PARAMETER FORMULATION AND ESTIMATION IN SYSTEM DYNAMICS MODELS

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ABSTRACT

The purpose of this paper is to convey the techniques and considerations normally involved in formulating and estimating parameters in system dynamics models. Ideally, model equations should be formulated so that the associated parameters each describe some unique observable characteristic of the real system. Thereby, translating observations and measurements below the level of aggregation of model structure (estimation from disaggregate data) into specific parameter values becomes very straightforward. Fewer assumptions about the structure of the system are needed than if the parameters were set by equation estimation or model estimation from data at the level of aggregation of model structure. Making additional assumptions provides more opportunities for systematic errors to creep into the parameter-setting process. Rather than using data at or above the level of aggregation of model structure to set parameters, such information might better be reserved for validity testing. When such data are not already used to set parameter values, the validity tests become simpler and depend upon fewer assumptions.

Parameters need only be set accurately enough to allow the model to fulfill its purpose. One time-saving research strategy is to determine, by using only roughly-set parameters at first, how accurately the parameters must be set before investing time and effort in setting them accurately. Then, sensitivity testing can identify the relatively small number of parameters whose values significantly alter the model behavior or response to policy changes. The model can then be reformulated, the policies redesigned, or the sensitive parameters reset by more elaborate and hopefully more accurate techniques.

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I. INTRODUCTION

System dynamics as a discipline diverges in several respects from more traditional scientific disciplies, such as economics or physics. The most apparent difference concerns the methods of selecting numerical values for model parameters. In economics or experimental physics, a significant portion of the total research effort is devoted to determining the precise values of the parameters that characterize the system under study. A significant part of professional communications in journals and conferences concerns measurement, data, and statistical technique. In contrast, the literature of system dynamics describes the complex structure of models, and devotes considerable space to analyzing the behavioral consequences of that structure. Description of the parameter-setting process is usually brief or nonexistent. It should not be surprising that practitioners of traditional disciplines incorrectly perceive glaring deficiencies in system dynamics models. Careful and laborious parameter setting, a part of research long presumed necessary, appears totally lacking in system dynamics models.

A. Purpose and Organization

The purpose of this paper is to convey the considerations and techniques used to formulate and estimate parameters in system dynamics models. Section II begins by discussing issues in formulation of equations and their associated parameters. The issues revolve around a parameter's dual purpose of accurately describing some real process, and lending itself to straightforward estimation. Estimation is divided into three broad categories, described respectively in Sections III, IV, and V: estimation from disaggregated data,

equation estimation, and model estimation. Section VI describes the procedures by which an initial model with roughly-estimated parameters is transformed into a reliable guide to policy-making: sensitivity testing isolates the parameters that require reformulation, reestimation, or policy redesign. Finally, Section VII summarizes the main points of the paper.

B. A Housing Model

This paper discusses various means of selecting parameter values in the context of a small model of an urban housing stock. Although very simple, the model illustrates most of the issues and problems which accompany parameter selection in more complex system dynamics models. Each parameter in a properly-formulated system dynamics model corresponds to some real process or processes. To set a parameter in a system dynamics model is therefore to characterize or describe some process with a numerical value. (Section III gives examples.) The problems and issues entailed in such a characterization are virtually the same, regardless of how many other processes also must be characterized--that is, regardless of the size of the model.

The model describes the aggregate structure of an urban housing market, and is designed to trace the broad history of housing growth and stabilization in a central-city area. Figure 1 shows a DYNAMO flow diagram of the model.[1] The level represents the total number of housing units H within a specified

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[1]The model leaves a large number of factors implicit within the formulation. For example, the model assumes that enough economic development occurs close to the residential area being modeled so that jobs will be available to support occupants of the housing units. For a more explicit treatment of a housing market within the context of an urban economy, see Forrester 1969 (Appendix A), Goodman 1974b (Exercise 12), or Alfeld and Graham 1976 (Chapters 6 and 7).

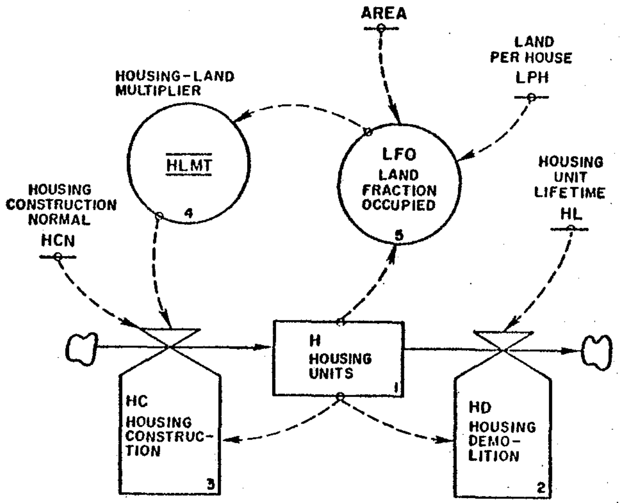

Figure 1. DYNAMO flow diagram

urban area. The rates are housing construction HC and housing demolition HD, both of which are measured in housing units per year. The equation that specifies the rate of housing demolition HD assumes some constant average housing unit lifetime HL. Sections III and IV describe the equations for this model in the context of estimating their associated parameters, with the exception of the level equation:

```
H.K=H.J+(DT)(HC.JK-HD.JK)                                1, L
H=HN                                                     1.1, N
HN=14000                                                 1.2, C
     H        - HOUSING UNITS (UNITS)
     HC       - HOUSING CONSTRUCTION (UNITS/YEAR)
     HD       - HOUSING DEMOLITION (UNITS/YEAR)
     HN       - HOUSING INITIAL (UNITS)
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II. CONSIDERATIONS IN PARAMETER FORMULATION

Much of the effort in system dynamics modeling is devoted to development of the appropriate equation formulations and their associated parameters. Only careful formulation can create parameters whose values can be set relatively straightforwardly. This section therefore discusses the considerations that enter into parameter formulation, as a prerequisite to the parameter-setting techniques discussed in Sections III, IV, and V.

A. Realism

A model is constructed to represent a set of real processes for a purpose. The ultimate test of a model's validity, therefore, is whether the characteristics of the representation agree closely enough with the characteristics of the real processes to allow the model to fulfill its purpose. The modeler can set a very high standard for the realism of a model structure by requiring that each equation in a model, and each parameter in each equation, correspond simply and directly to some specific characteristic of the real processes being modeled.[2]

For an example of direct correspondence to real characteristics, consider the equation for the rate of housing demolition HD in the simple housing model in Figure 1:

