OSCILLATIONS AND CHAOS IN ECOLOGICAL POPULATIONS.

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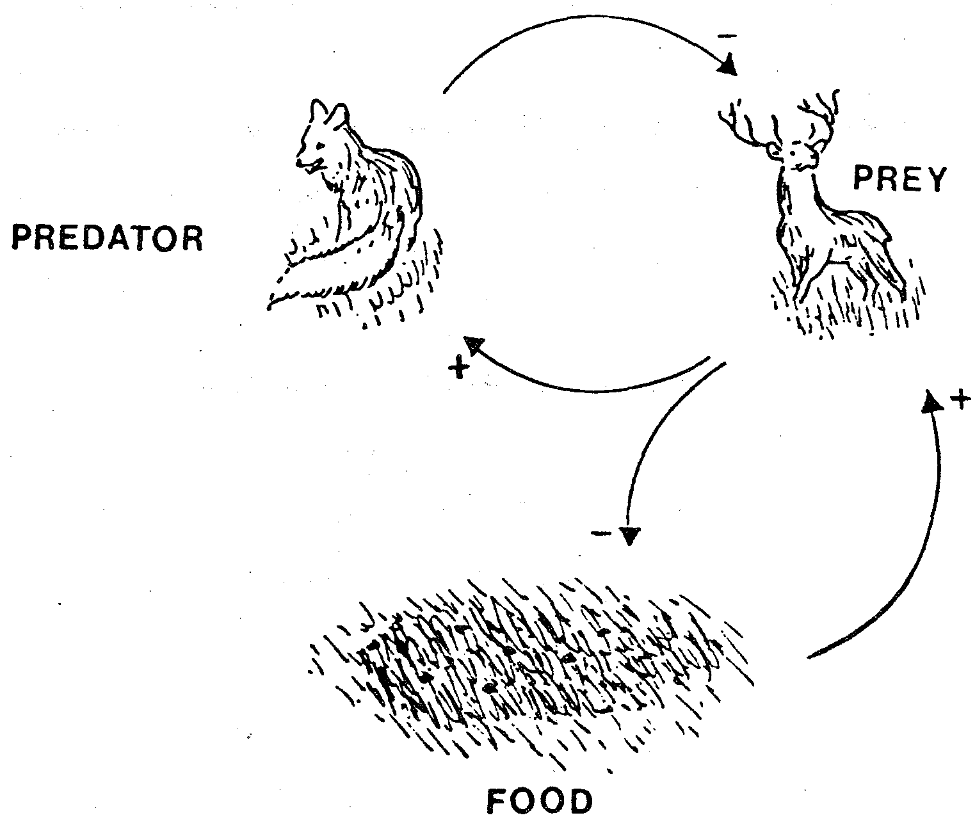
IN THIS POSTER WE ANALYSE THE CHAOTIC MOTION OF A MODEL WHICH DESCRIBES THE BEHAVIOR OF A PREY-PREDATOR--FOOD SYSTEM.

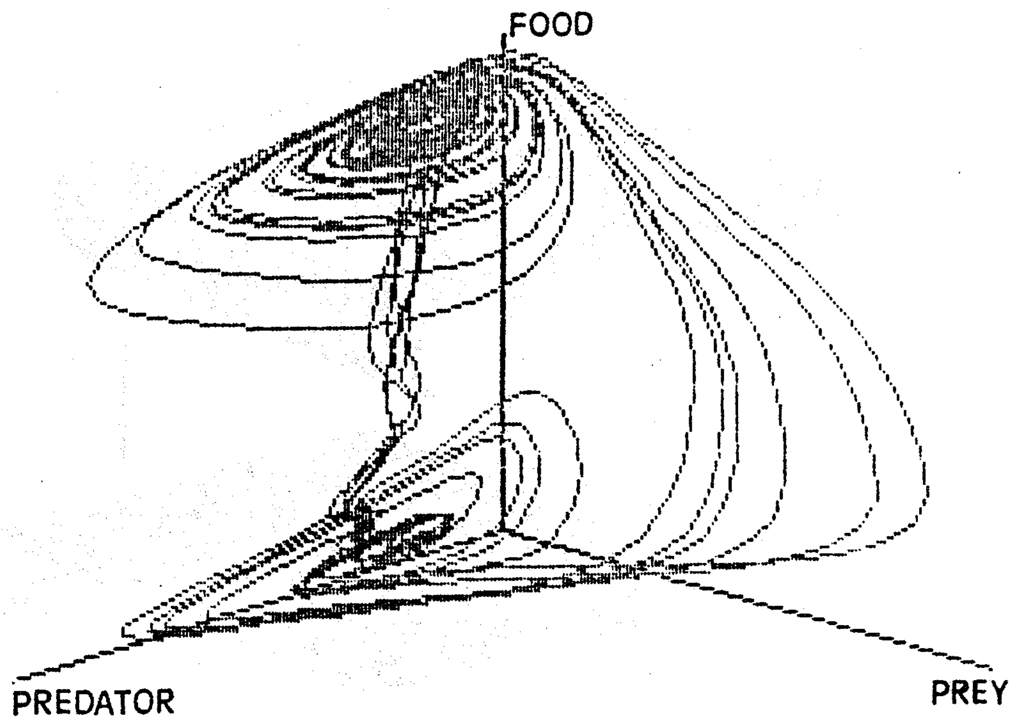


THIS SYSTEM CAN BE MODELED BY MIXING TWO WELL KNOWN MODELS: THE PREDATOR-PREY MODEL (HENIZE,1971) AND THE KAIBAB PLATEAU MODEL, WHICH COPES WITH THE PREY--FOOD PART OF THE MODEL (GODMAN,1974).

