THE PRINCIPLE OF CONSERVATION AND THE MULTIPLIER-ACCELERATOR THEORY OF BUSINESS CYCLES

Gilbert W. Low

System Dynamics Group
Alfred P. Sloan School of Management
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 USA

The principle of conservation states that physical quantities are confined to their own identifiable channels and can enter, circulate within, or depart from a system only by explicit processes. This paper applies the conservation principle to an analysis of the multiplier-accelerator theory of business cycles. Section I describes and critiques a well-known model of the multiplier-accelerator interaction. By ignoring accumulations of inventory and fixed capital investment, the model fails to observe the conservation of important physical flows. Section II proposes a system dynamics model that incorporates the multiplier and accelerator processes within a closed, conserved-flow framework. Section III uses computer simulation to portray the impact of conservation on the multiplier-accelerator interaction. Simulations of the system dynamics model reveal plausible long-term cycles, rather than the short-term fluctuations associated with traditional multiplier-accelerator models. At the end of Section III, the model is modified to account explicitly for labor, as well as capital, in the production process. This revised model produces both short-term and long-term oscillations when submitted to a noise input. The short-term oscillations, averaging about 5 years, reflect the attempt to adjust inventories by varying the labor input to production. The longer fluctuations in capital stock, averaging 19 years, reflect the management of investment in fixed capital. In all of the tests, the incorporation of conserved flows considerably reduces the sensitivity of system behavior to changes in parameter values. The simulations provide theoretical evidence for divorcing short-term business cycles from the interaction of the multiplier and accelerator.

TABLE OF CONTENTS

I. INTRODUCTION[1]

Economic processes, like the physical sciences, observe the principle of conservation. In real economic systems, physical flows such as production or the receipt of money income are conserved, or accumulated in physical stocks, such as inventories and money pools. In this fashion, things that move within a system--such as goods and money--are confined to their own identifiable channels and can enter, circulate within, or depart from the system only by explicit processes.

Many economic models fail to observe the principle of conservation. As a result, they sometimes generate misleading conclusions. This paper shows that one of the classic economic models, the multiplier-accelerator analysis first proposed by Samuelson in 1939,[2] fails to incorporate properly the conservation of physical flows. The Samuelson model and its later extensions[3] have been

[1]This paper draws on an earlier, unpublished paper entitled "A Systems Approach to the Multiplier-Accelerator Theory of Business Cycles," by G. W. Low and N. J. Mass (System Dynamics Group Working Paper D-1785-2, August 29, 1974). The model described in Section II closely resembles the model in the earlier draft. The simulations and conclusions are substantially different. I am grateful to Professor Mass for many aspects of the approach taken here, and refer specifically to his book, Economic Cycles: An Analysis of Underlying Causes (Cambridge: Wright-Allen Press, 1975), to which this paper relates.

[2]P. A. Samuelson, "Interactions Between the Multiplier Analysis and the Principle of Acceleration," Review of Economic Statistics, vol. 21 (May 1939), pp. 75-79.

[3]See, for example, J. R. Hicks, A Contribution to the Theory of the Trade Cycle (Oxford: Clarendon Press, 1950); R. G. D. Allen, "The Structure of Macro-Economic Models," Economic Journal (March 1960), pp. 38-51; R. M. Goodwin, "The Nonlinear Accelerator and the Persistence of Business Cycles," Econometrica, vol. 19 (January 1951), pp. 1-17; and A. Smithies, "Economic Fluctuations and Growth," Econometrica, vol. 25 (January 1957), pp. 1-52.

widely accepted as theoretical evidence linking the short-term business cycle with the interaction of the multiplier and acceleration principles.[4] This paper, however, shows that, when internal flows are conserved, a model of the multiplier-accelerator interaction does not produce short-term fluctuations, but does generate plausible long-term cycles. Moreover, the incorporation of conserved flows within a systems perspective considerably reduces the sensitivity of system behavior to parameter changes.

The purpose of revising the multiplier-accelerator model here is not to provide an alternative theory of the business cycle, but to correct the portrayal of a set of widely-acknowledged dynamic processes through a systems approach that is ignored too often in the economics literature.

A. Description of the Classic Multiplier-Accelerator Model

The basic model consists of three difference equations:[5]

[4]In Keynesian theory, a portion of each "round" of income payments in the circular flow of spending and production is devoted to consumption. Consumption spending encourages additional output, thereby bringing about expanded aggregate income and further subsequent spending. The multiplier describes the ultimate impact of this process on national income in response to an exogenous change in government or investment expenditures. The numerical value of the multiplier depends, in the simplest models, on the proportion of income (the "marginal propensity to consume") that is spent on current consumption rather than saved for future consumption. According to the accelerator, investment is proportional to the change in sales. The accelerator principle is based on the relationship between the flow of production and the stock of capital. As production, and therefore sales, expands, capital must rise to maintain productive capacity. Since capital (a stock) is linked directly to sales (a flow), the change in capital (investment) depends on the change in the flow of final product sales.

[5]This version of the model appears in W. L. Smith, Macroeconomics (Homewood, Ill.: Richard D. Irwin, 1970), p. 178.

$$Y_t = C_{t-1} + I_{t-1} + G_{t-1} \quad (1)$$

$$C_t = cY_t \quad (2)$$

$$I_t = K_t - K_{t-1} = a(Y_t - Y_{t-1}) \quad (3)$$

K - capital
Y - output
C - consumption
I - investment
G - government expenditures (a fixed quantity)
a,c - constant parameters

The (discrete) multiplier-accelerator model assumes that time is broken into finite periods of undefined duration. Equation (1) states that production in Period t equals expenditures in the previous time interval. The "output lag" between the time when aggregate spending takes place and the time when production stimulated by this spending subsequently occurs is one period.

In Equation (2), consumption is assumed to be a constant proportion of current income. In Keynes' terminology, this assumption implies a constant marginal and average propensity to consume out of income.

The investment function in Equation (3) is derived as follows:

> ...[C]onsider a model with an output lag in which businessmen produce in period t an amount equal to their sales in period t-1. Suppose further than they desire to have 'a' dollars of capital for each dollar of sales made in the previous period and that they always engage in an amount of investment sufficient to achieve this objective. That is,
>
> $$K_t = a * Sales_{t-1} = a * Y_t$$
>
> where K is the stock of capital and Y is gross national product.[6]

This assumption leads directly to the investment function:

[6] Ibid., p. 178.

$$I_t = K_t - K_{t-1} = DK_t - K_{t-1} = a * Y_t - a * Y_{t-1} = a(Y_t - Y_{t-1}),$$

where desired capital DK and actual capital are, by assumption, always equal.

By combining equations, the simple multiplier-accelerator model can be expressed as one second-order difference equation for gross national product Y:

$$Y_t = (c+a)Y_{t-1} - a * Y_{t-2} + G.$$ [7]

Depending on the values of the constants "a" and "c," income can increase without limit, move directly toward equilibrium, or exhibit convergent, steady, or divergent oscillations about the equilibrium level of Y.[8]

[7] Samuelson's equations are slightly different:

$$(1') \quad Y_t = g_t + C_t + I_t$$

$$(2') \quad C_t = cY_{t-1}$$

$$(3') \quad I_t = \beta(C_t - C_{t-1})$$

By substitution,

$$Y_t = g_t + c(1+\beta)Y_{t-1} - c\beta Y_{t-2}.$$

Equation (1') represents an accounting identify, or budget constraint. Equation (3') bases capital accumulation on the current change in consumption. "β" represents the "relation" between private investment and the current period change in consumption (Samuelson, p. 75), while "a" in the Smith version reflects the ratio between capital and total, rather than just consumer-goods, output. More important than the differences, however, is the fact that both sets of equations collapse into one second-order difference equation with two constant coefficients.

[8] Solving the difference equation with $Y = X^t$ yields:

$$X^2 - (c+a)X + a = 0 \quad (1)$$

$$(X - X_1)(X - X_2) = 0 \quad (2)$$

where X_1, X_2 are the two roots of the equation. Comparing (1) and (2), we see that $a = X_1X_2$. Therefore, if "a" is greater than 1, the system will be explosive. Also, X_1 and X_2 will be real numbers (and the system will not oscillate) if

$$[(c+a)^2 - 4a] > 0.$$

B. Critique of the Model

Real-world dynamic processes reflect the principle of conservation, which accumulates rates of flow in physical stocks. Fixed capital, for example, represents the accumulation, or integration, of net investment flows; money pools accumulate the difference between money inflows and payments. During periods of economic growth, physical additions to capital exceed discards, and the capital stock increases. Over a typical 3- to 7-year business cycle, money pools, product inventories, and other stocks rise and fall in response to variations in the flows that are being integrated over time.

These accumulations, in turn, influence the flow rates, through a variety of information and decision channels. For instance, an excessive accumulation of fixed capital will discourage continued investment; while excess product inventories may encourage reduced production, which augments inventories, and expanded shipments, which deplete inventories. In any dynamic system, such feedbacks between rates of flow and their accumulation in stocks determine the process of change over time.

