SYSTEMS DYNAMICS MODELING OF AIR WARFARE

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INTRODUCTION

The use of gaming to study contemporary warfare has evolved rapidly in recent years. Particularly important strides were taken during the 1960's and 1970's with advances in computer technology and the development of broader ranges of subject matter. The past two decades have seen a tremendous expansion in the role and influence of computer models as "policy assisting" devices -- first in the analysis of national security issues, and now for the analysis of problems arising throughout the entire affairs of Government.

While the scope and stakes of military modeling have escalated over centuries, reduced to its basic elements, the most sophisticated computer representation of a conflict has the same ingredients: two sides, a contest, rules and constraints, and a variety of scenarios that might be played out when the time comes to wage war in earnest. Since war is one of humanity's oldest activities, and one that involves great costs, it is not surprising that substitutes have always existed. With its outcome, either victory or defeat, come vast social and material changes. It is these that cause men to investigate methods by which the consequences of their actions may be predicted.

Policy assisting models differ from their engineering-accounting cohorts in ways that are subtle and yet nontrivial. The term "policy assisting" model connotes a computer model that 1) is used for the systematic examination or analysis of subjective problems (i.e. problems without a well-defined mathematical representation); and 2) is intended to influence high levels of government decision-making. It includes models that are used by agency and executive branch officials, as well as models that influence congressional debate and action. Above all, the term describes a model that deals with questions beyond the purview of rigorous scientific deduction.

SURVIVABILITY-ATTRITION CONCEPTS

Aircraft attrition is dependent on many factors including the susceptibility of the aircraft to detection and hit; the vulnerability of the aircraft once it is hit; the type, the number, and the placement of enemy defenses; and the tactics and countermeasures at its disposal. When an aircraft penetrates a specified threat scenario, its probability of survival P_S can be estimated from the aforementioned considerations. Then the probability that the aircraft gets killed P_k, or its attrition, is $1 - P_S$.

We begin the mathematical treatment of the subject by developing the expression for the cumulative number of sorties (CS) flown by an aircraft with survivability P_S after n scheduled sorties, which is given by the following geometric series:

$$CS = 1 + P_S + P_S^2 + P_S^3 + \dots + P_S^{n-1} \qquad \{1\}$$

After some manipulation and taking the limit as n approaches infinity, the expected number of sorties scheduled by an aircraft in its lifetime is

$$E(n) = \frac{1}{1 - P_s} \qquad \{2\}$$

Frequently, measures of effectiveness are expressed in terms of "exchange ratios". An example of an exchange ratio is the number of targets destroyed per aircraft lost TDPL, expressed as,

$$TDPL = E(n)\ TDPS \qquad \{3\}$$

where TDPS is the number of targets destroyed per sortie.

There are two significant shortcomings of the time invariant approach described in the previous section. First, knowing how fast that a given number of targets can be destroyed is as important as knowing the ultimate number to be destroyed. Second, attrition and the threat force size responsible for this attrition are not constant, but change over time. The first shortcoming is overcome by bringing time into the picture as follows:

$$n = SR\ t \qquad \{4\}$$

where n is the number of sorties flown, SR is the sortie rate, and t is the analysis time period. The sortie rate, a complex function involving maintenance considerations, repair time, reliability, crew ratio, etc., is assumed to be constant. From Eq. 1, the cumulative number of sorties flown by an aircraft with survivability P_s and sortie rate SR in t days is:

$$CS = \frac{1 - P_s^{SR\ t}}{1 - P_s} \qquad \{5\}$$

The cumulative number of sorties flown by a force of N aircraft, CS_N, is obtained by multiplying Eq. 5 by N:

$$CS_N = \frac{N\,(1 - P_s^{SR\ t})}{1 - P_s} \qquad \{6\}$$

The cumulative number of targets destroyed by a force of N aircraft, TN, is obtained by multiplying Eq. 6 by the target kill potential TDPS (targets destroyed per sortie):

$$TN = TDPS\ \frac{N\,(1 - P_s^{SR\ t})}{1 - P_s} \qquad \{7\}$$

Other parameters that can be obtained from the above relations are the fraction of force remaining FFR and the fraction of force lost FFL which are defined as follows:

$$FFR = P_s^{SR\ t} \qquad \{8\}$$

$$FFL = 1 - P_s^{SR\ t} \qquad \{9\}$$

Expressing the fraction of the force remaining FFR in alternate forms and taking the limit as n approaches infinity, leads to the following expression:

$$FFR = e^{-n\,P_k} = e^{-(1-P_s)\,SR\ t} \qquad \{10\}$$

Recognizing that FFR is $\$_t$, the number of aircraft at time t, divided by $\$_0$, the initial number of aircraft, Eq. 10 is really the solution of the differential equation,

$$\frac{d\$_t}{dt} = -(1 - P_s)\,SR\,\$_t \qquad \{11\}$$

Expressed as an integral equation,

$$\$_t = \$_{t-1} - \int_{t-1}^{t} (1 - P_s)\,SR\,dt \qquad \{12\}$$

which is equivalent to

$.K = $.J - (DT)(AR$.JK) {13}

where, AR$.JK, the attrition rate, is given by,

AR$.KL = (1 - S$.K) x SR$.K x $.K {14}

where S$.K is the sortie survivability of the aircraft, SR$.K is the sortie rate, and $.K is the number of aircraft.

The advantages of the systems dynamic formulation of the aircraft combat attrition phenomenon over the discrete-event representation of the preceding sections is that it permits the incorporation of much more realism. For example, attrition due to both surface and air threat platforms can be considered simultaneously. This is important because it recognizes that the threat, and therefore the attrition inflicted by it, will change from sorties to sortie. Indeed, many aircraft are especially designed for counter air missions to destroy combat aircraft with air-to-surface and air-to-air missiles.

AIRCRAFT SURVIVABILITY AND LETHALITY TRADEOFFS

Survivability related modeling can be thought of as spanning several tiers as shown in Table 1. In this context, engineering represents a bottom-up approach starting at the lowest tier, Level 1, and survivability management can be thought of as a top-down approach. Levels 2, 3, and 4 provide the mechanism relating the engineering/design parameters (e.g., speed, signatures, weight, excess energy, etc.), and the operational parameters (e.g., payload, range, flight profile, etc.) to the aircraft survivability. In Levels 5 and 6, the thrust area is in relating the aircraft survivability to the parameters that describe the overall force effectiveness so that the changes in technology, operations and tactics can be evaluated.

Eight models are being developed which will provide the means for systematically performing survivability/lethality tradeoffs for four types of aircraft missions: 1) surface attack, 2) fighter escort, 3) fleet air defense, and 4) multimission aircraft. These models are conceived with the Navy in mind due to the added constraints imposed by "carrier suitability" and because the advantages of mission flexibility on a carrier makes the case for multimission aircraft even stronger than for other services. Model conceptualization from decision variables to measures of merit for this phase of the research is shown in the causal diagram in Fig. 1.

EXAMPLE TRADEOFF STUDY (FIGHTER ESCORT)

For the fighter escort, a hypothetical aircraft, the CBFE (Carrier-Based aircraft for Fighter Escort) is used. Suppose that four modifications of a projected attack aircraft (i.e. a baseline design) are to be investigated using the aircraft effectiveness tradeoff methodology previously described. These modifications are changes to the baseline aircraft powerplant in order to appreciate the variations of thrust/weight on the four measures of effectiveness (MOE) previously described. The goal here is to choose the "best" airframe-powerplant configuration that yields the minimum acquisition cost ratio (ACR), minimum possible crew loss (PCL), and the highest campaign survivability (CS) for a given mission scenario. Several assumptions have to be made in order to simplify the problem and they are as follows: (1) the main airframe geometric characteristics are kept constant except those associated with the inlets size to accommodate the various mass flow rate requirements associated with each particular powerplant installation to be studied; (2) aerodynamic drag characteristics are varied to account for changes in inlet drag; (3) radar cross section (RCS) characteristics are also varied to account for changing inlet sizes; and (4) changes in aircraft weight are due to the different engines installed as well as the added weight of the airframe to support them. Items 1, 2, 5, and 8 in Table 2 constitute the principal inputs to investigate the new survivability and lethality of each modified aircraft.

Using the outputs of the model describing surface-to-air interactions and the tradeoff methodology previously described one obtains Tables 3 and 4 where two important tradeoff cases are depicted. In the first one (i.e., Table 3) a constant procurement run of 1000 aircraft is assumed for all the configurations studied and the idea is to determine the six measures of effectiveness (MOE's) defined by items 15-20 of Table 3.

