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Parallel Dual Problem of Optimization Embedded in Some Model Type System Dynamics

Elżbieta Kasperska, Damian Słota

Institute of Mathematics
Silesian University of Technology
Kaszubska 23, 44-100 Gliwice, Poland
e-mail: {e.kasperska, d.slota}@polsl.pl

Abstract
Some kind of dual problem of optimization, on the base of simulation on the model type System Dynamics, is presented. Authors refere to question of, so called, optimization embedded in simulation. Some new results of experiments with the comparison of the solution of the minimization of cost and maximization of profit in the firm are described. The generalization of formulation both problems in matrix form, on the example of described model type System Dynamics is proposed.

1 Introduction

The dual problem of optimization on economic, mathematical models is widely described in literature. Classical concept of maximization of profit and minimization of cost can be applied to simulation models type System Dynamics [1, 2, 5]. First attempts were undertaken by authors in paper [3]. Now the concept were generalized to formulate, so called, parallel dual problem. Generally speaking, in specific matrix form the profit and cost are conceived in one experiment. The results are compared with the individual experiments (only the maximization or only the minimization). Authors, in their experiments, use the simulation language Professional Dynamo 4.

2 Mathematical Model of the System

The model named DYNBALANCE(2-2-c-I) was chosen like the object of experiments. It was adappted from paper [3]. Its corrected main structure is illustrated on Figure 1.

