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ON RELATIONS BETWEEN FEASIBLE OBSERVATIONS AND DECISIONS

Abstract

The paper is an attempt at a theory of relations connecting feasible observations /or measurements/ and feasible decisions /or controls/ in general cybernetic systems. The theory gives a formal framework and a tool for quantitative analysis of the following facts:

1. An increase in observation possibilities, e.g. an increase of the precision of measurement, enlarging the scope of observation etc., results in an increase in decision possibilities by making more effective decisions possible. This works also in the other direction: if there are more feasible decision, new observations or measurements become available.
2. In the framework of a cybernetic model no decisions and/or observations which generate antinomies can be simultaneously feasible. This creates interesting and important constraints on measurements and decisions in systems which include man or where a human or automatic decision maker is an object of observation, and where the results of observation may be known to this decision maker.
3. The observation /measurement/ takes time and changes its object and thus the result of observation always refers to the past rather than to the present. This normally is due to physical effects though other phenomena, like psychological, may also be important depending on the nature of the object.

The facts of group 1 are in a sense opposite to those of groups 2 and 3. This leads to the existence of optimum decision-

-measurement possibilities. Conditions for this optimum to exist together with its significance for biological and technological systems will be discussed.

The subject of this paper is of interdisciplinary interest and has been studied, partially and from particular angles, within the framework of control theory /facts of group 1/, mathematical logic /theory of antinomies, principles of mathematics - mainly facts of group 2/, physics /theory of measurement, principles of quantum machanics - mainly facts of group 3/ and philosophy /the classic problems of free will and consciouness/. The relevance of the presented theory to these fields will also be discussed.

List of Figures.

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1. Introduction

The goal of this paper is to formulate some basic questions or problems connected with dynamic systems. We will consider these systems from the control-theoretical point of view.

The first question is to establish how the actual state of knowledge influences as a feedback on the foundations. Three classes of the systems will be discussed:

1. The systems for which there exists the concordance between identification and control,
2. The systems for which there exists antinomy between identification and control /decision/,
3. The systems with time delay between observations and controls.

2. Problems and examples

2.1. Antinomy between energy and accuracy

Fig.1. Let us consider the systems presented in Figure 1.

We assume the model of the system as a deterministic one described by the equation

$$\dot{y} = \frac{1}{T} u \qquad /1/$$

$$y(0) = y_0$$

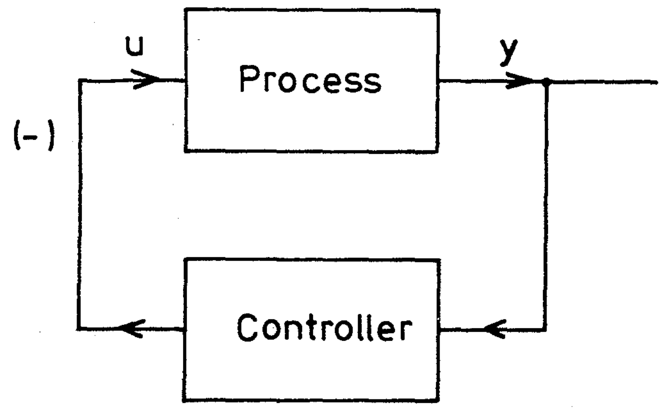

u
Process
y
(-)
Controller

Fig. 1.

