

Pension funds governance: combining SD, Agent based Modelling and fuzzy logic to address Dynamic Asset and Liability Management (ALM) problem

Ricardo Matos Chaim (rmchaim@yahoo.com.br)

Getúlio Vargas Foundation – Brasília

Av. L2 Norte, Quadra 602 – Brasília-DF, CEP 70.830-020 - Brasil

Tel: (61) 3225-9132/ 3226-0033

Fax: (61) 3226-0493 / 3225- 3960

Rosalvo Ermes Streit (rosalvo.streit@gmail.com)

Abstract

The governance is a system composed by a great number of interdependent entities, with different degrees of relationship. This article considers the governance of a social-economic and political environment under a pension fund's perspective as a complex system in which the interactions among the actors influence the governance and the governance can influence their interactions, in a recursive way. In order to cope with the peculiarities of complex systems, a system dynamics (SD) model, combined with an agent-based model is proposed to analyze population dynamics and the influence of credibility as a subjective factor over the expected adhesion of new participants. The behavior of the agents is modeled using fuzzy logic. This way, the article aims to evidence the power of a multi-paradigm model to study complex environments and to offer a way to address a dynamic ALM problem in order to manage solvency and liquidity risks in pension funds.

Key words: Pension funds, Dynamic Asset and Liability Management (ALM), pension schemes, fuzzy logic, agent based modeling, planning under uncertainty

Introduction

A dynamic ALM's model problem is how to manage credit, market, operational and image risks to estimate returns over long-term investments based on uncertain liabilities. Thus, planning under uncertainty requires reliable tools to get better dynamic financial analysis and to manage actuarial assumptions in order to set policies that assure good solvency and liquidity to pension funds.

Lifetime and demographic studies focus on the population dynamics of a pension fund that has, among others, rates of mortality, withdrawal, disability and retirement that must be considered in assessing pensions costs and to consider credibility in structuring a prospective cash flow. This way, the research conducted by the authors combine methods and techniques to study pension funds population models and the influence of subjective factors over it.

Thus, combining SD methods to agent-based modeling applies simulation to cope some aspects of the social-economic and political environment under the pension fund perspective, using fuzzy logic to model the behavior of the agents.

To place the issues into perspective, this paper has four sections. First, it observes the complexity inherent to pension funds and some information about its governance. Next, discusses a dynamic Asset and Liability Management (ALM) approach for pension funds. Follows fuzzy logic rules model the agent's behavior in a beliefs-desires-intentions (BDI) agent architecture. Finally, in the conclusion, considerations about the combination of system dynamics and agent based modeling, with summary comments about the combination of methods to address subjective factors.

1 Complexity and governance

Edmonds (2003) stated that when a study domain is quite complex, the approaches based on equations or on other analytical techniques are impracticable or even impossible to be applied. In complex systems, the interactions between the parts may cause relevant differences in system's performance. Wagner (1986) argues that the result of the combination of the uncertainty, the dynamic interactions, the subsequent events, and the complex interdependences among system variables difficult the analysis of a problem. According to Edmonds (2003), simulation is the only way to model the behavior of this type of systems.

In order to cope with the peculiarities of pension funds, we propose the use of an agent-based model to represent the behavior of the pension fund participants and the social-economic and political environment to provide deeper insights by simulation experiments. The agent-based models can help to clarify the agents' interactions and behaviors (micro level), e.g., the non-linear behaviors of the system that are difficult to be captured with mathematical formalisms. In this case, a multi-agent model combined with a SD model will aid to manage solvency and liquidity risks on a pension fund, called Dynamic Asset and Liability Management (ALM). Therefore, in this study the proposed pension fund model is a multi-paradigm simulation model. Each modeling approach supports some particular representation of it.

The study of multi-agent systems started in the field of the distributed artificial intelligence (DAI) about twenty years ago (Weiss, 1999). The precursor of these systems is the object-oriented programming (OOP). The OOP objects keep their own data structures and procedures (methods), and communicate to each other with messages. Artificial intelligence works with computational aspects of intelligence and focuses on systems that act separately.

The DAI, in turn, is the study, construction and the application of multi-agent systems, which are systems where some intelligent agents interact and aim to reach a set of goals or execute a sequence of tasks (Weiss, 1999). The term "intelligent agent" indicates object with flexible autonomous capacity. Streit (2002), for example, evidences the importance of the use of the DAI in social sciences study field and presents references of its use in the organizational area.

According to Lempert (2002), the agent-based models can represent important phenomena difficult to capture with mathematical formalisms. The author argues that these models are distinguished for relating the heterogeneous behavior of the agents (different information, different decision rules, and different situations) with the macro behavior of

the system. The agents have several interaction rules and, by simulation, it is possible to explore the emergent behavior along the time and the space. This modeling technique does not assume a unique component that takes decisions for the system as a whole. The agents are independent entities that establish their own goals and have rules for the decision making process and for the interactions with other agents.

The agents' rules can be sufficiently simple, but the behavior of the system can become extremely complex (Gilbert, 1995). Therefore, the complexity can emerge because of simple rules in the level of the individuals. The emergence “occurs when the interactions among objects at one level give rise to different types of objects at another level” (Gilbert, 1995, p.15).

The modeling process relates the representation of the system under analysis from the real world to a model capable to describe similar behavior. Figure one shows that the design of the computational model incorporates relevant aspects of the system that we want to know. It is a formal representation of a conceptual model. The conceptual model, in turn, is an abstraction of the real world under analysis.

