping point) for separation time cost, enabling the system to perform the transition to $x \approx 1$, describing a successful recycling initiative. In social systems, the reverse process is of minor interest, and the critical parameter value $P_0$ for the reverse transition is not important. Nevertheless, the hysteresis region $P_1 \ldots P_0$ exists.

*Structural instability*: $P_0$ and $P_1$ are the critical parameter values. If we do not know them or are even not aware about their existence, the phase transition from $x \approx 0$ to $x \approx 1$ comes as a surprise, turning a recycling initiative in a mere trial and error adventure.

*Abrupt change*: As soon as a certain *critical mass* $x_{crit}$ is reached resulting from the applied external force (garbage bag charge), the system changes fast, creating a fast growing amount of recovered waste, because the applied force (garbage bag charge) and the internal dynamics (social norm building process) mutually support each other.

If we abstract from all details of the stabilizing processes, we can understand the basic features of the dynamics by imagining two adjacent valleys with their floors at $x \approx 0$ and $x \approx 1$. The system state is represented by a light air-inflated ball rolling over this orography. External forces (garbage bag charge) drive the ball and might induce a transition from one to the other valley floor. A mathematical model for this generic process will be developed in the next section. We will show that this metaphor captures the essence of acceptance dynamics.

## 4. The mechanical light-ball model as a metaphor

The main generic features defined in section 2 and illustrated in section 3 are found in multistable systems with two or more local equilibria. The physical, social, biological or ecological processes stabilizing these local equilibria can be of very different nature. In vegetation dynamics, interactions between neighbors lead to quasistable patterns with two or more plant types (Gassmann et al. 2000, 2005) reminiscent of patterns formed in multicultural cities. In biological systems, symbiosis is an important stabilizing process based on the production and use of mutually useful substances. In closed physical systems, minimization of their total energy is the governing process in many cases.

To describe generic features of acceptance dynamics, according to our previous considerations in chapter 3, we basically need a bistable system allowing transitions from one stable state to another stable state, which are induced by external forces. To explain the generic behavior of systems with different equilibria (i.e., stable or unstable states), the metaphor of a ball rolling in a bowl or other nonlinear orography is often used in physics, chemistry, or biology, because it demonstrates a very basic phenomenon in a way everybody is able to relate to personal experience from everyday life (e.g., in Sterman 2000, p. 351, this metaphor is used for the explanation of *path dependence*). So, the light ball system discussed in the following is not directly related to acceptance, it is just another interpretation of the mathematical equations we use to describe a bistable system. We consider a light air-inflated plastic ball with mass m moving downhill with velocity u. Its dynamics can be formulated with the notion of *potential energy* V(x) (in the physical literature, V(x) is called *gravitational potential*) in the following way:

$$
\begin{aligned}
V(x) &= m \cdot g \cdot h(x) \\
m \cdot \frac{du}{dt} &= -\frac{dV}{dx} - \gamma \cdot u \qquad (1) \\
u &\equiv \frac{dx}{dt}
\end{aligned}
$$

where g is the gravitational acceleration, t is time, and h(x) describes the height of an imagined orography with one horizontal dimension x. Multiplication of the slope dh/dx by -mg gives the force -dV/dx accelerating the ball downhill*. The term $-\gamma \cdot u$ describes the frictional braking force of the air (according to Stokes' law, this frictional force is proportional to velocity u). Our experience tells us that such a light ball, after a short initial acceleration phase, rolls downhill at a constant speed, only

* Relations (1) are an approximation for small slopes. For an infinite slope, the force according to (1) becomes infinite, in contrast to the physical force for the free fall being -mg. However, this discrepancy for steep slopes does not disturb our metaphor, because in real applications, slopes are normally small.

depending on the slope. To simplify our dynamics (1), we neglect therefore the inertia term by setting

$$m \cdot \frac{du}{dt} = 0 \tag{2}$$

and find the approximative dynamics:

$$\boxed{u = -\frac{dV}{dx}} \tag{3}$$

The parameter □ has been set to unity because it does not affect the character of the solutions of (3). In the physical literature, this approximation is called *overdamped limit*, because the respective system approaches an equilibrium point gradually rather than with damped oscillations. This property can be demonstrated, e.g., for the equilibrium point in a quadratic potential $V=x^2$ situated at $x=0$. Introduction of this most simple nonlinear potential into (3) gives

$$u \equiv \frac{dx}{dt} = -\frac{d}{dx}x^2 = -2x \tag{4}$$

with the solution

$$x(t) = x_0 e^{-2t} \tag{5}$$

where $x_0$ is the initial position of the ball and t is time. (5) describes a trajectory approaching the equilibrium point $x=0$ gradually, without oscillations. Mathematically, the ball would need infinite time to reach $x=0$, but for practical applications, $t=3$ is already enough, giving a distance to zero of less than 1% of the initial value $x_0$.

For a multistable system, we need at least two stable equilibria, described by a double-well potential V(x) in form of a polynomial of 4th order:

$$\begin{aligned} V(x) &= ax^2\left\{x^2 - 2\square^2\right\} \\ -\frac{dV}{dx} &= -4ax\left\{x^2 - \square^2\right\} \end{aligned} \tag{6}$$

To prevent the ball escaping to infinity, we assume $a \geq 0$. At $x=0$ we find an unstable equilibrium, and two locally stable equilibria are located at $x=\pm\square$. Combined with (3), we get the following dynamics:

$$\frac{dx}{dt} = -4ax\left\{x^2 - \square^2\right\} \tag{7}$$

To assign simple meanings to the two parameters a and □, we define two new parameters □ and □ (their meanings will be explained below):

$$\begin{aligned} \square &= \frac{1}{8a\square^2} \\ \square &= a\square^4 \end{aligned} \tag{8}$$

and write the dynamics (6, 7) with these new parameters:

$$\frac{dx}{dt} = \frac{x}{2\tau}\left(1 - \frac{x^2}{8\Delta}\right) + F(t)$$
$$V(x) = \frac{x^2}{8\tau}\left(\frac{x^2}{8\Delta} - 2\right) \qquad (9)$$

