A Component Strategy for the Formulation of System Dynamics Models

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Abstract--*A component strategy to the development of system dynamics models is described. The approach concentrates on the formulation of the Forrester stock and flow diagram, and incorporates the concept of an interaction matrix to assist in the formulation of such models. The interaction matrix, together with an explicit sequence of steps for model development, are described. This description is followed by applications to illustrative problems. The strategy facilitates the determination of the quantities to be included as well as the existence of connectors between the quantities and the identities of the quantities and connectors. The paper is accompanied by a companion article [3] in this proceedings that formally derives the interaction matrix. The advantages of a component-based approach to model development include 1) reuse of the components, 2) diminished dependence upon the competence of the model-builder for the creation of quality models, 3) greater opportunity for managers/policy makers to build their own models from components rather than from scratch, and 4) a development strategy that can be partially automated.*

1. Introduction

A modular, component-like approach to the formulation of system dynamics models is not by any standard a new concept. Its use is evident in the many classic models of yesteryear-see Forrester [6, 7, 8, 9], and Goodman [11]. Credence to the modular, component-based approach is given by the multi-view approach taken by Eberlein's VENSIM tool [4]. Eberlein's "molecules" (available on the Vensim website) also provide a component-based approach to model development that is focused on reuse. Barry Richmond's identification and description of generic structures and sub-structures (published in the STELLA manuals [22]) is strongly supportive of a component-based approach as well. The component strategy has been advocated by Fitz and Hornbach [5] as a very utilitarian and intelligent method for formulation of system dynamics models. The main objective of the present paper is to make explicit the principles and concepts inherent within the component procedure while simultaneously inserting additional detail into the approach.

What is to be gained from a formal procedure whose guidelines are explicit? As currently conceived, the process of continuous dynamic simulation model formulation, regardless of the methodology employed, is an arduous, tedious, and time-consuming task (see Thesen [16]). This fact suggests that if the task could be partially algorithmatized, computer aids to facilitate model formulation could be developed to expedite the process. The strategy to be described, if algorithmatized, would divide the labor of modeling into human and computer portions where the part not requiring human judgment is automated. The computer aids would force the modeler to structure his model in a fashion consistent with the requirements of system dynamics methodology. Additionally, computer aids would allow for policy-maker participation in the model formulation process, thereby increasing his understanding of model assumptions and structure while lessening model development costs. The policy-maker participatory methodologies of Kane [13] and Warfield [18] have already demonstrated their usefulness. If applied to simulation model-building, the strategy might alleviate conventional problems of tedium, credibility, and validity.

In the component strategy, the quantities q_i to be included in the model and their associated interactions are generated simultaneously. This is in contrast to methods that start with a list of quantities and then use the list to determine the interactions between the quantities. Moreover, in the component approach the identity (the quantity types-stocks, rates, auxiliaries, parameters, etc. are technically defined in [3]) of each quantity is known at the moment the quantity is generated.

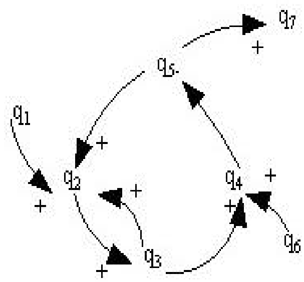
As will be observed, the component strategy to model fabrication can also be formulated so as to expeditiously make use of the notion of an interaction matrix as an aid to model formulation. Other similar model-building methodologies-see Kane [9], Wakeland [17], and Moll and Woodson [12]-utilize the matrix concept to advantage. One purpose of the paper is to explore whether such constructs would facilitate formulation of system dynamics models. Such matrices specify the interactions for all pairs (q_i, q_j) defined on the Cartesian product Q x Q, where Q is a set of quantities. If Q consists of n quantities, then the associated matrix is of dimension (n x n) and each entry m_{ij} in the matrix is either 0, 1 or –1, depending upon (1) whether a connector is directed from q_i toward q_j, and (2) what the sign of that connector is. Since there are just three possible insertions for any entry m_{ij}, such matrices are referred to in subsequent discussions as Square Ternary Matrices, abbreviated to STMs.

It is easy to show that the more conventional causal loop diagram (CLD) is fully isomorphic to the STM and conversely insofar as substantive content is concerned. On the other hand, neither the CLD nor the STM contains explicit information about the identities of the quantities (Q) involved. Such information is, however, exhibited in the Forrester-invented stock-and-flow diagram (SFD), along with all of the previous information contained in the CLD or equivalently, the STM. Isomorphic representations are shown in Fig. 1 below. When the connectors and quantities in the STM are labeled and identified, the resultant matrix is referred to as the Modified Square Ternary Matrix, or MSTM, which is also exhibited in Fig. 1.

As previously discussed, there are several ways to arrive at a causal loop diagram or its equivalent, an STM. One way is to list all quantities believed to be important to the problem of interest and then to systematically consider every pair (q_i, q_j), placing an entry into the corresponding m_{ij} within the STM to designate whether and what sign (i.e. positive or negative) interaction exists between q_i, q_j for a given relational context, as specified by R. The relation R might assume the form of a query posed to those who are substantive about the problem concerned, such as 'Does q_i influence or affect q_j in some way?', as suggested by Warfield [18].

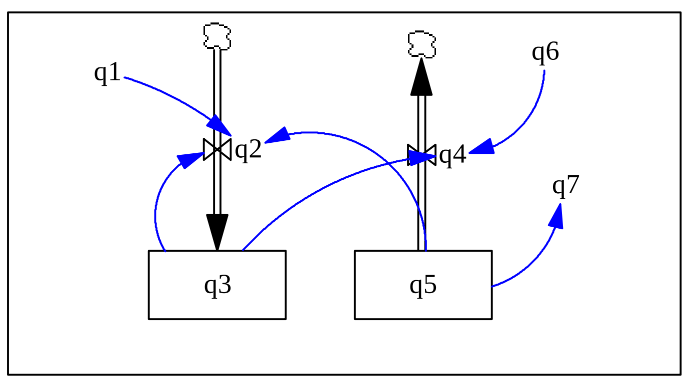
	1	2	3	4	5	6	7
1	0	1	0	0	0	0	0
2	0	0	1	0	0	0	0
3	0	1	0	1	0	0	0
4	0	0	0	0	-1	0	0
5	0	1	0	0	0	0	1
6	0	0	0	1	0	0	0
7	0	0	0	0	0	0	0

Square ternary matrix (STM)



Causal loop diagram (CLD)

		1	2	3	4	5	6	7
P	1	0	I	0	0	0	0	0
R	2	0	0	F	0	0	0	0
S	3	0	I	0	I	0	0	0
R	4	0	0	0	0	-F	0	0
S	5	0	I	0	0	0	0	I
P	6	0	I	0	I	0	0	0
O	7	0	0	0	0	0	0	0



Modified square ternary matrix (MSTM)

Figure 1. An Isomorphic Correspondence of the STM to the CLD and of the MSTM to the SFD of a Hypothetical System.