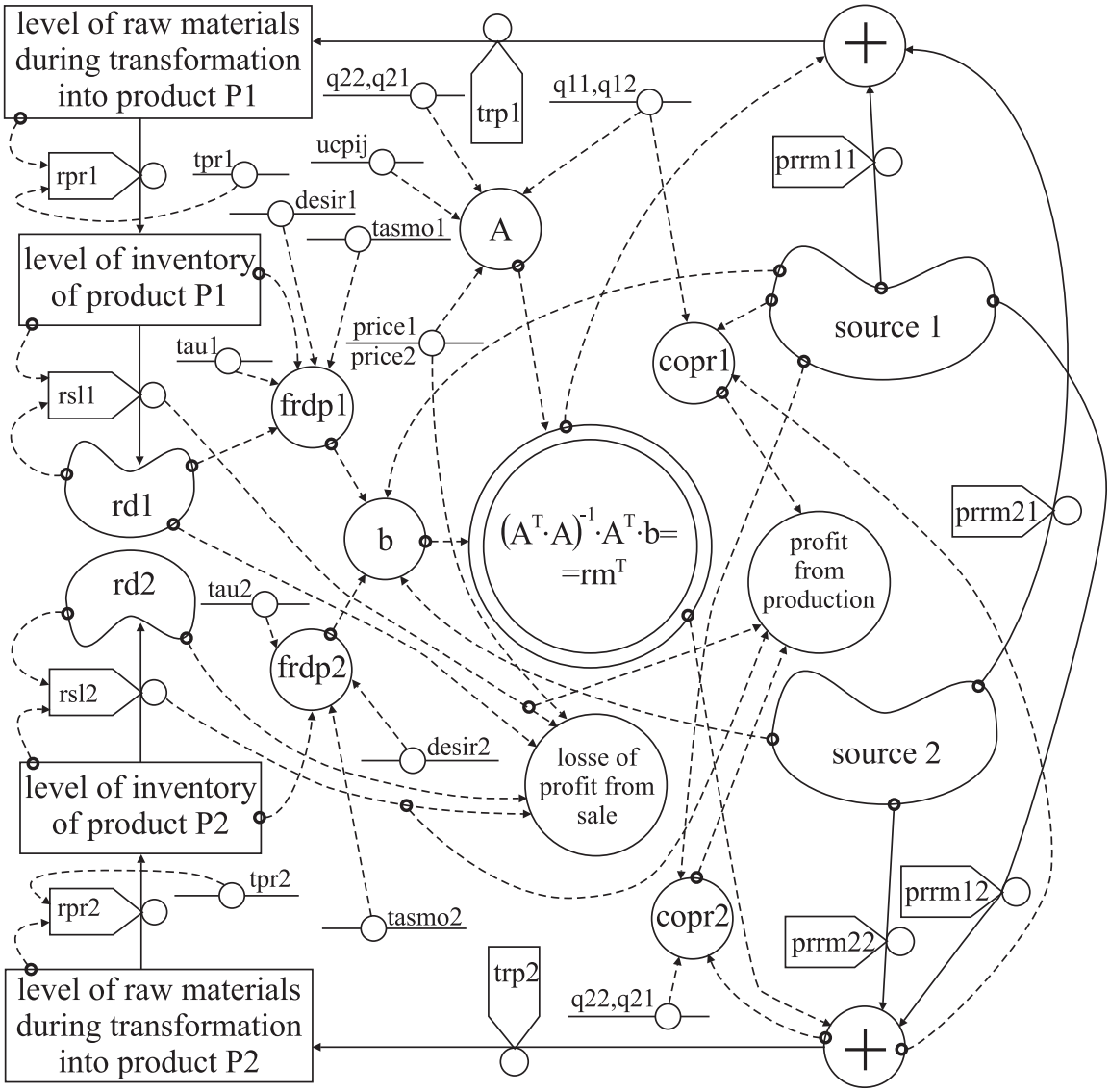

Figure 1: Structure of model DYNBALANCE(2-2-c-I)

Specific local matrix equation formulation is:

$$\begin{pmatrix} q11 & 0 & q12 & 0 \\ 0 & q21 & 0 & q22 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ ucp11 \cdot q11 & 0 & ucp21 \cdot q12 & 0 \\ 0 & ucp12 \cdot q21 & 0 & ucp22 \cdot q22 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} rm11 \\ rm12 \\ rm21 \\ rm22 \end{pmatrix} = \begin{pmatrix} frdp1(t) \\ frdp2(t) \\ sourc1(t) \\ sourc2(t) \\ b5 \\ b6 \\ b7 \\ b8 \\ b9 \\ b10 \end{pmatrix} . \quad (1)$$

The main idea was to achieve the solution which locally minimize the cost of production (the Euclidean norm of overdetermined system (1) was optimal in each step of simulation). The results of experiments for some taken scenarios was already described in paper [3]. Now we will try to formulate the "dual" problem for the presented model. Its mathematical form is as follows:

$$\begin{pmatrix} q11 & 0 & q12 & 0 \\ 0 & q21 & 0 & q22 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ price1 \cdot q11 & 0 & price1 \cdot q12 & 0 \\ 0 & price2 \cdot q21 & 0 & price2 \cdot q22 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} rm11 \\ rm12 \\ rm21 \\ rm22 \end{pmatrix} = \begin{pmatrix} frdp1(t) \\ frdp2(t) \\ sourc1(t) \\ sourc2(t) \\ b5 \\ b6 \\ b7 \\ b8 \\ b9 \\ b10 \end{pmatrix} . \quad (2)$$

The interesting idea occur. How to generalize the model, in order to achieve the simultaneously: minimization of cost of production and maximization of profit, in one model? So, the proposed form of such idea is below:

$$\begin{pmatrix} q11 & 0 & q12 & 0 \\ 0 & q21 & 0 & q22 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ ucp11 \cdot q11 & 0 & ucp21 \cdot q12 & 0 \\ 0 & ucp12 \cdot q21 & 0 & ucp22 \cdot q22 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ price1 \cdot q11 & 0 & price1 \cdot q12 & 0 \\ 0 & price2 \cdot q21 & 0 & price2 \cdot q22 \end{pmatrix} \begin{pmatrix} rm11 \\ rm12 \\ rm21 \\ rm22 \end{pmatrix} = \begin{pmatrix} frdp1(t) \\ frdp2(t) \\ sourc1(t) \\ sourc2(t) \\ b5 \\ b6 \\ b7 \\ b8 \\ b9 \\ b10 \\ b11 \\ b12 \end{pmatrix} , \quad (3)$$

where $b5$ and $b5$ are small numbers, and $b11$ and $b12$ are large numbers. The results of experiments both types (using formula (2) and formula (3) are presented in the next section and are compared with previously results described in paper [3], for formula (1).