```
HD.KL=H.K/HL                                             2, R
HL=66                                                    2.1, C
     HD       - HOUSING DEMOLITION (UNITS/YEAR)
     H        - HOUSING UNITS (UNITS)
     HL       - HOUSING UNIT LIFETIME (YEARS)
```

[2] See Forrester 1967, pp. 63-64, for a discussion of this subject in the context of managerial models.

The equation defines the annual rate at which houses are demolished, which is quite directly observable. HD is defined in terms of the number of housing units H in the area being modeled, and the average housing unit lifetime HL (the average age of housing units when they are demolished). Both quantities correspond to observable characteristics of the real system.

While an equation and its associated parameters may provide a good description of the real system, it may not be the best description. To avoid becoming fixated upon one set of parameters, the modeler must realize that, for the most part, parameters only describe the real system; they have no direct structural counterpart in the real system. For example, the housing-land multiplier table HLM exists only as a set of model parameters. In the real system, people buy and sell land, and sometimes erect buildings. HLMT merely describes those processes, and other descriptions are possible.[3]

Striving for a general description is sometimes more desirable and easier than creating a formulation that describes only a specific case. For example, suppose a model of a large corporation requires equations that describe pricing decisions. Is the price determined by supply and demand in a somewhat competitive market, or is the price determined by a traditional mark-up above costs in a somewhat oligopolistic market? The modeler could choose a pricing equation that reflects one or the other hypothesis, but a pricing equation capable of representing both hypotheses and every alternative in between (by means of different parameter values) would be clearly superior.

[3] See Mass 1974a and Miller 1975 for different, more detailed descriptions of the markets for urban land and housing.

One symptom of a non-general formulation is the presence of parameters whose values cannot be realistically set independent of the values of other parameters. If each parameter truly describes some unique, individual, and observable characteristic of some part of the system, then every combination of parameter values should have some plausible analogue in a real system. But it is quite possible to formulate a model in which only some combinations of parameters have a realistic interpretation, while other combinations give nonsensical results. As a relatively obvious example, a model of budget allocation within a firm could have parameters that could be set to continually allocate 200 percent instead of 100 percent of the firm's income.[4]

B. Effective Data Utilization

Every parameter in a system dynamics model must be assigned a specific numerical value before the system behavior can be simulated. Therefore, the equation formulations and their associated parameters should not only provide a realistic description of the real system, but should also facilitate parameter estimation.

What equation and paramter formulation best facilitates parameter estimation? The answer depends on the kinds of data to be used to estimate the parameter value. If the model parameters are to be set on the basis of detailed, firsthand observations, the model parameters should correspond simply and directly to observable characteristics of the real processes being represented. On the other hand, if an abundance of aggregate statistical information

[4] A superficial cure for such difficulties is to make algebraic constraints (such as allocating 100 percent of income) part of the model structure, either in initial computations or in the auxiliary equations. However, interdependent parameters (especially parameters describing allocations) often indicate missing levels or overaggregation. For example, a budget constraint is an aggregation of the feedback structure that surrounds the level of cash possessed by a firm or a household. (The feedback causes spending to increase when cash builds up and spending to decrease when cash is short.) The budget constraint is a behavioral consequence of that feedback structure.

is to be used to estimate the value, the model equations should be formulated to facilitate the necessary computations (even though such formulations may not match the features of the real system very closely).[5] These two types of data will be called (in this paper) data below the level of aggregation of model structure and data at the level of aggregation of model structure, respectively.

Data below the level of aggregation of model structure are observations and measurements of the processes whose aggregate is represented by a model equation. For example, consider the processes involved in housing demolition. One can observe the processes of aging and obsolescence which gradually render a housing unit less and less habitable. One can observe the other processes by which houses are destroyed such as fire or replacement by new construction in urban redevelopment. One can observe the details of the demolition of individual housing units, whose aggregate is represented in the model by the rate of housing demolition HD. If such observations are to be used to set model parameters, the parameters should directly correspond to observable characteristics, such as the average housing unit lifetime HL at the time of demolition.

The other kind of information that can be used to set model parameters is data at the level of aggregation of model structure. Such data closely correspond to model variables. For example, a model variable might be the

[5]The form of equations suitable for statistical estimation must often utilize relatively aggregate data, so that the equations usually do not depict the detailed processes through which the independent variables influence the dependent variable. Also, analytic tractability often restricts the form of such equations to be linear, with only one parameter per independent variable (even though in reality the independent variable may act upon the dependent variable linearly or nonlinearly through a variety of channels).

annual rate of housing demolition HD within an area, and the corresponding data then would be the number of housing units destroyed each year. Both the model variable and the associated data represent the aggregation of a number of objects: apartments, condominiums, single-family wooden houses, old houses, and so on. The model variable and the data also represent the effects of a number of processes: obsolescence of facilities within housing units, gradually-accumulating damage to the interior and structure of housing units, declining rent levels, declining maintenance expenditures, and condemnation proceedings, to name a few. The data and the model variable are therefore on the same level of aggregation. How can the modeler infer parameter values from data that correspond to model variables? The data alone do not suffice, since they describe the behavior of model variables, but not the model parameters. The modeler must also use a model equation or several equations to compute parameter values from data on model variables. One difficulty with data at the level of aggregation of model variables is that the computations require the use of assumptions about one or more equations. Such assumptions always constitute "more rope to hang yourself with." The more assumptions, the more opportunities for error. In contrast, setting parameters from data below the level of aggregation of model variables allows each parameter to be set and judged independently, without computations based on the rest of the equation which it helps to specify.[6]

[6]Econometricians are aware of an analogous situation in estimating simultaneous-equation models. Even though simultaneous-equation estimation methods theoretically deliver greater accuracy than multiple applications of single-equation methods, the simultaneous-equation methods are more sensitive to minor violations of assumptions (less robust) than single-equation methods. Similarly, paramter estimation from data at the level of aggregation of model variables is less robust than parameter estimation from data below the level of aggregation of model variables.

One potential hazard exists in using data below the level of aggregation of model structure. The hazard lies in formulating a model structure and parameters that are aggregated to the point where one cannot reliably observe the processes being characterized by the parameter values. For example, in the simple housing system, a variety of processes determine how long it takes the system to make a transition between growth and equilibrium--incentives to construct housing, supply and demand effects in the land market, and housing depreciation, for instance. In the model, a number of different parameters characterize these diverse processes. An alternative formulation of the model might have contained a single parameter that specified the time constant for the transition from growth to equilibrium. Urban experts may very well be willing to give estimates of such a quantity, but the number would be a conclusion or opinion drawn from their mental models of how the system behaves, rather than a report on direct observations of events in the city.