The second tradeoff study that can be derived from this methodology is the analysis of the economic implications that modifying a baseline design using tactical MOE's such as Campaign Survivability and Exchange Ratio. For the sake of illustration and following the same lines as the previous example suppose it is desired to know the procurement numbers (i.e. production run) necessary for each configuration to yield an equivalent Campaign Survivability to that of the baseline configuration.

Table 4 shows the results of such a study using the same values for encounter survivability, initial enemy force inventory, and T/W modifications. Since C_s is constant for the four modifications, it is necessary to express variables ER, PCL, and RC (replacement cost) in terms of the known quantities CS, P_{sss}, and $P_{k/s}$. The results of the Cost Effectiveness Tradeoff Methodology are shown in Table 4. A preliminary conclusion that can be drawn from this data is that to achieve the maximum effectiveness for the minimum counter-effectiveness (Modification 4 -- C=0.0857 & B=0.014) that 932 modified aircraft would be required at acquisition cost ratio of 1.073. That is to say that Modification-4 aircraft could cost up to 1.073 times as much as the baseline aircraft and still be favored for cost-effectiveness.

DESCRIPTION OF SCENARIO SPECIFIC SURVIVABILITY/LETHALITY MODELS

The scenario specific models developed for this research are written in DYNAMO III and address a typical air warfare scenario depicted in Fig. 2. Combat interactions occur between enemy ground forces (i.e., surface-to-air missile sites), enemy air forces, and friendly forces deployed from a carrier battle group (CBG). These models address individual aircraft and defense weapons sites in order to estimate the probabilities of survival and kill for each combat entity. The models are versatile and detailed to include various weapons for each aircraft, each site, and the use of dissimilar aircraft if desired. Each model requires six principal inputs from the user: 1) aerodynamic and propulsion characteristics of the attacking aircraft, 2) aircraft and ground sensor characteristics, 3) aircraft signatures, 4) aircraft trajectory definition, 5) defense weapon sites location, and 6) weapon kinematic envelopes.

The aircraft is modeled as a point mass, fifth-order system with motion constrained to the horizontal plane. State variables are the aircraft cartesian positions (X and Y), speed (V), mass (M), and heading angle (ψ). Control variables are the throttle setting (τ) and the bank angle (ϕ) as shown in Eqs. 15 through 17.

$$\frac{dY_i}{dt} = V_i \cos \psi_i \qquad \{15\}$$

$$\frac{dY_i}{dt} = V_i \sin \psi_i \qquad \{16\}$$

$$\frac{dV_i}{dt} = \frac{\tau_i \; T_i \; \cos(\alpha_i + \sigma_i) - D_i}{m_i} \qquad \{17\}$$

$$\frac{dm_i}{dt} = - c_i \, T_i - m_{w\,i} \, w_{lr\,i} \qquad \{18\}$$

and

$$\frac{d\psi_i}{dt} = \frac{\tau_i \; T_i \; \sin(\alpha_i + \sigma_i) + L_i}{m_i V_i} \; \sin(\phi_i) \qquad \{19\}$$

where, X_i and Y_i are position coordinates for the i th aircraft, V_i is the magnitude of the i th aircraft velocity vector, ψ_i is the heading angle, T_i is the engine installed thrust, τ_i the throttle setting, D_i the total aircraft drag, m_i is the i aircraft mass, $m_{w\,i}$ the particular masses of the weapons, $w_{lr\,i}$ the weapon load (units), c_i is the specific fuel consumption, α_i is the angle of attack, σ_i; the thrust offset angle, and ϕ_i the bank angle for the i th vehicle.