On the other hand, system dynamics methodology has always worked directly with the CLD. The graphic representation provided by the CLD is a device useful for cognitive and communication purposes.

Moreover, the CLD pictorially illustrates possible paths of redundant causality, a situation that occurs whenever a quantity q_j is reachable from a quantity q_i by more than one path. In fact, the CLD helps to eliminate paths of causality that are superfluous and clearly not realistic. If the interaction matrix M is specified without also considering the associated CLD, too little attention is given to paths that are redundant. For example, consider the relationship between births, deaths, and population. Do deaths in any way have an effect on births? Clearly, the response is affirmative, but the effect is through population.

Figure 2 shows two conceivable causal relationships between births, population, and deaths. In Fig. 2 (a) the effect of deaths upon births is through population. In Fig. 2 (b) there are two paths by which births may be reached from deaths. In this case the path of causality from deaths to births is through population, and the direct path from deaths to births is superfluous. However, without the use of the causal diagram, the user would be inclined to place an entry in the STM in the location that reflects the effect that deaths have on births.

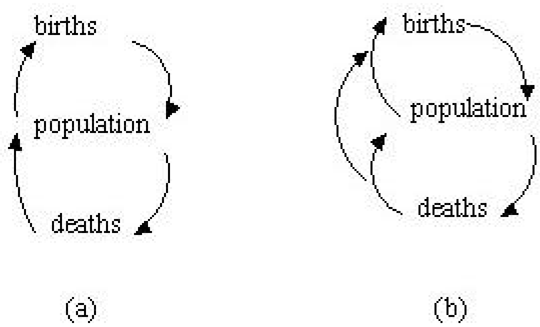


Figure 2. Conceivable causal relationships between births, population, and deaths.

Consider the interaction matrix M below. Corresponding to entry m_{13}, there is a temptation to insert a number other than zero, reflecting the influence that deaths have on births. However, no entry other than zero is causally correct, as the effect of deaths upon births is realistically modeled by the effect of deaths upon population and of population upon births.

	deaths	population	births
deaths	$m_{11}=0$	$m_{12}=-1$	$m_{13}=0$
population	$m_{21}=1$	$m_{22}=0$	$m_{23}=1$
births	$m_{31}=0$	$m_{32}=1$	$m_{33}=0$

Figure 3. Interaction (square ternary) matrix for the three quantities shown.

This would be indicated in the interaction matrix by non-zero entries in position m_{12} and m_{23}. This argument is sufficient to establish the claim that drawing the causal model helps the user establish paths of causality and eliminate interactions that are redundant or superfluous.

While the causal diagram can be a useful aid in the avoidance of errors of commission, its delineation should perhaps not be performed until after the user has systematically considered every entry in the interaction matrix M and entered an appropriate response. Kane [13] acknowledges ' a great benefit in going through the full book-keeping of all pairings of variables.' The setting of an entry in the interaction matrix M to zero should, he maintains, be a deliberately conscious act so as to avoid overlooking an important link in a feedback loop. Such a procedure would avoid possible errors of omission that an unsystematic and uncomprehensive procedure is likely to commit.

The interaction matrix is a construct that facilitates formulation of system dynamics models that are consistent with the primitives of systems dynamics. Once the interaction matrix has been determined, its corresponding SFD can be delineated, scrutinized, and changed if necessary. This suggested strategy could systematically eliminate possible errors of commission and omission while enabling portions of the formulation process to be algorithmatized.

In what follows, brief notation, together with the derived implications for the interaction matrix, are introduced in the next section, § 2. In fact, § 2 is a condensation of formal analyses provided in [3]. Section 3 describes the basic steps of the component development strategy. These basic steps are then illustrated through use of an example in section 4. Some textbook examples are then shown to fit the format of the interaction matrix in section 5. Finally, the limitations, alternatives, and advantages of the approach presented are discussed in the conclusion of the paper, section 6.

The companion paper [3] employs primitives (set-theoretic definitions and axioms) for the notions of system dynamics to prove theorems that describe what interactions are possible amongst the various quantity types in system dynamics. The implications of these theorems for the interaction matrix are also indicated. The specific format of the interaction matrix is rationalized using the theorems.

2. The interaction matrix

Before introducing the interaction matrix, we provide a brief discussion of the notation to follow and formally introduce the concept of a 'component'. In system dynamics five basic quantity types are distinguished: stocks, rates, auxiliaries, parameters and inputs, and outputs. Those quantities that are stocks can be thought of as possessing membership in the set of stocks, denoted by S. Individual members of this set are denoted by s_i. Similar conventions apply to the set of rates R, the set of auxiliaries V, the set of parameters and inputs PU, and the set of outputs O. In some applications it is useful to distinguish parameters P from inputs U, the latter being those points of influence that the policymaker can bring to bear upon the system of interest. The combined set of parameters and inputs PU is simply the union of sets P and U; that is PU = P U.

Next, the concept of a component is given a precise understanding. For our purposes a component will refer to a substructure (consisting of quantities and connectors) that can be associated with one and only one flow. Thus every flow defines a separate component. All stocks and rates that either accumulate or control the flow are included within the component. Auxiliaries are included within a component only if they affect and are affected by other quantities known to be contained within a particular component. Parameters and inputs are included only if they directly affect other quantities in the component, while outputs are included within the component if they are functions exclusively of quantities within that component.

The quantities in system dynamics can interact only by means of connectors that are one of two types -- flow F or information I. This is to say that a connector (q_i, q_j) directed from q_i toward q_j is either an information connector or a flow connector. In what follows, it is useful to speak of the interactions directed from one set of quantities to another set, say from the set of stocks S to the set of rates R. These interactions can be specified by means of a submatrix denoted (S, R) and are defined on the Cartesian product $S \times R$. It will also be useful to use the notation S_1, R_1 to denote specifically those stocks and rates within component 1. Finally, the set of quantities that affect a particular quantity q_i are referred to as the 'affector set' in subsequent discussions while the set of quantities that q_i affects are referred to as the 'effector set' (see [3] for formal definitions).