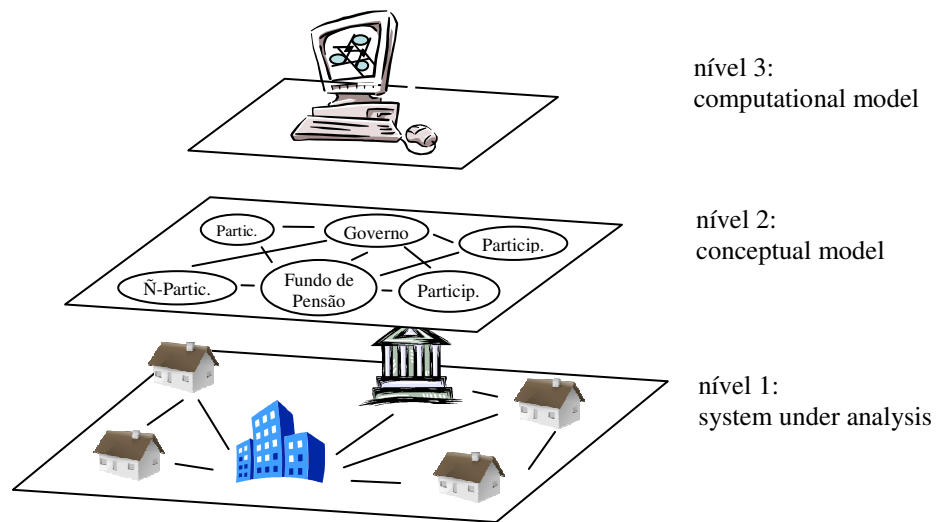


Figure 1 - The modeling process.

The modeling process of an agent-based model defines its individual components, as a bottom-up approach. The definition of the agents' behaviors is extremely important for a good representation of the system under analysis. Besides, there must be a very good equivalence between the system under analysis and the conceptual model to guarantee great consistency to the agent-based model and reliability from the simulation results.

Figure 2 presents a conceptual model to study pension funds governance. Streit (2006) developed this model for regulatory governance analysis of sectors under regulation. The conceptual model is generic and, consequently, it is useful to structure different pension funds scenarios.

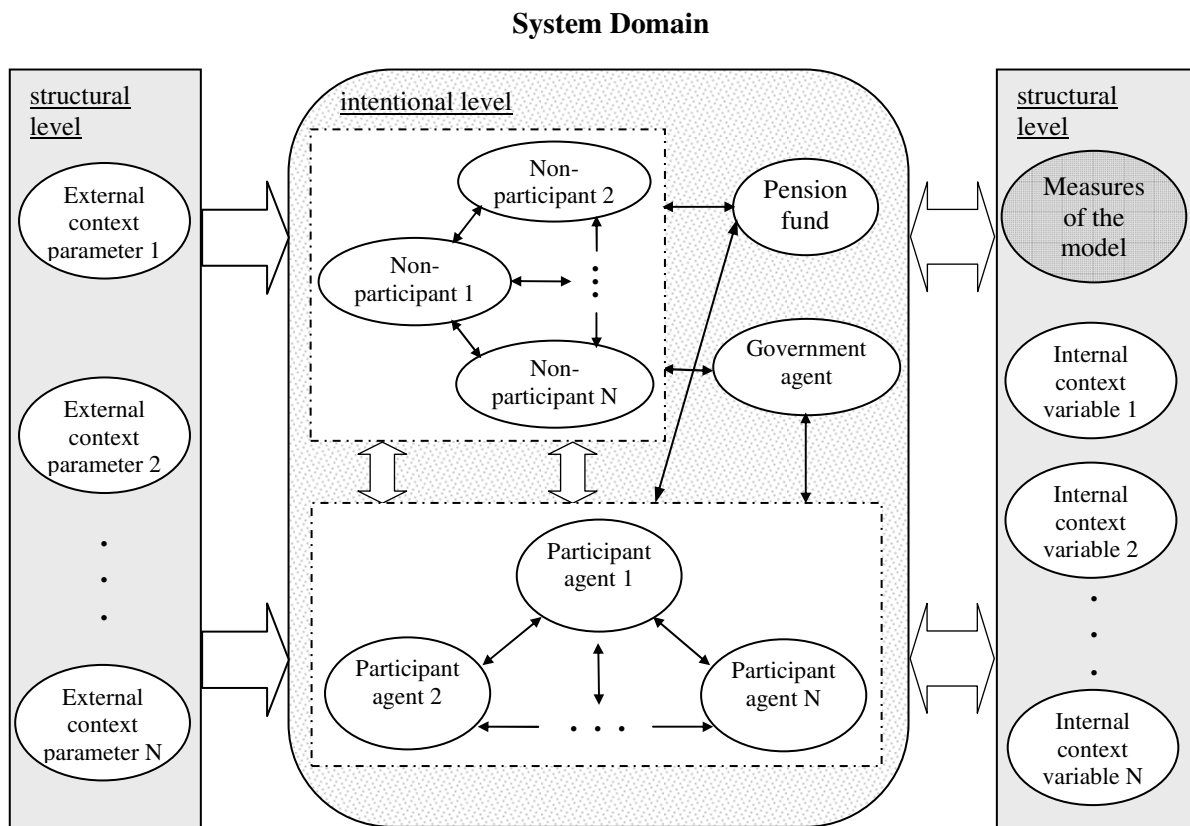


Figure 2 – Generic conceptual model to study pension funds governance.