THIS MODEL HAS PREVIOUSLY BEEN INTRODUCED IN (TORO AND ARACIL, 1988).

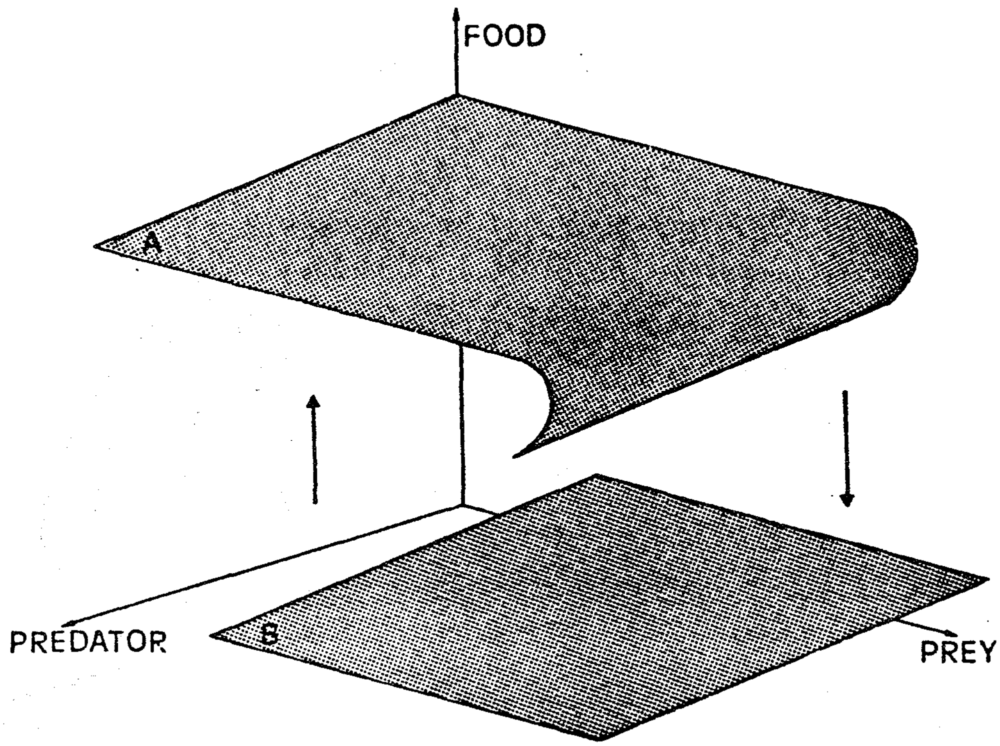
THE DYNAMO EQUATIONS OF THE MODEL ARE GIVEN IN THE APENDIX.

FOR CERTAIN VALUES OF THE PARAMETER "MAXIMUM FOOD CONSUMED PER PREY (MFCPPY)" THIS MODEL EXHIBITS CHAOTIC BEHAVIOR, FOR INSTANCE FOR MFCPPY = 7.4. A THREE DIMENSIONAL PICTURE OF THE TRAJECTORY IS SEEN AS:

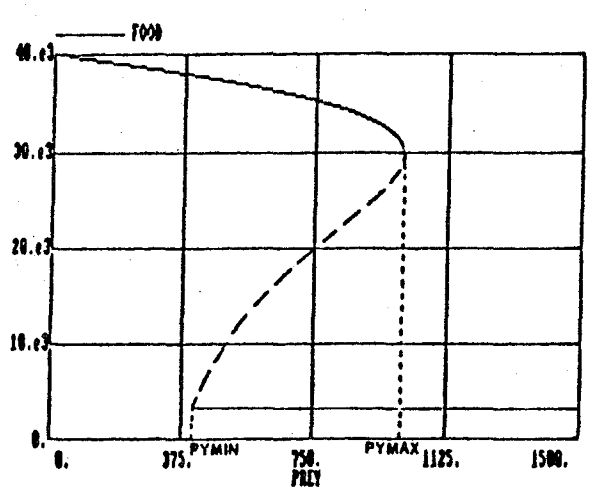


THIS IS VERY COMPLEX OSCILLATORY BEHAVIOR. WE ARE GOING TO SHOW THAT THIS BEHAVIOR IS ACTUALLY CHAOTIC AND APPEARS ABRUPTLY WHEN THE PARAMETER MFCPPY CHANGES FROM 7.41 TO 7.4. THERE EXIST A ATTRACTOR POINT FOR MFCPPY $\geq$ 7.41 , BUT FOR MFCPPY $\leq$ 7.4 A CHAOTIC ATTRACTOR APPEARS.