In addition, an external force F(t) has been introduced. The stable equilibria (with F=0) are now located at

$$x_s = \pm\sqrt{8\Delta}$$

The parameter Δ **is the height of the "activation potential"** (e.g. the unstable equilibrium) with its top at $x_u$=0 lying in between the two stable equilibria at $x_s$, as can easily be verified:

$$V(x_u) - V(x_s) = 0 - \frac{8\Delta}{8\tau}\left(\frac{8\Delta}{8\Delta} - 2\right) = \Delta \qquad (10)$$

**τ is the endogenous time constant of the system** near its stable equilibria $x_s$. This can be verified by linearization of the dynamics around $x_s$. To this purpose, we replace x by the new coordinate ξ being the distance from $x_s$:

$$\xi = x - x_s \qquad (11)$$

After introducing (11) into (9), we linearize the dynamics for small ξ and get the approximative differential equation for the trajectory in the neighborhood of the stable equilibria

$$\frac{d\xi}{dt} \approx -\frac{1}{\tau}\xi \qquad (12)$$

with the solutions

$$\xi(t) \approx \xi_0 e^{-\frac{t}{\tau}} \qquad (13)$$

$\xi_0$ is the initial position of the ball at t=0. By definition, τ is the time constant for the relaxation of the system to its equilibrium point ξ=0, what had to be shown.

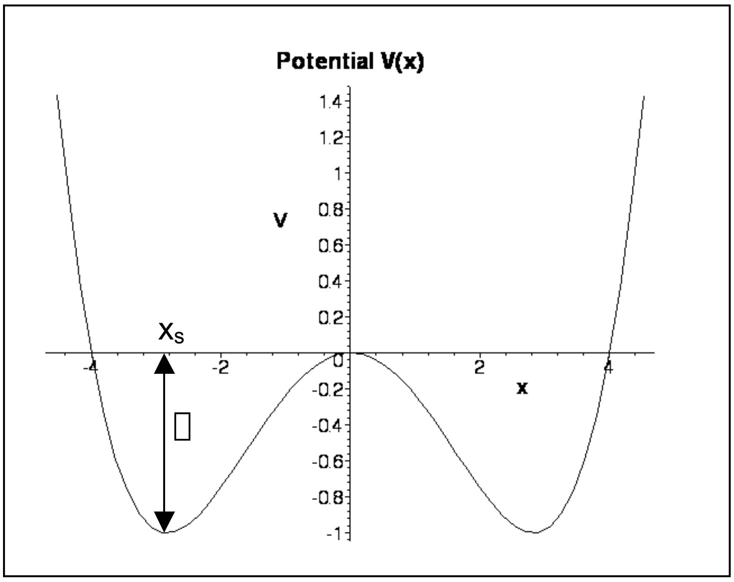


**Fig. 4:** Potential V(x) according to (9) for Δ=τ=1. The stable equilibria are at $x_s$=±2.828, the unstable equilibrium is at $x_u$=0. The potential is symmetric against the axis x=0.

The potential V(x) according to (9) is shown in Fig. 4 in graphical form. To illustrate one of the basic phenomena arising from such a double-well potential, namely hysteresis, we investigate the trajectory resulting from a periodic sinusoidal force F(t) = $F_0\sin(2\pi t/T)$ with period T and amplitude $F_0$. After an initial aperiodic sequence, the system will find a periodic trajectory, i.e., the system state x(t) will be a (bivalent) function of F(t). An example of such a *hysteresis curve* for a **slowly varying force F(t)** is shown in Fig. 5. "Slowly varying" means that the period of the external force (T=60) has been chosen much longer than the internal time constant of the system ($\tau$=1), resulting in a *quasi-stationary hysteresis curve*, i.e., the system is in many time points t near the equilibrium it would settle in when the variation of the external force would be stopped at time t, i.e., F(t')=F(t) for all t'>t. The verification of this behavior in Fig. 5 is especially simple for F(t)=0: The two cutting points of the hysteresis curve with the vertical axis (F=0) indeed lie very near to the stable equilibria $x_s$=±2.828 (see Fig. 4). As soon as the external force is able to move the system past the steepest slope of V(x), halting this force would drive the system to the vicinity of the equilibrium point $x_s$ lying ahead of the trajectory. According to the first inequation of (17), the critical force is ±0.544, i.e., the force-intervals from -0.544 to +2 (=+$F_0$), and from +0.544 to -2 (=-$F_0$), indicated in bold in Fig. 5, are the quasi-stationary parts of the hysteresis curve. The remaining dynamical (steepest) parts of the hysteresis curve cannot be understood by stationary reasoning. The force range between ±0.544 is the bistable region, i.e., for forces within this interval, two different stable equilibria exist.