Another example of confusing observations with conclusions occurs in the field of international trade, where experts needed to predict the time it would take for the volume of trade to adjust to the Smithsonian currency realignment. Junz and Rhomberg 1973 show that statistical estimates of this delay time differ from the expert opinions by about a factor of two. That a difference exists is not surprising. That the difference is only a factor of two <u>is</u> surprising. Consider: the experts were attempting to predict, on a purely intuitive basis, the behavior resulting from a very high-order, nonlinear, multiple-loop feedback system, involving a wide diversity of processes, including marketing, inventorying, production, hiring, financing, and

pricing.[7] If one formulates an equation with parameters that characterize the result of a complex set of interactions by a single number, then one must of course experience great difficulty in obtaining reliable expert opinion or other data below the level of aggregation of model structure: the model structure is aggregated well above the point where a person can reliably witness the workings of its components.[8]

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[7] There is an epistomological difficulty associated with the distinction between parameters that describe system behavior and parameters that describe processes within the system. The difficulty is that ultimately, all that one ever observes is behavior. For example, the observed average lifetime of a housing unit discussed above could be considered as the behavioral result of a more detailed system of interactions among rent levels, maintenance and capital costs, population and income levels, social traditions about housing, and many other variables. So regardless of how detailed one makes a model, in a strict philosophical sense, one always has parameters that are descriptions of the outcome of processes not explicitly represented. These descriptions are adequate for the purposes of the model provided that the outcomes of the processes not explicitly represented do not change significantly as a result of the dynamics being modeled. For example, one could ask whether the average lifetime of a housing unit changes as the system makes the transition from abundant land to scarce land.

[8] There are two possible courses of action when one has formulated a model whose parameters are too aggregated to be set reliably from available data below the level of aggregation of model structure. One course is to restructure (usually disaggregate) the model so that its parameters do correspond directly to observable unchanging characteristics of processes within the system. The disaggregation will usually involve not only subdivision of levels into more levels, but also explicit addition of feedback loops that control the levels. For example, Mass 1974a and Miller 1975 disaggregate the relationship between land availability and urban housing construction discussed in Section III.B of this paper. The other course of action when a parameter is too aggregated to be set reliably from available data below the level of aggregation of model structure is to use another estimation technique and data at the level of aggregation of model structure--usually statistical techniques. It seems unwise, however, to attempt to estimate a simple relationship if the actual system is complex enough to render expert opinion unreliable. In the foreign-trade example above, for instance, the aggregated delay between exchange rates and trade volume could (for some model purposes) be completely inadequate: numerous other variables impinge on that part of the system, including, among others, forward exchange rates; availability of arbitrage capital; relative interest rates; expected exchange rates; availability of capital, labor, and financing; and transport costs. For many model purposes, the total aggregate delay time must be regarded as an endogenously-determined dynamic variable, and not as a constant parameter.

The final consideration about effective data use concerns model validation. After a model has been formulated and the parameter values set, validation tests demonstrate whether or not the model's structure and behavior agree (are consistent with) available data about the system being modeled. The more a model's equations, parameter values, and behavior resemble the known characteristics of the real system, the more valid the model.

If parameters are computed from data at the level of aggregation of model structure, the model is to some extent forced to replicate the characteristics of those data. These characteristics can range from phase and magnitude relationships between individual variables to the entire system's behavior mode. But such a replication cannot increase confidence in the validity of the model. The method of parameter estimation forces the model behavior and real behavior to agree.

One possible strategy for data use would be to use aggregate numerical data (data at the level of aggregation of model structure) to set parameter values. The information left over for testing the validity of the model is then the data below the level of aggregation of the model structure (that is, observations of the details of the processes being modeled), plus whatever aggregate information remains to be extracted by more elaborate statistical tests. This strategy seems to provide many opportunities for error. It is too easy to declare parameter values reasonable and characteristic of the real system after the fact, even when they may not be (especially since a model whose parameters are set by statistical or numerical procedures may not have paraemters that correspond directly to observable characteristics of the real system; see footnote 5). Statistical testing procedures do not seem appropriate for validity testing, both because of logical limitations (discussed in Mass

and Senge 1976), and because the tests are predicated upon many assumptions, which when false can cause the tests to indicate good results spuriously (see Senge 1975a).

Another strategy for data use, more commonly employed in system dynamics models, is to reserve the more aggregate numerical information for validity testing, and to set model parameters from data below the level of aggregation of the model structure. This strategy maximizes the opportunities for unbiased utilization of data below the level of aggregation of model structure. By excluding the use of aggregated data from parameter setting, this strategy also maximizes the amount of aggregate data that can legitimately be used to test model validity, without resorting to complex and non-robust statistical tests.

To summarize this section, realistic equation formulation should provide a recognizable yet general description of some real process, in which each parameter describes some independent characteristic of the process being modeled. Formulation at the proper level of aggregation can facilitate parameter setting and validity testing, by allowing parameters to be set from data below the level of aggregation of model structure. This reduces the number of assumptions made in parameter settings, and allows the more aggregate data to be used in validity testing.

III. ESTIMATION FROM DISAGGREGATE DATA

A. Overview of the Three Categories of Parameter Estimation Techniques

Each of the next three sections describes one category of parameter setting technique. (Several examples of each type are given.) In order to delineate the differences between the types, we will discuss all the types in this subsection.

This section describes estimations from disaggregate data or, more precisely, data below the level of aggregation of model structure. As an example, the text has already discussed how the average housing unit HL can be estimated by observing the age of individual housing units when they are demolished. Estimations from disaggregate data may or may not involve computation, but the computations never involve the actual model equations.

Section IV describes equation estimation, which uses data at the level of aggregation of model structure in a computation based on the equation that contains the parameter being estimated. For example, assuming that Equation 2 defining housing demolition HD is correct, dividing the number of housing units by the number of housing units in a year within an area yields an estimate of average housing unit lifetime HL.

Section V describes model estimation, which uses data at the level of aggregation of model structure in a computation based on the entire set of

model equations.[9] For example, one could simulate the housing model with various values of the average housing unit lifetime HL until the model behavior approximately matches observed historical behavior.

B. Estimating Between Limits

There are several variations in the technique of estimation from disaggregate data; the example of estimating the average housing unit lifetime HL is the simplest variety, where a single parameter is directly set equal to an easily-measured quantitative characteristic of the real system. A slightly more complex situation arises when not enough observations are available to set a single unique value. Even so, the modeler can obtain a parameter estimate by considering upper and lower limits, which the parameter values should not approach.

For example, Equation 3 defines the rate of housing construction HC as the product of the number of housing units H, the housing construction normal HCN, and the housing-land multiplier HLM.