Economic models that fail to capture the integration processes are often inadequate for describing the disequilibrium characteristics of economic activity. The multiplier-accelerator model, for example, attempts to explain disequilibrium behavior. But the model ignores the accumulations that occur when the real economy is in disequilibrium. For example, income, production, and sales are different concepts which have equal values when the system is in equilibrium, but do not have equal values under disequilibrium conditions.

Income represents the transfer of purchasing power to the factors of production. Production represents the creation of goods and services. Sales involve the transfer of produced goods and services. The difference between

production and sales must accumulate somewhere in order to conserve the physical flows through the production-distribution channel. Conservation, therefore, requires a level of inventory to uncouple these two distinctly different rates of flow.

By ignoring inventories, the multiplier-accelerator model neglects dynamic processes that are important to explaining the business cycle. When demand is growing, for example, inventories decline and production must rise more than in proportion to demand in order to redress the balance between inventories and sales. Conversely, when demand declines, production may fall below the volume of purchases in order to permit a runoff of unwanted inventories. The influence of inventory, therefore, will affect the swings in income induced by the interaction of the multiplier and the accelerator.

The traditional model also fails to conserve investment flows in an explicit level of capital. Equation (1) in the model described previously specifies production at time t (Y_t) independently of capital and, therefore, of the capacity to produce. However, the model implies a relationship between capacity and output in the form of a capital-output ratio (the coefficient "a"). Because there are no constraints on investment, actual capital always equals the desired amount. The relative availability of capital goods inventories, however, affects investment and thereby the accumulation of productive capacity. A model portraying the multiplier-accelerator processes, therefore, must track capital stock so as to show properly the dynamic impact of changing demand on system behavior.

The conservation of investment flows is also required if we wish to portray the real processes of capital accumulation and runoff through obsolescence. While the traditional model represents <u>net</u> investment (Equation (3)), which is zero in equilibrium, a more realistic model would incorporate the asymmetric process by which capital actually grows or declines. In a cyclical upswing, for example, capital accumulates the positive difference between gross additions and discards. In the downswing, however, capital runoff is limited by the rate of capital depreciation.[9]

II. A SYSTEM DYNAMICS REVISION OF THE MULTIPLIER-ACCELERATOR MODEL

The traditional multiplier-accelerator model fails to conserve physical flows. Yet the "physics" of real economic processes requires conservation to link such rates as production and consumption with capacity and available output. This section offers a system dynamics alternative to the usual versions of the multiplier-accelerator model.[10] The revised model retains the multiplier and accelerator principles but adds the structure required to represent conservation. The consequences of adding conservation to the multiplier-accelerator interaction are explored in the simulations described in Section III.

The revised multiplier-accelerator model, consisting of 16 equations, is shown in the flow diagram in Figure 1. Equation 1 represents sales S as the sum of three components--consumption C, investment I, and government purchases G.

---

[9] Jan Tinbergen raised this issue even before the publication of Samuelson's model, in "Statistical Evidence on the Acceleration Principle," Economica (May 1938), pp. 164-176.

[10] The system dynamics approach to modeling is described in J. W. Forrester, Industrial Dynamics (Cambridge: MIT Press, 1961), and Principles of Systems (Cambridge: Wright-Allen Press, 1968).

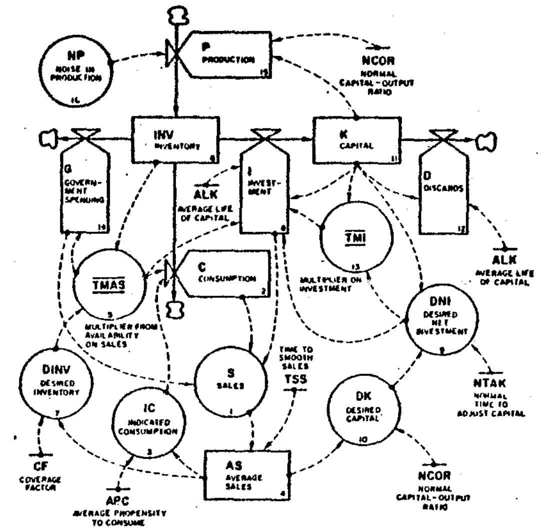

Figure 1. DYNAMO flow diagram for the revised multiplier-accelerator model

```
S.K=C.JK+I.JK+G.JK                                          1, A
    S       - SALES (OUTPUT UNITS/YEAR)
    C       - CONSUMPTION (OUTPUT UNITS/YEAR)
    I       - INVESTMENT (CAPITAL UNITS/YEAR)
    G       - GOVERNMENT PURCHASES (OUTPUT UNITS/YEAR)
```

Consumption C is defined in Equation 2 as the product of indicated consumption IC and the multiplier from availability on sales MAS.

```
C.KL=IC.K*MAS.K                                             2, R
    C       - CONSUMPTION (OUTPUT UNITS/YEAR)
    IC      - INDICATED CONSUMPTION (OUTPUT UNITS/YEAR)
    MAS     - MULTIPLIER FROM AVAILABILITY ON SALES
                (DIMENSIONLESS)
```

Indicated consumption IC, appearing in Equation 3, is proportional to average sales AS and therefore resembles the consumption function that appears in the traditional multiplier-accelerator treatment. The constant term, average propensity to consume APC, represents the fraction of total income (0.65 in the model) normally spent on personal consumption as opposed to being taxed by the government or saved for private investment.[11] The link between average sales AS and indicated consumption IC implies that the real purchasing value of total sales is distributed over time to owners of productive factors (for example, labor and capital services), who, in turn, spend their income on further consumption.

```
IC.K=APC*AS.K                                    3, A
APC=.65                                          3.1, C
    IC      - INDICATED CONSUMPTION (OUTPUT UNITS/YEAR)
    APC     - AVERAGE PROPENSITY TO CONSUME (FRACTION)
    AS      - AVERAGE SALES (OUTPUT UNITS/YEAR)
```

Average sales AS is defined in Equation 4. The averaging process subsumes delays in paying factor inputs, perceiving information, and changing consumer habits. In the model, average sales AS is an exponentially smoothed value of current sales. Current consumption consequently depends on the sales (transferred into income) of all past periods. For example, with a time to smooth sales TSS equal to 2 years, roughly 63 percent of the sales level of 2 years ago is included in current "operational" or "permanent" income (and therefore influences current consumption); however, sales of 8 years ago exert almost no impact on today's spending.[12] The formulation reflects the continuous process by which consumption habits and standards are gradually adapted to the levels dictated by

---

[11] In recent years, personal consumption expenditures have equaled about 63% of the gross national product (see Statistical Abstract of the United States 1974, p. 373).

[12] The two-year time to smooth sales TSS is probably on the short side of the real value. According to Mass, for example, empirical work by Friedman in 1957 would imply a value of 2.65 years for TSS (Mass, op. cit., p. 85).

current income.[13] It may be contrasted with the consumption function in the earlier model where expenditures are determined solely by current income or by income averaged over some "current" period.

```
AS.K=AS.J+(DT/TSS)(S.J-AS.J)                               4, L
AS=1000                                                    4.1, N
TSS=2                                                      4.2, C
    AS      - AVERAGE SALES (OUTPUT UNITS/YEAR)
    TSS     - TIME TO SMOOTH SALES (YEARS)
    S       - SALES (OUTPUT UNITS/YEAR)
```

When inventory deviates from the desired level, the multiplier from availability on sales MAS compels consumption C to depart from the indicated quantity. Equation 5 defines the multiplier as a nonlinear function of the ratio of inventory INV to desired inventory DINV. The table that specifies the relationship, shown in Figure 2, approximates a complex underlying structure of pricing, distribution, and consumption decisions. When inventories are abundant, for example, consumers can be induced to spend up to 12 percent more than the normal indicated amount, thereby helping to bring inventories down to desired levels; when inventories are scarce, consumers can be forced to purchase less than normal. For example, during a period of rising prices, which in turn may respond to inventory shortages, a given amount of nominal consumer expenditure will purchase less in real terms--that is, in terms of physical output. The multiplier from availability on sales MAS modifies government and investment spending as well as consumption.

---

[13] Theories of consumption abound in the economics literature. Planned consumption PC here corresponds to the "permanent" portion of consumption in Friedman's formulation:

$$\text{Permanent consumption} = C_p = \lambda b^t Y_{t-1}$$

where $0 < b < 1$ and λ is the average propensity to consume. See M. Friedman, A Theory of the Consumption Function (Princeton: Princeton University Press, 1957).