Combat interactions between aircraft and defense sites are modeled using a Lanchester-type square law with variable attrition coefficients and aimed targeting scheme. The attrition equations are solved numerically and parallel to the aircraft equations of motion using a second order Runge-Kutta integrating scheme. Denoting CBF as the friendly force side (i.e., the attacking aircraft) and DWS as the aggressor force (i.e., the defense weapon sites), the attrition rate vectors $\dot{\overline{CBF}}(t)_i$ and $\dot{\overline{DWS}}(t)_j$ are functions of the time-varying systems effectiveness attrition coefficients of every element on each force and the number of adversaries.

$$\dot{\overline{CBF}}(t)_i = - F[\overline{DWS}(t)_{SE\;ji}, \overline{DWS}(t)_j] \qquad \{20\}$$

$$\dot{\overline{DWS}}(t)_j = - F[\overline{CBF}(t)_{SE\;ij}, \overline{CBF}(t)_i] \qquad \{21\}$$

where, $\overline{DWS}(t)_{SE\;ji}$ is the defense weapon site system effectiveness matrix (n by m) containing the attrition coefficients from each j th defense ground site as it affects the i th aircraft in the friendly force, $\overline{CBF}(t)_{SE\;ij}$ represents the time-varying aircraft system effectiveness matrix (m by n) containing the attrition coefficients from each i th aircraft as it affects the j th defense weapon site in the enemy force, and $\overline{DWS}_j$ and $\overline{CBF}_i$ are the system states at time t.

Changes in thrust-to-mass ratio, aircraft radar signature, aircraft flight altitude, aircraft ingress speed, and the use of electronic countermeasures were performed to see their impact on the aircraft probability of survival (P_s), aircraft lethality (i.e., probability of defense weapon sites being killed - $P_{k/s}$), and the sortie exchange ratio, ER_s.

Table 1. Survivability Analysis Heirarchy

Level	Description	Key Parameters
1	Vulnerability Assessment Munitions Effectiveness Assessment	Aircraft Single Shot Prob. of Kill, Pssk(A/C) Threat Single Shot Prob. of Kill, Pssk(Threat)
2	Encounter Survivability Assessment Encounter Lethality Assessment	Aircraft Survivability/Encounter, Ps/e Aircraft Lethality/Encounter, Ps/e
3	Sortie Survivability Assessment Sortie Lethality Assessment	Aircraft Survivability/Sortie, Ps/s Aircraft Lethality/Sortie, Pk/s
4	Mission Attainment Survivability/Effectiveness Determination	Campaign Survivability, Ps/c Mission Attainment Measure, MAM Measure of Mission Success, MOMS
5	Survivability Cost-Effectiveness Tradeoffs	Relative Exchange Ratio, RER Aircraft Replacement Cost, RC Possible Crew Loss, PCL
6	Resource Allocation Tradeoffs	**Budget to R&D, O&M, and Procurement**

Table 3. Aircraft Effectiveness Tradeoff Methodology.

DESCRIPTION	FORMULATION	BASE	MOD. 1	MOD. 2	MOD. 3	MOD. 4
1) Threat Inventory	X_0 = Goal	2000	2000	2000	2000	2000
2) Thrust-to-Weight Ratio		0.85	0.55	0.70	1.00	1.10
3) Acquisition Cost Ratio	ACR=ACM/ACB	N/A	N/A	N/A	N/A	N/A
4) Production Run	$\$_0M = \$_0B(ACR)$	1000	1000	1000	1000	1000
5) Encounter Sortie Surv.	P_{S/ϵ_S}	0.682	0.631	0.658	0.702	0.722
6) No. to Encounter Sorties	n_{ϵ_S}	19	19	19	19	19
7) Sortie Survivability	$P_{S/S}$	0.984	0.982	0.983	0.985	0.986
8) Effectiveness of X	B	.016	.018	.017	.015	.014
9) Encounter Sortie Surv.	P_{S/ϵ_X}	0.155	0.160	0.158	0.150	0.143
10) No. to Encounter Sorties	n_{ϵ_X}	9	9	9	9	9
11) Sortie Lethality	$P_{K/S}$	0.0845	0.0840	0.0842	0.0850	0.0857
12) Effectiveness of $	C	0.0845	0.0840	0.0842	0.0850	0.0857
13) Surviving No. of $	$\$_t$	497	348	433	547	593
14) No. of X Destroyed	$X_0 - X_t = X_0$	2000	2000	2000	2000	2000
15) Campaign Survivability	CS	0.497	0.348	0.433	0.547	0.593
16) Fraction Force Lost	FFL	0.503	0.652	0.567	0.453	0.407
17) Exchange Ratio	ER	3.979	3.070	3.528	4.412	4.910
18) Rel. Exchange Ratio	RER	1.989	1.535	1.764	2.206	2.455
19) Possible Crew Loss	PCL	503	652	567	453	407
20) Replacement Cost	RC	N/A	N/A	N/A	N/A	N/A

Table 4. Aircraft Cost-Effectiveness Tradeoff Methodology.