```
HC.KL=(H.K)(HCN)(HLM.K)                                    3, R
HCN=0.07                                                   3.1, C
    HC      - HOUSING CONSTRUCTION (UNITS/YEAR)
    H       - HOUSING UNITS (UNITS)
    HCN     - HOUSING CONSTRUCTION NORMAL (FRACTION/YEAR)
    HLM     - HOUSING-LAND MULTIPLIER (DIMENSIONLESS)
```

The number of housing units H indicates the size and inherent ability to engender growth of the developing community or city being modeled. The housing

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9 Single-equation methods in econometrics (such as the family of least-squares estimators) are equation estimations. The econometric full-information maximum-likelihood (FIML) techniques and the control engineering full-information maximum-likelihood via optimal filtering (FIMLOF) techniques are both model estimations. For descriptions of FIMLOF techniques, see Peterson 1976, Peterson 1975, Peterson and Schweppe 1974, or Schweppe 1974, Chapter 14.

construction normal HCN is the proportion of additional new houses a community or city can build under some defined set of normal conditions. Under such normal conditions, the third factor, the housing-land multiplier HLM, assumes a value of 1.0. HLM represents the effects of deviations from the set of normal conditions on construction by exceeding or falling below 1.0.[10]

What values are appropriate for housing construction normal HCN? One set of answers comes from observations of houses being built and neighborhood expansions. If normal conditions are defined to apply when the community can still experience substantial growth, an HCN value of 0.01 is too small. Such a value would imply that a community of 100 houses, despite the availability of acceptable construction sites, would have only one more house built in it in a year, which is not substantial growth. At the other extreme, a value of HCN of 1.0 is clearly too large. With such a value, every year, new housing units would be constructed in numbers equal to the size of the housing stock at the beginning of that year. Neighborhoods seldom, if ever, grow so rapidly. How fast do neighborhoods grow? A realistic value of HCN must lie somewhere between 0.01 and 1.0. Simply choosing a value for HCN somewhere between these two extremes may suffice for the purpose of the model. Alternatively, the modeler may have to seek more precise information to further narrow the range of possible parameter values. (Section V discusses methods of determining how accurately parameters must be set.)

[10]The normal conditions chosen for this model are the conditions that occur when the growing housing stock first occupies 80 percent of the land area being modeled. "Normal" is used here in the scientific sense of normalized quantities, rather than in the sense of either "typical" or "healthy." Alfeld and Graham 1976 (Section 5.3) and Graham 1974 further discuss the use of normal conditions in model formulations.

C. Estimating Table Functions with Extreme Conditions, Normal Points, and Smooth Curvature

Table functions seem to constitute a formidable estimation problem, since they are typically specified by 5 to 15 numbers. But the problem can be broken into subproblems: estimating the value and the slope of the function at one extreme, at the normal value, and at the other extreme. The remaining subproblem is connecting the known values and slopes with a smooth curve.

For example, consider Equation 4, which specifies the housing-land multiplier HLM as a table function of the land fraction occupied LFO.