With these brief concepts it is possible to introduce formally the interaction matrix. For simplicity the interaction matrix prescribing a two-component system is considered, as shown in Fig. 4. Thus for a two-component system the interaction matrix can be partitioned into two component matrices and two interconnection matrices. The interconnection matrices specify interactions between components, while the component matrix specifies the interactions within a particular component. Specifically, the i j interconnection matrix specifies the interaction directed from component i toward component j, and conversely for the j i interconnection matrix. The component and interconnection matrices are each discussed in what follows.

	component i	component j
Component i	Component matrix i	Interconnection matrix i j
Component j	Interconnection matrix j i	Component matrix j

Figure 4. Interaction matrix for a two-component system (no quantities between components).

The component matrix. For convenience, the component matrix is indexed by the sets *S*, *R*, *V*, *PU*, and *O* as shown in Fig. 5 below. The possible interactions in the component matrix are indicated by a ± *I* or ± *F*, where ± *I* denotes possible information connectors, while ± *F* denotes possible flow connectors between members of the associated sets. Submatrices (such as the cell indexed by *S*,*S*) that are blank

component j

component i	S	R	V	PU	O
S		± I	± I		± I
R	± F				
V		± I	± I		± I
PU		± I	± I		± I
O					

Figure 5. Possible interactions in the component matrix.

represent situations in which direct interactions cannot occur without producing a violation of the basic rules of system dynamics. For example, no entries can be placed in the (*S*, *S*) or (*S*, *PU*) submatrices because, according to the rules of system dynamics, it is impossible for stocks to directly affect stocks or to affect parameters and inputs. Conversely, the matrix does suggest that it is possible for stocks to directly affect rates, auxiliaries, and outputs by means of information connectors. Similar statements could be made for the remaining four rows of the component matrix. The matrix is formally derived from the basic principles of system dynamics (see Forrester, [5, 6, 7, 8],) in [3].

The interconnection matrix. A similar situation exists for the interconnection matrix as shown in Fig. 6 below. Clearly, many submatrices do not contain any interactions. By virtue of the definition of 'component', it is virtually impossible for a rate within one component to affect a stock within another (see Theorem 5, [3]). Thus the (*R*, *S*) submatrix is vacant in the interconnection matrix. Since rates can affect stocks only, all the remaining submatrices along the row associated with *R* are also vacant. The matrix is formally derived in [3].

component j

	S	R	V	PU	O
S		± I			
R					
V					
PU					
O					

component i

Figure 6. Possible interactions directed from quantities of component *i* toward quantities of component *j*.

It is possible for some quantities to be incapable (by our conventions) of being associated with any one component. Such quantities appear at the interface between two or more components and include auxiliaries V_b, parameters and inputs PU_b, and outputs O_b. These between-component quantities Q_b must also be included in the interaction matrix in situations where they are required. The interaction matrix for two components with between-component quantities included appears in Fig. 7 below. The entire matrix shown in Fig. 7 is rationalized in [3].

Matrices labeled *A* in the interaction matrix depicted in Fig. 7 are simple component matrices previously described, while matrices labelled *B* will be recognized as interconnection matrices. Matrix *B* is the matrix of information connectors directed from component *i* directly (i.e. there are no between component quantities) toward component *j*, while matrix *B* is the matrix of information connectors directed from component *j* directly toward component *i*. Matrix *E* is the matrix exhibiting the possible interactions amongst the between-component quantities. Matrices labelled *C* exhibit where and how quantities within components *i* and *j* can affect or influence between-component quantities, while matrices labelled *D* exhibit where and how between-component quantities can influence quantities within components *i* and *j*. Matrices *C* and *D* could also be thought of as interconnection matrices, but to avoid possible confusion with matrices *B*, these matrices will be referred to as interface matrices. Note that the only matrices in which flow connectors can occur are in the component matrices labeled *A* -- all remaining matrices contain exclusively information connectors.

If the interaction matrix were used in a computer-assisted model formulation exercise, an interrogation would be performed by the computer for each of the entries in which a possible interaction could take place. In this fashion the computer would insist upon a comprehensive consideration of all feasible interactions that are consistent with the rules of system dynamics.

		Si	Ri	Vi	PUi	Oi	Sj	Rj	Vj	PUj	Oj	Vb	PUb	Ob
				Qi					Qj				Qb	
	Si		±I	±I		±I	±I					±I		±I
	Ri	±F												
Qi	Vi		±I	±I		±I								
	PUi		±I	±I		±I								
	Oi													
	Sj	±I						±I	±I		±I	±I		±I
	Rj						±F							
Qj	Vj							±I	±I		±I			
	PUj							±I	±I		±I			
	Oj													
	Vb		±I					±I				±I		±I
Qb	PUb											±I		±I
	Ob													

	Qi	Qj	Qb
Qi	**A**	**B**	**C**
Qj	**B'**	**A**	**C**
Qb	**D**	**D**	**E**

Where

A =

	Si	Ri	Vi	PUi	Oi
Si		±I	±I		±I
Ri	±F				
Vi		±I	±I		±I
PUi		±I	±I		±I
Oi					

Figure 7a. Detailed interaction matrix for two-component system (between component quantities included).