THE MODEL SHOWS TWO TIME SCALES: ONE FAST AND, THE OTHER SLOW. THIS FACT HELPS ONE TO GET A GEOMETRICAL INSIGHT INTO THE BEHAVIOR MECANISM OF THE SYSTEM. THE SLOW MOTION OF THE MODEL TAKES PLACE ON SHEETS A AND B IN THE FIGURE:



THE PROJECTION OF THESE SURFACES ON THE FOOD-PREY PLANE IS:

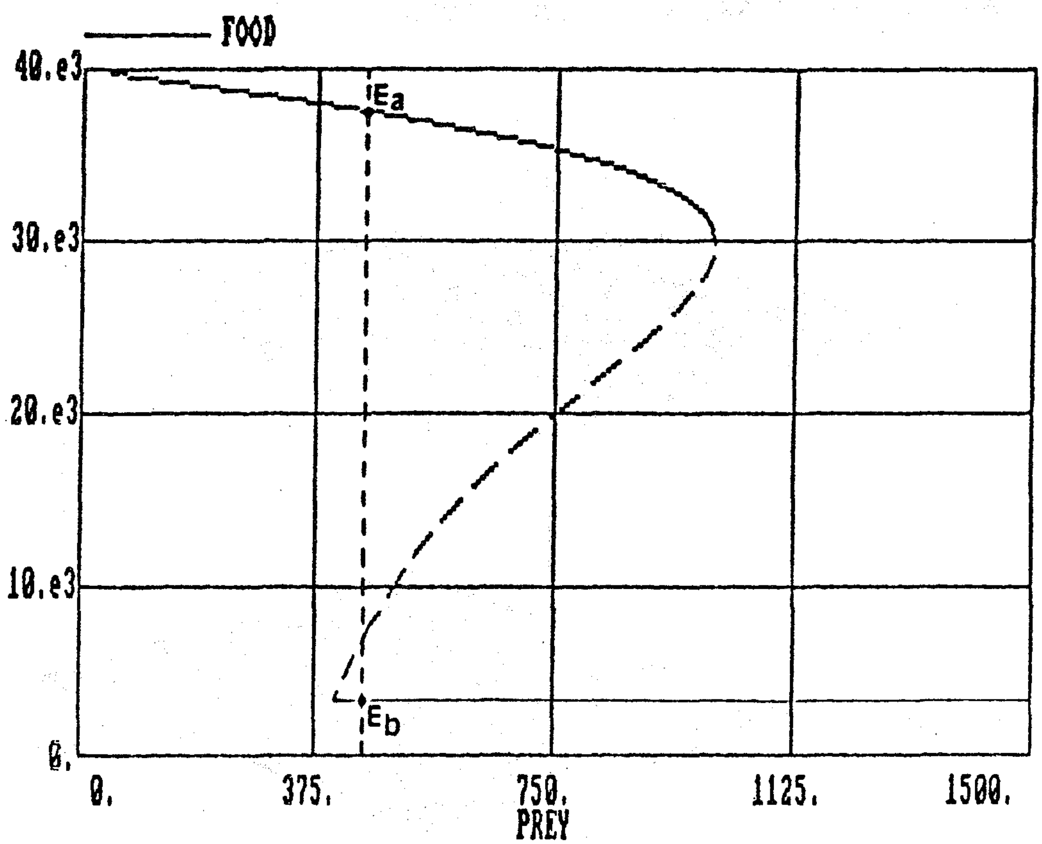


IT CAN BE INTERPRETED ECOLOGICALLY THAT MOTION IN SHEET A CORRESPONDS TO THE SYSTEM EVOLUTION WITH ENOUGH RESOURCES FOR THE PREY.

IN THE SAME WAY, MOTION IN SHEET B CORRESPONDS TO BEHAVIOR UNDER BAD FOOD CONDITIONS.

WHEN THE MOTION ON SHEET A REACHES THE FOLD (PREY = PYMAX) OR THE MOTION ON SHEET B DECREASES TO THE LOWER BOUND (PREY = PYMIN) FAST MOTION PRODUCES A JUMP FROM ONE SHEET TO THE OTHER.

FOR PARAMETER MFCPPY = 7.41 THE MODEL HAS TWO EQUILIBRIA E_a AND E_b (RESPECTIVELY ON SHEETS A AND B).