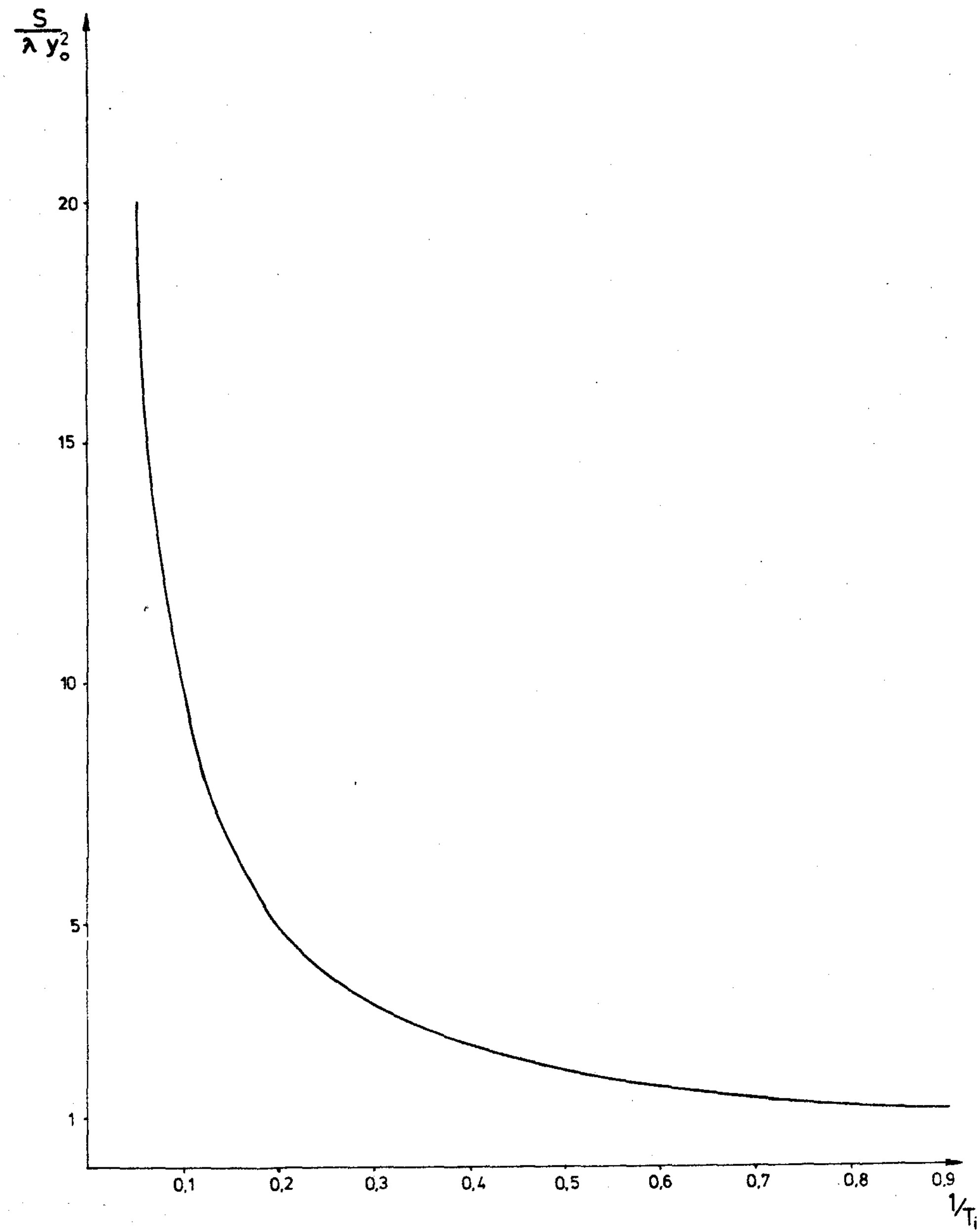

$\frac{S}{\lambda y_o^2}$
20
15
10
5
1
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
$1/T_i$

Fig. 2.

where y - output

u - decision

and the criterion function as

$$I = \int_0^{\infty} (y^2 + \lambda^2 u^2)\,dt \qquad /2/$$

where λ - constant coefficient

Application of the Bellman equation leads to the determination of the optimal controller in the sense of minimization of functional /2/.

In the general case, the system is described by the vector equation

$$\dot{\underline{y}} = \underline{f}(\underline{y}, \underline{u}, t) \qquad /3/$$

and the criterion function is

$$I = G(\underline{y}_f) + \int_0^{t_f} f_o(\underline{y}, \underline{u}, t)\,dt \qquad /4/$$

We denote the minimal value of I by:

$$S(t) = \min_{u \in U} G(\underline{y}_f) + \int_t^{t_f} f_o(\underline{y}, \underline{u}, s)\,ds \qquad /5/$$

U admissible region of decision

Then the Bellman equation is [1]

$$-\frac{\partial S}{\partial t} = \min_{u \in U} \left\{ f_o(\underline{y}, \underline{u}, t) + \sum_{i=1}^{n} \frac{\partial S}{\partial y_i} f_i(\underline{y}, \underline{u}, t) \right\} \qquad /6/$$

From /5/ we have for $t = t_f$

$$S(\underline{y}_f\ ,\ t_f) = G(\underline{y}_f) \qquad /7/$$

Assuming that the minimum of S with respect to $\underline{u}$ is inside the region U we can search for it using the equation obtained by the differentiation of equation /6/

$$\frac{\partial f_o(\underline{y},\underline{u}^*,t)}{\partial \underline{u}^*} + \frac{\partial S}{\partial \underline{y}^*}^T \frac{\partial \underline{f}(\underline{y}^*,\underline{u}^*,t)}{\partial \underline{u}^*} = 0 \qquad /8/$$

In our case, from equations /1/ and /2/ we see that:

$$\left.\begin{aligned} G(y_f) &\equiv 0 \\ f_o(\underline{y},\underline{u},t) &= y^2 + \lambda^2 u^2 \\ f(\underline{y},\underline{u},t) &= -\frac{1}{T_i} u \end{aligned}\right\} \qquad /9/$$

From /6/ we have

$$0 = \left(y^{*2} + \lambda^2 u^{*2} + \frac{\partial S}{\partial y}\ \frac{1}{T_i}\ u^*\right) \qquad /10/$$

and from /8/

$$2\lambda^2 u^* + \frac{\partial S}{\partial y^*}\ \frac{1}{T_i} = 0 \qquad /11/$$

Elimination of $\frac{\partial S}{\partial y}$ from /11/ and /10/ leads to:

$$0 = y^{*2} - \lambda^2 u^{*2} \qquad /12/$$

The optimal decision in the feedback form is

$$u^{*} = -\frac{1}{\lambda} y^{*} \qquad /13/$$

returning to /1/ gives

$$\dot{y}^{*} = -\frac{1}{\lambda T_i} y^{*} \qquad /14/$$

the solution of which is

$$y^{*}(t) = y_o\, e^{-\frac{t}{\lambda T_i}} \qquad /15/$$

and the minimal value of the functional is

$$S = \int_o^{\infty} \left(y^2 + \lambda^2 u^2\right) dt = \lambda T_i y_o^2 = \frac{\lambda}{\frac{1}{T_i}} \cdot y_o^2 \qquad /16/$$

Fig.2.