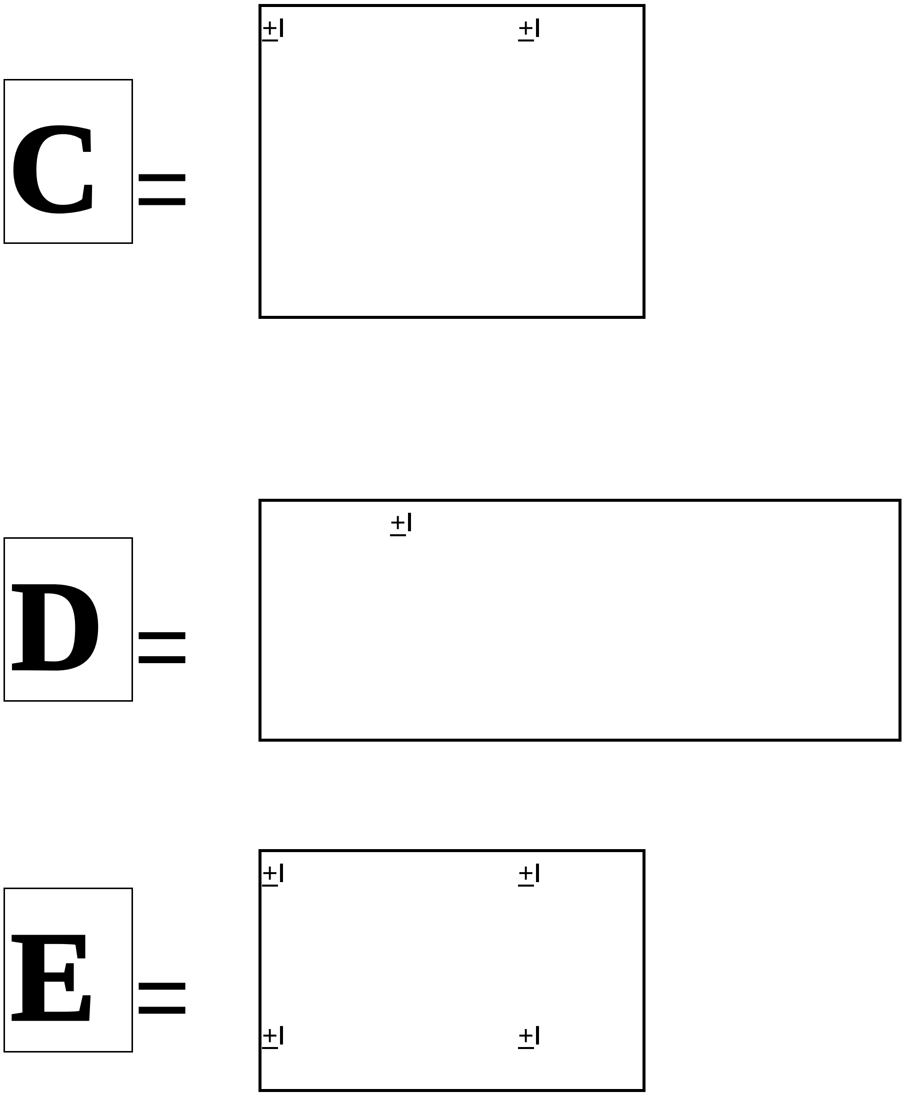


Figure 7b. Detailed interaction matrix for two-component system (between component quantities included).

3. Component strategy: overview

The general procedure of the component strategy is intended to both motivate and facilitate simultaneous specification of the set of quantities *Q*, the set of connectors *C*, and the identities of all quantities and connectors, by a user who has moderate background in system dynamics. After completion of the approach, the stock-and-flow diagram can be delineated by means of an equivalent interaction matrix (see Fig. 1) that could be used in computer-aided applications to print and store the information contained in the stock-and-flow diagram.

In computer-aided applications the algorithms would request the necessary information by means of queries. The specific order and form of queries that might be employed would depend on the specific algorithm chosen. Although no specific algorithms will be discussed, the overall approach consists of the following steps. Steps 1 to 7 will be illustrated in the conceptualization exercise, section 4. Steps 10 and 11 were illustrated by Burns [1,2].

Step 1. Familiarization with the problem and approach. In this step the user familiarizes himself with the literature describing the issue context and then discusses the problem with substantive experts and decision makers.

Step 2. Determination of system components. The subsystems of the problem considered will be identified as the system 'components'. Using the definition for a component, each relevant flow is determined first. A separate component is assigned to each flow. Each component will involve quantities related to controlling and accumulating the flow. For example, one flow may consist of money (the financial component), another of people (the population component), another of goods (the product component), still another of natural resources (the natural resources component). This step will result in the specification of the rough boundary of the system considered. And, the major components (subsystems) will have been identified.

Step 3. Determination of interactions among components. Through an iterative process, interaction among different component pairs will be determined in this step. This step will depict the user's initial understanding of the interactant structure of system components. Components that drive other components would be identified at this stage. The user will thus develop an aggregated causal loop diagram of the system component interaction using the square ternary matrix to facilitate the development.

Step 4. Determination of stock-rate interactions within components. This step will involve first the compilation of a list of the stock variables and rate variables for each component. The connectors between stocks and rates within each component are then identified. The interaction matrix for each component could, for clarification, be printed by interactive computer algorithms at the completion of this step.

Step 5. Determination of interacting stock-rate pairs between components. The information linkages directed from stocks of one component to the rates of another component will be determined at this step. The model-maker would start by examining the system component interaction delineated at step 3. Assuming all of the interactions were detected in step 3, all that remains to be performed within this step is to identify the adjacent stock-rate pairs associated with each interaction. These interactions might then be inserted in the interaction matrix for storage within the computer's memory.

Step 6. Determination of between-component quantities. The only quantity types that, by our definition of components, are allowed between components are auxiliaries, parameters, inputs, and outputs. This is formally rationalized in [3]. Iterating through each linkage defined in step 5, auxiliaries in-between different components are determined either manually or through computer interrogation. Affector and effector sets of these auxiliaries, also determinable through user response to computer interrogation, will define all the between-component auxiliaries, parameters, inputs, and outputs of the system. These mechanisms would then be incorporated into the interaction matrix.

Step 7. Determination of within-component support structure. Within-component quantities that have not yet been determined include auxiliaries, parameters, inputs, and outputs. These comprise the necessary supporting structure for the stock-rate mechanisms already determined affector sets of the rates of each component will guide the determination of the within-component quantities. The interaction matrix delineated at the end of step 6 is updated by the insertion of additional quantities and linkages within each component as necessary. Steps 6 and 7 could conceivably surface an additional component that was not perceived under step 2. Should this happen, it is advisable to return to step 3 in order to fill in the missing details associated with the newly created component.

Step 8. Insertion of delays where appropriate. In this step each of the information links are examined to see if a perception or transmission delay exists in the channel, and the user is also asked to designate whether delays in the various component flows are necessary to adequately represent the system. Then the user inserts smoothing functions in those situations where rates (or decision processes) use averaged information about other rates.

Step 9. Empirical verification of structure. In this step the model-builder subjects his structure to critical review and evaluation by those who are familiar with the Forrester methodology and the problem being considered.

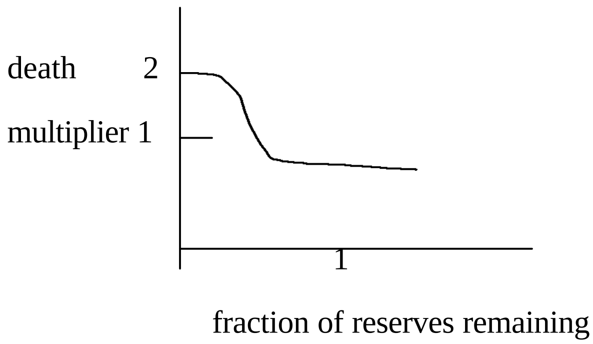
Step 10. Determination of the equations from the interaction matrices. Burns [1,2] illustrates how algorithms would be fully capable of formulating equations from interaction matrices, when the identity and dimensionality of each quantity is known.