THE EIGENVALUES IN EACH EQUILIBRIUM ARE:

FOR E_a:

$\lambda_1 = +0.14 + 3.3i$

$\lambda_2 = +0.14 - 3.3i$

$\lambda_3 = -4.9$

FOR E_b:

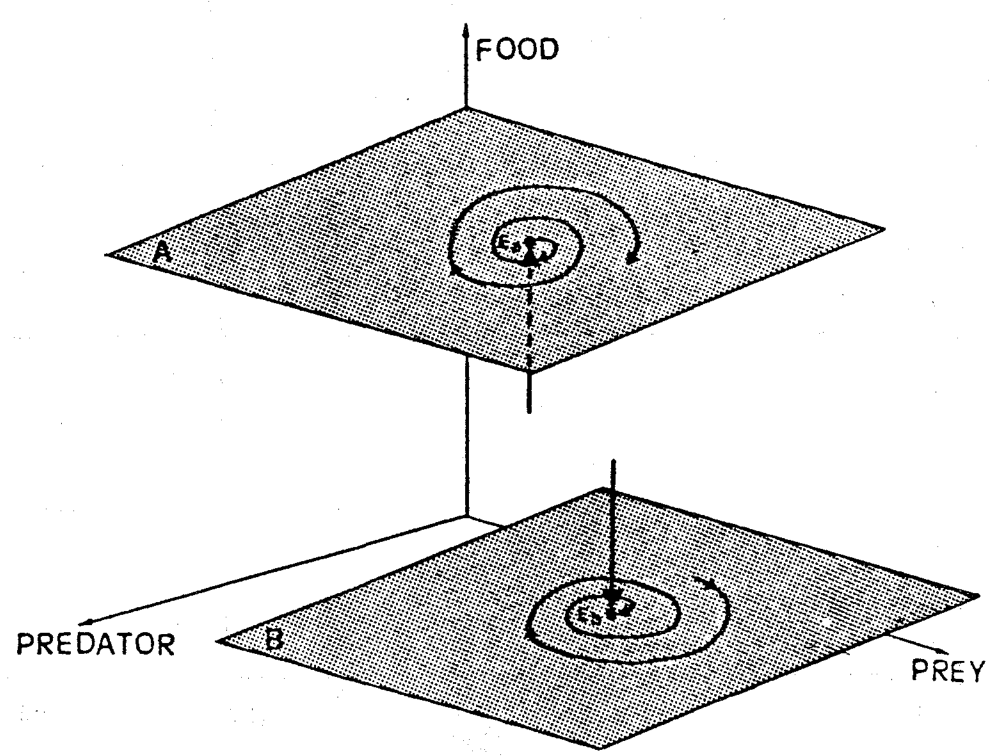
$\lambda_1 = -0.88 + 0.40i$

$\lambda_2 = -0.88 - 0.40i$

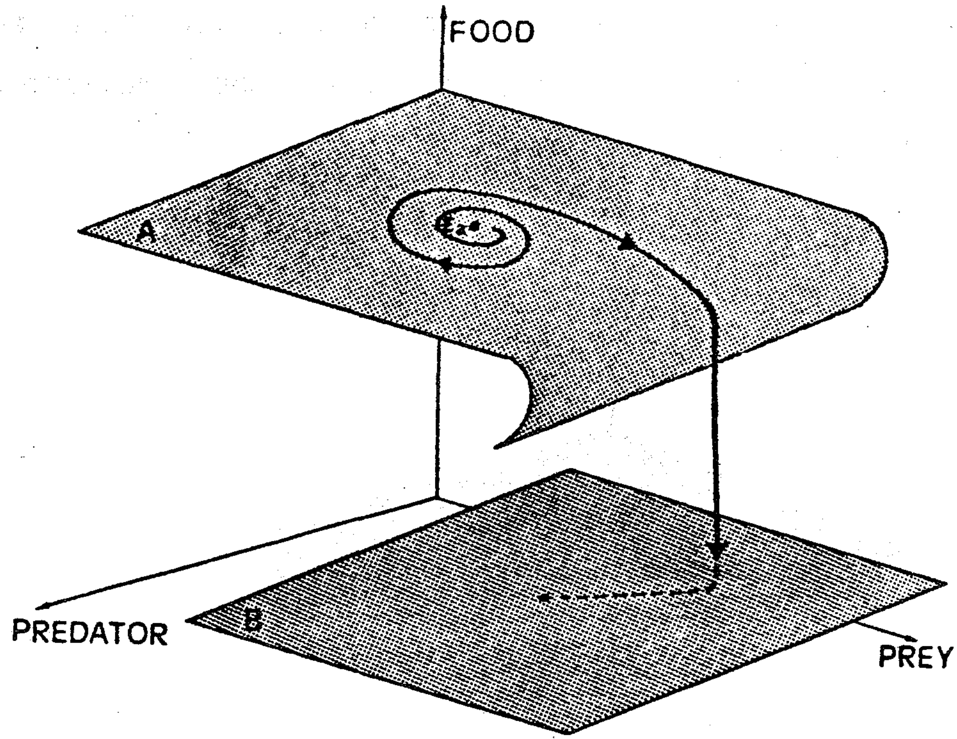
$\lambda_3 = -0.18$

SO, E_a IS A SADDLE POINT AND E_b IS A STABLE EQUILIBRIUM.