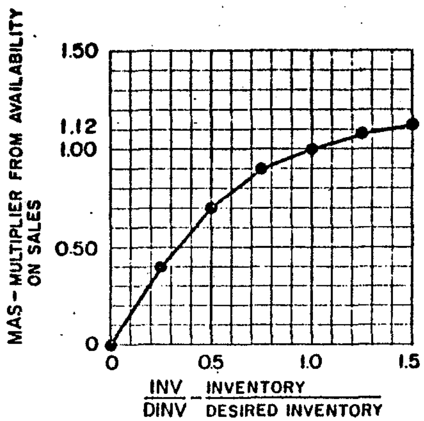

Figure 2. Table for multiplier from availability on sales

```
MAS.K=TABHL(TMAS,INV.K/DINV.K,0,1.5,.25)                5, A
TMAS=0/.4/.7/.9/1/1.08/1.12                             5.1, T
    MAS    - MULTIPLIER FROM AVAILABILITY ON SALES
               (DIMENSIONLESS)
    TMAS   - TABLE FOR MULTIPLIER FROM AVAILABILITY ON
               SALES
    INV    - INVENTORY (OUTPUT UNITS)
    DINV   - DESIRED INVENTORY (OUTPUT UNITS)
```

The level of inventory INV is defined in Equation 6. In this simple model, all production, whether consumer goods or capital equipment, flows into one aggregate inventory, to be distributed subsequently to purchasers of consumer goods (as "output units") or capital goods (as "capital units"). Inventory conserves the flow of goods in the production-distribution channel, thereby permitting an analysis of disequilibrium behavior, when the two ends of the channel (production and sales) are not equal.


```
INV.K=INV.J+DT*(P.JK-C.JK-I.JK-G.JK)                    6, L
INV=DINV                                                6.1, N
    INV    - INVENTORY (OUTPUT UNITS)
    P      - PRODUCTION (OUTPUT UNITS/YEAR)
    C      - CONSUMPTION (OUTPUT UNITS/YEAR)
    I      - INVESTMENT (CAPITAL UNITS/YEAR)
    G      - GOVERNMENT PURCHASES (OUTPUT UNITS/YEAR)
    DINV   - DESIRED INVENTORY (OUTPUT UNITS)
```

Equation 7 defines the desired level of inventories. In order to maintain continuity in production scheduling and distribution operations, producers are assumed to desire an inventory equal to approximately 3.5 months of average sales AS.[14]

```
DINV.K=CF*AS.K                                          7, A
CF=.3                                                   7.1, C
    DINV   - DESIRED INVENTORY (OUTPUT UNITS)
    CF     - COVERAGE FACTOR (YEARS)
    AS     - AVERAGE SALES (OUTPUT UNITS/YEAR)
```

Investment I is defined in Equation 8. Although consumer goods are no longer accounted for after they flow out of inventory, capital goods flow directly from the level of inventory INV to the stock of capital K (see Figure 1). The rate that links the two levels, investment I, usually equals desired net investment plus discards. If desired and actual capital balance, then investment just offsets discards, and capital remains in equilibrium.

Investment plans are not always realized, however. For one thing, gross investment, unlike net investment, cannot decline below zero. If desired net investment DNI is negative, gross investment is constrained by the multiplier on investment MI (see Equation 13) to a value greater than or equal to zero.

The second influence on investment I, the multiplier from availability on sales MAS, introduces a significant feature of conservation in the revised

---

[14] This estimate for the coverage factor CF is consistent with data presented in Moses Abramovitz, Inventories and Business Cycles (New York: National Bureau of Economic Research, 1950), p. 132.

model. Inadequate inventories constrain the rate of capital accumulation. The flow of capital from finished goods inventory into the stock of productive capital links the levels of inventory and capital. Capital accumulation, in turn, governs the economy's capacity to produce and, thereby, to replenish the inventory of finished goods for further investment (or consumption). The introduction of one conserving level (inventory), therefore, requires another level (capital) to conserve the flow of investment goods and to relate current capacity to the production of more capacity.

```
I.KL=(DNI.K+K.K/ALK)*MI.K*MAS.K                          8, R
     I       - INVESTMENT (CAPITAL UNITS/YEAR)
     DNI     - DESIRED NET INVESTMENT (CAPITAL UNITS/YEAR)
     K       - CAPITAL (CAPITAL UNITS)
     ALK     - AVERAGE LIFE OF CAPITAL (YEARS)
     MI      - MULTIPLIER ON INVESTMENT (DIMENSIONLESS)
     MAS     - MULTIPLIER FROM AVAILABILITY ON SALES
                 (DIMENSIONLESS)
```

Equation 9 defines desired net investment DNI as the difference between desired and actual capital, divided by an adjustment time. The normal time to adjust capital NTAK of 2 years reflects the period of planning and organization required to make changes in operating capacity.[15]

```
DNI.K=(DK.K-K.K)/NTAK                                    9, A
NTAK=2                                                   9.1, C
     DNI     - DESIRED NET INVESTMENT (CAPITAL UNITS/YEAR)
     DK      - DESIRED CAPITAL (CAPITAL UNITS)
     K       - CAPITAL (CAPITAL UNITS)
     NTAK    - NORMAL TIME TO ADJUST CAPITAL (YEARS)
```

---

[15] Despite the lack of substantial empirical evidence, some investment studies suggest a lag of about 6 quarters between appropriations and expenditures (M. K. Evans, Macroeconomic Activity (New York: Harper and Row, 1969), p. 101). To this lag one should add some period for observing past activity and for planning investment. As with the time to smooth sales TSS, therefore, the normal time to adjust capital NTAK probably is also on the short side.

Like the investment equation in the traditional multiplier-accelerator model, desired capital DK, defined in Equation 10, is proportional to past sales, in this case average sales AS rather than a discrete one-period lag.

```
DK.K=AS.K*NCOR                                           10, A
NCOR=2.25                                                10.1, C
     DK      - DESIRED CAPITAL (CAPITAL UNITS)
     AS      - AVERAGE SALES (OUTPUT UNITS/YEAR)
     NCOR    - NORMAL CAPITAL-OUTPUT RATIO (CAPITAL UNITS/
                 OUTPUT UNIT/YEAR)
```

In Equation 11, capital K accumulates the difference between investment I and discards D.

```
K.K=K.J+DT*(I.JK-D.JK)                                   11, L
K=IK                                                     11.1, N
IK=2250                                                  11.2, C
     K       - CAPITAL (CAPITAL UNITS)
     I       - INVESTMENT (CAPITAL UNITS/YEAR)
     D       - DISCARDS (CAPITAL UNITS/YEAR)
     IK      - INITIAL CAPITAL (CAPITAL UNITS)
```

Discards D, defined by Equation 12, assumes a constant average life of capital ALK of 15 years.[16] The discard equation states that a constant fraction of existing capital becomes technically obsolete or otherwise uneconomical each year.

```
D.KL=K.K/ALK                                             12, R
ALK=15                                                   12.1, C
     D       - DISCARDS (CAPITAL UNITS/YEAR)
     K       - CAPITAL (CAPITAL UNITS)
     ALK     - AVERAGE LIFE OF CAPITAL (YEARS)
```

Equation 13 and Figure 3 show the formulation for the multiplier on investment MI, which constrains investment as desired net investment DNI becomes increasingly negative. When desired net investment DNI is zero or positive,

---

[16] A 15-year average lifetime for total capital stock (including equipment and structures) is consistent with the depreciation guidelines given in "Asset Guideline Classes and Periods, Asset Depreciation Ranges," Paragraph 220, in 1971 Depreciation Guide (New York: Commerce Clearing House, 1971).

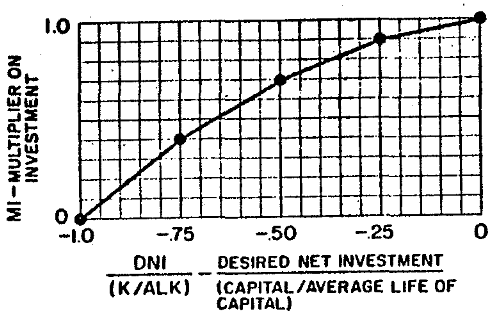

Figure 3. Table for multiplier on investment

the multiplier has no impact on investment; in that case, the only constraint on expanding capital by the desired amount is the already-described multiplier from availability on sales MAS. When desired net investment DNI is negative, however, the multiplier constrains (gross) investment I, eventually forcing investment to zero when firms in the aggregate want capital stock to decline by an amount equal to or greater than the rate of discards (by Equation 12, discards D = K/ALK). As we have seen already, discards in the model are determined by the fixed average life of capital ALK. Therefore, capital cannot decline fractionally by more than 1/ALK, even if businessmen want capital to decline faster. When the ratio of DNI to K/ALK falls between -1 and 0, the multiplier on investment MI causes (gross) investment I to fall below its usual value of DNI + K/ALK (see Equation 8). In effect, while the normal time to adjust capital NTAK is 2 years, the actual adjustment time falls

below 2 years[17] as businessmen become increasingly pessimistic and permit capital to decline through discards even more rapidly than would be warranted by desired net investment DNI alone.

```
MI.K=TABHL(TMI,DNI.K/(K.K/ALK),-1,0,.25)                     13, A
TMI=0/.6/.9/1/1                                              13.1, T
     MI      - MULTIPLIER ON INVESTMENT (DIMENSIONLESS)
     TMI     - TABLE FOR MULTIPLIER ON INVESTMENT
     DNI     - DESIRED NET INVESTMENT (CAPITAL UNITS/YEAR)
     K       - CAPITAL (CAPITAL UNITS)
     ALK     - AVERAGE LIFE OF CAPITAL (YEARS)
```

As shown in Equation 14, the rate of government spending G is determined outside of the system, except to the extent that it is constrained by the multiplier from availability on sales MAS.

```
G.KL=(IG+STEP(SG,TSG))*MAS.K                                   14, R
IG=200                                                         14.1, C
SG=20                                                          14.2, C
TSG=1                                                          14.3, C
     G       - GOVERNMENT PURCHASES (OUTPUT UNITS/YEAR)
     IG      - INITIAL GOVERNMENT PURCHASES (OUTPUT UNITS/
                 YEAR)
     SG      - STEP IN GOVERNMENT PURCHASES (OUTPUT UNITS/
                 YEAR)
     TSG     - TIME TO STEP GOVERNMENT PURCHASES (TIME)
     MAS     - MULTIPLIER FROM AVAILABILITY ON SALES
                 (DIMENSIONLESS)
```

---

[17] It can be shown algebraically, for example, that when

$$\frac{DNI}{K/ALK} = -0.5 \text{ and } -0.75,$$

the actual adjustment times equal 1.82 years and 1.76 years, respectively.