Step 11. Implementation of the equations is the form of a simulation computer program. Once the equations have been formulated, algorithms would be entirely capable of organizing the equations into the form of a subroutine capable of being called by other routines which would perform the numerical integration and store the resultant-generated time-series trajectories for later plotting (see Burns [1,2]).

4. A conceptualization exercise

This section illustrates the concepts of the component strategy through the use of an example problem. The problem chosen for this purpose is the following description involving interaction between a growing population and a limited natural gas supply.

A country's growing population is consuming ever greater quantities of natural gas each year. The following parameters and initial conditions are known at time t_0: birth rate normal BRN, death rate normal DRN, gas usages per-capita per-year normal GUN, the initial population P_0, and the initial reserves of natural gas. It is known that natural gas usage is proportional to (a) the size of the population and (b) the fraction of reserves remaining. It is also known that deaths increase when natural gas shortages occur in accordance with the following schedule:

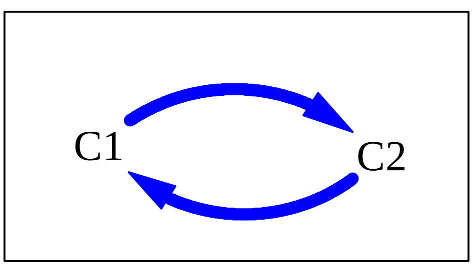


The Forrester schematic of this system (along with the associated interaction matrix) will be developed following the steps of the component strategy. An iteration through the steps of component strategy yields the following results.

Step 1. Familiarization with the problem and approach. The country considered has limited natural gas supplies upon which it is heavily dependent. It has not found a readily available substitute, resides in a cold climate, and cannot import gas from other sources. Since the country is isolated, there are negligible changes in population due to migration.

Step 2. Determination of the components that comprise the system. Two components or subsystems are identified for the problem. They are population (C_1) and natural gas (C_2). These two components delineate a rough boundary for the model being formulated, and are sufficient to generate the symptoms of the problem.

Step 3. *Determination of interaction among components.* Both components affect each other in a strong sense (see Harary *et. al.* [11]). Population size is dependent upon the gas reserves, while gas reserves are dependent upon population. Specifically, there is a negative cycle between the subsystems as shown below in Fig. 8.



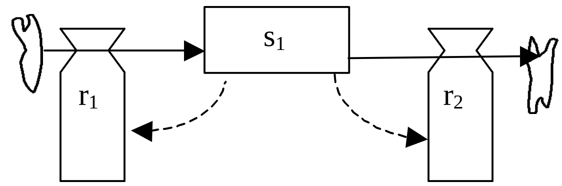
	C_1	C_2
C_1		-1
C_2	1	

Figure 8. Interactions between population and gas reserves

Step 4. Determination of stock-rate interactions within components. For the demographic or population component, there is a flow involving people from births to deaths. The number of persons alive at any time in the country can be accumulated in one level or stock variable. For simplicity, the population stock variable is represented by s_1. Symbolically, $S_1 = \{s_1 :$ population$\}$. The rate variables, R_1, that control the flow of people into and out of the population stock variable s_1 are birth rate (r_1) and death rate (r_2) respectively. In set notation, R_1 is the set of rates in component $S_1 = \{r_1$: birth rate, and r_2 : death rate$\}$. Since both of these rates are increasing or decreasing the population as a certain percentage of the number of capita each year, the stock (population) affects both of the rate variables. These interactions among the stocks and rates of the demographic component lead to the following schematic and ssociated modified square ternary matrix, abbreviated MSTM.

	s_1	r_1	r_2
s_1		I	I
r_1	F		
r_2	-F		

State-rate MSTM of component 1



State-rate SFD of component 1

Figure 9. Stock-rate interactions within demographic component.

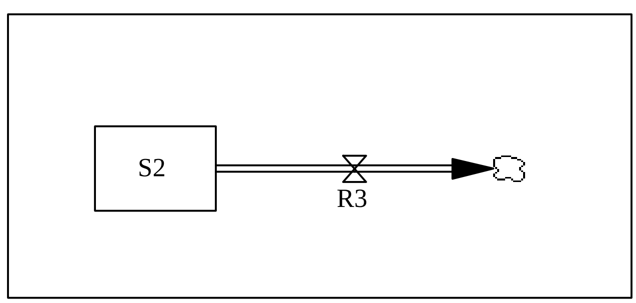
A consideration of the gas reserves component leads to the understanding that total gas reserves remaining (proven and unproven) is the only significant stock variable of this component. Thus S_2 = {s_2 : gas reserves remaining}. There is only one rate in the component--gas consumption rate--as the rate at which natural gas is created by nature is negligible for the time interval of interest. Symbolically, R_2 = {r_3 : gas consumption rate}.

Having determined what stocks and rates are contained within component, the interactions between these are specified next. It should be apparent that gas consumption rate depletes the gas reserves remaining whereas the problem stocks that gas consumption rate is proportional to the fraction of gas reserves remaining (to be taken up later).

The MSTM and SFD depicted in Fig. 10 characterize the stock-rate interactions for the gas reserves component.

	s_2	r_3
s_2		0
r_3	(-)F	

Stock-rate MSTM of component 2



Stock-rate SFD of component 2

Figure 10. Stock-rate interactions within gas reserves component.

Step 5. Determination of interacting stock-rate pairs between components. In step 3 the interactions between components were specified. From Axiom A3 ([3] section 1) it is known that components cannot be isolated and that the interactions between components are via information links. The only information links possible are cross-component stock-rate interactions, since these are the only quantities considered at this level of the approach. A rate-stock connector between components is impossible by Theorem T5 of [3] §7.2; therefore, position (r_3, s_1) is blank, as shown in Fig. 14.

From the problem description, clearly population s_1 affects gas consumption rate r_3. Consequently, there are information links in the (s_1, r_3) and (s_2, r_2) positions of the MSTM. Note that at this point there is little concern about whether the cross-component information links are direct or indirect, as this issue will be taken up in the next step.

Step 6. Determination of between-component quantities (auxiliaries, outputs, parameters and inputs). The previous step disclosed two between-component information links: (s_1, r_3) and (s_2, r_2). The (s_2, r_2) link is really a (s_2, v_2) and (v_2, r_2) link. In this step of the component strategy the between-component information links are each considered separately with the intent of determining if intermittent auxiliaries with adjacent inputs, parameters, or outputs are part of the information path. Should this step generalize between-component quantities that are believed to be of types other than auxiliaries, parameters or inputs (such as rates or stocks), then a new component will have been identified and the user must return to step 3. Each of the linkages found in step 5 are considered separately in the following discussion.