Simulation along the time is the strategy to analyze the emergent phenomenon of the model. The intentional level (action level), where the interactions among the agents occur, is differentiated from the structural or contextual level. The structural level indicates the contexts where the interactions happen, e.g., the circumstances that limit, amplify and determine the interactions among the agents and with the environment. Moreover, structural level is the level where the emergent phenomenon takes place. It is a higher level comparing to the intentional level where the agents interact. The basic principle that guides the model is that all interactions have an intention or a set of intentions. For a better understanding, follows the main components of the generic conceptual model:

- **Measures of the model:** they are the results of the model that make possible the study of the phenomenon for which the model is developed;
- **External context parameters:** indicate external aspects that may contextualize the model. They are not influenced from the behavior of the model (unidirectional arrow from the structural level to the individual level), but they can influence the interactions among the agents who act in the intentional level of the model (example: international indicators). The external context parameters define the external environment of the system under analysis;

- Internal context variables: represent important external aspects. These variables influence the interactions among the agents and are influenced by them. Thus, during the simulations, the values of these variables are modified depending on the interactions at the intentional level of the model. They express, in its totality, relative external situations of the system under analysis in the environment;
- Government agent: it represents the government at the individual level of the model. The government agent defines the regulation policies of the pension funds;
- Participant agents: they represent the agents who participate in a pension fund. The participant agents are directly influenced by the regulations and situations that impact the pension fund sector. The amount of “participant agents” in the model will depend on the type of analysis and abstraction desired;
- Non-participant agents: they interact with other agents at the individual level, but they do not participate in a pension fund. The non-participants agents are indirectly influenced by the regulations of the sector and they can indirectly influence the agents who regulate the sector.

The “internal context variables” and the “external context parameters” belong to the structural level of the model (macro level).

2 A dynamic Asset and Liability Management (ALM) approach for pension funds

Pension funds need to produce a high-income return to correspond to actuarial expectations and to pay different kind of benefits. Because of its long-term obligations, an ALM model of a Pension Fund must consider a large planning horizon. ALM must control the solvency of the fund by acceptable investments and contribution policies. The process requires a great amount of information about the organization, its operations and market performance. It comprises: (1) better understanding of the wealth of the organization by evaluating balance sheet; (2) actions to control credit, liquidity and market risks (3) statistical and mathematical methods to predict, forecast or foresee how the future should be or define a finite number of scenarios to model uncertainty.

One of SD’s paradigms is that the structure determines behavior and events are snapshots of that behavior. One step back from events is the idea of behavior patterns as something that connects together a long series of events over time. They show sources of pressure and imbalances that cause things to change. Pension funds have to decide periodically how to allocate the investments over different asset classes and what the contribution rate should be in order to fund its liabilities.

Risk and uncertainty are key features of most pension funds and need to be understood to made rational decisions. A problem has many uncertainties and they are materialized in various elements or factors in a risk analysis model as ALM. There are basic principles that an ALM model concerns to:

- Deterministic modeling involves using a single “best guess” estimation of each variable within a model to determine the models outcomes;
- Sensitivities determine how much that outcome might vary via what-if scenarios. Every possible value that each variable could take is weighted by the probability of its occurrence to achieve this. Each uncertain variable has a probability distribution that needs to be considered in an ALM model;
- Within a risk analysis model, available data and expert opinions are the two sources of information used to quantify the uncertainty.

Computer simulations, among others, give to the analyst a way to generate data or optimize the model to give the parameters that will materialize the uncertainty. The analyst must revise the data he has available and assure they are both reliable and as representative of the true uncertainty as possible.

Many techniques try to fit theoretical distribution to observed data and to give the dynamic model ways to foresee or forecast the possible results via estimators and probabilities. To do this, many authors has a common sense that each variable is correlated with, or a function of, another variable within the model. System dynamics gives a way to explore causation between variables and feedback loops that are responsible for problems in a considered context. As stated by Sterman (2000, pg.141), “correlation among variables reflect the past behavior of a system. Correlations do not represent the structure of the system (...) correlations among variables will emerge from the behavior of the model when you simulate it”. Professor Sterman also states “confusing correlation with causality can lead to terrible misjudgments and policy errors”.

Engert and Lansdowne (1999), states that “risks are events or occurrences that prevent a program from achieving its cost, schedule, or performance objectives. This way, Chaim (2006) and Chaim (2007) applied system dynamics principles to ALM (Asset and Liability Management) models, in the specific case of pension funds. The author did a research with actuaries and financial managers of 20 Brazilian pension funds. Figure 3 shows the results materialized in a causal loop diagram that represents the complexity of a benefit plan of a pension and that include population dynamics as a way to reduce pension costs.