Equation 15 defines production P as proportional to the amount of capital K, except when disturbed by a noise term[18] or by the multiplier from inventory on production MIP.[19] The multiplier will permit certain simulation tests discussed in Section III, but it is not considered a part of the basic model described here and, therefore, does not appear in the flow diagram in Figure 1.

```
P.KL=(K.K/NCOR)*MIP.K*NP.K                          15, R
     P      - PRODUCTION (OUTPUT UNITS/YEAR)
     K      - CAPITAL (CAPITAL UNITS)
     NCOR   - NORMAL CAPITAL-OUTPUT RATIO (CAPITAL UNITS/
                OUTPUT UNIT/YEAR)
     MIP    - MULTIPLIER FROM INVENTORY ON PRODUCTION
                (DIMENSIONLESS)
     NP     - NOISE IN PRODUCTION (DIMENSIONLESS)
```

---

[18] Noise in production NP, defined in Equation 16, approximates random disturbances with short-term autocorrelation by smoothing normally-distributed random values generated for each solution interval over a one-year time to smooth noise in production TSNP. The mean value of the distribution is 1. Setting the standard deviation in noise in production SDNP to some value greater than zero introduces noise and, thereby, permits one to study phasing and natural frequency characteristics. The noise input is used in two of the simulations described in the next section.

```
NP.K=NP.J+(DT/TSNP)(NORMRN(1,SDNP)-NP.J)            16, L
NP=1                                                16.1, N
TSNP=1                                              16.2, C
SDNP=0                                              16.3, C
     NP     - NOISE IN PRODUCTION (DIMENSIONLESS)
     TSNP   - TIME TO SMOOTH NOISE IN PRODUCTION (YEARS)
     SDNP   - STANDARD DEVIATION IN NOISE IN PRODUCTION
                (DIMENSIONLESS)
```

[19] The equation for MIP relates production to the relative level of inventory. All of the values in the table equal 1 (TMIP = 1/1/1/1/1) in the basic model.

```
MIP.K=TABHL(TMIP,INV.K/DINV.K,0,2,.5)               17, A
TMIP=1/1/1/1/1                                      17.1, T
     MIP    - MULTIPLIER FROM INVENTORY ON PRODUCTION
                (DIMENSIONLESS)
     TMIP   - TABLE FOR MULTIPLIER FROM INVENTORY ON
                PRODUCTION
     INV    - INVENTORY (OUTPUT UNITS)
     DINV   - DESIRED INVENTORY (OUTPUT UNITS)
```


III. MODEL SIMULATIONS

Explicit representation of fixed capital and product inventory significantly alters the effects of the modeled multiplier-accelerator interaction. The first simulation test presented here depicts the influence of capital accumulation in isolation, by ignoring the conservation of production flows in inventory. The remaining tests reflect the impact of both inventory and capital accumulation on system behavior. The simulations suggest that the interaction of the multiplier and accelerator has little to do with the generation of short-term business cycles.

A. Conservation of Capital Investment Flows

For the first simulation experiment, inventory is removed from the production-distribution channel so that the system dynamics model conforms very closely to the original multiplier-accelerator model. Production in this case still varies in proportion to capital stock, but the model assumes that production is sufficient to permit the realization of desired purchases at all times. These considerations eliminate the influence of inventory and production, leaving essentially the structure displayed in Figure 4.[20]

---

[20] Inventory INV is ignored by setting the table for multiplier from availability on sales TMAS to 1 (TMAS = 1/1/1/1/1/1/1). Since government purchases G and consumption C now are not affected by inventory INV, the two rates simply equal the indicated, or unconstrained, quantities. Therefore, indicated consumption IC (which equals consumption C here) appears in the figure as a component of sales S; and government purchases G appears simply as an auxiliary variable used as a test input.

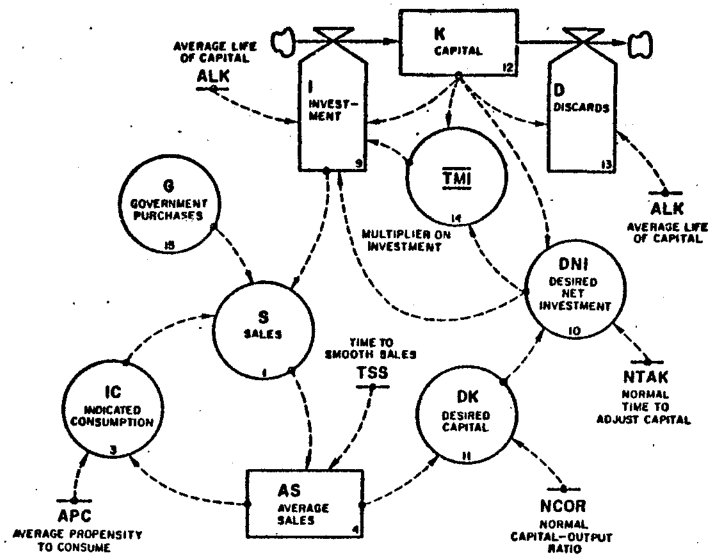

Figure 4. Revised version of the system dynamics model

To generate the computer plots (shown in Figure 5), the model is disturbed from equilibrium[21] by a 10-percent step in government purchases (SG = 20). Two characteristics of the resulting behavior bear particular notice--the tendency for the model variables to overshoot their eventual equilibrium values, and the

[21] In equilibrium, all three level variables remain constant. Inventory INV = desired inventory DINV = 300; capital K = 2550; production P = sales S = 1000; and investment I = discards D = 150. Government purchases G start at 200 and then step to 220 (SG = 20) at time 1 (TSG = 1). These proportions closely correspond to data for the United States in recent years; during the period 1969-1973, personal consumption equaled 63 percent of GNP, government purchases of goods and services equaled 22 percent and gross private domestic investment equaled 15 percent (Statistical Abstract 1974, p. 373).

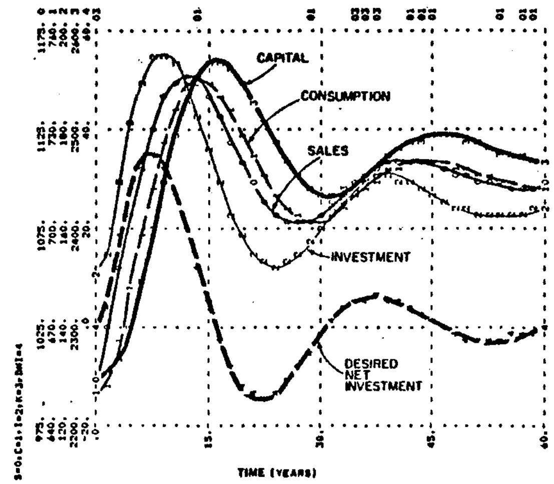

Figure 5. Behavior of the basic model without inventory accumulation

periodicity of oscillation.

Overshoot is produced by combining the accelerator (the dependence of investment on the change in sales) with the multiplier (the dependence of consumption on (average) sales). The multiplier alone embodies the feedback loop shown in Figure 6. The causal polarities in the figure are all positive, which normally would suggest self-sustaining, rather than self-correcting, behavior. Yet the loop is goal-seeking, because the gain around the loop is less than 1 (the gain equals the average propensity to consume APC, here equal to 0.65). As sales and average sales increase, for example, because of an initial step in government purchases, consumption grows by less than the full amount. Eventually, with investment held constant, consumption would rise

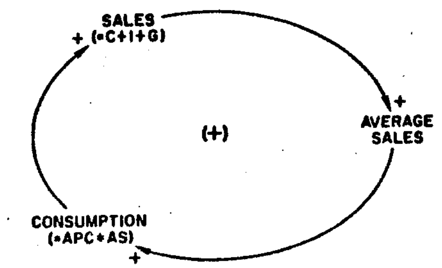

Figure 6. The multiplier loop

without overshoot to a new equilibrium value of 687, over a period determined by the time constant used in smoothing sales.[22]

Overshoot occurs, however, because investment I, instead of remaining constant, varies in response to average sales AS. Investment, like consumption, also is linked to average sales AS in a positive feedback loop, as shown in Figure 7. But the investment-sales loop does not necessarily produce goal-seeking

[22]Viewed in discrete period terms, the incremental step of 20 in government purchases produces an initial incremental increase in sales of 20 and causes consumption to rise to $0.65 * 1020 = 663$. Now sales equals 1033 and consumption rises again to $0.65 * 1033 = 671.45$. Consumption continues to grow at a declining rate and approaches a new equilibrium, which is computed as follows:

$$S = C + I + G = 0.65\,S + 150 + 220 = 1057.13$$

$$C = 0.65 * 1057.13 = 687.12$$

In the model, time to smooth sales TSS equals 2 (years), so consumption is within 5 percent of its new equilibrium after 3 time constants, or six periods (years).

behavior, since the gain (equal to 1.125 in the model[23]) can be greater than 1. The combination of the multiplier and the accelerator leads to new equilibrium values,[24] as well as oscillatory behavior.