```
HLM.K=TABLE(HLMT,LFO.K,0,1,0.1)                              4, A
HLMT=.4/.7/1/1.25/1.45/1.5/1.5/1.4/1/.5/0                    4.1, T
    HLM    - HOUSING-LAND MULTIPLIER (DIMENSIONLESS)
    HLMT   - HOUSING-LAND MULTIPLIER TABLE
    LFO    - LAND FRACTION OCCUPIED (DIMENSIONLESS)
```

Under the extreme condition of very low land occupancy, incentives for construction should be appreciably lower than under normal conditions. When the land fraction occupied LFO approaches zero (near the left side of the curve in Figure 2), the area being modeled is mostly vacant land. The area's viability as an urbanizing entity has not yet been demonstrated. Developers cannot count on continuing demand for the housing units they construct. Many services taken for granted in more heavily-settled areas must be installed new neighborhood—by—new neighborhood: roads, sewers, electricity, gas, and schools. These services will by no means be complete in an area too sparsely-settled to make even city water or sewers an economical proposition, let alone public transportation. So the housing-land multiplier HLM should be well below 1.0 when LFO is 0.0; Figure 2 gives HLM a value of 0.4 when LFO equals 0.0 (Point A).

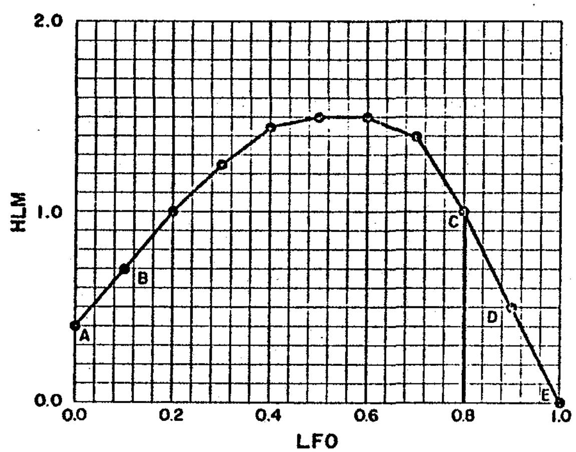

Figure 2. Housing-land multiplier table

Adding housing units to a sparsely-settled area gradually makes more and more urban services economical, paying propositions, thereby making housing construction more attractive and more obviously profitable. But adding a few houses cannot pay for the infrastructure--schools, roads, libraries, utilities--necessary to deliver a complete ensemble of urban services. The curve for the housing-land multiplier table should slope upwards, but not very steeply from where LFO equals 0.0. (See the line segment between Points A and B on Figure 2.)

Now consider the normal condition, defined as the condition that occurs during the normal period, when the land fraction occupied LFO equals 0.8. By definition, under normal conditions, the rate of housing construction HC equals the product of housing units H, and housing construction normal HCN, Therefore, when LFO equals 0.8, HLM must exert no influence on HC, and must equal 1.0 (Point C on Figure 2).

Consider the other extreme condition in which the land fraction occcupied LFO equals 1.0. The land area within the community, city, or district being modeled is fully and totally occupied. Even the least desirable sites have been built upon. Regardless of whatever incentives exist to construct housing, no housing can be constructed within the area being modeled until there is some physical space available upon which to build--until LFO ceases to equal 1.0. So the housing-land multiplier HLM should equal 0.0 when LFO equals 1.0, which establishes Point E on the graph of the housing-land multiplier table HLMT in Figure 2.

If the land fraction occupied LFO was not 1.0 but close to 1.0 (nearly full land occupancy), urban services such as sidewalks, schools, libraries, roads, and public transportation would already be installed and fully developed. To be sure, the crowding and lack of desirable construction sites implied by an LFO close to 1.0 would not permit housing construction to take place so rapidly under the normal conditions. Nonetheless, any small reduction of LFO from 1.0 opens up the possibility of appreciable housing construction. Therefore, the curve of the housing-land multiplier should probably be fairly steeply-sloped as LFO approaches 1.0. (See the line segment between Points D and E on Figure 2.)

So far we have estimated the values at and near two extreme conditions and at the normal condition. Now all that remains is to draw a curve through the estimated points. Any sharply bent or kinked curve is probably not very realistic. A bend or kink implies something special and unique about the exact conditions at which the bend or kink occurs. Since the housing-land multiplier table HLMT represents a very large number of processes, the probability is vanishingly small that all of the processes represented would show major changes under a single unique set of conditions. Accordingly, the curve for HLMT (and in general, all highly-aggregated relationships) should change smoothly, without kinks or bends.

Solving the subproblems of extreme conditions, normal conditions, and connecting known points with smooth curves, allows the modeler to estimate a nonlinear table function with a high degree of confidence. The estimated table summarizes observations of a large number of processes below the level of aggregation of model structure. HLMT is the aggregate representation of these processes and their effect on housing construction.[11]

D. Calculating a Parameter Estimate from Disaggregate Numerical Data

The modeler can combine numerical estimates or observations of processes below the level of aggregation of the model structure into values for model parameters. For example, consider an equation in an ecological model which specifies the birth rate of rabbits BR (measured in rabbits per month) as the product of the total rabbit population RP and some constant function, the rabbit birth fraction RBF:

R BR.KL = RP.K*RBF

The average person may not seem to have enough information to specify a value for RBF, but most people in fact know enough about the biological characteristics of rabbits to specify at least an approximate value. Suppose that a mature

[11] HLMT can be considered to be the composite of two nonlinear functions, one of which represents the simulating effect of infrastructure development on housing construction. The other represents the inhibiting effect of low land availability on housing construction. In general, if a curve becomes any more complex than the hump-shaped HLMT curve, it should be broken into components and its components each estimated separately. Customarily, most multipliers in system dynamics models have a simple monotonic form.

female rabbit litters about every 5 months, and that about 4 babies per litter survive. Since about half the mature rabbit population is female, that makes

$$\frac{\text{1 litter}}{\text{5 months}} \times \frac{\text{4 babies}}{\text{1 mature female}} \times \frac{\text{1 mature female}}{\text{2 mature rabbits}}$$

$$= 0.4 \text{ babies / mature rabbit / month.}$$

But not every rabbit is mature. If rabbits live about 4 years or 48 months, and require about 6 months to mature, and if the rabbit population is evenly distributed, then (48 - 6)/48, or 0.875 of the rabbit population will be mature. If the rabbit population is growing, then there will be proportionately more young rabbits in the population, which reduces the fraction of mature rabbits. Assume that the rabbit population being modeled is growing rapidly. Therefore, the fraction of mature rabbits should be less than 0.875, say around 0.5. For the whole rabbit population, there are:

$$\frac{\text{0.4 babies}}{\text{mature rabbit—month}} \times \frac{\text{0.5 mature rabbits}}{\text{rabbit}}$$

$$= 0.2 \text{ babies/month/rabbit}$$

So, setting the rabbit birth fraction RBF equal to 0.2 should be fairly close to the value that would be derived by direct observation.[12] (Senge 1975b discusses a more complex computation in a managerial model.)

[12] The parameter estimation uses only the author's impressions of the biological characteristics of rabbits. The reader may wish to check the parameter value derived above against more detailed observations and measurements. An encyclopedia should have information about the average longevity, maturation time, gestation period, and litter size and frequency of rabbits. (Don't forget to account for infant mortality when carrying out the computation, and make sure the lifetimes and infant mortality rates apply to a growing rabbit population.)

IV. EQUATION ESTIMATION

Estimation from disaggregate data employs data below the level of aggregation of model structure, and never uses a model equation to compute a parameter value. In contrast, equation estimation employs data at the level of aggregation of model structure, and must always use a model equation to compute a parameter value.[13] Subsection III.A described estimating the average housing unit lifetime HL by using the equation for housing demolition HD and data on HD and housing units H. The following subsection describes a slightly more complex example.

A. Estimating a Normal Fractional Rate of Flow

The format for many rate equations in system dynamics models is

$$\text{Rate} = \text{Level} * \text{Normal Fraction} * \text{Multipliers}$$

By simple algebra, the value for the normal fraction is given by:

$$\text{Normal Fraction} = \frac{\text{Rate}}{\text{Level} * \text{Multipliers}}$$

Under the normal conditions (at whatever time period it is defined), the multipliers, by definition, assume values of 1.0. So the normal fractional flow rate can be computed by dividing the observed rate by the observed level, both measured during the period of normal conditions. For example, suppose that one defines the year 1960 as the normal period for the urban

[13] Only in rare instances can the modeler use simple manipulation of the model equation to estimate more than one parameter. Single-equation econometric techniques routinely estimate many parameters simultaneously, with correspondingly more stringent requirements for specification and data accuracy.

area being modeled. Then, if the data are available, one can divide the number of housing units constructed in the area during 1960 by the number of housing units in the area in 1960 to obtain a value for housing construction normal HCN.

Equation estimation requires several assumptions--in this case, that the data apply to the normal period, that the data are accurate, and that the equation is accurate. These assumptions provide opportunities for errors. One example occurred in an attempted revision of the Urban Dynamics model (Forrester 1969) in Babcock 1970. Babcock attempted to set normal constants using data on levels and rates of flow, but neglected to use data only for the normal period. He used data for cities near equilibrium also. The simple housing model presented here can show what happened as a result. The housing model reaches equilibrium after the housing stock grows until a shortage of land suppresses further housing construction. Because the normal conditions in the model are growth conditions, the housing-land multiplier HLM must suppress housing construction by going well below 1.0. Suppose we divided the actual rate of housing construction HC_a by the actual number of houses H_a to obtain a computed value for the housing construction normal HCN_c. If the model equations are accurate, using equilibrium data to compute HCN:

$$HCN_c = \frac{HC_a}{H_a} = \frac{H_a * HCN_a * HLM_a}{H_a} = HCN_a * HLM_a$$

which means that

$$HCN_c < HCN_a .$$

Using the computed value of HCN in a model reduces the model's impetus to grow, and thus reduces the extent to which HLM must drop to bring the model into equilibrium. Similarly in Urban Dynamics, growth ceases when land shortage and unfavorable internal conditions (principally a job shortage and predominance of lower-income groups) depress construction. Using data from near-equilibrium to compute normal fractions considerably reduces the extent to which internal conditions in the model must decline to halt growth. In fact, the model will no longer reproduce and account for depressed urban conditions. Babcock's modified Urban Dynamics model therefore no longer even fulfills its purpose, merely because the implicit assumptions used in parameter setting were violated.

B. Estimating a Conversion Factor

A large number of parameters are conversion factors, which convert quantities from one dimension to another. For example, land per house LPH converts housing units to an equivalent number of acres. Equation 5 uses LPH in the definition of land fraction occupied LFO:

```
LFO.K=(H.K*LPH)/AREA                                     5, A
LPH=0.1                                                  5.1, C
AREA=9000                                                5.2, C
    LFO    - LAND FRACTION OCCUPIED (DIMENSIONLESS)
    H      - HOUSING UNITS (UNITS)
    LPH    - LAND PER HOUSE (ACRES/UNIT)
    AREA   - AREA (ACRES)
```

Equation 5 could be manipulated to compute LPH as a function of LFO, housing units H, and AREA. The only difficulty with such a computation lies in making sure that the definitions of the data used are appropriate for the model. For example, land per house LPH must include not only the land directly beneath each housing unit, but also the associated land used for yards, sidewalks, roads, garages, driveways, and schools and stores serving the neighborhood. The modeler might suppose that the land per house LPH for a particular area could be calculted from the land area zoned for residential use (minus the area of vacant lots), divided by the number of dwelling units within the area. However, many cities have land that is zoned for both residential and commercial use; some fraction of that land must be included in the residential land area as well. Once the definitional considerations have been laid to rest, conversion factors are relatively straightforward. Schroeder and Strongman 1974 describe the use of such procedures to adapt the Urban Dynamics model (Forrester 1969) to a real city.

V. MODEL ESTIMATION

As just described, equation estimation consists of manipulating one model equation to compute a parameter value. In contrast, model estimation consists of manipulating all of the model equations to compute a parameter value. For example, the housing construction normal HCN could be estimated by finding the value of HCN that causes housing growth to fit the observed rate of growth. The fitting could either be performed with repeated simulations or (if possible) by an ad hoc computation. For example, say that the stock of housing grew at 4.0 percent per year under normal conditions. Also suppose that, from observation of housing demolition, the housing unit lifetime HL is estimated to be 66 years--that is, 1/66 of the houses are demolished each year. If the model equations are assumed to be correct, then the housing construction normal HCN must exceed 1/66 by 0.04 to produce the observed rate of growth during the normal period. Therefore, HCN can be inferred to be $1/66 + 0.04 \cong 0.07$. As another example, suppose a real system exhibits fluctuations of some specific period. The modeler can choose the magnitudes of time constants of the system so as to produce oscillations near the real period. (Forrester 1968, Chapter 10, derives a simple rule of thumb: for a system with two time constants τ_1 and τ_2, their geometric average approximately equals the period divided by 2π: $\sqrt{\tau_1\tau_2} \cong P/2\pi$.)

One danger of model estimation is misattributing the observed behavior to the value of a particular parameter. In the oscillation example just cited, if τ_1 is inaccurate, model estimation will compute an inaccurate value for τ_2 as well in order that $\sqrt{\tau_1\tau_2} \cong P/2\pi$. As a subtler example, the Urban Dynamics model was once being modified to match the historical growth and decline of Lowell, Massachusetts. A period of rapid growth early in the city's history was being modeled by altering a table function similar to the housing-land multiplier HLM. The table, arrived at through repreated simulations, had about the same values at the extremes and normal points as the curve in Figure 2, but Point B was well above 1.0. Although the altered curve allowed the model to reproduce the historical behavior quite accurately, it no longer constituted a realistic representation of the true cause-and-effect relationships within the city.[14]

The modeler can choose one parameter value over another merely because it yields model behavior closer to real system behavior. But such a technique

[14] Errors in model estimation are often detected by checking the results with other data. In the urban example above, the table function was deemed incompatible with day-to-day observations on the process of industrial development (which is data below the level of aggregation of model structure). This use of two independent sets of data is equivalent in principle to the long-standing econometric practice of estimating parameters with data from one time period and evaluating the parameter estimates with data from another time period.

presumes that the entire model structure is correct, which is equivalent to making the maximum possible number of assumptions. Then, the falsity of any one of the assumptions can in principle cause serious problems. The modeler might better avoid making chains of assumptions, where possible, by setting parameter values from easily-observed characteristics of the processes being modeled (data below the level of aggregation of model structure). A model is more credible if each formulation and each parameter value stands independently as a plausible and realistic representation of a real process.[15]

VI. PLANNING PARAMETER ESTIMATION EFFORTS

The preceding sections have discussed considerations in parameter formulation (Section II) and a variety of techniques for estimating parameter values (Sections III, IV, and V). Those discussions cover the parameter-related issues involved in arriving at an initial model, the accuracy of whose parameter values may or may not suffice to allow the model to fulfill its purpose. What are the appropriate next steps?

[15] This is not to say that model estimation techniques cannot increase one's confidence in a model. If one has a means of detecting errors in model estimations, such estimations can be quite useful in formulating and validating a model. For example, Peterson 1975 used statistical consistency checks and strong prior parameter values to uncover flaws in developing a model of energy demand. After the flaws were corrected, confidence in the model was much increased when model estimation (which is rather sensitive to specification problems) failed to indicate further problems. If, however, one does not have an independent means of checking the model estimation, the resulting parameter values seem highly likely to contain systematic errors.

A. Strategies

The modeler could devote considerable time and effort to estimating realistic and accurate parameter values, and defer further work on model testing and refinement of formulations. Or, one could continue the development of the model formulation through model testing, and defer the parameter-estimation effort. (Forrester 1961, Chapter 13, and Mass and Senge 1976 further discuss general model testing.) What strategy best allows the model to fulfill its purpose? Most system dynamics models do not have the purpose of precise numerical prediction. Instead, they are usually aimed at replicating the causes of an undesirable behavior mode, and investigating policies that diminish or eliminate the undesirable behavior.[16]