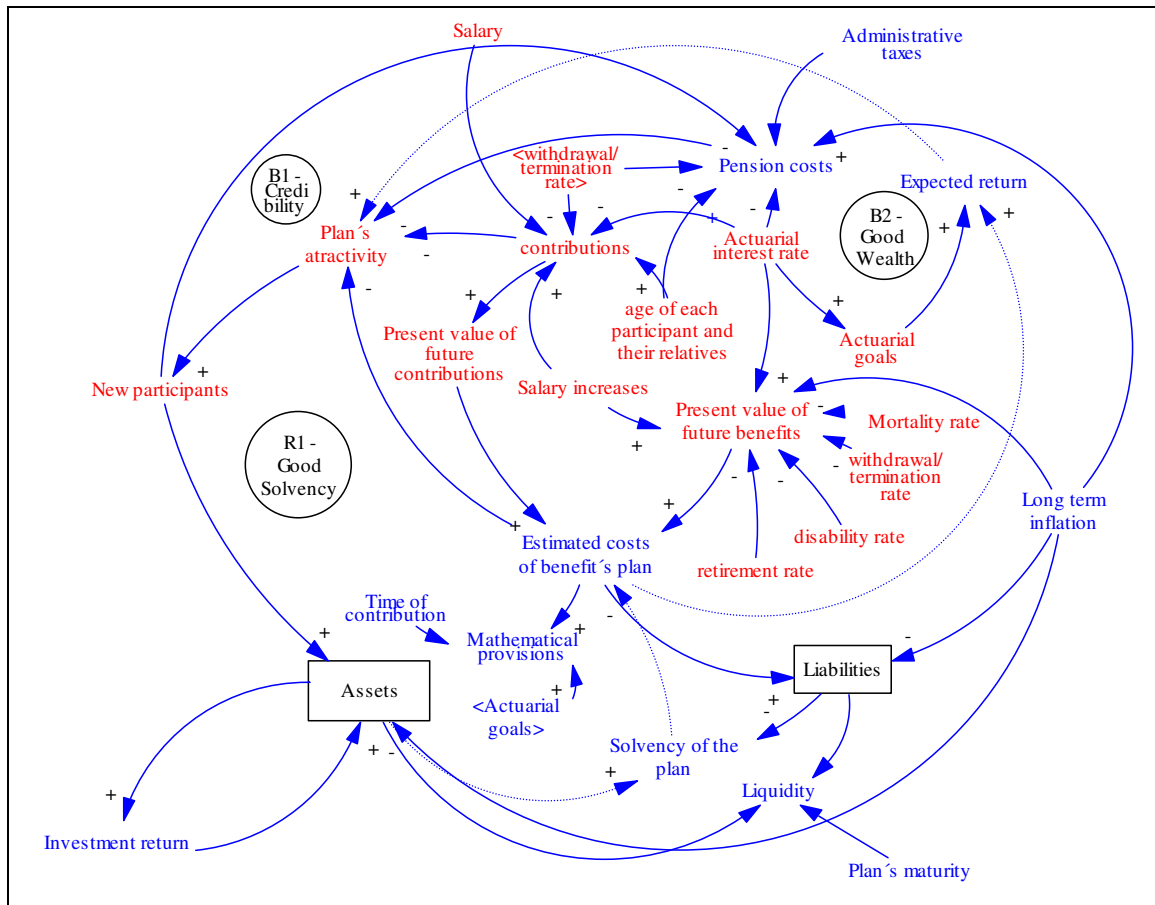


Figure 3: A broad causal loop diagram obtained by expert opinion
Source: Adapted from Chaim (2007).

Three rounds of Delphi technique gave conditions to structure this causal loop diagram and to analyze causation between factors. These factors were obtained by many declarations of actuaries and practices statements from financial managers. Among others, there are three feedback loops that constitute the main hypothesis about the causes of dynamics under investigation by the research in progress:

R1 – Good Solvency: more attractive plans may attract more participants or more sponsors that may generate more accumulation. This way, the solvency tends to get better by the reduction of the estimated costs of the plan;

B1 – *Credibility*: more attractive plans may obtain more participants and then the costs tend to be lower, because they share the staff of the plan and material resources;

B2 – Good Wealth: more attractive plans may attract more participants or more sponsors that may generate more accumulation. This way, the solvency tends to get better by the reduction of the estimated costs of the plan and, thus, enhancing the attractiveness of the plan.

To constrain the scope of the article, attractive plans will consider just credibility factor in population models. These article will not consider the calculating of pension costs and mathematical provisions as in Rodrigues (2004). The value of mathematical provisions of benefits to be paid of a participant with age x is represented by the equation $MP_x = PVFB_x - PVFC$ and its value consider population factors and they will be determined by the equation:

$$MP_x = FCS.[S_x.(1 + CS)^{r-x} \cdot {}_{r-x}p_x^{aa} \cdot \ddot{a}_r \cdot v^{r-x} - (S_{x \rightarrow r} \cdot CN(\%) \cdot \ddot{a}_{x:r-x} \neg)] \{x < r\}, \text{ where}$$

MP = Mathematical provisions;

($PVFB$ = Present value of future benefits):

FCS = Capacity factor of salary. It reflects inflation.

CS = Salary enhancement;

$S_x \cdot (1 + CS)^{r-x}$ = salary of one participant, projected to the retirement age r

$r - x$ = For a participant of age x , the time remaining between the assessment date and the retirement date (r)

${}_{r-x}p_x^{aa}$ = the probability of a participant of age x to be alive and active when reaching the age x of retirement

\ddot{a}_r = factor of anticipated actuarial income related to the participant when initiating the retirement

v^{r-x} = discount factor considering the interval between ages r and x

($PVFC$ = Present value of future contributions):

$S_{x \rightarrow r}$ = All salaries between ages x and r

$CN(\%)$ = Taxes that represents the cost of the plan

$\ddot{a}_{x:r-x} \neg$ = factor of an anticipated actuarial income, temporary, related to activity period of the participant.

Actuarial literature does not consider subjective factors since like credibility in ALM analysis. For example, the uncertain parameters identified by Rocha (2001) were interest rates, administrative taxes, capacity factor of salaries and benefits and the rates of salaries increase, all of them economical factors. Lifetime and demographical studies also do not consider subjective factors.

Causal loops relations may represent the uncertainty and may predict the impact of each of it in the system as a whole. Chaim (2006) noticed that the use of system dynamics in combination with asset-liability management model (ALM) represents an opportunity to amplify its capability to become risk oriented. Streit (2006) indicate the use of subjective factors using agent based models. Thus, macroeconomics, biometrics and actuarial classes of variables must holistically consider the incorporation of risk factors, subjective factors and constraints (shortfalls) into the model.