The affect of population s_1 upon gas consumption rate r_3 is direct since the consumption rate is described as being directly 'proportional' to the population. Consequently there are no interface quantities in this information channel. On the other hand, the affect of gas reserves s_2 upon death rate r_2 is known to be attenuated through a table function specified in the problem description. Thus an auxiliary or multiplier resides in the path from reserves remaining to death rate and this information link is indirect. We denote this auxiliary by v_2: reserves remaining to death rate multiplier.

Now consider the auxiliary v_2 itself. The user must address the question of whether there are parameters and inputs which affect v_2. Clearly, information about the initial gas reserves is required to know the fraction reserves remaining. Therefore, a second between-component quantity is required: gas reserves initial, p_5. These two between-component quantities comprise the between-component set Q_b.

The matrix of between-component quantities (matrix E) can now be delineated and appears as shown in Fig. 11.

	v_2	p_5
v_2		
p_5	(-)I	

Figure 11. Between-component matrix for example problem.

Step 7. Determination of within-component parameters, inputs, outputs, and auxiliaries. In this step the necessary support structure within each component is determined. Component S_1, the population component, is considered first. A consideration of what other variables affect the rates and are required to construct dimensionally consistent rate equation leads to the understanding that, according to the problem description, two known parameters must also be included. These are birth rate per capita per year and death rate per capita per year, denoted by p_1 and p_2 respectively.

The manner in which these quantities interact with the remaining quantities contained in component S_1 is prescribed by the component matrix, Fig. 5. The result is shown in Fig. 12 below. For this component involving 25 pairs, interaction between 17 of them are not possible. A consideration of the remaining eight (non-blank) pairs leads to the interaction indicated in Fig. 12, where the associated flow diagram is also shown. This completes the structural description for the demographic component, S_1.

	s_1	r_1	r_2	p_1	p_2
s_1		(+)I	(+)I		
r_1	(+)F				
r_2	(-)F				
p_1		(+)I	0		
p_2		0	(+)I		

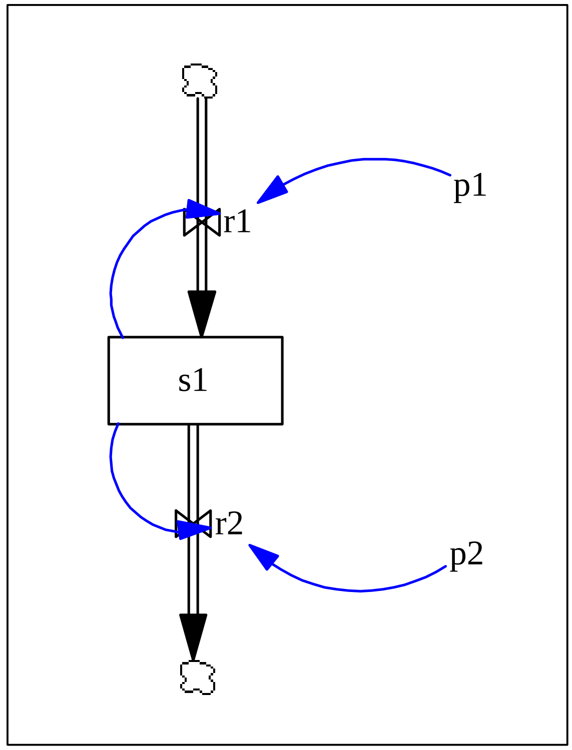


Figure 12. Structural description for the population component.

	s_2	r_3	v_1	p_3	p_4
s_2		0	(+)I		
r_3	(-)F				
v_1		(+)I			
p_3		(+)I	0		
p_4		0	(-)I		

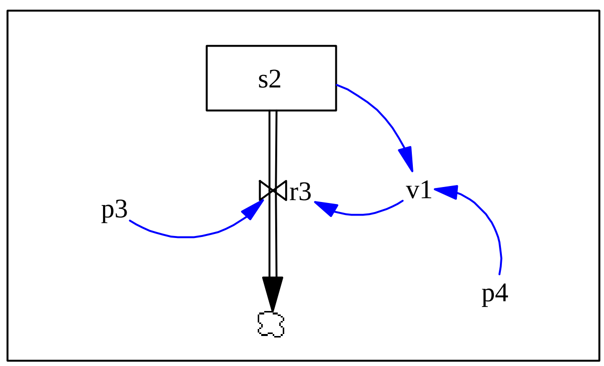


Figure 13. Structural description for the gas reserves component

	s_1	r_1	r_2	p_1	p_2	s_2	r_3	v_1	p_3	p_4	v_2	p_5
s_1		(+)I	(+)I				(+)I				0	
r_1	(+)F											
r_2	(-)F											
p_1		(+)I	0									
p_2		0	(+)I									
s_2							0	(+)I			(+)I	
r_3						(-)F						
v_1							(+)I					
p_3							(+)I	0				
p_4							0	(-)I				
v_2		0	(+)I				0					
p_5											(-)I	