Such a purpose allows the system dynamics model to capitalize upon a remarkable fact: system dynamics models usually represent nonlinear, high-order, multi-loop feedback systems, whose responses infrequently show sensitivity to a parameter variation.[17] System dynamicists capitalize on this fact by constructing models using very rough, very quick parameter estimates. The completed model itself can then serve to assess the model's need for accurate parameter values: by testing model behavior when parameters are changed, the modeler sees whether or not altering the parameter value after

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[16] Forrester 1961, pp. 123-128, describes this distinction in terms of predicting a future system state versus predicting the system behavior.

[17] There appear to be four structural causes of parameter insensitivity. One, minor negative feedback loops; they tend to compensate for parameter changes within them. Britting and Trump 1975 further discuss this subject. Two, structure outside dominant loops; usually, only a relatively small number of feedback loops (the dominant loops) produce the system behavior. Parameters that characterize processes not involved in any of the dominant loops cannot have much affect on behavior. Three, redundancy; a feedback loop can have several branches, so that parameter changes that inactivate one branch cannot prevent the feedback from functioning. Four, numerical insignificance; for example, doubling the time constant or a relatively short delay in a series of delays does not significantly change the overall response time.

a laborious redetermination of the value could possibly have an effect on the outcome.

Usually, only a few parameter values significantly influence the outcome. Those parameters alone warrant further effort in formulation and estimation. The initial rough estimates of most of the parameters are accurate enough for the purpose of the model. The following subsection discusses the simulation tests that distinguish the sensitive from the insensitive parameters.

B. Sensitivity Testing

Uncertainties and inaccuracies in parameter values may affect either the model behavior or the policy recommendations derived from the model. Testing behavior sensitivity requires a comparison of two simulations: a reference simulation, and a simulation with an altered parameter value.[18] For an example of behavior sensitivity, suppose that minor parameter variations cause the housing model to exhibit several distinct modes of behavior. Perhaps the behavior of real urban areas depends critically on the processes represented by the sensitive parameters. If each of the several model-behavior modes

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[18] A model can be subjected to two types of parameter variations. One type is to evaluate model behavior or policy impact only over the range of values that the parameter could realistically assume. For example, if housing construction normal HCN could plausibly lie only between 0.04 and 0.4, then 0.04 and 0.4 would be the extreme values of HCN tested. The other type of parameter variation is to raise or lower a parameter value progressively to find the point at which the parameter variation substantially alters the model behavior or policy impact. For example, lowering HCN far enough would cause the rate of housing demolition HD to exceed the rate of housing construction HC so that the number of housing units H would shrink instead of grow. Both types of parameter variation are appropriate for either behavior or policy sensitivity testing; the former variation gives more information about the realism of a model or workability of a policy, and the latter variation gives more information about the possible behavior modes of the system.

corresponds to a real situation or example, the parameter sensitivity builds confidence that the model captures the essential features of the real system. However, if the model exhibits a behavior sensitivity that does not correspond to the behavior sensitivity of real urban areas, then the model requires careful reexamination. The sensitive parameter indicates an area that requires either reformulation or reestimation.

Parameter variation may also alter or reverse the impact of simulated policy changes. The model user needs to know whether a policy yielding favorable results with one set of parameters can also yield unfavorable results with a different set of parameters. Susceptibility of policy results to parameter changes is called policy sensitivity. Passing a policy-sensitivity test builds confidence in the policy recommendation, while passing a behavior-sensitivity test builds confidence in a model structure.

A policy-sensitivity test requires at least four simulations: a reference simulation, a simulation of the policy change, a reference simulation with an altered parameter value, and a policy simulation with the same altered parameter value. Figure 3 shows the procedure for comparing simulations. First, determine the impact of the policy change by comparing the reference simulation and the policy simulation. Second, determine the impact of the policy change under the conditions depicted by the altered parameter value: compare the reference simulation with the altered parameter value to the policy simulation with the altered parameter value. At this point, the modeler knows the impact of the policy change upon the original model and upon the model with an altered parameter value. Comparing the two policy impacts provides a measure of whether or not the given parameter variation affects the desirability of the policy--the policy sensitivity.

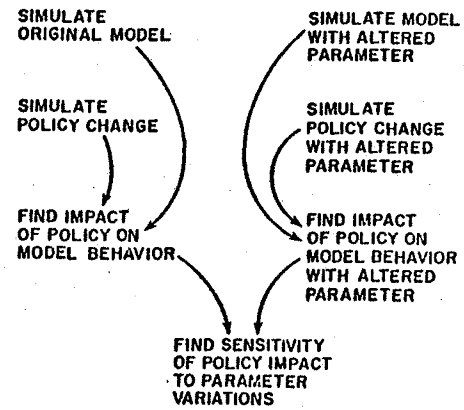

Figure 3. Procedure for testing policy sensitivity

In performing various types of model tests, the modeler might be tempted to avoid thoroughly analyzing the model behavior and examine only the end result: whether or not the overall behavior changes in response to a parameter value change. Especially for a model suspected of being faulty, or during numerous sensitivity tests, one might not take the time to analyze exactly why the model behaves as it does in each simulation. (Senge and Mass 1976 give an example of model analysis.) Such a purely technical analysis, however, provides several benefits. First, a technical analysis simplifies model testing. If the structural causes of a system's insensitivity are known (see footnote 17), one can immediately identify whole areas of the model structure that are not important (in the behavior mode being tested). Moreover, one can identify the areas in the model structure that are important to the behavior mode being tested, and that require further investigation. So, contrary to initial suspicions, performing a technical analysis may shorten, instead of lengthening, the testing process.