Following the same way we find for the process described by the equation

$$y^{o}(t) = -\frac{1}{T} y + \frac{K}{T} u \qquad /17/$$

$$y(o) = y_o$$

$$y^{*}(t) = y_o e^{-\sqrt{1+\left(\frac{K}{\lambda}\right)^2}\ \frac{t}{T}} \qquad /18/$$

$$u^{*}(t) = 1 - \sqrt{1+\left(\frac{K}{\lambda}\right)^2}\ \frac{y^{*}}{K} \qquad /19/$$

and the minimal value of the functional is

$$S = \frac{\left(\frac{K}{\lambda}\right)^2 + \left(1 - \sqrt{1+\left(\frac{K}{\lambda}\right)^2}\right)^2}{\left(\frac{K}{\lambda}\right)^2 \sqrt{1+\left(\frac{K}{\lambda}\right)^2}} \cdot \frac{Ty_o^2}{2} \qquad /20/$$

Fig.3.

There exists concordance between attainable accuracy and control. If the action $\frac{1}{T_i}$ rises, then accuracy also rises. The total ISE decreases. See Fig. 2 and Fig. 3.

2.2. Antinomy between identification time and control time

Let us assume that our process is a priori not known, and for its recognition we need an interval of time equal τ. After that we can control by the time equal Θ which in particular may be equal to infinity. Now we analyse our system /see Fig. 1/, but we must take into account that between the process and the controller there exists some

Fig. 4. delay /see Fig. 4/.

<u>The first interval /Identification/</u>

Let us assume that during the interval τ of time we have recognized that our process can be described by the equation