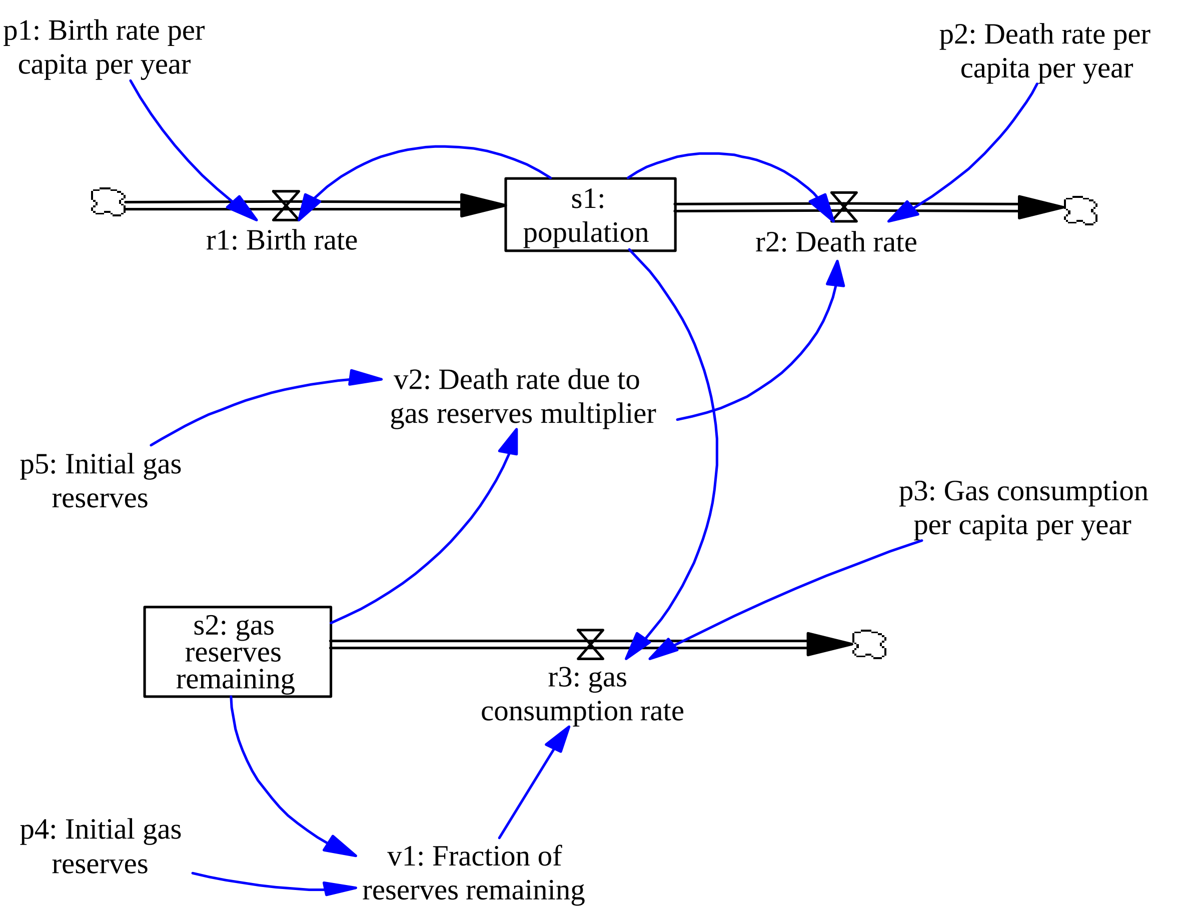


Figure 14. Complete interaction and associated flow diagram for example problem considered.

For the gas reserves component, C_2, a similar strategy is applied. In addition to the stock-rate interaction already determined, certain quantities are required to complete the construction. The problem stipulates that gas consumption rate is proportional to population and to the fraction of gas reserves. Thus a constant of proportionality p_3, gas consumption per capita per year, is required. The fraction of gas reserves remaining is an auxiliary v_1 that is dependent upon the actual gas reserves s_2 and the initial gas reserves p_4. The interactions are determined from a consideration of the component matrix, Fig. 5. The resultant matrix and associated SFD are shown in Fig. 13. For this component as for the previous one, 17 of the 25 possible pairs are inferred as non-interactant (all that are blank in Fig 13). Thus only eight pairs (those which are non-blank) permit interactions consistent with the rules of system dynamics.

Algorithms within the computer would utilize the previously obtained information about the problem of interest to complete the interaction matrix depicted in Fig. 7. The resultant interaction matrix for the example problem is shown in Fig. 14 together with the associated flow diagram. Blank entries were inferred, while zero entries represent situations where interactions were feasible but nonetheless not representative of the problem considered.

5. A textbook model

To demonstrate the faithful representation of the interaction matrix to the conventions of system dynamics, this section exhibits a typical textbook model taken from Goodman [11] in interaction matrix form. The model chosen is intended to characterize the growth and collapse of the deer population on the Kaibab Plateau (the north rim of the Grand Canyon). The model is adapted from that described on pages 377 to 388 of Goodman [11]. The three-component model depicted on page 382 is reduced to a two-component model, for convenience, by replacing the predator population component with a simple parameter-p_1, predator population.

Table 1. List of the Quantities and their Designations for the Kaibab Plateau Example.

Symbol	**Name**
s_1	Deer Population
r_1	Deer net growth rate
r_2	Deer predation rate
v_1	Deer kill rate
v_2	Deer density
p_1	Predator population
p_2	Land area
s_2	Food Supply
r_3	Food generation rate
r_4	Food consumption rate
v_3	Food generation time
p_3	Food capacity
p_4	Food needed per deer
v_4	Food ratio
v_5	Growth rate factor
v_6	Food per deer
v_7	Food consumption per deer

A list of the quantities included in the model together with their symbolic designation is provided in Table 1. The stock-and-flow diagram and interaction matrix are depicted in Fig. 15.

From the stock-and-flow diagram, it is easy to distinguish the component quantities Q_1, Q_2 as well as the between-component quantities Q_b for this model. Since Q_1 is the set of quantities within component S_1, the deer population component, it includes the set of quantities $\{s_1, r_1, r_2, v_1, v_2, p_1, p_2\}$. The between-component quantities Q_b can also be distinguished from the stock-and-flow diagram. They are p_4, v_4, v_5, v_6, v_7. The remaining quantities are contained in Q_2.

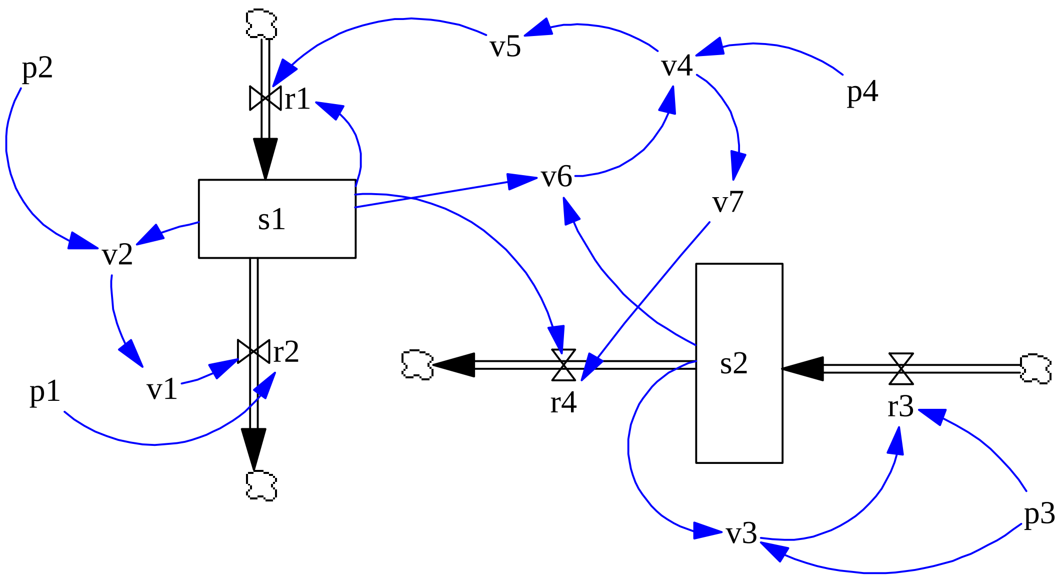
The interaction matrix for this two-component system is exhibited so that entries that were inferred can be distinguished from those which are not. Consistent with the previous example, blank entries represent inferred non-interactions. Zero entries represent situations where non-zero entries could have been inserted without violating the rules of system dynamics, but were not because they do not characterize the issue at hand. For this problem, approximately 75% of the entries in the interaction matrix were inferred non-interactions.