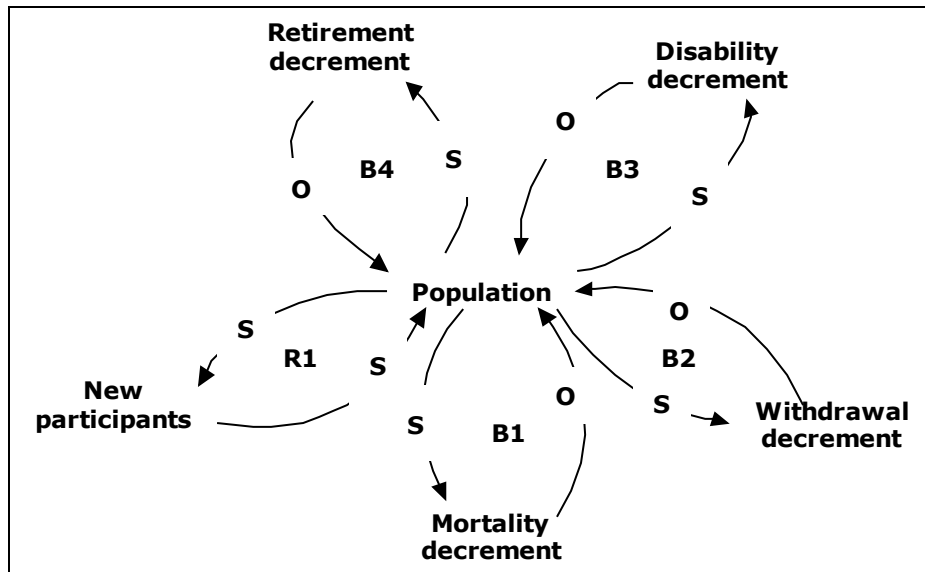


Figure 3: Populational decrements causal loop diagram.

Because pension funds are typically a multi-decrement environment (WINKLEVOSS, 1977, p. 10-22), the causal loop diagram of figure 3 shows the dynamics of a benefit plan. Credibility influences new adhesions made by word of mouth and ad campaigns (R1); many decrements since like mortality (B1), withdrawal (B2), disability (B3) and retirement (B4) are the balancing way to reduce this population and thus the costs of the benefit plan (WINKLEVOSS, 1977, p. 10-22). Follows details about each one:

R1 – *Credibility*: means that people become more and more interested in adhering to a benefit plan of a pension fund. It means more assets coming from the participants and the organizations that are sponsors of the benefit plan. Credibility generates confidence that tends to foster new adhesion of new participants;

B1 – *Mortality decrements*: “among active employees prevents the attainment of a retirement status and hence the receipt of a pension benefit, while mortality among pensioners acts to terminate the payment of their pension benefit” (WINKLEVOSS, 1977, p. 12);

B2 – *Withdrawal decrements*: this decrement is also called termination decrement and like the mortality decrement, “prevents employees from attaining retirement age and receiving benefit under the plan (...) there are a multitude of factors entering into the determination of employee termination rates, but two factors consistently found to have significant relationship are age and length of service. The older the employee and/or the longer his period of service, the less likely it is that he will terminate employment” (WINKLEVOSS, 1977, p. 15-16). Accordingly to Winklevoss (1977, p. 18), “disability among active employees, like mortality and withdrawal, prevents qualification for a retirement benefit and, in turn, lowers the cost of retirement”.

B3 – *Disability decrements*: “a typical disability benefit might provide an annual pension, beginning after a waiting period, based on the employee’s benefits accrued to date, or on his projected normal retirement benefit. When disability benefits are provided outside the pension plan, it is common to continue crediting the disabled employee with service until normal retirement, at which time the auxiliary plan’s benefits cease and the employee begins receiving a normal pension” (WINKLEVOSS, 1977, p. 18-19);

B4 – *Retirement decrement*: “the retirement decrement among active employees initiates the pension payments” (WIKLEVOSS, 1977, p. 21).

Figure 4 shows the stock and flow diagram to manage the population dynamics of a pension fund. Credibility is a factor that influences new adhesions and used a lookup table based on parameters obtained by expert opinions.

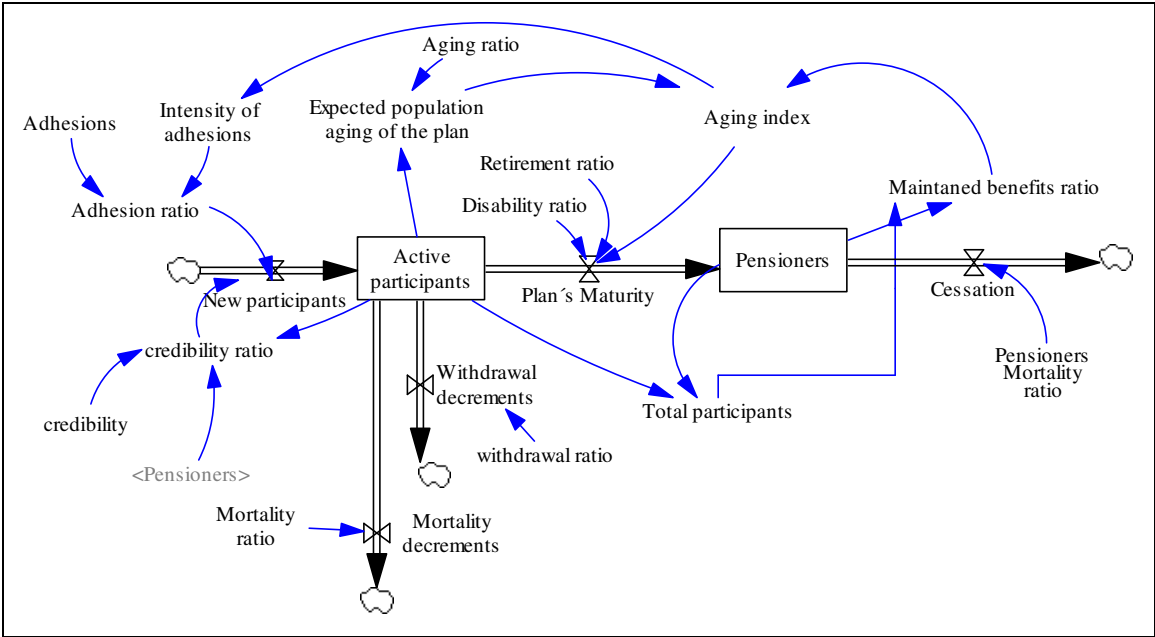


Figure 4: Population dynamics stock and flow diagram.