$$\dot{y}(t) = +\frac{1}{T_i} u(t-\tau) \qquad /21/$$

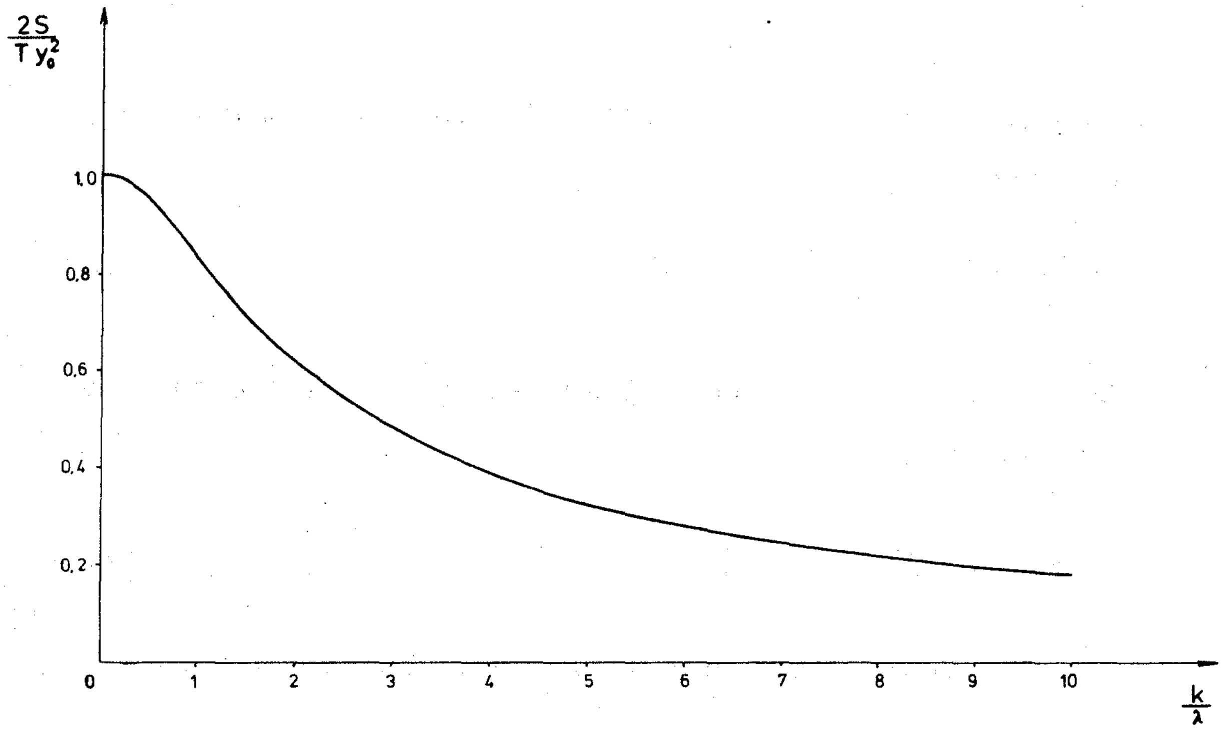
$$y(o) = y(\tau) = y_o$$

and

$$u(t) = 0 \quad \text{for} \quad t \leq \tau$$

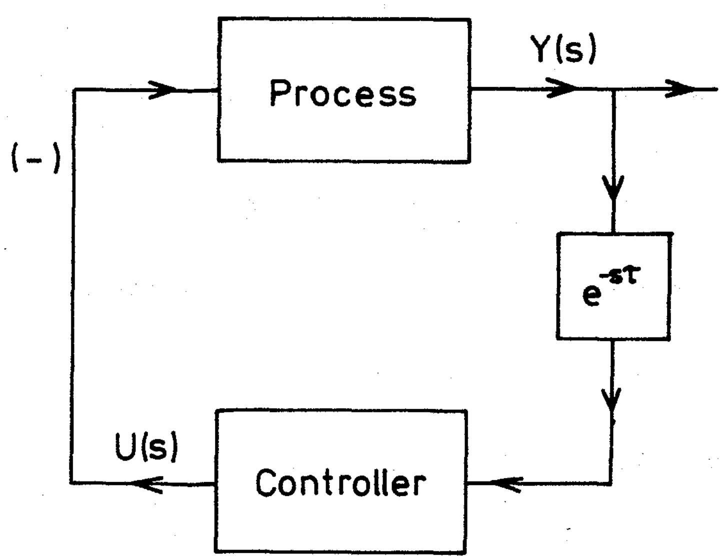
As the criterion functional we assume

$$S(u) = \int_0^{\infty} \left(y^2(t) + \lambda^2 u^2(t)\right) dt \qquad /22/$$



$\frac{2S}{T y_0^2}$
1,0
0,8
0,6
0,4
0,2
0
1
2
3
4
5
6
7
8
9
10
$\frac{k}{\lambda}$

Fig. 3.



Process
Y(s)
(−)
$e^{-s\tau}$
U(s)
Controller

Fig. 4.

During the interval of identification, according to /21/ we have

$$S(u) = \int_0^{\tau} y^2(t)\,dt = y_o^2\,\tau \qquad /23/$$

After this time the controller can be informed about the process and we have

$$S(u) = y_o^2\tau + \int_{\tau}^{\infty}\left(y^2(t) + \lambda^2 u^2(t-\tau)\right)\,dt \qquad /24/$$

Applying Bellman's procedure leads to the relations

$$0 = y^2(t) + \lambda^2 u^2(t-\tau) + \frac{\partial S}{\partial y}\left(\frac{1}{T_i}\,u(t-\tau)\right) \qquad /25/$$

and

$$0 = 2\,\lambda^2 u(t-\tau) + \frac{1}{T_i}\frac{\partial S}{\partial y} \qquad /26/$$

Elimination of $\frac{\partial S}{\partial y}$ from equations /25/ and /26/ gives:

$$y^2(t) - \lambda^2 u^2(t-\tau) = 0 \qquad /27/$$

We realize negative feedback so we choose

$$u(t-\tau) = -\frac{1}{\lambda}\,y(t) \qquad /27/$$

which inserted to equation /21/ yields

$$\left.\begin{aligned} \dot{y}(t) &= -\frac{1}{\lambda T_i}\,y(t) \\ y_o' &\neq 0 \end{aligned}\right\} \qquad /28/$$

The solution of /28/ is

$$y(t) = y_o e^{-\frac{t}{\lambda T_i}} \qquad /29/$$

and

$$u(t-\tau) = -\frac{y_o}{\lambda} e^{-\frac{t}{\lambda T_i}} \qquad /30/$$

The value of the functional is obtained from /24/, /29/ and /30/

$$S = y_o^2 \lambda T_i \left(\frac{\tilde{\tau}}{\lambda T_i} + e^{-2\frac{\tilde{\tau}}{\lambda T_i}} \right) \qquad /31/$$

Minimum of S as a function of τ is attained for

$$\tau^{*} = \frac{\lambda T_i}{2} \ln 2 \qquad /32/$$

and

$$S^{*} = \frac{\lambda T_i}{2} y_o^2 (1 + \ln 2) \qquad /33/$$

It is evident from the relations /32/ and /33/ that there is an antinomy between the time of identification τ and control action $\frac{1}{T_i}$. For that reason it is not possible to obtain an arbitrarily high accuracy. The product of time delay τ and control $\frac{1}{T_i}$ is constant

$$\tau \cdot \frac{1}{\lambda T_i} = \frac{1}{2} \ln 2 \qquad /34/$$

Similarly for process

$$\dot{y}(t) = -\frac{1}{T} y(t) + \frac{K}{T} u(t-\tau) \qquad /35/$$

$$y(o) = y(\tau) = y_o \text{ , and } u(t) = 0 \text{ for } t \leq \tau$$

we have

$$u(t-\tau) = \left(1 - \sqrt{1+\left(\frac{K}{\lambda}\right)^2}\right)\frac{y(t)}{K} \qquad /36/$$

which with the equation /35/ gives

$$\dot{y}(t) = -\frac{1}{T}\sqrt{1+\left(\frac{K}{\lambda}\right)^2}\; y(t) \qquad /37/$$