Of importance here is the simple observation that all linkages for the model do occur only in those submatrices where, according to the interaction matrix depicted in Fig. 7, linkages can occur. Even though the interaction matrix has been rigorously derived in [3], its format has been tested through consideration of several textbook models.

6. Epilogue and conclusion

In this paper a new perspective for a somewhat conventional approach to the formulation of system dynamics models is described. The new perspective concentrates on the development of the Forrester stock-and-flow diagram and incorporates the concept of an interaction matrix to assist in the formulation of such models. The interaction matrix, together with an explicit procedure for model formulation, was described in the paper. In this format the approach could be algorithmatized in which the procedure comprised the algorithm and the interaction matrix comprised the data structure. The strategy was found to facilitate the determination of the important quantities to be included as well as the existence of connectors between the quantities, and the identities of both connectors and quantities, in a systematic fashion.

As mentioned in the introduction, other computer-aided strategies for formulating Forrester models are possible. Such strategies may usefully employ many of the concepts introduced in [3] and the main text of the paper--continuity, the interaction matrix, set-theoretic definitions and axioms, even the theorems. In the absence of empirical and experimental comparison of various strategies in controlled model formulation exercises, it is difficult to ascertain what strategy seems most appropriate for the broadest class of problems and users. Certainly each would have advantages and disadvantages. Noteworthy is the dual approach taken by Burns. In this strategy users start by generating a list of unidentified quantities to be included and from this list construct a causal loop diagram (i.e. determine the connectors). Burns showed that the structure of the causal loop diagram imposed certain



			s_1	r_1	r_2	v_1	v_2	p_1	p_2	s_2	r_3	r_4	v_3	p_3	p_4	v_4	v_5	v_6	v_7
	S_1	s_1		I	0	0	I				0	I				0	0	(-)I	0
		r_1	F																
	R_1	r_2	(-)F																
Q_1		v_1		0	I		0												
	V_1	v_2		0	0	I													
		p_1		0	I	0	0												
	PU_1	p_2		0	0	0	(-)I												
	S_2	S_2		0	0						(-)I	0	I			0	0	I	0
		R_3								F									
Q_2	R_2	R_4								(-)F									
		V_3									(-)I	0							
	$^{Q}W_2$	P_3									I	0	(-)I						
	PU_b	P_4														(-)I	0	0	0
		V_4		0	0						0	0					I	0	I
Q_b		V_5		I	0						0	0				0		0	0
		V_6		0	0						0	0				I	0		0
	V_b	V_7		0	0						0	I				0	0	0	

Figure 15. Interaction matrix for a textbook problem: the kaibab plateau deer population [11].

constraints upon the identities of the quantities and connectors. As a consequence, the causal loop diagram is machine-translatable into a stock-and-flow diagram; that is, the identities of the quantities and connectors could be determined by computer just by inspection of the STM. From the stock-and-flow diagram or its equivalent, the MSTM, Burns showed that equations could be composed (by computer) for each of the quantities, provided the dimensions or units associated with each quantity were known. The approach taken by Burns [2] assumes no *a priori* understanding of system dynamics on the part of users; as a result some 'uncertainties' may enter the model. This method, by contrast, does assume some understanding of system dynamics on the part of the user. Therefore, it is reasonably safe to surmise that the component strategy is more appropriate for users with minimal understanding of the notions of system dynamics, whereas, the approach taken by Burns is appropriate for users with no understanding of system dynamics.

Finally, the use of interaction matrices in connection with system dynamics models was explored in the paper. By use of the interaction matrix, it is possible to infer many of the interactions as non-existent because of their incompatibility with the rules of system dynamics. Thus the interaction matrix focuses user attention upon just those interactions which are consistent with the notions of system dynamics and forces construction of a model that is rigorously adherent with such notions as a first cut. It was discovered that, by consideration of every feasible interaction in the interaction matrix, a systematic process for preventing errors of omission to occur results. It was also discovered that roughly 5/6 of the interaction matrix can be filled with inferred zeros (blanks), thereby preventing errors of commission to occur in which a link which is incompatible with the rules of system dynamics is inserted.

The advantages of the component-based strategy are many. First, the development strategy prevents the inclusion of structures that are inconsistent with the basic presuppositions of system dynamics. Second, the strategy results in the development of reusable components. These components can be catalogued, documented and placed in a model repository where they can be used again and again (i.e., reused). Subsequent component-based modeling initiatives do not have to start from scratch, therefore. The correct component can be identified, copied from the repository and inserted into the developer's model. Many studies [19, 20] in the software engineering literature have documented the substantial reduction in development time that comes from reuse of components. These reductions can be as much as an order-of-magnitude [19, 20]. Second, reuse results in significant improvements in the overall quality of the software system because the components are largely free of defects and bugs. The components we have defined and described here are exceptionally good at encapsulation and information hiding. All of the structure associated with a single flow have been encapsulated and "hidden" so that it con be used context free. On the other hand, no mention was ever made of inheritance, since the authors regard that characteristic as undesirable (it creates a path dependency structure that makes debugging a nightmare).

A study at NASA's Software Engineering Laboratory considered ten projects that vigorously pursued reuse aggressively (McGarry, Waligora, and McDermott[21]). The initial projects weren't able to take much of their code from previous projects because previous projects hadn't established a sufficient code base. In subsequent projects, however, the projects that used functional design were able to take about 35 percent of their code from previous projects.

Projects that used component-based design were able to take more than 70 percent of their code from previous projects. This is germane to the concept of component-based model development—it should increase the reusability of model structures and improve the overall quality and productivity of system dynamics model development.

To ideally foster a “culture” of component reusability, a repository of reusable components should be created. Model developers can then deposit their components, along with documentation, in the repository. A reward system should then be instituted that encourages modelers to reuse existing components instead of reinventing them from scratch. Indeed, developers who place their well-designed and documented components into the repository should also receive a reward each time one of their components is reused.

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