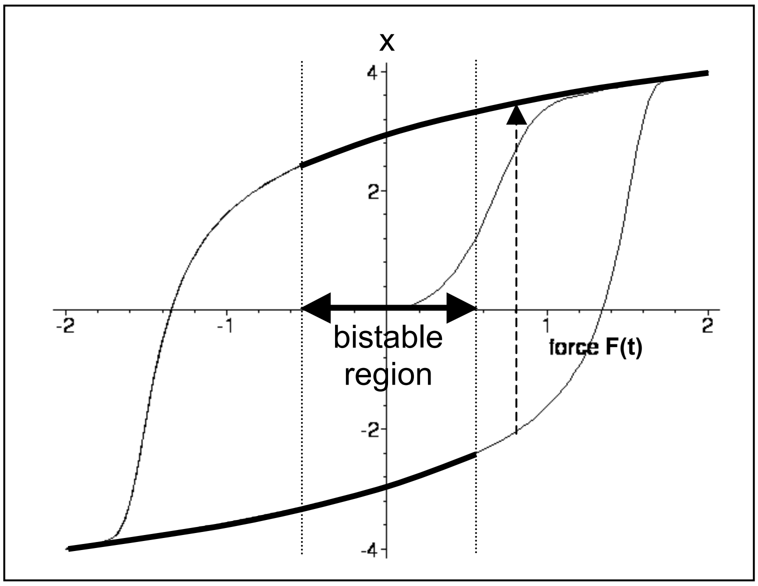


**Fig. 5:** Quasi-stationary hysteresis curve for the potential given in Fig. 4 with T=60>>$\tau$=1. The simulation started at (0/0), giving an aperiodic initial trajectory, before the system response is periodic. Note the ordinate intersection points (0/±2.828) showing that the system is able to follow the variation of the external force F(t). Only the bold portions of the hysteresis curve are quasi-stationary. Halting the force on the other portions of the curve would drive the system to the respective stationary part, as indicated by the dashed arrow for F=0.8. The interval of forces between ±0.544 is the bistable region (double arrow), i.e., for forces within this interval, two different stable equilibria exist.

For **fast varying external force F(t)** with period T in the order of $\tau$, the hysteresis curve changes shape: The system cannot longer follow the variation of F(t) and oscillates around one of the stable equilibria as shown in Fig. 6. The important message is that a force of the same strength ($F_0$=2) as applied in Fig. 5 does not induce a state transition when it varies at a time scale comparable to the system's internal (endogenous) time scale, because the internal time constant acts as a kind of inertia keeping the system near its actual equilibrium point.

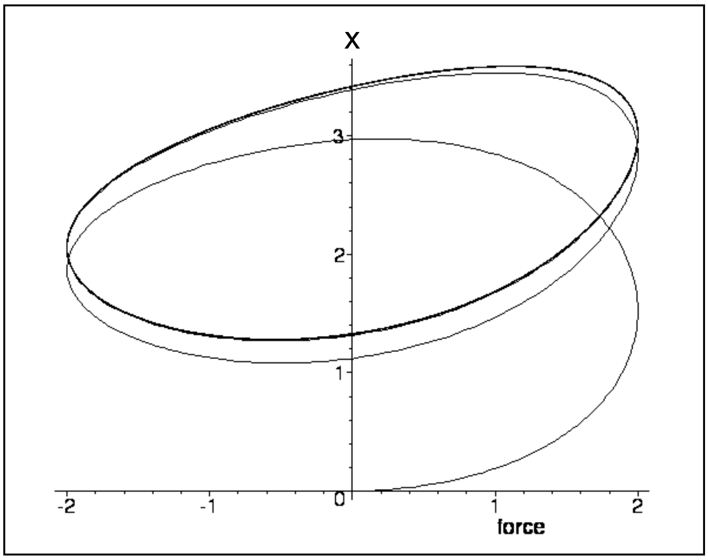


**Fig. 6:** Dynamic hysteresis curve with $F_0$=2, T=4, □=□=1 showing oscillation around the equilibrium point at x=+2.828. The trajectory circles around the positive equilibrium of the potential V(x), because it started at x=0 with the force increasing from zero to positive values.

To induce transitions between the two equilibrium states, a fast varying force has to be much larger as shown in Fig. 7.

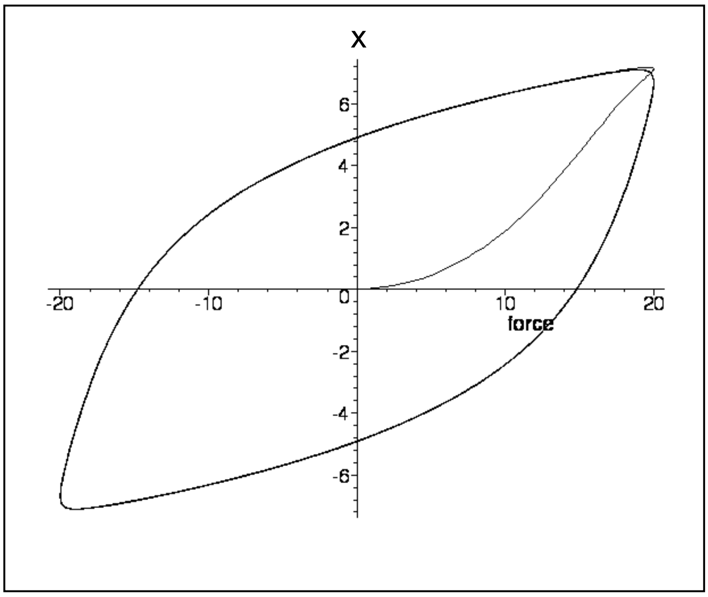


**Fig. 7:** Repetition of the simulation shown in Fig. 6 but with the force increased tenfold ($F_0$=20, T=4, □=□=1). The resulting dynamical hysteresis curve is different from the quasi-stationary curve shown in Fig. 5, because the system follows the variation of the external force F(t) with a non negligible delay resulting from its internal time constant □=1 being in the same order of magnitude as the period of the external force T=4.