The period of oscillation is 27 years, considerably longer than the 3- to 7-year period that characterizes short-term business cycles. The long period reflects mainly the influence of the 15-year average life of capital, which contributes considerable inertia to the processes of varying the capital stock over time. For example, once investment has augmented the stock of fixed

[23]Assuming MAS = MI = 1, investment $I = D + DNI = \frac{K}{ALK} + \frac{DK - K}{NTAK} = \frac{K}{ALK} + \frac{NCOR * AS - K}{NTAK}$. Therefore, the gain around the loop shown in Figure 7 is $\frac{NCOR}{NTAK} = \frac{2.25}{2} = 1.125$. Lower values of normal time to adjust capital NTAK increase the gain in the loop and, thereby, the instability of the system.

[24]The multiplier relationship determines the ultimate (equilibrium) change in output produced by a step in an exogenous component of aggregate spending (here government purchases G). The multiplier value for the entire system can be derived as follows:

In equilibrium,

Production P = Sales S = Average Sales AS

Investment I = Discards D = Capital K / Average Life of Capital ALK

Capital K = Desired Capital DK

From the equations described in Section II, we obtain (in equilibrium):

$$P = S = C + I + G$$
$$= APC * S + DK/ALK + G$$
$$= APC * S + NCOR * S/ALK + G$$

Therefore,

$$(1 - APC - NCOR/ALK) * S = G$$

$$\rightarrow S = \frac{1}{1 - APC - (NCOR/ALK)} * G$$

The coefficient of G is the multiplier. With APC = 0.65, NCOR = 2.25, and ALK = 15, the multiplier equals 5. Since government spending G steps up from 200 to 220 units per year, the new equilibrium value of sales S is $S * 220 = 1100$. The new equilibrium values for consumption and investment are 715 ($= 0.65 * 1100$) and 165 ($= 2.25 * 1100/15$), respectively.

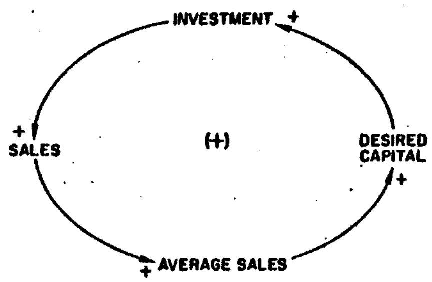

Figure 7. The accelerator loop

capital, substantial time must pass before an excess over the desired level can be eliminated.[25]

[25]Other responses to the 10-percent step in government spending, in which the normal time to adjust capital NTAK and the time to smooth sales TSS are varied over wide ranges, still exhibit long periods of oscillation. The following chart reveals the period (in years) resulting from 3 different values for each of the two parameters. (When TSS = 3, the system is highly damped; the values shown below are approximations on the short side derived from observing the computer plots.)

TSS ↓ \ NTAK →	1	2	3
1	65	24	25
2	19	27	38
3	25	35	42

B. Conservation of Capital Investment and Production Flows

By conserving the flow of capital goods and permitting the regular discard of obsolete capital, the revised model exhibits a frequency that far exceeds the period of short-term business cycles. However, inventory in the first simulation (Figure 5) has no impact on sales and is not constrained from going below zero.[26] The second simulation, therefore, introduces the effect of conservation in the production-distribution channel by activating the multiplier from (inventory) availability on sales MAS.

The addition of inventory has an important influence on the growth potential of the entire system. This influence can be seen best in terms of the positive loop portrayed in Figure 8. Whereas in the first simulation test, capital always adjusted to higher levels of desired capital over the normal time to adjust capital, inadequate inventory now prevents capital from adjusting to higher desired levels over the specified time. For example, a drop in inventory restrains investment, which causes capital to decline or prevents it from rising to a higher desired level. Restrained capital stock, in turn, holds back production, which prevents inventory from rising as fast as it otherwise might. The conservation of capital investment takes on added significance when linked in this positive loop to the conservation of production flows in inventory.

[26]For the run displayed in Figure 5, inventory INV equilibrates at about -130, an impossible result for a model designed to reflect real macro-economic behavior.

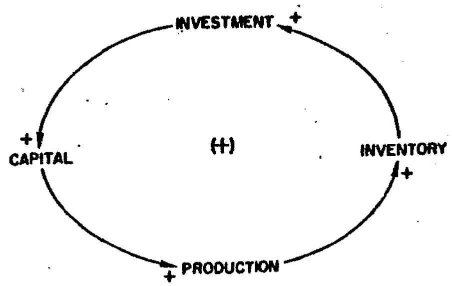

Figure 8. A positive loop linking capital and inventory

As shown in Figure 9, a step in government purchases produces an initial upswing in sales and investment, followed by a slow and steady decline over the 20-year simulation. The negative loops between inventory and the sales components stabilize system behavior. The positive loop from Figure 8 is responsible for the decline; that is, Investment I is insufficient to replace depreciating capital, and both fixed capital and production drop away from their initial values.

Although physical values no longer can fall below zero in this simulation, the results hardly offer an adequate portrayal of real macro-economic behavior. What the experiment shows, however, is that the multiplier and accelerator processes, when combined in a closed, conserved-flow system, produce behavior that

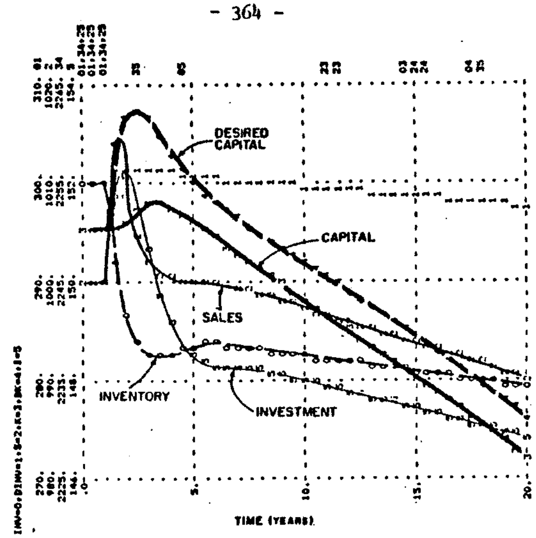

Figure 9. Behavior of the basic model with inventory accumulation

bears little resemblance to the short-term cycles generated by traditional business-cycle models.[27]

Figure 9 revealed the overdamped (non-oscillatory) behavior of the basic model. However, an inherent frequency of the system can be observed by stimulating the structure with a random noise input, rather than with a step in government purchases. The structure selects and amplifies certain frequency components of the incoming noise signal,[28] thereby exhibiting behavioral

[27] Only when both the normal time to adjust capital NTAK and the time to smooth sales TSS are reduced to below 1.1 years does production show one or more cycles in response to the step in government spending (still in an overall decline mode). In that case, the period of oscillation is reduced to roughly 5 years. Under certain conditions, therefore, the conserved-flow version of the multiplier-accelerator interaction can generate short-term cycles. The short time constants required to produce this result, however, do not seem reasonable in light of the evidence about lags in investment and consumption activity.

[28] For a discussion of the response of social system models to noise inputs, see Forrester (1961), Appendix F.

tendencies that were obscured in the earlier experiment. To perform a test of this sort, production is subjected to (smoothed) random noise with a 5-percent standard deviation.

The most striking feature of the experiment, displayed in Figure 10, is the easily discernible long-term fluctuations in capital stock. Over the 80 years of the run, the time between peaks ranges from 16 to 24 years and averages 19 years.[29] In comparison with the first simulation, the conservation of production flows in an inventory level appears to damp overall behavior and shorten the system's natural frequency. But the periodicity still far exceeds the usual length of short-term business cycles and is tied, as before, to the management of fixed capital investment.[30]

C. The Basic Model with a Variable Capital-Output Ratio

So far, an important assumption underlying the original Samuelson model has not been altered. That is, the model developed here has retained a constant capital-output ratio, thereby implying a fixed relationship between capital and other factor inputs (for example, labor). The assumption of a constant ratio may be relaxed in several ways.[31] One simple way to represent

---

[29] The short fluctuations in production P reflect the 1-year smoothing in noise in production NP and are not produced by the rest of the system structure.

[30] The inherent frequency is characteristic of the 15- to 20-year Kuznets cycle in the rate of growth of capital, production, and other variables. Mass (1975, op. cit.) argues in more detail that the management of fixed capital investment underlies the so-called Kuznets cycle. He attributes short-term cycles to the management of inventories and labor, rather than to fixed capital investment.

[31] For example, production may be constrained when the economy approaches full employment, thereby affecting the ratios of fixed capital to output and to labor. The availability of labor to meet production needs is not treated in this paper but is an important avenue for future system dynamics research into the dynamics of economic systems.