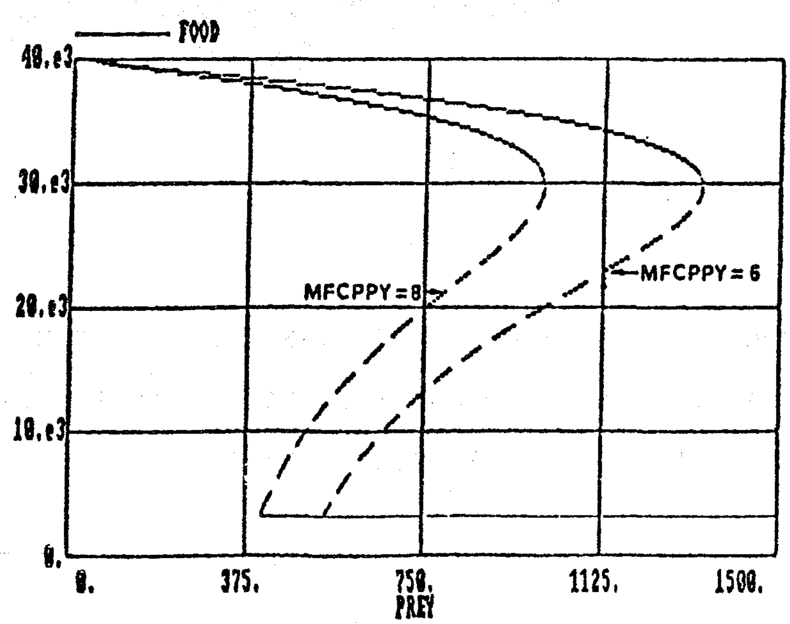
THE LOCAL BEHAVIOR ABOUT THE EQUILIBRIA E_a AND E_b HAS THE SHAPE SHOWN IN THE FIGURE:



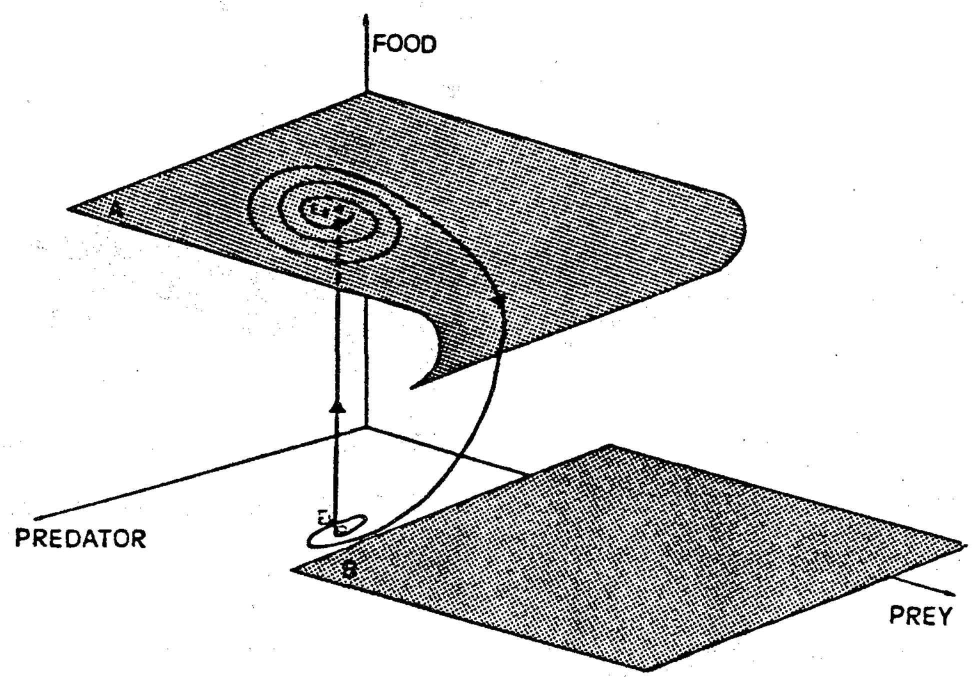
ONCE THE TRAJECTORY ON SHEET **A** REACHES THE FOLD, AND LEAVES THE SHEET, IT IS CAPTURED BY EQUILIBRIUM E_b.



WHEN PARAMETER MFCPPY DECREASES, THE SHAPE OF THE PROJECTION OF THE SLOW MOTION SURFACE ON THE FOOD PREY PLANE EVOLVES AS IS SHOWN IN THE FIGURE:

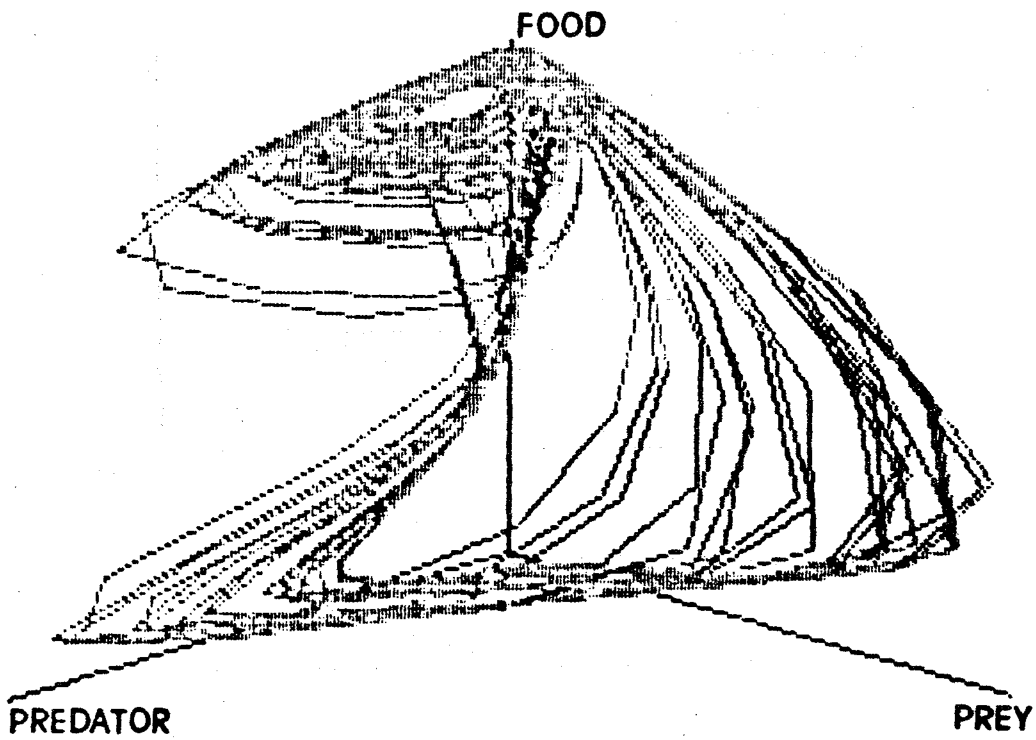


IN THIS WAY FOR ONE VALUE OF MFCPPY (7.41) THE EQUILIBRIUM E_b DISAPPEARS. IN SUCH A CASE THE TRAJECTORY COMES BACK TO SHEET A, CAPTURED BY THE STABLE MANIFOLD (THE INSET) OF E_a, AND THE GLOBAL TRAJECTORY SHOWS THE SHAPE OF THE FIGURE:



IT SHOULD BE NOTICED THAT EVEN IF EQUILIBRIUM E_b HAS DISAPPEARED THE FIELD SEEMS TO BE TRYING TO FIND IT (WE HAVE SOME KIND OF GHOST EQUILIBRIUM).

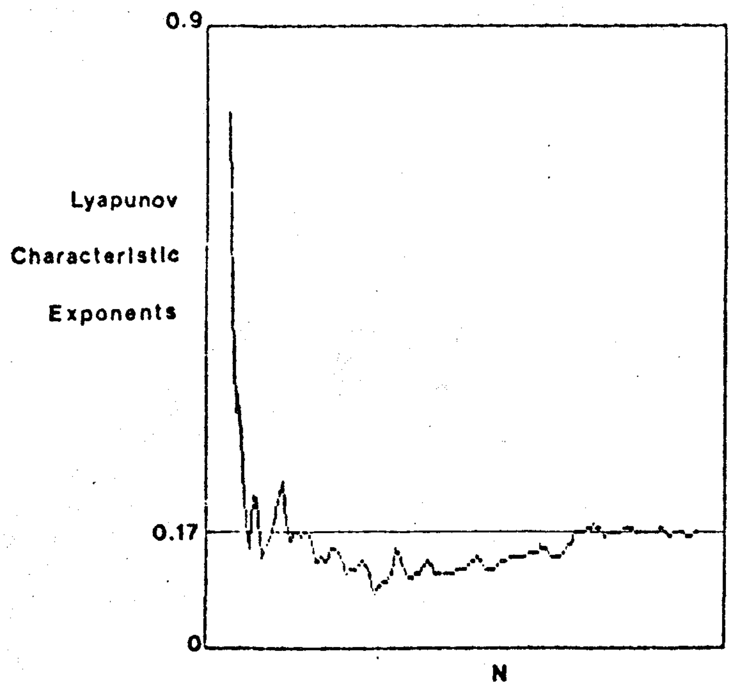
AS THE VALUES OF PARAMETER MFCPPY DECREASE THIS SEARCH FOR THE MISSING EQUILIBRIUM IS LESS APPARENT. FOR INSTANCE FOR MFCPPY = -5 THE TRAJECTORY HAS THE SHAPE:



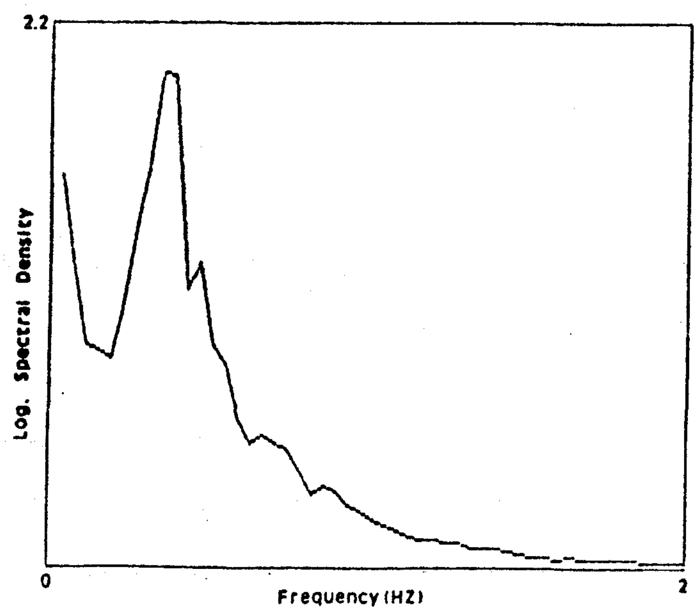
IN THIS WAY WE GET AN INTUITIVE INSIGHT INTO HOW THE OSCILLATIONS ARE GENERATED.

TO SHOW THAT FOR VALUES OF PARAMETER MFCPPY < 7.4 THE BEHAVIOR IS ACTUALLY CHAOTIC SOME COMPUTATIONS ARE NEEDED. THESE COMPUTATIONS HAVE BEEN MADE FOR MFCPPY = 7.4

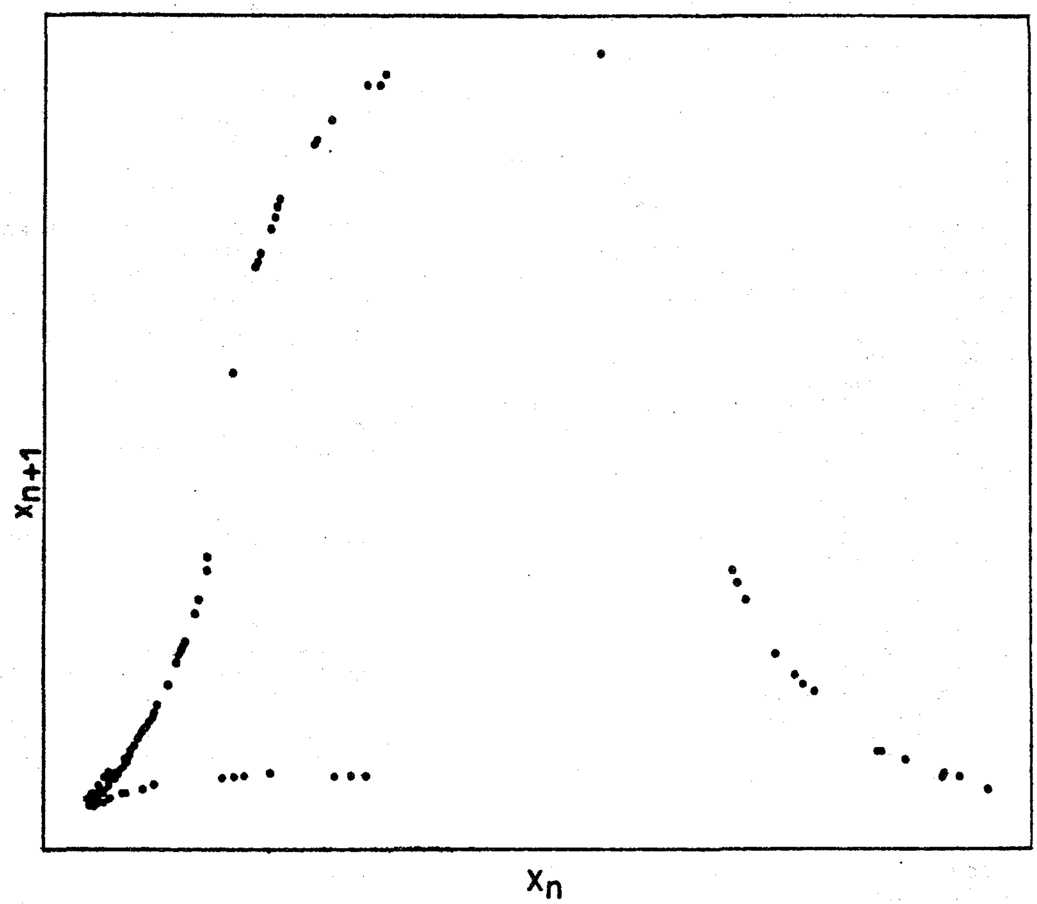
THE VALUE OF THE LYAPOUNOV EXPONENT IS 0.17±0.08 BIT/YEAR WHICH CONFIRMS THE CHAOTIC NATURE OF THE ATTRACTOR. THE CONVERGENCE OF THE LYAPOUNOV EXPONENT IS QUITE GOOD, AS SHOWN BY THE FIGURE:



THE POWER SPECTRUM HAS THE SHAPE:



THE POINCARE MAP PLOTS THE RELATIVE MAXIMUM OF X VERSUS SUBSEQUENT RELATIVE MAXIMUM OF X.