3 The Results of Experiments

The most interesting point of view is to compare the important (for estimation the fitting of the balances (1), (2) and (3)) variable like *norm*. Its characteristics in the whole horizon of simulation (two years) are presented on Figure 2.

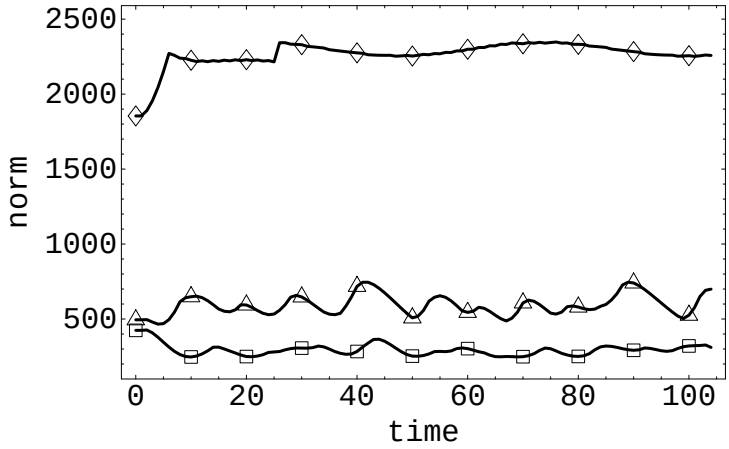

Figure 2: The characteristic of variable *norm* with the formulas of balance $A\,x = b$ given by equation (1) (◊), (2) (□) and (3) (△)

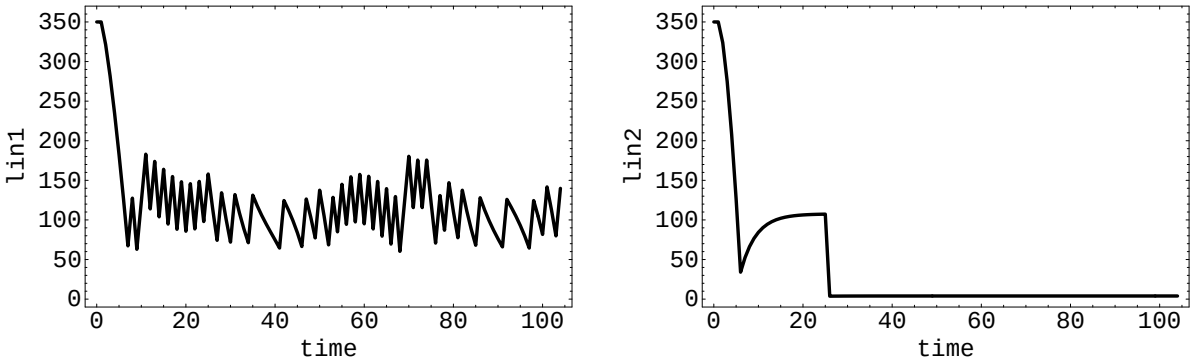

Figure 3: The characteristic of variables: $lin1$ and $lin2$ in minimization experiment