## 5. Generic acceptance model

The simulations presented in the previous section illustrate the generic dynamics of a light ball in a double-well potential submitted to periodic external forces. For the investigation of acceptance dynamics, we are not interested in periodic trajectories. Rather, we ask for aperiodic trajectories representing state transitions beginning in the left equilibrium point and leading to the right one. First, we apply a **small constant**

**force $F_0$** and ask for the deviation □ from the force-free equilibrium point at position $x_s=-(8□□)^{1/2}$:

$$□ = x + \sqrt{8□□} \qquad (14)$$

We substitute x in (9) by □ according to (14) and ask for the stationary solution by setting the time derivative to 0. This leads to the following relation between $F_0$ and □:

$$□\left\{□^2 □ 3□\sqrt{8□□} + 16□□\right\} = 16□^2□F_0 \qquad (15)$$

For small □, the bracket in (15) reduces to the constant term and we get approximately:

$$\boxed{□ □ F_0 \cdot □} \qquad (16)$$

An example for a dynamical simulation with $F_0$=0.1 is given in Fig. 8. Realize that the bracket of (15) reads for □=□=1 and □=0.1: {0.01 - 0.85 + 16}. Obviously, the first two terms are negligible against the constant third term!

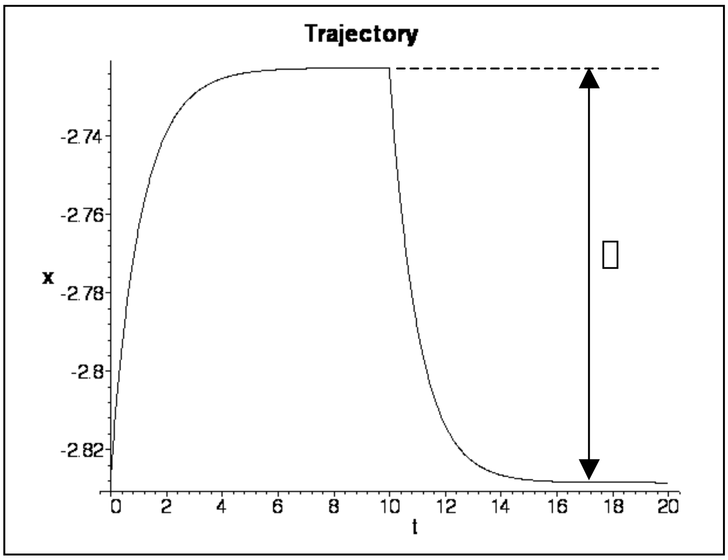


**Fig. 8:** Trajectory beginning at the left equilibrium ($x_s$=-2.828) for a constant external force $F_0$=0.1 in the time interval t=0...10. For t>10 $F_0$=0. A new equilibrium position is found after a few □ (□=1) with a distance □ from the stable force-free equilibrium point of approximately F· □=0.1. The simulated distance between the two equilibria is about 0.105 as indicated by the double arrow.

In the social sciences, the reaction of a social system on small variations of prices, taxes, subsidies, etc. are often described by elasticities e, i.e., the variables are supposed to be related by a power law $y=cx^e$. Our relation between force $F_0$ and deviation □ is linear, equivalent to an elasticity e=1, and our constant □ is the proportionality coefficient (c in the relation $y=cx^e$). Note that in our light-ball model, □ describes also the endogenous relaxation time, i.e., □ is at the same time a system-internal time constant and a proportionality coefficient for the limit of small forces. This is a generic result, because every potential V(x) can be approximated by a polynomial of order 2 around each equilibrium point: $V(□)=□^2/(2□)$ with □ being the distance from the respective equilibrium point $x_s$. From this approximation, both meanings of □ follow immediately, without recurrence to our full $4^{th}$ order potential. Realize also that the system always relaxes to the original equilibrium (here to

$x_s$=–2.828) when all external forces are relieved, i.e., the system does not learn nor is there any paradigm change in this fully reversible case.

For **large forces**, the above explained approximation does not longer hold and analytical calculations become difficult due to the non-linearities of V(x). Here, simulations come into play and help us to understand the basic dynamics of the system. By imagining a light ball pushed by an external force over the central hill of the potential V(x) (see Fig. 4), it is obvious that a transition from the negative to the positive attractor (reminiscent of a paradigm change, acceptance, double-loop learning) cannot happen unless the external force F brings the system at least to the local maximum of the potential at x=0. From there, the internal dynamics drives the system downhill to either attractor without any force applied. To reach x=0 by application of a constant force $F_0$ during a time interval T, two conditions must be fulfilled: $F_0$ must exceed the largest opposing force exerted by the potential V(x) and T must be long enough to let the system reach x=0. The respective necessary conditions are the following:

$$F_0 > \max\square\frac{dV}{dx}\square = \sqrt{\frac{8\square}{27\square}} \qquad (17)$$
$$F_0 T > \sqrt{8\square\square}$$

The second condition represents the fact that for a flat potential (V(x)=0), the system state x would have to be shifted over the distance $\square x=(8\square\square)^{1/2}$, i.e., from the left equilibrium point to zero. As $u=F_0$ for this simplified system with V(x)=0, $\square x=uT=F_0T$. Due to the opposing force for a system with non-zero potential, $F_0T$ must be larger than $\square x$ as formulated in (17). Fig. 9 shows the simulated product $F_0 \cdot T$ (called transition cost, because it is the product of the applied force, e.g., subsidies, taxes, with the time during which the force must be active) as a function of the applied force that is necessary to bring the system to the critical point x=0. The graph clearly indicates when the relations (17) are good approximations: The minimum force necessary to induce a transition, described by the first inequality, holds for **small forces** (for this case, the second inequality gives far too low transition costs). For **large forces**, the potential gets negligible and the zero-potential limit (second inequality in (17)) becomes a good approximation (in this case of large forces, the first inequality is fulfilled anyway). Fig. 9 clearly demonstrates that the cheapest way (i.e. resulting in lowest cost) to induce a transition is to apply large forces (realistically around 4 to 6 times the minimum force). As soon as the critical point of the system (at x=0) is passed, its internal dynamics will drive it into the second equilibrium, and no external force is needed any more.