The figure assumes a pension fund with an expected credibility rate that influences new adhesions. People who are active participants are exposed to mortality, disability, withdrawal and retirement risks. A pensioner is exposed just to mortality risks. An expected population-aging ratio uses historical data and calculates the aging index. This dynamic is essential an ALM model estimate the total assets and the flow of liabilities, maintain a good solvency and prevent against liquidity risks.

In a SD way, a lookup table aids to model credibility as a factor that influences accumulation and liabilities. It may enhance or reduce the adhesions of new participants over time. According to Edmonds (2003), in cases where the field of study is sufficiently complex, it is impractical or even impossible to rely only on mathematical models. Therefore, the construction of an agent-based model appears to be the most suitable way to

assess the impact of the social-economic and political issues on the pension funds participants and non-participants.

For the analyst uninitiated in risk analysis modeling, it's difficult to find explicit techniques that will produce an accurate model of the problem in hand. To fill that gap, this article proposes the combination of SD, agent based modeling and fuzzy logic.

3 Agent model and fuzzy logic

One of the major concerns in the process of developing agent-based models is with the nature of the agents themselves and the definition of their behavior, that is, how they interact with other agents and with their environment (Edmonds, 1998). The agents modeling at the intentional level will define the way they take decisions, their behavior and attitudes during the simulation experiments. The author argues that the purpose of modeling the agents is to reveal the emergent behavior of the system. In the literature, it is possible to identify diverse types of agent frameworks that have been conceived for various types of analyses.

The agent model used in the research was based on the BDI (beliefs-desires-intentions) architecture. BDI has been used for the modeling of different types of agent behavior, and adopted in numerous fields. BDI agent architecture was introduced by the philosopher Michael Bratman (1999), who proposed a framework for understanding ways of characterizing mental attitudes and rational actions in human beings, in terms of their intentions. The principles of Bratman's work have been fundamental for the theoretical formalization of computational agents with rational behavior, and for the development of formal agent architectures.

In BDI, agents are described as a set of beliefs, desires and intentions. The agents' decision-making process occurs during analysis of beliefs relative to their desires, according to the precepts of this approach. Beliefs are items of information held by the agent about himself and about the environment in which he is active. They correspond to the informative component of the agent's status and may be subject to uncertainties and errors. Desires, in turn, are objectives the agent adopts and attempts to achieve. In terms of BDI architecture, an agent's desires are essentially the 'options' or 'possibilities' available to the agent (Wooldridge and Parsons, 1988). The theoretical model of the BDI architecture also employs the concept of intentions, which represent courses of action chosen by the agents to achieve their goals (desires). The agent's actions are organized into plans. In the process of deliberation, after the selection of an intention, an agent's plan is chosen and initiated. Thus, intentions correspond to the agent's plans under execution.

Since the relevant beliefs, desires and intentions of agents are of a subjective nature, the specification of the agents in this research employed a fuzzy-extension BDI agent model (Shen et al., 2004). The basic idea behind the use of the fuzzy extension for modeling multi-agent systems is the specification and description of the agent behavior by means of fuzzy rules. The inference of these rules can be understood as the mapping between a set of inputs and a set of outputs. Thus, the inference of these rules during simulation establishes the dynamic behavior of each agent in the system and, as a consequence, the behavior of the system as a whole. The practical reasoning of the agent consists of two principal activities (Shen et al., 2004; Schut et al, 2004): (i) deliberation,

where the agent decides what to do (which intention to carry out); and (ii) planning, which is the decision of how to carry out the intention.

In order to simplify the model proposed, the agent's deliberation (what to do) and planning (how to do it) processes have been combined into one process. In this case, the agent's practical reasoning mechanism consists in choosing a pair <objective, plan> for execution, that is, the intention the agent can adopt and the plan of action for carrying out such intentions. This simplification was suggested in the work of Hsieh et al. (2004). The figure 5 represents the internal model proposed for the agents and indicates its principal components (Streit, 2006). These components can be described as:

- Perceptions: refers to the means by which the agent perceives the environment;
- Agent's status: refers to the agent's current set of beliefs about its environment and by the intention it is currently pursuing;
- Database pair <objective,plan>: data structure storing the possible space state of an agent's pair <objective,plan>;
- Database 'Beliefs': stores the agent's beliefs about its environment;
- Components 'revises beliefs' and 'selects <objective,plan>': they are components that carry out the procedures for the selection of the agent's intentions and action plans. These components constitute the agent's decision-making process, along with the component 'deliberation control';
- Action: component that executes the actions for carrying out the current intention or the new intention selected by means of the fuzzy logic;
- Action outlet: refers to the means by which the agent transmits messages to the environment and to the other agents. It is the outlet for the outcome of the agent's inference process.