DESCRIPTION	FORMULATION	BASE	MOD. 1	MOD. 2	MOD. 3	MOD. 4
1) Threat Inventory	X_0 = Goal	2000	2000	2000	2000	2000
2) Thrust-to-Weight Ratio		0.85	0.55	0.70	1.00	1.10
3) Acquisition Cost Ratio	ACR=ACM/ACB	1.000	0.937	0.965	1.033	1.073
4) Production Run	$\$_0M = \$_0B(ACR)$	1000	1067	1036	968	932
5) Encounter Sortie Surv.	P_{S/ϵ_S}	0.682	0.631	0.658	0.702	0.722
6) No. to Encounter Sorties	n_{ϵ_S}	19	19	19	19	19
7) Sortie Survivability	$P_{S/S}$	0.984	0.982	0.983	0.985	0.986
8) Effectiveness of X	B	0.016	0.018	0.017	0.015	0.014
9) Encounter Sortie Surv.	P_{S/ϵ_X}	0.155	0.160	0.158	0.150	0.143
10) No. to Encounter Sorties	n_{ϵ_X}	9	9	9	9	9
11) Sortie Lethality	$P_{K/S}$	0.0845	0.0840	0.0842	0.0850	0.0857
12) Effectiveness of $	C	0.0845	0.0840	0.0842	0.0850	0.0857
13) Surviving No. of $	$\$_t$	497	530	515	481	463
14) No. of X Destroyed	$X_0 - X_t = X_0$	2000	2000	2000	2000	2000
15) Campaign Survivability	CS	0.497	0.497	0.497	0.497	0.497
16) Fraction Force Lost	FFL	0.503	0.503	0.503	0.503	0.503
17) Exchange Ratio	ER	3.979	3.726	3.838	4.108	4.266
18) Rel. Exchange Ratio	RER	1.989	1.989	1.989	1.989	1.989
19) Possible Crew Loss	PCL	503	537	521	487	469
20) Replacement Cost	RC	1.00	1.00	1.00	1.00	1.00

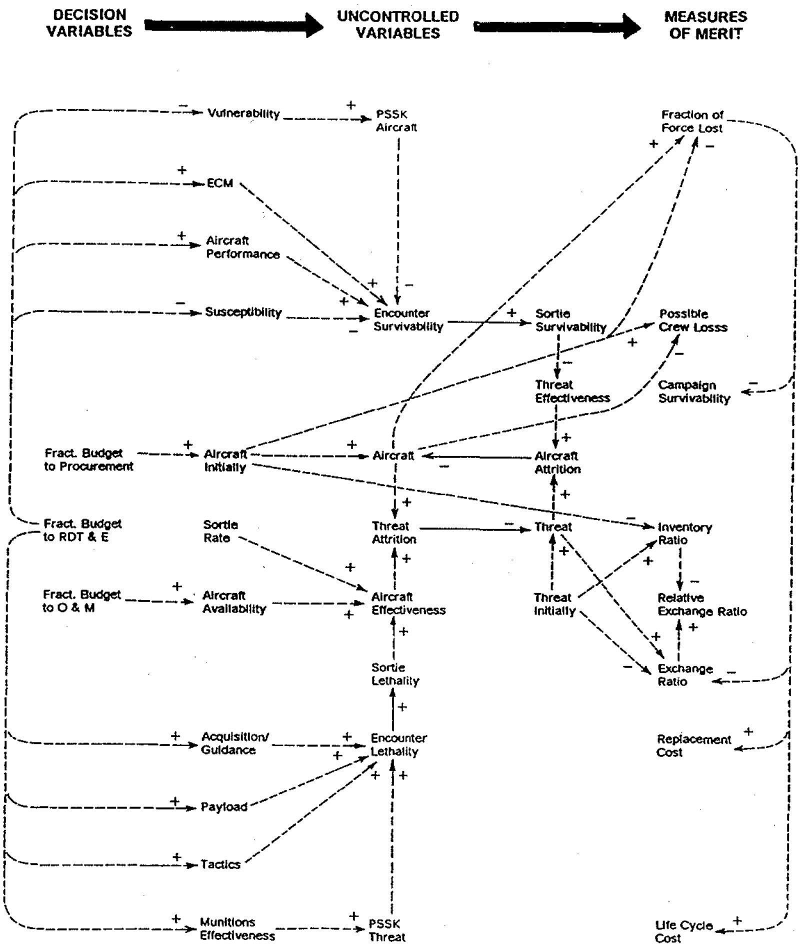

Figure 1. Combat Aircraft Tradeoff Methodology.