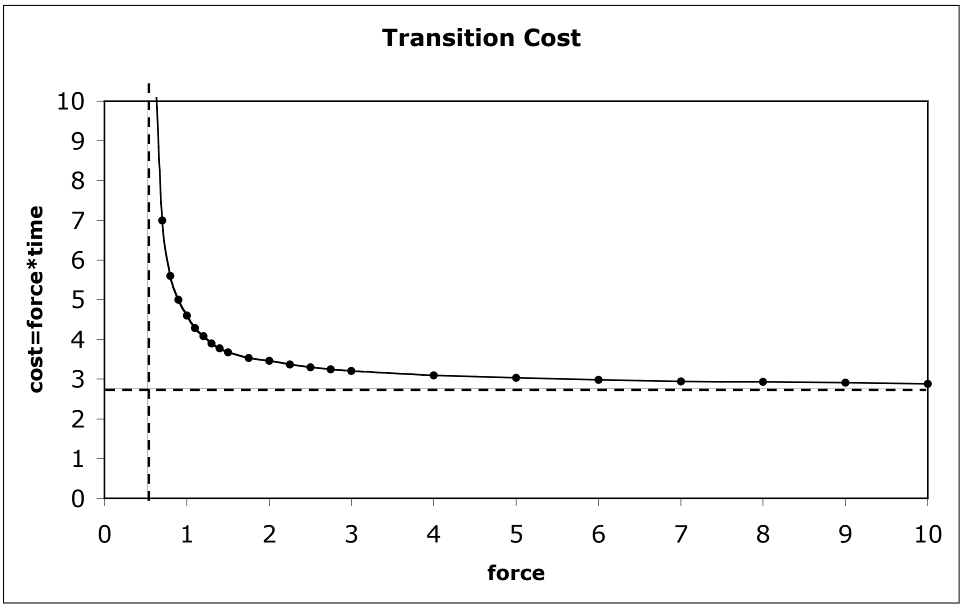


**Fig. 9:** Transition cost ($F_0 \cdot T$) versus force ($F_0$) for □=□=1 necessary to bring the system to the critical point x=0. The broken vertical line shows the first and the broken horizontal line the second condition of (17). To induce a state transition, a minimum force of about 0.5 is needed; the lowest transition cost of nearly 3 is approximated when large forces are applied.

## 6. The generic acceptance model as a simplified waste recycling model

To illustrate the realization of our potential V(x) within the framework of a „normal“ system dynamics model, we simplified the waste recycling model (cf. section 3) by lumping all functionally related parameters and reduced the number of groups from 4 to 2: adopters x and non-adopters y. The structure of this simplified model is shown in Fig. 10.

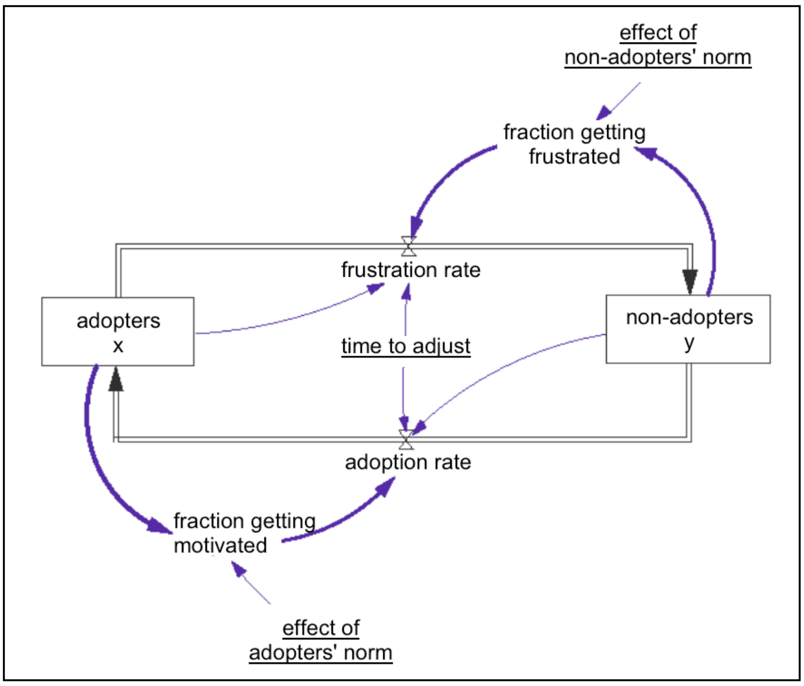


**Fig. 10:** Structure of the simplified version of the waste recycling model presented in section 3. Underlined text refers to parameters.

The 50 parameters and 33 nonlinear functions of the original model have been reduced to the two parameters P and $\tau$, and the two functions f(x) and g(y) with the following meanings:

P = overall population
$\tau$ = time to adjust
f(x) = influence of the adopters' norm on non-adopters
g(y) = influence of the non-adopters' norm on adopters

The dynamical equations of the simplified model are

$$\frac{dx}{dt} = p(x,y) - q(x,y)$$
$$\frac{dy}{dt} = q(x,y) - p(x,y) \qquad (18)$$

with the condition

$$P = x + y \qquad (19)$$

and the two functions being defined by

$$p(x,y) = \frac{f(x)/P}{\tau} y$$
$$q(x,y) = \frac{g(y)/P}{\tau} x \qquad (20)$$