Furthermore, purely technical analysis of model behavior also begins to address the problem of sensitivity to multiple parameter changes. It is almost feasible to test the effect of all single parameter variations for most models. However, the number of possible combinations of multiple parameter changes in medium-sized or large models is far too large for the modeler or a computer to test all possible combinations of parameter changes.[19] But if a tested theory is available to explain the underlying causes of the model's behavior and its insensitivity to parameter variations, the modeler can distinguish between the multiple parameter changes that have a significant impact and those that do not. Exhaustive testing is therefore not necessary in such cases.

C. Dealing with Sensitive Parameters

The identification of behavior-sensitive and policy-sensitive parameters can help to guide model reformulation, parameter estimation, and policy development, as shown in Figure 4. The upper part of the figure illustrates behavior-sensitivity testing (1) as a means of evaluating the realism of the

[19] It is not clear at all, however, that changes in large numbers of parameters are reasonable tests. Assuming a Bayesian viewpoint, assign each parameter value a probability, and assume that the parameters are independently distributed. Changing a single parameter away from its most probable value reduces its probability by some factor. For example, assume the factor is 0.5. The probability of the entire set of parameters is reduced in proportion to the product of the reductions in the individual probabilities. The probability of the entire set of parameters therefore is reduced by 0.5. Then a multiple parameter change reduces the probability density of each parameter by 0.5. So making four such parameter changes reduces the aggregate probability by 0.0625. Investigating the consequences of such an unlikely event as four such parameter changes does not seem very worthwhile.

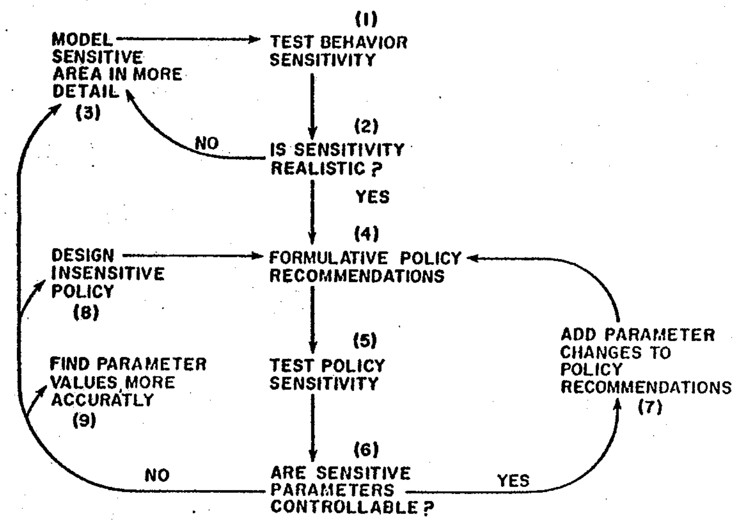

Figure 4. Sensitivity testing in policy analysis

model. If the behavior sensitivity of the model does not correspond to the behavior sensitivity of the real system (2), the model formulation must be refined until they do correspond (3). The model then can be used to identify policies that improve the behavior of the system (4).

To establish confidence in the policy recommendations, the policies should be tested for their sensitivity to parameter variations (5). Suppose variations in a parameter influence the desirability of a policy. The first question to pose is whether or not the model user can control or influence the real processes represented by the sensitive parameter (6). For instance, suppose a policy of encouraging business expansion is sensitive to the average housing unit lifetime HL.

To some extent, policy-makers may be able to manipulate the average lifetime of housing by altering assessment and property-tax practices or zoning. Adjusting the values of controllable parameters should be incorporated into the policy recommendations (7). Of course, the policy of encouraging business expansion and altering housing lifetimes then requires further testing for sensitivity to other parameters (5).

If policies are sensitive to uncontrollable parameters, model users have three options. First, perhaps the easiest option is to model in more detail the processes represented by the sensitive parameters (3). Model parameters describe the aggregate effects of processes below the level of aggregation of model structure. These processes may occur over time periods much shorter or longer than the time horizon the model is intended to portray. For example, the housing-land multiplier table HLMT implicitly represents both the purchase and sale of parcels of land and the elevation of land prices when unoccupied land becomes scarce. HLMT gives the longer-term, aggregate results of these short-term processes: building construction slows down as the land approaches full occupancy. If the feedback loops that regulate land use are explicitly represented, the revised model may show significantly less parameter sensitivity than the original representation. (Mass 1974a and Miller 1975 give more detailed models of land use.)

The second option is to use the model to search for combinations of policies not sensitive to model parameters (8). Single policy changes which produce only moderate improvements and are fairly insensitive to parameter variations occasionally may be combined into a potent, insensitive policy. (Mass 1974b and Forrester 1969, pp. 227-237, give examples.) Third and finally, if model reformulation and policy redesign both fail, the modeler must resort to

some form of empirical research to determine more accurately the value of the sensitive parameter (9).

VII. CONCLUSION

This paper discusses parameter-related issues that span the process of modeling, from initial model formulation to final policy recommendations. Of necessity, there are a large number of specific conclusions, principal among which are:

(1) Each parameter should describe a separate and independent characteristic of the real processes being modeled.

(2) The equation formulation should be general enough to allow parameter values to describe many different cases.

(3) Preliminary parameter estimation should utilize data below the level of aggregation (estimation from disaggregate data) where possible.

(4) If parameters are to be estimated from data below the level of aggregation of model structure, the model structure should be disaggregated enough to allow the parameters to be based on reliable observations of relatively unchanging characteristics of the elements of the system, rather than on (possibly ill-founded) conclusions or opinions about the dynamic behavior of some subsystem.

(5) Equation estimation and model estimation should be used as secondary techniques if at all, since they are much more vulnerable to error than estimation from disaggregate data.

(6) Data at the level of aggregation of model structure should be reserved for validity testing.

(7) Testing a model's behavior sensitivity and policy sensitivity can help to identify the parameters and equations that require further estimation or reformulation.

(8) Technical analysis of model behavior identifies the structural causes of parameter sensitivity, and diminishes the need to test every parameter and combination of parameters.

(9) There are three ways to derive workable policy recommendations in the presence of uncontrollable policy-sensitive parameters: reformulate the model to reduce the sensitivity, redesign the policy recommendation to reduce its sensitivity, or reestimate the parameters in question with more accuracy.

The number of techniques, even nonstatistical techniques, for setting parameter values is very large. The appropriateness of each technique depends on the needs of the model for accurate parameter values to fulfill its purpose, the information available, and the strategy followed for model construction and validity testing. These same considerations motivate both the traditional uses of data (in experimental physics or economics, for example) and the typical system dynamics use of data described here. Researchers should respond to different purposes, models, and availability of data by choosing a method of setting parameters appropriate to the problem being investigated.

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