Table 2. Summary of Modifications for Baseline Aircraft.

		ALTERNATIVE A/C			
DESCRIPTION	BASE	MOD. 1	MOD. 2	MOD. 3	MOD. 4
1) Engine Thrust SSL (kg)	8131	5260	6840	9473	10525
2) Aircraft Mass (kg)	19300	17602	18473	20143	20869
3) Thrust/Weight	0.85	0.55	0.70	1.00	1.10
4) ΔC_{D0} (counts)	0.0	-30.60	-14.25	12.90	23.40
5) Engine Capture Area (sq. m)	0.363	0.472	0.567	0.653	0.723
6) Engine Diameter (m)	0.95	0.76	0.87	1.02	1.07
7) Radar Cross Section (sq. m)	5.00	3.23	4.08	5.98	6.88
8) Normalized RCS	1.00	0.646	0.816	1.196	1.376

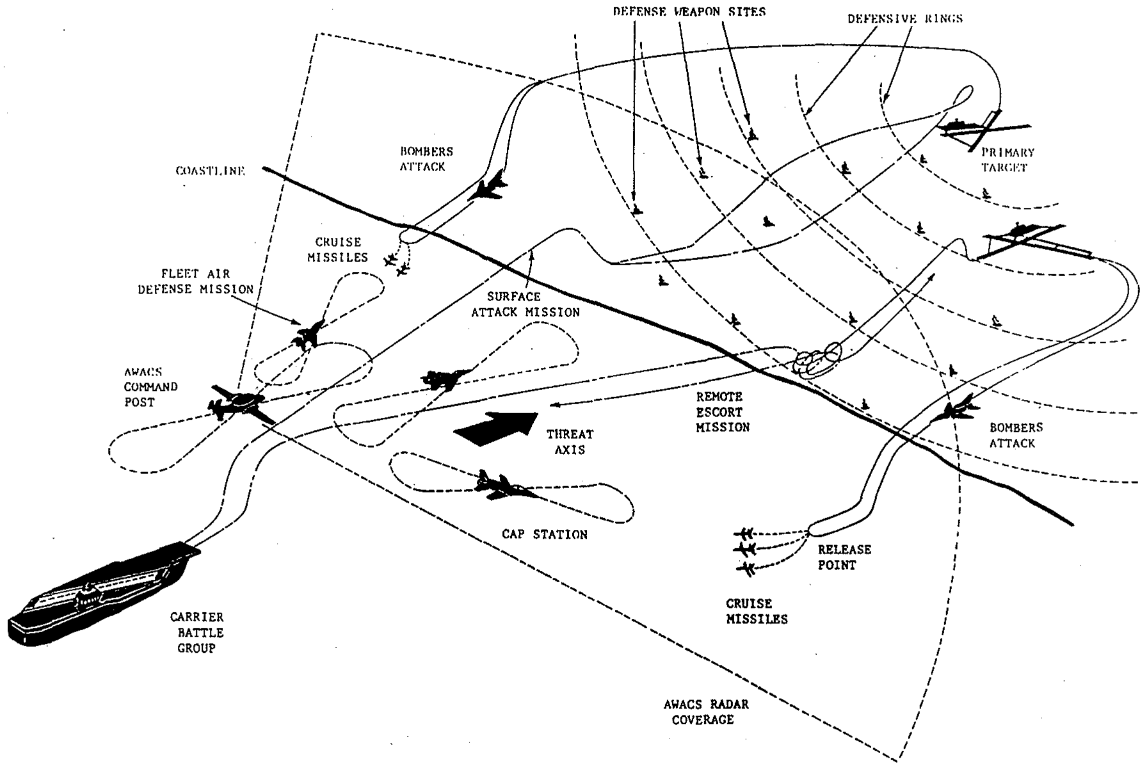

Figure 2 **ASALT Models Combat Scenario.**