The two dynamical equations (18) for the two population groups x and y, together with the condition (19), can be expressed by one dynamical equation for x, describing the balance of the adoption rate p(x,y) and the frustration rate q(x,y). These two rates are defined symmetrically with the functions f(x) and g(y), and involve the time constant $\tau$ (20). The influence f(x) of the adopters' norm on non-adopters vanishes for x=0, because there is no adopters' norm established without adopters. With only a few adopters, their influence is still negligible, suggesting a horizontal tangent f'(0)=0. With an increasing number of adopters, however, their influence gets important. The most simple functions f(x), and analogously g(y), that fulfill the three conditions, are quadratic polynomials:

$$f(x) = \alpha_1 \cdot x^2$$
$$g(y) = \alpha_2 \cdot y^2 \qquad (21)$$

The two new parameters $\alpha_1$ and $\alpha_2$ describe the strength of the effect of the adopters' norm and the non-adopters' norm, respectively. We apply the following normalization to further simplify the equations:

$$x' = \frac{x}{P}$$
$$y' = \frac{y}{P} = 1 - x' \qquad (22)$$
$$\alpha_1' = P \cdot \alpha_1$$
$$\alpha_2' = P \cdot \alpha_2$$

With the substitutions (22), the dynamical equations (18) take a simple form. For convenience, the dashes are omitted in the following:

$$\frac{dx}{dt} = \frac{\beta_1 + \beta_2}{\tau} x(1 - x)\left(x - \frac{\beta_2}{\beta_1 + \beta_2}\right) \tag{23}$$

Equation (23) can be transformed into the form of our generic acceptance model (without external force $F_0$)

$$\frac{dx}{dt} = -\frac{dV(x)}{dx} \tag{24}$$

giving

$$V(x) = \frac{1}{12\tau} x^2 \left\{6\beta_2 - 4\left(\beta_1 + 2\beta_2\right)x + 3\left(\beta_1 + \beta_2\right)x^2\right\} \tag{25}$$

This double-well potential (a polynomial of fourth order) has the following extremes:

$$\begin{aligned} V(0) &= 0 \\ V(1) &= \frac{\beta_2 - \beta_1}{12\tau} \\ V\left(\frac{\beta_2}{\beta_1 + \beta_2}\right) &= \frac{1}{12\tau}\left(\frac{\beta_2}{\beta_1 + \beta_2}\right)^3 \left(2\beta_1 + \beta_2\right) \end{aligned} \tag{26}$$

The first two extremes at x=0 and x=1 are stable minima and the third is an unstable maximum in between them. In general, the potential (25) is asymmetric, because the minimum at x=1 is above or below the x-axis, if $\beta_2$ is larger or smaller than $\beta_1$, respectively. We find the symmetric situation investigated in the previous section for $\beta_1 = \beta_2 = \beta$. **For this special case, the simplified waste recycling model becomes identical with the generic acceptance model** (with the exception of a non-important linear transformation of the x-coordinate) with the following choice of its parameters:

$$\begin{aligned} \alpha_g &= \beta / \tau \\ \gamma_g &= \frac{1}{32\, \beta / \tau} \end{aligned} \tag{27}$$

Symbols with subscript g refer to the generic model described in the previous section. (27) shows that the effective time constant $\beta/\tau$ is the only relevant parameter for this special symmetric case of the simplified waste recycling model. For the more general asymmetric case, the minimum of the potential at x=1 (waste is recycled) gets higher (i.e. less stable than the minimum at x=0, staying at V=0), and the "hill" separating the two minima increases and moves towards x=1, with growing $\beta_2$. This is plausible because, with increasing effect of the non-adopters' norm $\beta_2$, the recycling mode gets less stable and harder to achieve, and so it would need a larger external force (garbage bag charge) to reach and stabilize the recycling mode.

Our transformation of the waste recycling model into the generic form, that can be interpreted as the dynamics of a light ball rolling over a potential V(x), has shown that **the minima of V(x) are created by stabilizing feedback-loops**. In our case, the *perceived social norm*, involving the parameters $\beta_1$ and $\beta_2$, is the process shaping the potential: If most people separate waste, the non-separators are motivated to do so

(□$_1$); if most people do not separate, the separators loose their motivation and stop separating their waste (□$_2$).

Our analysis shows further, that at least one of the two functions f(x) and g(1-x), describing the effect of the perceived social norms, must be nonlinear to be able to lead to two simultaneously stable minima of the potential V(x). If both functions are assumed linear, the respective potential is a third order polynomial having only one global minimum at x=0 or at x=1 for the slope of f being smaller or larger than the slope of g, respectively. For this case with linear functions f and g, the basic character of the system would be different: As soon as the effect of the adopters' norm would have a larger slope than the effect of the non-adopters' norm, the system would undergo a transient from x=0 (not recycling) to x=1 (recycling), without any external force (garbage bag charge) needed. For the case that the slope of the effect of the adopters' norm would be smaller compared to the one of the non-adapters' norm, a garbage bag charge would push the system towards x=1, but no paradigm change would occur, stabilizing this state: As soon as the charge would be relieved, the system would fall back to x=0. This analysis demonstrates, that **one of the most important decisions during the modeling process is the choice of the shape of the norm-functions f and g**. In the model validation process, observational evidence suggesting linear or nonlinear norm-functions would therefore be of prime importance.

A last remark concerns the discrepancy of the numbers of parameters and functions between the full model and the generic model. In every model useful for practical purposes, a large number of parameters are needed, because the important effective parameters (in our case □, □$_1$ and □$_2$) must be related to practically relevant input parameters. The strength of the generic model, however, is not its application to simulate observed processes, but to help us understand its basic behavior and to give us an idea of its solution manifold. It contains only a very limited number of effective parameters and functions, and so shows us **the relevant combinations of parameters and functions** defining the trajectories to be expected.