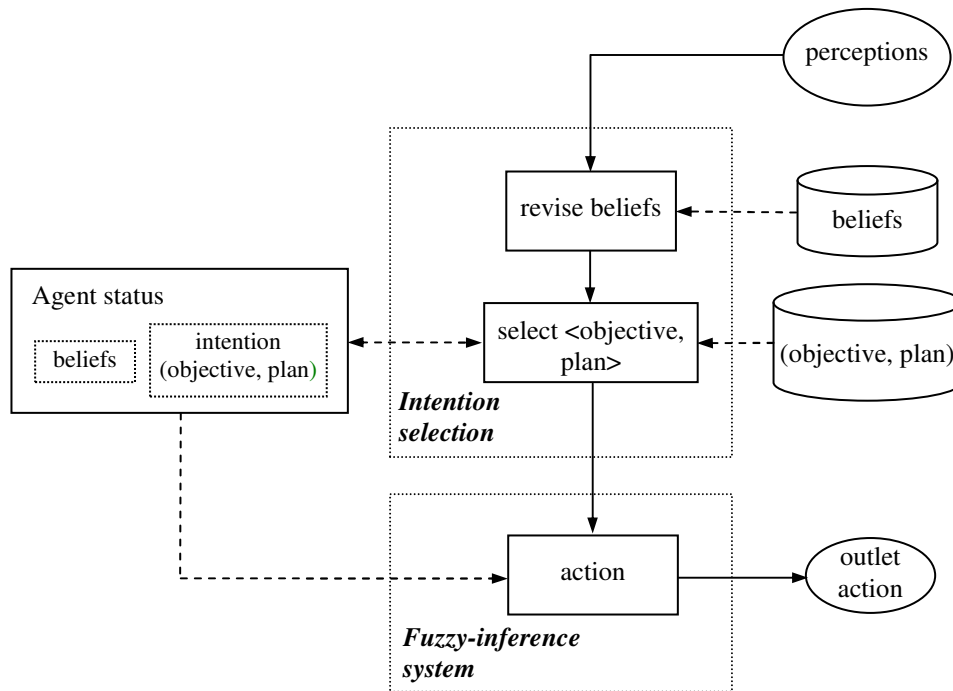


Figure 5: Agent's internal model.

As presented in figure 5, the process of carrying out the intentions is based on fuzzy logic. This process is circumscribed by the component denominated 'fuzzy-inference system'. In this case, the fuzzy rules execute the agent's actions following the BDI framework. The agent's beliefs are defined on the antecedent term of the fuzzy rules (IF side), while the term relative to the agent's deliberation is found on the consequent side (THEN side). The main definition step of the model is associated with the selection of the production rules to model the agents' behavior. For instance, the fuzzy rule "IF inflation is high AND inflation variation increases THEN exert moderate pressure for interest rate reduction " indicates, for example, that there is an agent belief that inflation is high and, also, there is a tendency towards increased inflation. Then, the agent deliberation will exert a moderate pressure into the 'monetary authority' agent for an interest rate reduction. The value resulting from the pressure will depend on the degree of truth of the input variables 'inflation' and 'inflation variation' to the fuzzy sets 'high' and 'increases', respectively.

The notion of a fuzzy set was introduced by Zadeh (1965 apud Rizzi et al., 2003, p. 365), in the decade of 60. The objective is to represent mathematically uncertainties and to supply formal tools to deal with the inherent imprecision of many problems. The main idea is the revision of the classical theory of the sets. The traditional way of representing elements u of a set A is through the characteristic function (Kasabov, 1998):

$$\mu_A(u) = 1, \text{ if } u \text{ is an element of set } A, \text{ and}$$

$$\mu_A(u) = 0, \text{ if } u \text{ is not an element of set } A,$$

that is, an object u either belongs or does not belong to a given set. In fuzzy sets theory an object can belong to a set partially. The degree of membership is defined through a generalized characteristic function called membership function: $\mu_A(u):U \rightarrow [0 \ 1]$, where U is the universe and A is a subset of U . The values of the membership function are real numbers in the interval $[0 \ 1]$, where 0 means that the object is not a member of the set and 1 means that it belongs entirely.

The fuzzy logic has been considered useful when the process (system under analysis) is difficult to forecast or model using traditional methods (Mohammadian and Kingham, 2004). This paradigm allows the modeling of complex systems by the use of simple rules that are defined with linguistic variables and terms. The fuzzy logic is versatile because it allows the modeling and manipulation of vague and inexact information mathematically. This type of information is natural in the human language, as the information supplied by the specialists (not mathematicians) (Amendola et al., 2004). This feature, according to Berg et al. (2004), it is an important advantage, because it allows the linguistic interpretability of the model results and the comparison to the specialists knowledge. The use of fuzzy-inference mechanisms is an interesting option for modeling the reasoning and behavior of the agents. It makes possible to describe the agents' behaviors semantically using production rules (IF-THEN).

In addition to the work of Shen et al. (2004), other studies in the literature demonstrate the advantages of using fuzzy logic in the development of agent models (Bossomaier et al., 2005; Li et al, 2004; Hsieh et al, 2004; Shajari and Ghorbani, 2004). Fuzzy logic has been employed in the agent decision-making process and in the definition of agent behavior.

4 Combining SD and agent-based model

The main stage of the agent-based model definition in this study is the production rules selection to model the agents' behavior. The criteria that can be used to the production rules delimitation is based on the variables used in the dynamic model and the agent-based model.

The figure 6 presents the main components of the model and simulation techniques discussed in this article.

The research is multidisciplinary and interdisciplinary by nature and the article presents part of the literature review.

The research is in progress. The authors identified the main actors and the methodology to proceed the modeling recommendations identified on the literature review. The software to be produced will consider ages, mortality, withdrawal and mortality rates, assets, liabilities, investments and many other factors from the database of a important Brazilian pension fund company.

REFERENCES

AMENDOLA, Mariângela; SOUZA, Anderson Luiz; BARROS, Laécio Carvalho. Manual do uso da teoria dos conjuntos Fuzzy no Matlab 6.5. Versão 2005 do manual apresentado no Ciclo de Palestras/2004, FEAGRI/UNICAMP, 2004. 46 p.

BERG, Jan van den; KAYMAK, Uzay; BERGH, Willem-Max van den. Financial markets analysis by using a probabilistic fuzzy modelling approach. *International Journal of Approximate Reasoning*, vol. 35, n. 3, p. 291 – 305, 2004.

BLAKE, David. Pension schemes as options on pension fund assets: implications for pension fund management. *Insurance: Mathematics and Economics*, Amsterdam, n. 23, p. 263-286, sep. 1998.

BOSSOMAIER, Terry; AMRI, Siti; THOMPSON, James. Agent-based modelling of house price evolution. Centre for Research in Complex Systems, Charles Sturt University, Bathurst NSW, Australia, 2005.

Boulier, Jean François, Michel, Stéphane, Wisnia, Vanessa. Optimizing Investment and Contribution Policies of a Defined Benefit Pension Fund. AFIR colloquium, Germany, 1996.