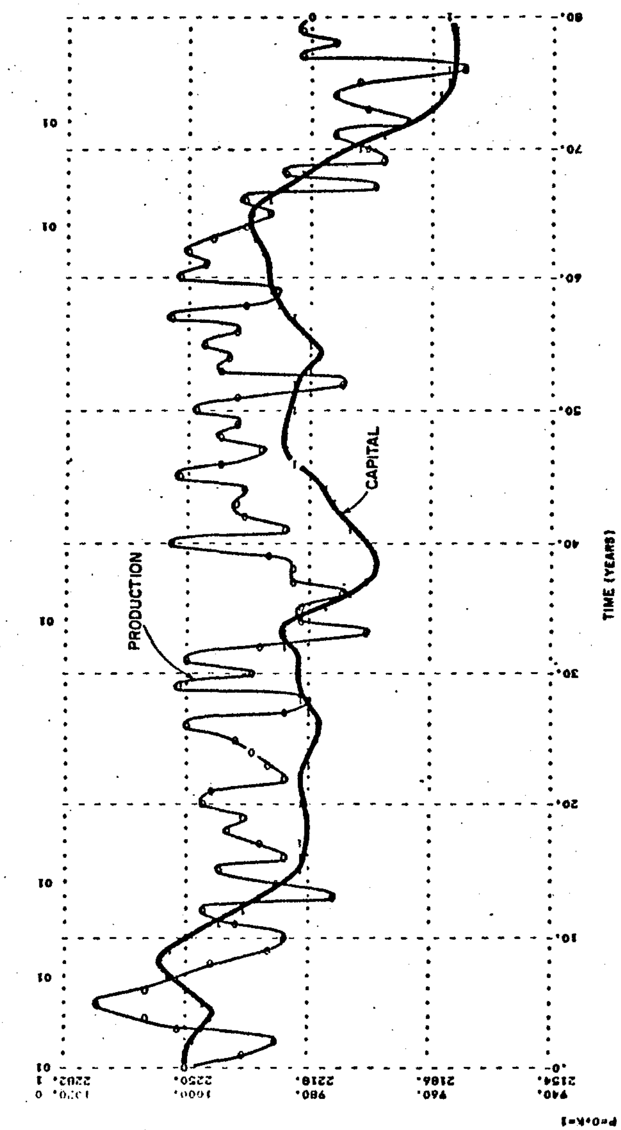

Figure 10. Behavior of the basic model with noise in production

an independent, non-capital input is to retain the basic model but to allow production to vary in response to influences other than changes in fixed capital. A more detailed approach is to add an explicit flow of workers who move, like capital, within a conserved-flow channel. The simulations that follow explore the implications of these two possibilities.

In the first case, the capacity to expand or contract labor independently of capital stock is implied by changing the multiplier from inventory on production MIP (Equation 17). Previously, the table equaled 1 over all possible ranges of the ratio of inventory INV to desired inventory DINV. Now the function describes a negative slope, as shown in Figure 11.

This new slope activates the negative feedback loop shown in Figure 12. The additional feedback implies that business firms can adjust labor more rapidly than capital; the accelerator no longer assumes the full burden of capacity adjustment. Declining inventory, for example, reveals growing demand and encourages firms to increase production, and thereby inventory, by using

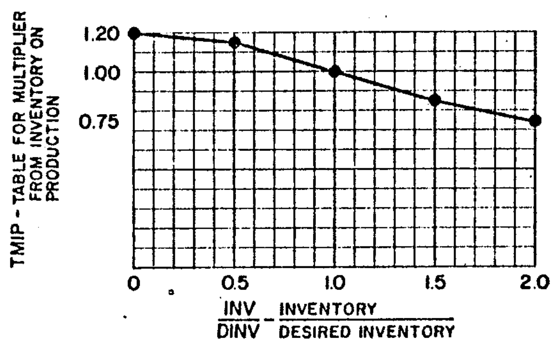

Figure 11. Table for multiplier from inventory on production

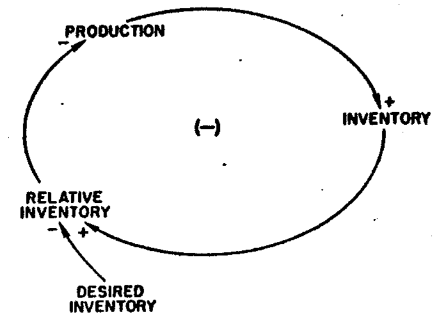

Figure 12. The inventory-production loop

capital more intensively than usual. Whereas the normal capital-output ratio remains fixed at 2.25, the actual ratio of capital to production now can decline to a low of 1.69 (= 0.75 * 2.25).

With the added negative feedback between inventory and production, the system is not locked into slow decline, as was the case with the previous experiment. Figure 13 shows that the early gap in relative inventories, produced by the step in government purchases, still inhibits investment and the other sales components. While investment is restrained by inadequate inventory, however, additional production is encouraged. Thus production initially rises faster than capital and helps to reduce the discrepancy between inventory and desired inventory. Investment, therefore, continues to rise and the accumulation of net investment in capital increases as well.

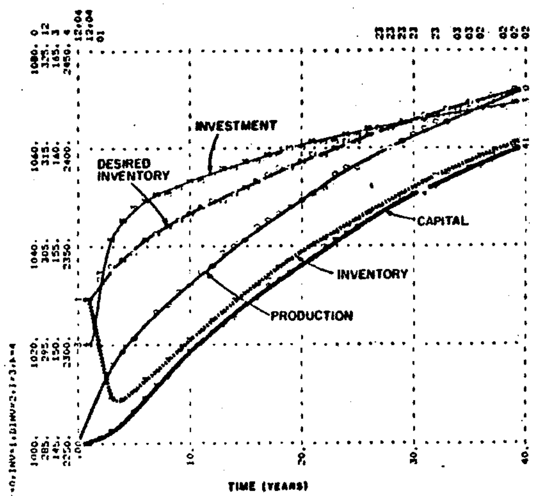

Figure 13. Behavior of the basic model with inventory-production feedback

After the initial expansion phase, the model variables slowly approach new equilibrium values. As was shown previously, the new equilibria for production and investment are 1100 and 165. However, after 40 years, production has only closed 70 percent of the gap between its initial and final values, thereby revealing the persistent constraint of relative inventories on fixed capital accumulation. The long duration of adjustment suggests that equilibrium analysis of the system, which yields the end result of a step in government purchases, tells only a small part of the story. Disequilibrium analysis, on the other

hand, focuses on the process of change which can persist for a very long time.[32]

Figure 14 compares production P in six different simulations of the basic model with production P from the previous base run. The purpose of this exercise is to reveal the relative insensitivity of model behavior to

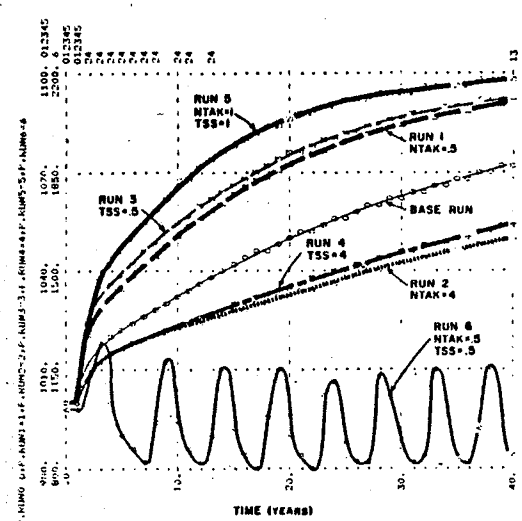

Figure 14. Comparative plots of production from the basic model with changes in two parameters, normal time to adjust capital NTAK and time to smooth sales TSS

---

[32]In another test not shown here, the model used to produce Figure 14 was subjected to random noise in production (SDNP = 0.05). Despite the overdamped response of the system to a step input, the noise run revealed long-term oscillations in capital stock. The periodicity of fl· ·tions ranged between 15 and 25 years, thereby closely resembling the ·e run of the basic model that contained a fixed capital-output ratio (se· ·gure 10).

changes in the two parameters, normal time to adjust capital NTAK and time to smooth sales TSS. Varying either parameter value over a range of 0.5 to 4 (Plots 1-4) has no significant impact on the model's behavior, although shorter adjustment times permit a more rapid approach to the final equilibrium value. Even cutting both parameters together by 50 percent (Run 5) fails to produce an overshoot in production along the adjustment path. In fact, one must reduce both constants to one-fourth of their original values in order to generate sustained cycles (Run 6). The period of these cycles, about 5 years, is indeed characteristic of the observed short-term business cycle. However, with this version of the multiplier-accelerator model, the parameter values required to produce the short period are implausibly short.

D. An Extended Model with Explicit Labor Flows

The variable capital-output ratio discussed in the preceding section implied that labor could be adjusted independently of capital. The adjustment, however, was instantaneous and ignored the conservation of labor flows as people move through different employment categories. The final simulation test, therefore, adds an explicit labor input to the production process. The hiring of labor, like the initiation of production, can be augmented when inventories are low or reduced when inventories are excessive. The added structure extends the multiplier-accelerator model beyond the usual analysis and begins to explore the relative impacts of labor versus capital management on disequilibrium behavior.