## 7. The Bass model as a simplified acceptance model

Generic models can be useful to detect common properties of seemingly unrelated models. We demonstrate this property of generic models by showing that our generic light-ball acceptance model comprises, e.g., the widely used Bass model (Bass et al. 2000). We define our system state x as the number of items sold until time t and concentrate on the portion of the potential V(x) between x=0 and $x=x_s+F_0$□ ($x_s$=positive stable equilibrium for $F_0$=0). The system dynamics with small external force $F_0$>0, according to (9), is the following:

$$\frac{dx}{dt} = \frac{x}{2□□}\left(1□\frac{x^2}{8□□□}\right)+F_0 \qquad (28)$$

For small x, (28) reduces to $dx/dt=F_0$ leading to a linear growth of x. At larger x, for $x^2$<<8□□ and $F_0$<<x/(2□), we find approximately an exponential growth of x with time constant 2□:

$$x(t) \approx x_0 e^{\frac{t}{2\square}} \quad (29)$$

For $x_m$ with maximum slope of V(x), $F_0$ can be neglected and we find:

$$x_m \approx \sqrt{\frac{8\square\square}{3}}$$
$$\frac{dx}{dt} \approx \sqrt{\frac{8\square}{27\square}} \quad (30)$$

Finally, for x near the equilibrium $x_s+F_0\square$, we find an exponential approximation with time constant $\square$ according to (12).

The **Bass model** is generally presented in the following form:

$$\frac{f(t)}{1-F(t)} = p + qF(t) \quad (31)$$

where f(t) stands for the sale rate of a product and F(t) for the total amount of items sold until time point t. p is called *coefficient of innovation* and q is the *coefficient of imitation*. If we use the relation f=dF/dt, we can write the Bass model in the form:

$$\frac{dF}{dt} = (p + qF)(1-F) \quad (32)$$

For small F (near t=0), the dynamics reduces to

$$\frac{dF}{dt} \equiv f \approx p \quad (33)$$

giving a constant sale rate f=p and a linear increase of the total amount F of items sold. For a time interval, where qF>>p and F<<1, the approximate dynamics is

$$\frac{dF}{dt} \equiv f \approx qF \quad (34)$$

leading to an exponential growth of both, F and f with time constant 1/q:

$$F(t) \approx F_0 e^{qt}$$
$$f(t) \approx qF_0 e^{qt} \quad (35)$$

The maximum slope of F (maximum selling rate f) is found from (32) at $F_m$ lying near 50% of the ultimate market potential (which is normalized to 1) for the majority of situations characterized by p<<q:

$$F_m = \frac{1}{2}\left(1 - \frac{p}{q}\right) \approx \frac{1}{2}$$
$$f_m \approx \frac{q}{4} \quad (36)$$

For F near to unity, we find from the approximated dynamics

$$\frac{dF}{dt} \approx q(1-F) \quad (37)$$

the trajectory

$$F(t) \approx 1 - e^{-qt} \tag{38}$$

i.e., an exponential approximation of the ultimate market potential with a time constant 1/q.

With the above stated relations (28)-(38), it is straightforward to bring our light ball model into approximative coincidence with the Bass model for p<<q. We identify

the normalized number of items sold with the system state: $F = x$
and the selling rate with the velocity of the ball: $f = u = dx/dt$
$x_s+F_0\tau = 1$ with the ultimate market potential set to 1: $\varepsilon = \frac{(1-F_0\tau)^2}{8\tau}$
This condition follows from (28) and (16).
the coefficient of innovation with the external force: $p = F_0$
the coefficient of imitation
with the inverse of the internal time constant: $q = 1/\tau$

With these equivalences, the constant sale rate (33) for small t gets identical in both models. This linear growth of the normalized number of items sold is followed by an exponential growth in both models with the only difference, that our ball model (29) has a time constant twice as large as compared to the Bass model (35), i.e., the ball model grows more slowly than the Bass model. The position of the maximum slope gets $1/3^{1/2} \approx 0.58$ in the ball model (30), which is slightly larger than in the Bass model, where the maximum slope is found near 0.5 (36). This difference affects maximum sale rates: Comparison of (30) and (36) shows that maximum sale rates are $27^{1/2}/4 \approx 1.3$ times smaller in the ball model than in the Bass model. For the market saturation phase (i.e. F=x near to unity), both models show an exponential approximation of the ultimate market potential with the same time constant $q=1/\tau$ (comparison of (12) and (38)). Fig. 11 shows respective trajectories for both models for p=0.01 and q=0.1.

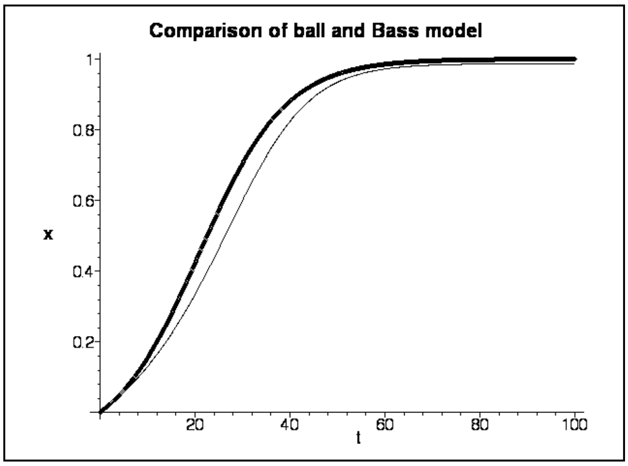


**Fig. 11:** The ball model (thin line) is compared to the respective Bass model (thick line) for p=0.01 and q=0.1. As explained in the text, the ball model is somewhat lower after the initial linear growth phase.

We would like to add that the two models could be made *exactly identical* by replacing the 4th order potential V(x) of the ball model by the 3rd order potential

$$V(x) = \frac{x}{\tau}\left[\frac{x^2}{3} - \frac{x}{2}\left(1 - F_0\tau\right) - F_0\tau\right] \tag{39}$$

Here, $F_0$ would not any longer be an external force, but an additional parameter shaping the potential V(x). (39) is proven to be correct by the substitutions x—>F, $1/\tau$—>q, $F_0$—>p and comparing dF/dt=-dV(F)/dF with (32).