BRATMAN, Michael E. Intention, plans, and practical reason. CSLI Publications, 1999. 200 p.

CHAIM, Ricardo Matos. Combining ALM and System Dynamics in Pension Funds. In: 24th International Conference of System Dynamics Society, 2006a. Proceedings of the 24th International Conference The Netherlands: Wiley Inter Science, 2006. Available at: <<http://www.systemdynamics.org/conferences/2006/proceed/papers/CHAIM315.pdf>>. Accessed in: 30 de setembro de 2006.

CHAIM, Ricardo Matos. 2007. Gestão das informações sobre riscos de ativos e passivos previdenciários em fundos de pensão: associação entre a Dinâmica de Sistemas e o Asset and Liability Management (ALM). PhD dissertation, Information Science School, University of Brasilia, Brazil.

CHANG, Shih-Chieh; CHENG, Hsin-Yi. Pension Valuation under Uncertainties: Implementation of a Stochastic and Dynamic Monitoring System. *Journal of Risk & Insurance*, vol. 69, n. 2, 2002, p. 171-192.

EDMONDS, Bruce. Simulation and complexity: How they can relate. Centre for Policy Modelling Discussion Papers, CPM Report No.: CPM-03-118, 2003.

EDMONDS, Bruce. Modelling socially intelligent agents. *Applied Artificial Intelligence*, vol. 12, n. 7, p. 677 – 699, 1998.

GILBERT, Nigel. Simulation: An emergent perspective. Draft Paper, Department of Sociology, University of Surrey, 1995.

HSIEH, Luke; LIU, Alan; YU, Shao-En, HSU, Harry C. S. A method in social reasoning mechanism for intelligent agents using fuzzy inference. In: *International Computer Symposium (ICS2004)*, Taipei, Taiwan, 2004. Proceedings of... Taiwan, 2004.

KASABOV, Nikola K. *Foundations of Neural Networks, Fuzzy Systems, and Knowledge Engineering*. Cambridge, MA: MIT Press, 1998, 550 p.

LEMPERT, Robert. Agent-based modeling as organizational and public policy simulators. *Proceedings of the National Academy of Sciences of the United States of America (PNAS)*, vol. 99, suppl. 3, p. 7195 – 7196, 2002.

LI, Yifan; MUSILEK, Petr; WYARD-SCOTT, Loren. Fuzzy logic in agent-based game design. In: *The 2004 Annual Meeting of the North American Fuzzy Information Processing Society*, Banff, Alberta, Canadá, 2004. Proceedings of... Alberta, Canadá, 2004.

MOHAMMADIAN, M.; KINGHAM, M. An adaptive hierarchical fuzzy logic system for modelling of financial systems. *Intelligent Systems in Accounting, Finance and Management*, vol. 12, n. 1, p. 61 – 82, 2004.

RIZZI, Lorenzo; BAZZANA, Flavio; KASABOV, Nikola; FEDRIZZI, Mario; ERZEGOVESI, Luca. Simulation of ECB decisions and forecast of short term Euro rate with an adaptive fuzzy expert system. *European Journal of Operational Research*, vol. 145, n. 2, p. 363 – 381, 2003.

ROCHA, Cleide Barbosa da. *Análise do modelo estocástico do passivo atuarial de um fundo de pensão*. 2001. Dissertação (Mestrado em Administração) - PUC-RJ, Rio de Janeiro, 2001.

RODRIGUES, José Angelo. *Gestão do risco atuarial em fundos de pensão*. Rio de Janeiro: Previ/BB-GECAT, 2004.

SCHUT, Martijn; WOOLDRIDGE, Michael; PARSONS, Simon. The theory and practice of intention reconsideration. *Journal of Experimental & Theoretical Artificial Intelligence*, vol 16, n. 4, p. 261 – 293, 2004.

SHAJARI, Mehdi; GHORBANI, Ali A. Application of belief-desire-intention agents in intrusion detection & response. In: *The Second Annual Conference on Privacy, Security and Trust*, University of New Brunswick Fredericton, New Brunswick, Canada, 2004. *Proceedings of...* New Brunswick, Canada, 2004.

SHEN, S.; O'HARE, G.M.P.; COLLIER, R. Decision-making of BDI agents, a fuzzy approach. In: *The Fourth International Conference on Computer and Information Technology (CIT2004)*, Wuhan, China, 14-16 September 2004. In: *Proceedings of...* IEEE Publishers, 2004.

STERMAN, John D. *Business Dynamics: Systems Thinking and Modeling for a Complex World*. Boston, Irwin McGraw-Hill, 2000.

STREIT, Rosalvo E. Um modelo baseado em agentes para a análise da governança regulamentar do sistema financeiro. PhD dissertation, Management School, Federal University of Rio Grande do Sul, Brazil, 2006.

STREIT, Rosalvo Ermes. Técnicas de Inteligência Artificial Aplicadas à Pesquisa Social. In: *XXXVII Assembléia do Conselho Latino-Americano de Escolas de Administração*, 2002, Porto Alegre. *Anais ... Porto Alegre*, 2002. CD-ROM.

WAGNER, Harvey M. *Pesquisa Operacional*. 2. ed. Rio de Janeiro: Prentice-Hall do Brasil Ltda, 1986. 851 p.

WEISS, Gerhard. *Multiagent systems: A modern approach to distributed artificial intelligence*. Cambridge: MIT, 1999. 619 p.

WINKLEVOSS, Howard. *Pension Mathematics*. University of Pennsylvania, 1977

WOOLDRIDGE, Michael; PARSONS, Simon. Intention reconsideration reconsidered. In: *5th International Workshop on Agent Theories, Architectures, and Languages (ATAL-98)*, Paris, France, 1998. *Proceedings of the...* Springer, 1998.

ZADEH, Lotfi. Fuzzy sets. *Information and Control*, vol. 8, p. 338 – 353, 1965.