The solution of it is

$$y(t) = y_o\, e^{-\sqrt{1+\left(\frac{K}{\lambda}\right)^2}\,\frac{t}{T}} \qquad /38/$$

and

$$u(t-\tau) = \frac{y_o}{K}\left(1 - \sqrt{1+\left(\frac{K}{\lambda}\right)^2}\right)\cdot e^{-\sqrt{1+\left(\frac{K}{\lambda}\right)^2}\,\frac{t}{T}} \qquad /39/$$

Finally using /38/ and /39/ we can calculate the minimal value of the functional:

$$S(u) = \left\{\frac{\tau}{T} + \frac{e^{-2\sqrt{1+\left(\frac{K}{\lambda}\right)^2}\,\frac{\tau}{T}}}{2\left(\frac{K}{\lambda}\right)^2\sqrt{1+\left(\frac{K}{\lambda}\right)^2}}\left[\left(\frac{K}{\lambda}\right)^2 + \left(1-\sqrt{1+\left(\frac{K}{\lambda}\right)^2}\right)^2\right]\right\} T y_o^2 \qquad /40/$$

Fig.5.

The functional $S(u)$ as the function of relative time delay $\frac{\tau}{T}$ attains its minimum for

$$\left(\frac{\tau}{T}\right)^* = \frac{1}{2\sqrt{1+\left(\frac{K}{\lambda}\right)^2}}\ \ln\frac{\left(\frac{K}{\lambda}\right)^2 + \left(1-\sqrt{1+\left(\frac{K}{\lambda}\right)^2}\right)^2}{\left(\frac{K}{\lambda}\right)^2} \qquad /41/$$

Fig. 6. see Fig. 6.

The minimal value is equal to:

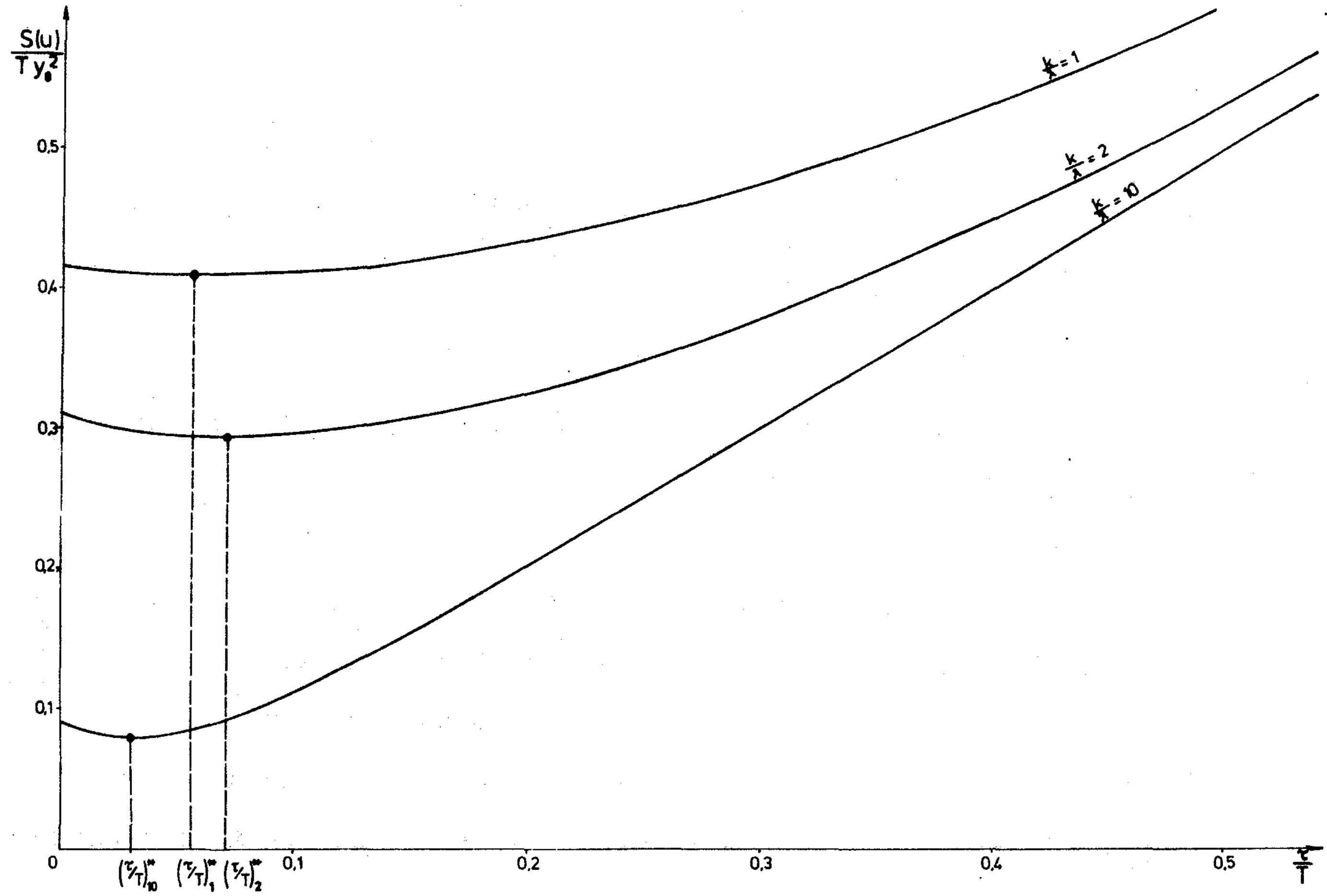

$\frac{S(u)}{T y_0^2}$
0,5
0,4
0,3
0,2
0,1
0
$(\tau/T)^{**}_{10}$
$(\tau/T)^{**}_{1}$
$(\tau/T)^{**}_{2}$
0,1
0,2
0,3
0,4
0,5
$\frac{\tau}{T}$
$\frac{k}{\lambda} = 1$
$\frac{k}{\lambda} = 2$
$\frac{k}{\lambda} = 10$

Fig. 5.

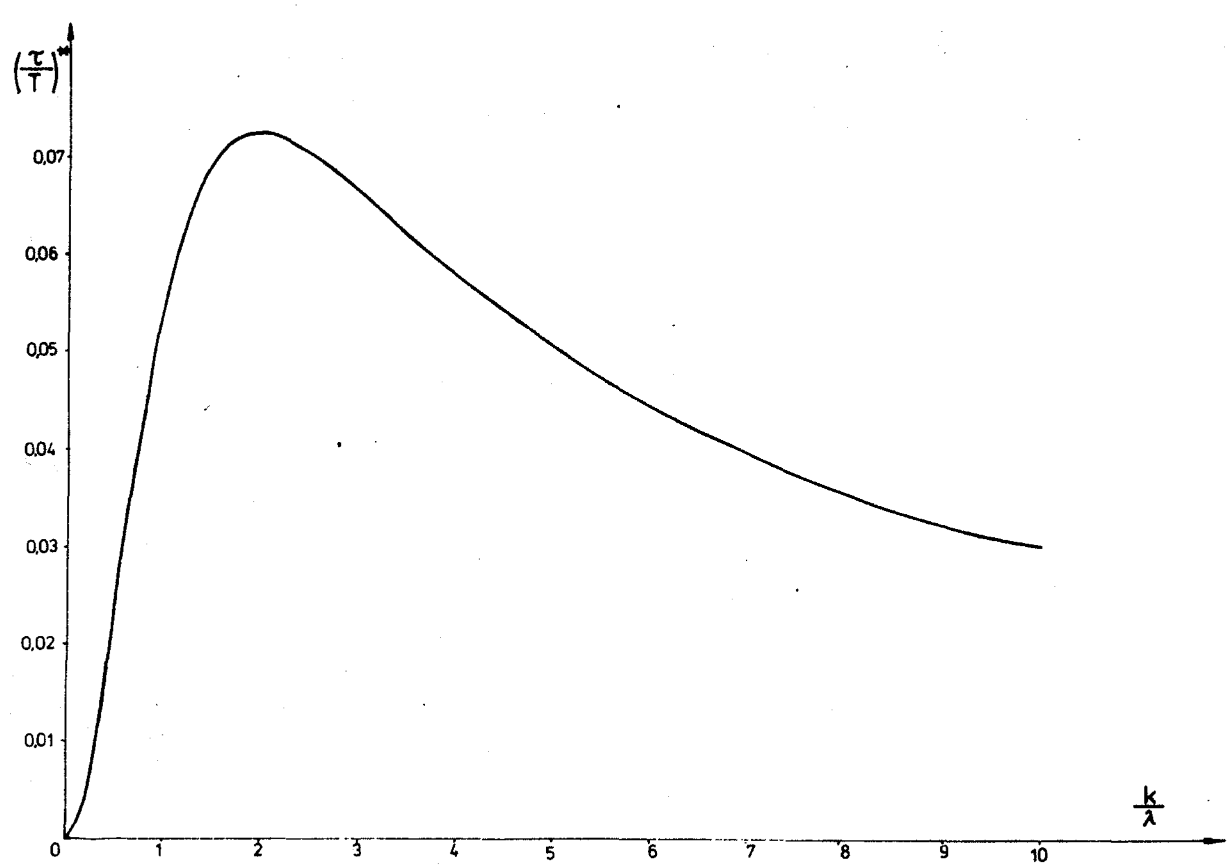

$\left(\frac{\tau}{T}\right)^{**}$
0,07
0,06
0,05
0,04
0,03
0,02
0,01
0
1
2
3
4
5
6
7
8
9
10
$\frac{k}{\lambda}$

Fig. 6.

$$S^{*}(u) = \left[1 + \ln \frac{\left(\frac{K}{\lambda}\right)^2 + \left(1 - \sqrt{1+\left(\frac{K}{\lambda}\right)^2}\right)^2}{\left(\frac{K}{\lambda}\right)^2}\right] T y_o^2 \qquad /42/$$

Fig.7.