## 8. Discussion and research implications

We realized that the term *generic* is associated with somewhat different meanings in different scientific communities. Our interpretation of a generic model throughout this paper is compatible with Paich 1985 and states that it should give the basic qualitative properties of a phenomenon by abstracting from all less important specific properties. However, the separation of important from less important properties cannot be made by a procedure based on first principles and always includes some subjective freedom or "grey zone". We would like to illustrate our point of view with the example of dancing steps. On the most basic level, dancing can be described by *****....*, where * stands for a group of steps. On the next level, * can be structured in the following way, L and R referring to the left and right foot respectively:

| | |
|---|---|
| * = LR | generic March |
| * = RLRLRL | generic Waltz |
| * = RLRLRLRL | generic Tango |
| * = LRL0RLR0 | generic Salsa (0 stands for a pause or a "tip" with the foot that follows) |

We show with this example that *generic* can be understood on different levels. The higher the level is the more realistic, the more interesting and colorful the description becomes, but also the more different types arise. The differentiation process can be continued leading to a tree-like structure. On each higher level, more and more specific descriptors will be necessary. For the example of Salsa, the next level would consist of generic Salsa puertorriqueña, generic Salsa cubana, etc. The characteristic feature of this differentiation is an ever growing difficulty to distinguish between the different types the higher the level is. From a certain level on, we speak of *experts* that are still able to distinguish between different types. As we all know, also the term *expert* is relative and can be attributed to persons attaining different levels. Our dancing-example is intended also to be reminiscent of the basic structure of scientific thinking. This remark should make clear that the question towards generic models lies at the very heart of modeling science and helps structuring the "modeling landscape". We would like to remember that this same idea was expressed by Jay W. Forrester at the end of his banquet talk at the 1989 System Dynamics Conference at the University of Stuttgart (Germany): "Whether we think of pre-college or management education, the emphasis will focus on *generic structures*. A rather small number of relatively simple structures will be found repeatedly in different business, professions, and real-life settings."

Another open question concerns our choice of a physical model (a light ball rolling downhill) to describe a social phenomenon. It is a fact (but also an unanswered philosophical question) that most simple mathematical equations can be used to describe a physical phenomenon. As it is a good modeling practice to start with a simple model for acceptance dynamics, it is of no surprise that such an approach can be interpreted in a physical way. The advantageous side of this circumstance is our deep understanding of light-ball behavior based on our experience gained from childhood, which helps us to directly understand the solution manifold of the differential equation defining our acceptance model. It is this understanding that helped us to conjecture the basic equivalence of the Bass diffusion model with our generic acceptance model.

The physical analogy helps us even further, namely, to extend the presented model to include additional phenomena. A natural extension would be to introduce the acceleration term of the Newtonian equation, m·du/dt, that was neglected for a first version of the model. This term would allow overshoot and damped oscillations and the physical analogon would be a *heavy* sphere rolling downhill. Another extension would be the introduction of a stochastic external force or *noise* (in the physical literature, this would be called a "coupling to a heat bath with temperature T" defining the variance of the fluctuations). This extension would replace the description of an average population by the description of an ensemble of individuals being subjected to numerous external influences pushing into all directions (see, e.g., Rahn 1985). Again, physics would guide our intuition to anticipate the range of phenomena we could expect by this extension. Among other effects, we would expect the following phenomena (for an overview on noise phenomena see the introduction of Gassmann 1997a):

> *noise-induced state transitions*: a part of the population (expressed by a probability) would "cross the hill towards the other state" even in the absence of a constant external force F. Within the innovation theory, this part is called "innovators". In chemistry, this generic effect leads to chemical equilibria (mixture of educts and products of a reaction) which depend on temperature. The (light or heavy) ball analogy makes this dependence plausible and understandable without first deducing it from the mathematical equations. We understand, e.g., that the fraction (probability) of the population sitting in the lower valley would be higher than the fraction in a higher positioned valley (for the case of an asymmetrical potential V(x)): The chemical reactions run from higher internal energy to lower internal energy, the height of the activation potential (i.e., the hill separating the two valleys) together with temperature defining the reaction rate (Arrhenius-law). In the framework of social behavioral models, noise would establish a link to choice models as, e.g., the well-known logit model.
>
> *noise-induced oscillations*: in the heavy ball model, noise would activate the system to oscillate in its eigenfrequency around a stable equilibrium.
>
> *stochastic resonance*: The effect of a small external force is amplified by the presence of noise. This counter-intuitive effect makes some marine animals hear very weak sound signals in the presence of large background noise produced by wave-induced turbulence (Sutera 1981, Dykman et al. 1995).

The above given examples for natural extensions of our generic acceptance model show some far reaching effects resulting from the introduction of simple new terms into the governing equation. Further research will show which extensions make most sense for applying them within a socio-economic framework.

## 9. Conclusions

We have shown that the simple dynamics dx/dt=-dV(x)/dx+F, representing a light ball rolling over a double-well potential V and being influenced by an external force F, is able to describe

the linear reaction of the system resulting from small forces

the hysteresis-behavior resulting from large periodic forces

the acceptance-behavior resulting from large transient forces

the dependence of transition cost on the magnitude of the force

the waste recycling model of Ulli-Beer (2006)

the Bass diffusion model

and helps to make the class of acceptance phenomena turn into a more tractable issue. In the discussion we made clear that this generic model can be extended to higher levels of detail. It has the potential to inspire model development and to generate new research questions, e.g., for the systematic investigation of acceptance dynamics in innovation systems, leading to a better understanding of technological change processes.

## 10. Acknowledgments

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