APENDIX: DYNAMO EQUATIONS.

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* PREDATOR-PREY-FOOD MODEL
NOTE----------
NOTE   PREY
NOTE----------
L PY.K=PY.J+(DT)(PYB.JK-PYD.JK-PYK.JK)              PREY
N PY=PYIN
C PYIN=100
R PYB.KL=(PY.K)(PYBN)(PYBM.K)                       PREY BIRTHS
C PYBN=3                                            PREY BIRTH NORMAL
A PYBM.K=TABPL(PYBMT,H*CFR.K,0,3000,300)            PREY BIRTH MULTIPLIER
C H=1500
T PYBMT=.8/1/1.2/1.1/1/.9/.8/.6/.5/.4/.3/0/0/0/0/0/0/0/0/0/0/0
R PYD.KL=(PY.K)(1/PYALT)(PYDM.K)                    PREY DEATH RATE
C PYALT=1                                           PREY AVERAGE LIFETIME
A PYDM.K=TABPL(PYDMT,H*CFR.K,0,3000,300)            PREY DEATH MULTIPLIER
T PYDMT=.9/.9/1/1.05/1.1/1.15/1.2/1.35/1.75/2/2.25/0/0/0/0/0/0/0/0/0/0/0
R PYK.KL=(PD.K)(PYKPDN)(PYKM.K)                     PREYS KILLED
C PYKPDN=500                                        PREYS KILLED PER PREDATOR NORMAL
A PYKM.K=TABPL(PYKMT,H*DD.K,0,3000,300)             PREY KILL MULTIPLIER
T PYKMT=0/.25/.5/.75/1/1.1/1.2/1.3/1.4/1.5/1.55/0/0/0/0/0/0/0/0/0/0/0
A CFR.K=(PY.K)(MFCPPY)/(FOOD.K)                     CONSUMPTION FOOD RATIO
C MFCPPY=5                                          MAXIMUM FOOD CONSUMED PER PREY
A DD.K=PY.K/AREA                                    PREY DENSITY
C AREA=1500
A PYCR.K=(PYK.KL)(FPYKC.K)                          PREY CONSUMPSION RATE
A FPYKC.K=TABPL(TFPYKC,PYK.KL/(PD.K*PDCN),0,2,.20)
T TFPYKC=1/1/1/1/1/1/.9/.85/.8/.75/.7/0/0/0/0/0/0/0/0/0/0/0
C PDCN=400                                          PREDATOR CONSUMPTION NORMAL
NOTE --------------
NOTE    PREDATOR
NOTE --------------
A APDCR.K=PYCR.K/(PD.K*PDCN)                     AVERAGE PREDATOR CONSUFTION RATIO
L PD.K=PD.J+(DT)(PDB.JK-PDD.JK)(PDBN)            PREDATOR
N PD=PDIN
C PDIN=2
R PDB.KL=(PD.K)(CRBM.K)                          PREDATOR BIRTHS
C PDBN=1.5                                        PREDATOR BIRTH NORMAL
A CRBM.K=TABPL(CRBMT,APDCR.K,0,1,.1)             CONSUMPTION RATIO BIRTH MULTIPLIER
T CRBMT=0/.15/.3/.45/.6/.85/.95/1/1.05/1.1/1.15/0/0/0/0/0/0/0/0/0/0/0
R PDD.KL=(PD.K)(ALFA)(CRFM.K)                    PREDATOR DEATH RATE
C ALFA=0.25                                      PREDATOR AVERAGE LIFETIME
A CRFM.K=TABPL(CRFMT,APDCR.K,0,1,.1)             CONSUMPTION RATIO DEATH MULTIPLIER
T CRFMT=20/15/10/7/5/2.5/1.5/1/.8/.75/.7/0/0/0/0/0/0/0/0/0/0/0
NOTE ----------
NOTE    FOOD
NOTE ----------
L FOOD.K=FOOD.J+(DT)(FRR.JK-FCR.JK)(K)           FOOD
N FOOD=FOODI
C FOODI=4000
C K=5
R FRR.KL=(FOODM-FOOD.K)/FRT.K                    FOOD REGENERATION RATE
C FOODM=40000                                    FOOD MAXIMUM
A AUX.K=FOOD.K/FOODM
A FRT.K=TABPL(TFRT,AUX.K,0,1,.25)                FOOD REGENERATION TIME
T TFRT=13.333/7.5/3.333/1.25/0.60302/0/0/0/0/0
R FCR.KL=FOOD.K*FFC.K                            FOOD CONSUMPTION RATE
A FFC.K=TABPL(TFFC,CFR.K,0,1.2,.1)               FRACTION OF FOOD CONSUMED
T TFFC=0/.1/.2/.3/.4/.5/.6/.7/.8/.9/.97/.99/1/0/0/0/0/0/0/0/0/0/0/0/0
SAVE PY,PD,FOOD
SPEC DT=0.05/LENGTH=40/SAVPER=0.4
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