In the extended version, the multiplier from inventory on production MIP has been dropped, and several new equations have been added. Figure 15

exhibits a flow chart for the additional structure.[33]

In the extended version of the model, a Cobb-Douglas production function generates the flow of production P.[34] Instead of varying in proportion to capital K, production P now depends on both labor L and capital K. The new Equation 15 takes the form:

$$P = \beta \cdot K^{1-\alpha} \cdot L^{\alpha} = \beta \cdot K \cdot \left(\frac{L}{K}\right)^{\alpha},$$

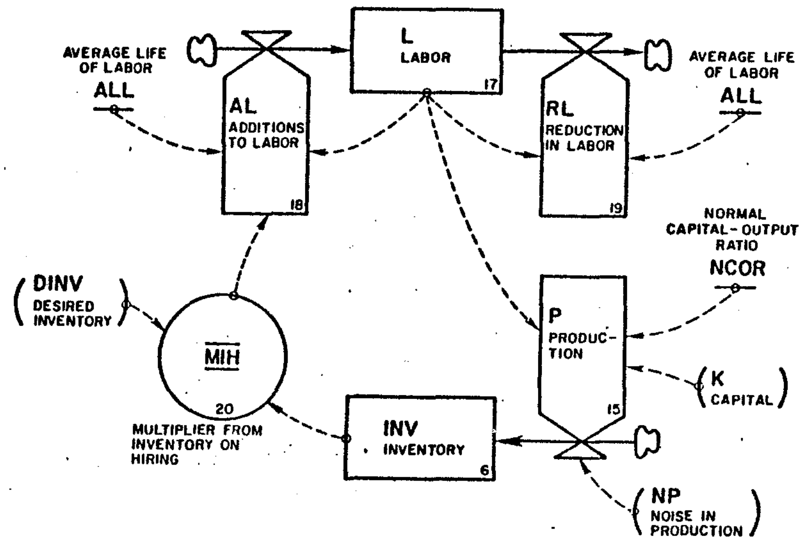

Figure 15. Extension to the basic system dynamics version of the multiplier-accelerator model

[33] The equation numbers refer to the new program, RMA2.DYNAMO, shown in the Appendix. Inventory is defined as before, but appears in the figure in order to complete the labor-inventory loop.

[34] The Cobb-Douglas function exhibits constant returns to scale and diminishing marginal returns to each factor unit. A commonly-used formulation, the Cobb-Douglas function [illegible] described elsewhere in detail. (See, for example, J. M. Henderson and [illegible], _Microeconomic Theory: A Mathematical Approach_ (New York: McGraw-Hill, 1971), pp. 80-89.)

where $\beta = \frac{1}{NCOR}$ and $\alpha = 0.65$.[35] The assumption that labor L and capital K have the same numerical value at the initial equilibrium permits a simple equivalent of β in terms of NCOR. The noise term (NP) appears for testing purposes.

```
P.KL=(1/NCOR)*K.K*EXP(.65*LOGN(L.K/K.K))*NP.K        15, R
     P       - PRODUCTION (OUTPUT UNITS/YEAR)
     NCOR    - NORMAL CAPITAL-OUTPUT RATIO (CAPITAL UNITS/
                 OUTPUT UNIT/YEAR)
     K       - CAPITAL (CAPITAL UNITS)
     L       - LABOR (LABOR UNITS)
     NP      - NOISE IN PRODUCTION (DIMENSIONLESS)
```

Labor L, like capital K, is expressed in Equation 17 as a level variable.

```
L.K=L.J+DT*(AL.JK-RL.JK)                                17, L
L=IL                                                    17.1, N
IL=2250                                                 17.2, C
     L       - LABOR (LABOR UNITS)
     AL      - ADDITIONS TO LABOR (LABOR UNITS/YEAR)
     RL      - REDUCTION IN LABOR (LABOR UNITS/YEAR)
     IL      - INITIAL LABOR (LABOR UNITS)
```

Additions to labor AL appears in Equation 18 as labor L divided by the average life of labor ALL, multiplied by the influence of relative inventories. The term in parentheses represents the hiring rate required to replace normal labor turnover. Whereas the normal turnover of capital (expressed in the average life of capital ALK) is 15 years, the average life of labor ALL is only 2 years.[36]

---

[35]The use of the EXP and LOGN operators in DYNAMO expresses the function

$$P = \beta \cdot K \cdot e^{0.65 \ln(L/K)}.$$

The exponential term is of the form $y = x^b$. Taking the natural logarithm of each side gives

$$\ln y = b \ln x$$

or

$$y = e^{\ln y} = e^{b \ln x}$$

[36]Two years is roughly the average turnover in manufacturing over recent years, including both voluntary and involuntary quits, as shown in the Statistical Abstract (1974), p. 344.


```
AL.KL=(L.K/ALL)*MIH.K                                   18, R
     AL      - ADDITIONS TO LABOR (LABOR UNITS/YEAR)
     L       - LABOR (LABOR UNITS)
     ALL     - AVERAGE LIFE OF LABOR (YEARS)
     MIH     - MULTIPLIER FROM INVENTORY ON HIRING
                 (DIMENSIONLESS)
```

Equation 19 expresses the reduction in labor RL as a two-year exponential delay of additions to labor AL. That is, the average unit of labor remains employed for two years.

```
RL.KL=L.K/ALL                                           19, R
ALL=2                                                   19.1, C
     RL      - REDUCTION IN LABOR (LABOR UNITS/YEAR)
     L       - LABOR (LABOR UNITS)
     ALL     - AVERAGE LIFE OF LABOR (YEARS)
```

The final equation in the extended model is the multiplier from inventory on hiring MIH, which appears in Equation 20 as a table function of the ratio of actual to desired inventory.[37] When inventory exceeds the desired level, the multiplier suppresses hiring and, thereby, reduces the labor input to production. Inadequate inventories reflect tight market conditions and encourage firms to expand production by hiring additional labor. At the extremes, the hiring rate can be expanded by 20 percent over the replacement amount or can be reduced by as much as 25 percent.

```
MIH.K=TABHL(TMIH,INV.K/DINV.K,0,2,.5)                   20, A
TMIH=1.2/1.15/1/.85/.75                                 20.1, T
     MIH     - MULTIPLIER FROM INVENTORY ON HIRING
                 (DIMENSIONLESS)
     TMIH    - TABLE FOR MULTIPLIER FROM INVENTORY ON
                 HIRING
     INV     - INVENTORY (OUTPUT UNITS)
     DINV    - DESIRED INVENTORY (OUTPUT UNITS)
```

---

[37]The table function (TMIH) displays the same values with respect to the ratio INV/DINV as the table for multiplier from inventory on production TMIP, shown in Figure 11.

Figure 16 reflects a noise input to the extended model version. Comparing the behavior of production, labor, and capital reveals an important implication of adding conserved labor flows to the basic model. Production, which reflects the combination of capital and labor as well as the noise in production, exhibits 2- to 3-year oscillations with a few noticeable swings of longer duration. The short oscillations reflect the one-year smoothing in the random noise input rather than structural components of the model itself. Labor displays oscillations of a longer periodicity, ranging generally between 3 and 7 years and averaging 5 years. Capital moves over a considerably longer cycle, with a period of between 15 and 22 years.

The period of oscillations in the pool of employed labor reflects the 2-year average lifetime of labor. Like fixed capital, labor accumulates in a level of employed people which can be adjusted in response to changing market conditions only over some appreciable period of time. Companies are seldom disposed to hiring and firing labor overnight, especially if social, legal, or union bargaining pressures impose penalties on too rapid an adjustment. On the other hand, labor can be adjusted, at least in the United States, far more rapidly than capital, whose average lifetime in the model is 15 years. Therefore, when market conditions change, as signalled by variations in relative inventory levels, firms rely mainly on adjustments in labor to manage production and inventories over the short term.