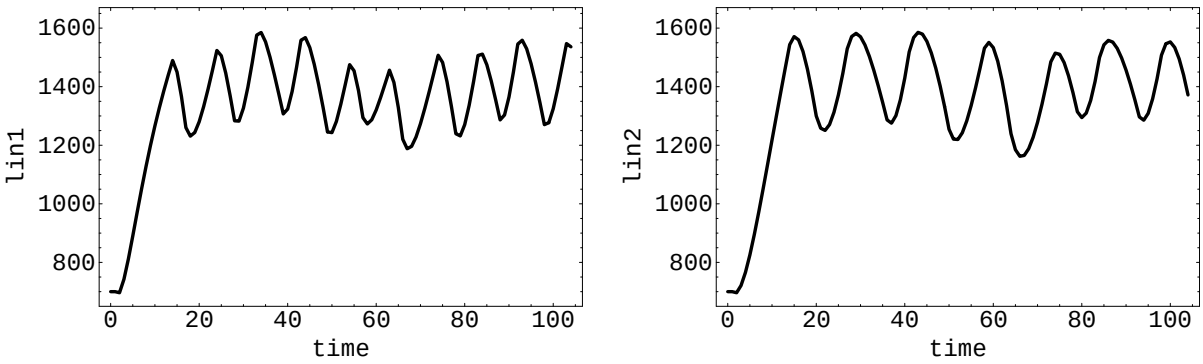

Figure 4: The characteristic of variables: $lin1$ and $lin2$ in maximization experiment

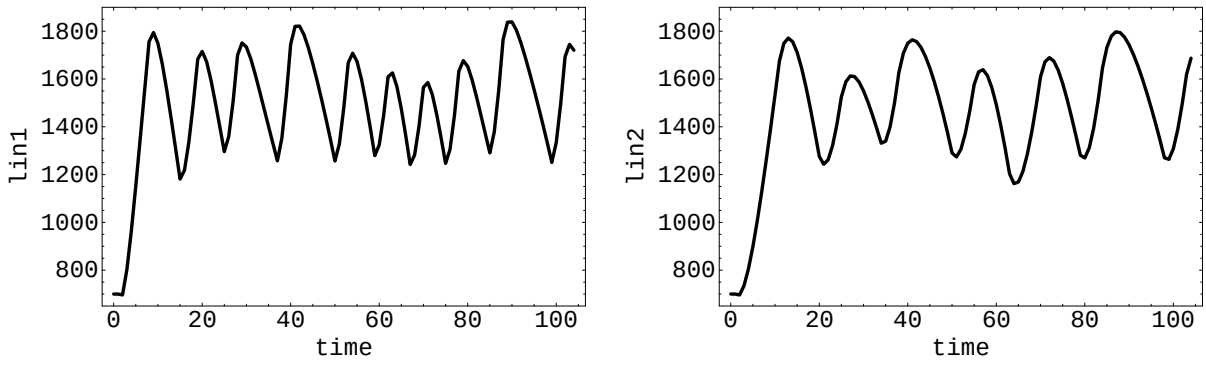

Figure 5: The characteristic of variables: $lin1$ and $lin2$ in parallel experiment

It is also interesting to compare the dynamics of such important variables like: inventories of product $P1$ and $P2$ ($lin1$ and $lin2$, respectively), in these three types of experiments.

The three experiments: minimization (formula (1)), maximization (formula (2)) and parallel (formula (3)) were undertaken under the some condition about the demand for product $P1$ and $P2$. The systems have reacted in such a way that minimize the Euclidean norm (see [4]). The rates of raw materials: $rm11$, $rm12$, $rm21$, $rm22$, were optimal to fit the balances $A\,x = b$ (given by equations (1), (2) and (3) respectively).

4 Final Remarks and Conclusions

The purpose of the paper was to present some results of experiments considered dual problem of optimization embedded on model type System Dynamics. These results were compared with previously results achieved in paper [3], given the base for estimation the goodness of pseudosolution of the overdetermined systems (1), (2) and (3), in fitting balances type $A\,x = b$, embedded in model Forresters type. Final conclusions are as follows:

- the generalization of formulation of the dual problem of optimization on model type System Dynamics, seems to be interesting way to achieve in parallel form, the best fitting of balance of raw materials (the balance models the interesting aspect of dynamics behaviour of system);

- the opportunity for analysing the consequences of locally optimal solutions, for the whole system (its dynamics), can't be overestimated for complex, dynamic and multilevel systems;

- the fruitfull Forresters idea (classical models of type System Dynamics) can be embedded or linked with method of classical numerical analysis, giving new dimension for investigation (not only in economic area).

References

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