The dependence of $\left(\frac{\tau}{T}\right)^{*}$ and $S^{*}(u)$ as the functions of $\left(\frac{K}{\lambda}\right)$ is shown in Fig. 6 and 7 .

The conclusion is evident.

In the presence of time delay it is impossible to obtain an arbitrarily small error, even using the optimal controller.

The analysed controller has the disadvantage that in the case of over estimation of the value of time delay the whole closed system may be unstable. To prevent this it is necessary to maintain stability condition.

Let us finally analyse the same process, but using a conventional controller.

We have in the interval of identification τ

$$\left.\begin{aligned} & y(o) = y(\tau) = y_o \\ \text{and} \\ & u(t) = 0\,, \quad t \leq \tau \end{aligned}\right\} \qquad /43/$$

In the period of control we have the equation

$$\dot{y}(t) = -\frac{1}{T_i}\, u\,(t-\tau) \qquad /44/$$

The functional see [2,3] is equal to

$$J_2 = \int_o^{\infty} y^2(t)\,dt = \frac{y_o^2 T_i}{2}\;\frac{\cos\frac{\tau}{T_i}}{1-\sin\frac{\tau}{T_i}} = \frac{y_o^2\,\tau\cos\frac{\tau}{T_i}}{2\,\frac{\tau}{T_i}\left(1-\sin\frac{\tau}{T_i}\right)} \qquad /45/$$

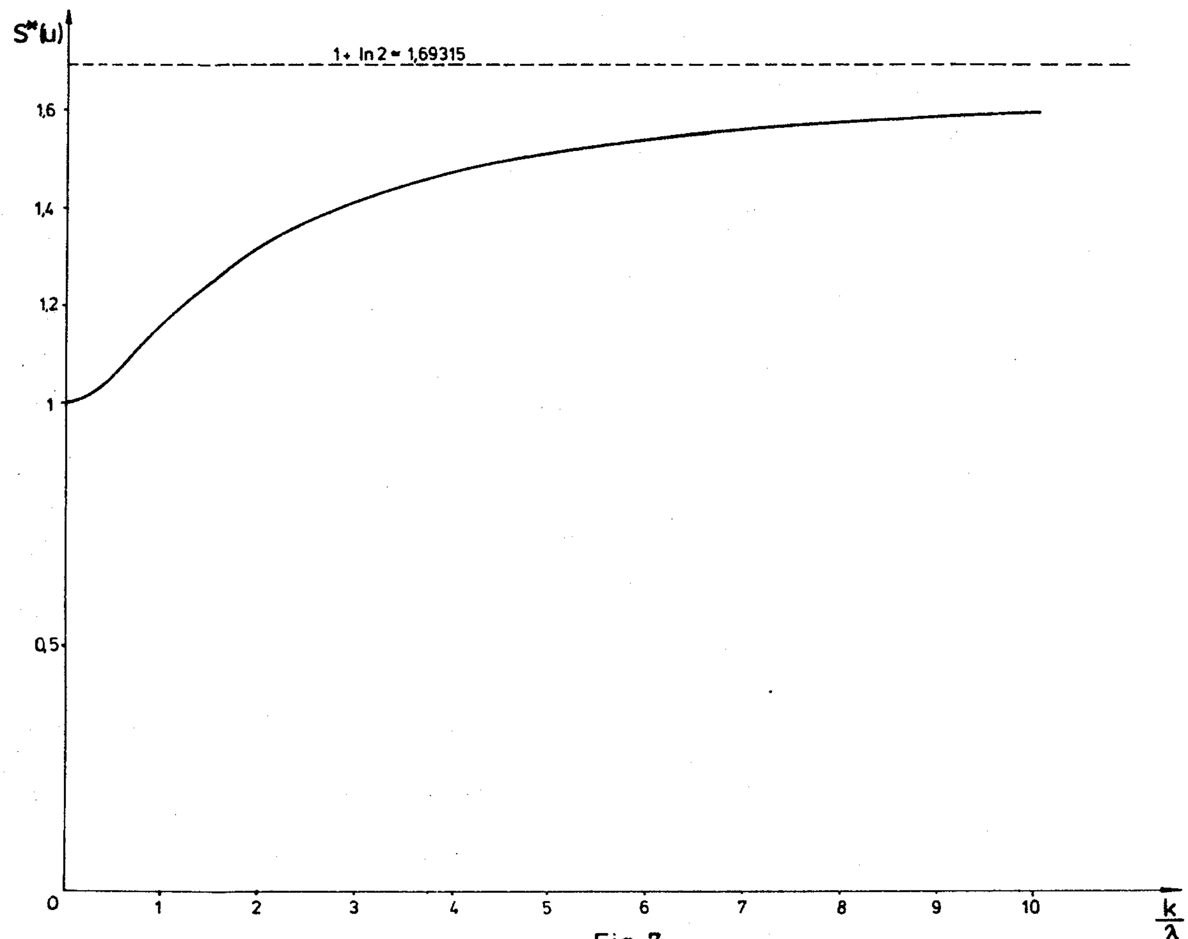


Fig. 7.