With the inclusion of an explicit level of labor, the revised multiplier-accelerator model now appears to select two distinct frequencies from the random noise input. The shorter of the two inherent frequencies, with an average period of about 5 years, reflects the economy's capacity to adjust labor and inventories fairly quickly. The relatively long period of fluctuations in

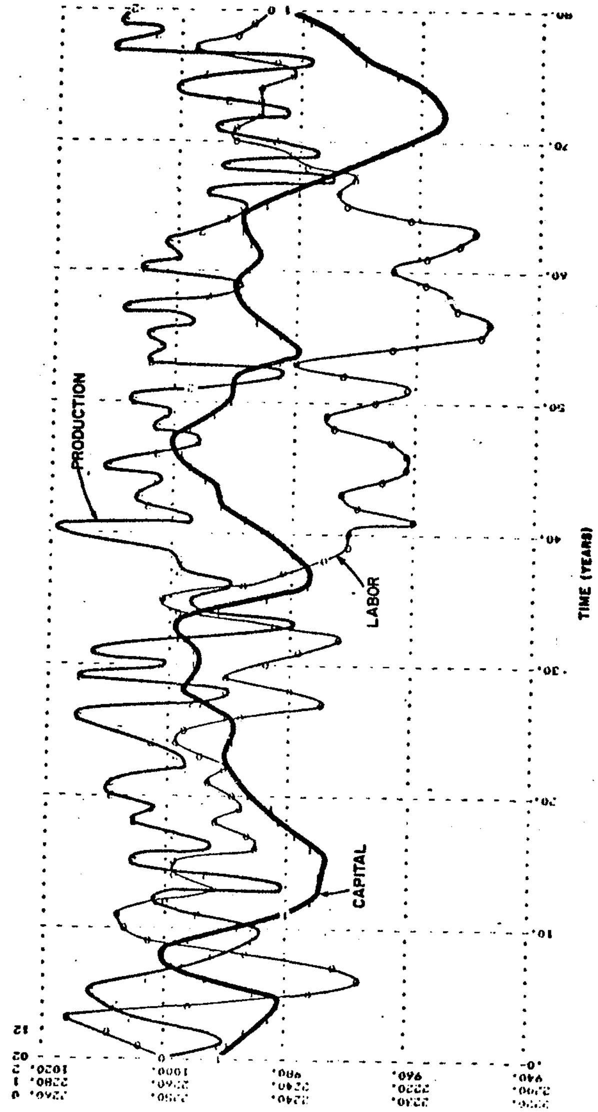

Figure 16. Behavior of the extended model (including labor) with noise in production

productive capital, on the other hand, reveals considerably greater inertia in the stock of fixed capital.

By focusing on the response of investment to changes in business activity, the usual multiplier-accelerator models claim that the management of fixed capital underlies short-term business cycles. However, the extended system dynamics model described in this section suggests that labor and inventory management can produce short-term cycles independently of the long-term fluctuations observed in fixed capital.[38]

By introducing the principle of conservation and gradually increasing model realism, the work described in this paper provides theoretical evidence for divorcing short-term cycles from the interaction of the multiplier and accelerator. Application of the systems approach to economic analysis is continuing[39] and, eventually, may lead to greater understanding of the business cycle and other economic processes.

---

[38] The work presented here leads to conclusions that are consistent with those of Mass (op. cit., 1975), and may serve as an introduction to his more detailed analysis.

[39] A large-scale project is currently underway at MIT to apply the concepts of system dynamics modeling to macroeconomic analysis. See J. W. Forrester, N. J. Mass, and C. J. Ryan, "The System Dynamics National Model: Understand-Socio-Economic Behavior and Policy Alternatives" (System Dynamics Group Working Paper D-2248-3, 1976).

APPENDIX A. EQUATION LISTING FOR THE REVISED MULTIPLIER-ACCELERATOR MODEL

```
 RMA1.DYNAMO
00001 * RMA1 (9/2/76)
00010 A S.K=C.JK+I.JK+G.JK
00020 R C.KL=IC.K*MAS.K
00030 A IC.K=APC*AS.K
00031 C APC=.65
00040 L AS.K=AS.J+(DT/TSS)(S.J-AS.J)
00041 N AS=1000
00042 C TSS=2
00050 A MAS.K=TABHL(TMAS,INV.K/DINV.K,0,1.5,.25)
00051 T TMAS=0/.4/.7/.9/1/1.08/1.12
00060 L INV.K=INV.J+DT*(P.JK-C.JK-I.JK-G.JK)
00061 N INV=DINV
00070 A DINV.K=CF*AS.K
00071 C CF=.3
00080 R I.KL=(DNI.K+K.K/ALK)*MI.K*MAS.K
00090 A DNI.K=(DK.K-K.K)/NTAK
00091 C NTAK=2
00100 A DK.K=AS.K*NCOR
00101 C NCOR=2.25
00110 L K.K=K.J+DT*(I.JK-D.JK)
00111 N K=IK
00112 C IK=2250
00120 R D.KL=K.K/ALK
00121 C ALK=15
00130 A MI.K=TABHL(TMI,DNI.K/(K.K/ALK),-1,0,.25)
00131 T TMI=0/.6/.9/1/1
00140 R G.KL=(IG+STEP(SG,TSG))*MAS.K
00141 C IG=200
00142 C SG=20
00143 C TSG=1
00150 R P.KL=(K.K/NCOR)*MIP.K*NP.K
00160 L NP.K=NP.J+(DT/TSNP)(NORMRN(1,SDNP)-NP.J)
00161 N NP=1
00162 C TSNP=1
00163 C SDNP=0
00170 A MIP.K=TABHL(TMIP,INV.K/DINV.K,0,2,.5)
00171 T TMIP=1/1/1/1/1
00172 SPEC LENGTH=60/PLTPER=1.5/DT=.0625/SAVPER=0
00173 PLOT S/C/I/K/DNI
```

APPENDIX B. EQUATION LISTING FOR THE EXTENDED VERSION (INCLUDING LABOR) OF THE MULTIPLIER-ACCELERATOR MODEL

```
 RMA2.DYNAMO
00001 * RMA2 (9/2/76)
00002 NOTE
00003 NOTE -- RMA2 IS RMA1 WITH ADDITION OF LABOR AS A LEVEL THAT
00004 NOTE -- ENTERS A COBB-DOUGLAS PRODUCTION FUNCTION AND WHOSE
00005 NOTE -- HIRING IS AFFECTED BY RELATIVE INVENTORIES.
00006 NOTE
00010 A S.K=C.JK+I.JK+G.JK
00020 R C.KL=IC.K*MAS.K
00030 A IC.K=APC*AS.K
00031 C APC=.65
00040 L AS.K=AS.J+(DT/TSS)(S.J-AS.J)
00041 N AS=1000
00042 C TSS=2
00050 A MAS.K=TABLE(TMAS,INV.K/DINV.K,0,1.5,.25)
00051 T TMAS=0/.4/.7/.9/1/1.08/1.12
00060 L INV.K=INV.J+DT*(P.JK-C.JK-I.JK-G.JK)
00061 N INV=DINV
00070 A DINV.K=CF*AS.K
00071 C CF=.3
00080 R I.KL=(DNI.K+K.K/ALK)*MI.K*MAS.K
00090 A DNI.K=(DK.K-K.K)/NTAK
00091 C NTAK=2
00100 A DK.K=AS.K*NCOR
00101 C NCOR=2.25
00110 L K.K=K.J+DT*(I.JK-D.JK)
00111 N K=IK
00112 C IK=2250
00120 R D.KL=K.K/ALK
00121 C ALK=15
00130 A MI.K=TABHL(TMI,DNI.K/(K.K/ALK),-1,0,.25)
00131 T TMI=0/.4/.7/.9/1
00140 R G.KL=(IG+STEP(SG,TSG))*MAS.K
00141 C IG=200
00142 C SG=0
00143 C TSG=1
00150 R P.KL=(1/NCOR)*K.K*EXP(.65*LOGN(L.K/K.K))*NP.K
00160 L NP.K=NP.J+(DT/TSNP)(NORMRN(1,SDNP)-NP.J)
00161 N NP=1
00162 C TSNP=1
00163 C SDNP=0
00170 L L.K=L.J+DT*(AL.JK-RL.JK)
00171 N L=IL
00172 C IL=2250
00180 R AL.KL=(L.K/ALL)*MIH.K
00190 R RL.KL=L.K/ALL
00191 C ALL=2
00200 A MIH.K=TA[illegible](TMIH,INV.K/DINV.K,0,2,.5)
00201 T TMIH=[illegible].15/1/.85/.75
00202 SPEC [illegible]PLTPER=.5/DT=.0625
00203 PLOT S,P/INV,DINV/K,DK/MAS,MI/C/I
```


APPENDIX C. SPECIFICATIONS FOR MODEL SIMULATIONS

```
RUN 1 (FIGURE 5)
     PLOT S/C/I/K/DNI
     T TMAS=1/1/1/1/1/1/1
RUN 2 (FIGURE 9)
     PLOT INV,DINV/S/K,DK/I
     C LENGTH=20
     C PLTPER=.5
RUN 3 (FIGURE 10)
     PLOT P(940,1020)/K(2154,2282)
     C LENGTH=80
     C PLTPER=1
     C SDNP=.05
RUN 4 (FIGURE 13)
     PLOT P/INV,DINV/I/K
     C LENGTH=40
     C PLTPER=1
     T TMIP=1.2/1.15/1/.85/.75
RUNS 5-10 (COMPARATIVE PLOTS IN FIGURE 14)
     CP LENGTH=40
     CP PLTPER=0
     CP SAVPER=1
     TP TMIP=1.2/1.15/1/.85/.75
     RUN RUN0
     C TAK=.5
     RUN RUN1
     C TAK=4
     RUN RUN2
     C TSS=.5
     RUN RUN3
     C TSS=4
     RUN RUN4
     TAK=1
     TSS=1
     RUN RUN5
     C TAK=.5
     C TSS=.5
     RUN RUN6
     PLOT P.RUN0,P.RUN1,P.RUN2,P.RUN3,P.RUN4,P.RUN5/P.RUN6(800,2200)
     C PLTPER=1
     C SAVPER=0
RUN 11 (FIGURE 16)
     PLOT L/K/P
     C LENGTH=80
     C PLTPER=1
     C SDNP=.05
```