The minimum of this functional with respect to the parameter $\frac{1}{T_i}$ of the controller is at the point for which

$$\cos \frac{\tau}{T_i} = \frac{\tau}{T_i} \qquad /46/$$

This optimal value is

$$\frac{\tau}{T_i} = 0,739 \qquad /47/$$

The minimal value of

$$J_2 = \frac{y_o^2 \tau}{2} \; \frac{1}{1 - \sqrt{1 - 0,739^2}} \qquad /48/$$

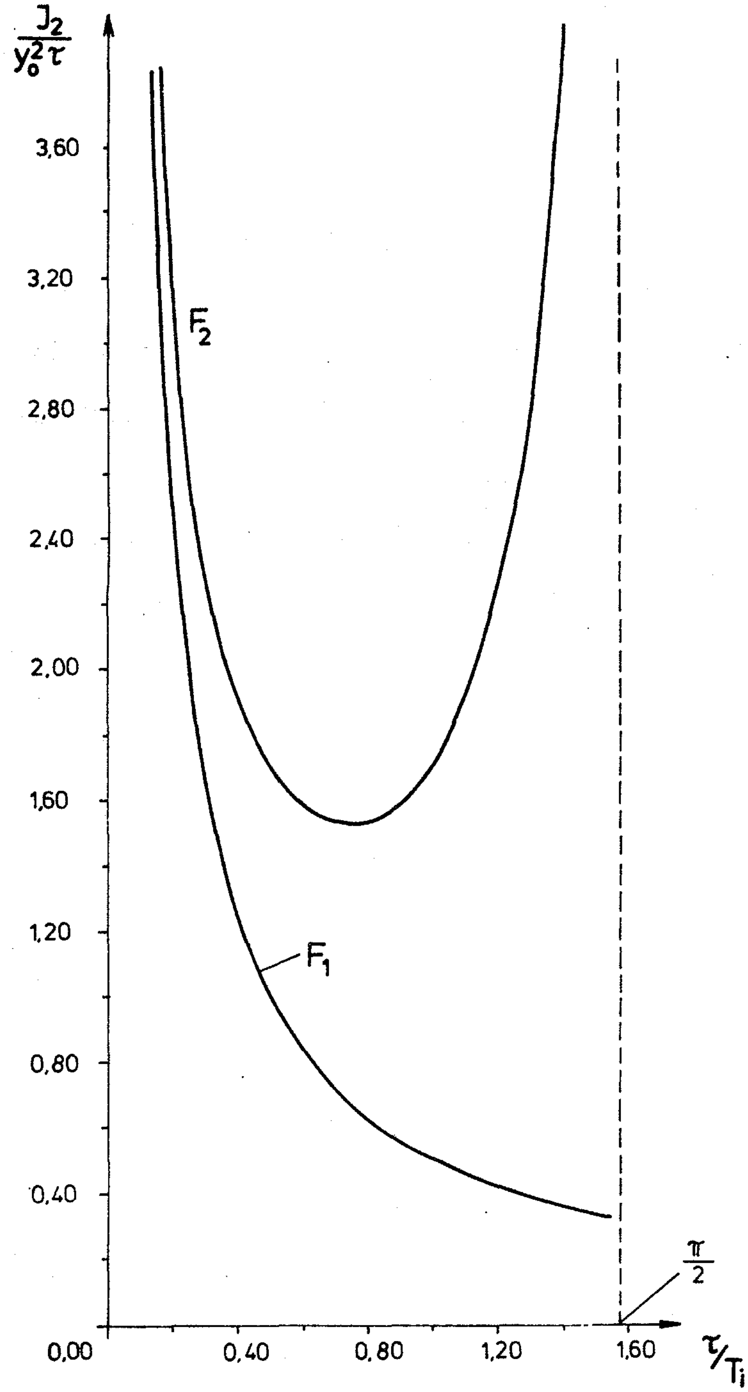
The condition of stability is

$$\tau \cdot \frac{1}{T_i} < \frac{\pi}{2} \qquad /49/$$

Fig. 8. In figure 8 there is shown the dependence of the functional on $x = \tau \cdot \frac{1}{T_i}$ for the system with time delay /curve F_2/, and for the system without time delay /curve F_1/

$$J_2 = \frac{y_o^2}{2 \frac{1}{T_i}} = \frac{y_o^2 \tau}{2 \frac{\tau}{T_i}}$$

It is evident that the antinomy between time of identification and control yields the limited accuracy given by equation /48/.



$$F_1 = \frac{1}{2\frac{\tau}{T_i}}$$

$$F_2 = \frac{\cos\frac{\tau}{T_i}}{2\frac{\tau}{T_i}\left(1-\sin\frac{\tau}{T_i}\right)}$$

Fig. 8.

References

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