A NEW KIND OF SENSITIVITY TESTING IN SYSTEM DYNAMICS MODELING FOR SENSITIVE RESULTS FROM AGGREGATION ASSUMPTIONS

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ABSTRACT

This study has illustrated that simulating an aggregate model, using the same data set at the same level of aggregation, can lead to different model conclusions when different aggregation criteria are applied. This study's conclusion to the effect that aggregation of individuals can have significant influence on the results of the model is expected to have direct implications for system-dynamics modeling.

For the field of system-dynamics modeling, the study has identified a kind of model sensitivity that can not be tested by the methods of sensitivity testing presently used. For future research in the field, the concept of aggregation of invididuals has to be clearly established and differentiated from the concept of aggregation of variables before general rules for this type of sensitivity testing can be identified. Similar sensitivity testing should be adopted in the system-dynamics modeling technique. If this has not been done, this simulation approach should be interpreted conservatively. This paper also discusses the problem of whether a universal aggregation scheme is the only highest aggregation scheme.

Introduction

The aggregate computer-simulation approach has been widely adopted for use in various techniques in diverse policy areas. Three kinds of aggregation are used in this

approach: aggregation of variables, aggregation of individuals, and aggregation of time periods. Hannan has described the differences between the aggregation of variables and the aggregation of individuals:

> There is a subtle but important distinction between aggregation variables in a stochastic specification of a function relationship and aggregating the entire stochastic specification. . . . the important distinction is that between aggregation of variables in relations and aggregation of entire empirically specified functional relationships. (Hannan, 1971, pp. 16-17)

In other words, aggregation of variables concerns variables within the model structure and the aggregation of individuals concerns an entire studied population. In explaining the less-well-known third type of aggregation, Theil gives a short example of aggregation of time period, in which "the demand for labour during a quarterly period is a function of the quantity sold during the same and the preceding quarterly periods" (Theil, 1965, p. 4).

This study concerns questions related to the aggregation of individuals.

The concept of aggregate analysis is commonly accepted in the computer-simulation modeling approach as well as in other quantitative techniques. In the area of data analysis, discussion of the use of the aggregate analysis approach is advanced. Examples of different aggregation schemes leading to different solutions can be found in many sources, for example: Taylor, (1968) and Borgatta & Jackson, (1980). In the computer-simulation modeling approach (except in techniques such as input-output analysis), more attention has been placed on the aggregation of variables than on that of individuals. Testing possible model sensititivity to the assumption of aggregation of individuals is not included in the regular modeling procedures.

This situation leads to the following questions concerning the appropriateness of the present approach: Once the aggregate level of individuals is decided, what aggregation scheme should be decided upon to represent the level of aggregation? Will the criteria used to aggregate individuals have significant impact on simulation results? If the policy results are sensitive to the aggregation scheme used, what does such sensitivity mean to the model conclusions and to the validity of this approach? If such sensitivity exists, are there general rules for aggregating individuals, based on the possible relationships between the aggregation scheme and the simulation results?

These are all legitimate and interesting questions for the aggregate simulation approach. This study explores the above questions by testing one aggregate simulation model in the field of system dynamics. By doing so, this study asks: will changing the aggregation schemes (i.e., the method of population aggregation) while holding the model structure constant affect the conclusions of the model? This paper, consequently, investigates the status of the subject problem in the field of system dynamics, introduces the sample model, explains the research design, presents the results of the study, and discusses its implications.

The Status of Aggregation of Individuals in the Field of System Dynamics

In the field of system dynamics, the concept of aggregation of individuals is not well differentiated from aggregations of variables; thus, the problem of aggregation of individuals is seldom discussed directly. This phenomenon can be observed in the methods of sensitivity testing used in the field.

Because of the characteristics of this modeling approach, sensitivity testing is an important modeling procedure in system-dynamics modeling. Forrester and Senge

(1980) list a series of 17 available tests for building confidence in system dynamics. The tests are included within the broader categories of model structure, model behavior, and policy implications. The boundary-adequacy tests in all three categories are related to the problem of this study, but the sensitivities of variables and individuals are not differentiated.

In this type of sensitivity testing, criticisms of Forrester's Urban Dynamics and World Dynamics models and Mass's business-cycle study were listed as examples (pp. 215, 220, 222). In the first example, the urban dynamics model is criticized for ignoring city-surburban interactions. The second example criticizes the world-dynamics model, noting that it ignores the distinction between the developed and the underdeveloped countries in the world. The third example, criticizing the business-cycle study, illustrates the influence on the behavior of the model of incorporating an endogenous consumer demand into the business-cycle model. Among these three examples, the first and third are examples of aggregating more model structures that are problems of the aggregation of variables. The second example, which discusses different components of the aggregated world and the methods of presenting the world, is a problem of the aggregation of individuals.

The cases of aggregation of variables have been tested by adding more structures. The case of aggregation of individuals, unfortunately, is regarded as a model-purpose problem, rather than one of model sensitivity. Within this discussion, in addition to the mixed concepts concerning aggregation of variables and individuals, the acceptance of presumptions on the aggregation of individuals can be observed:

> Forrester's World Dynamics model . . . has often been criticized for failing to distinguish developed from underdeveloped countries. When one looks deeper, one sees that these criticisms generally stem from an interest in regional development rather than an interest in growth and transition for world society as a whole. Hence, they should be seen as criticism of model purpose . . . rather than boundary adequacy. Hence, the evaluator must continually distinguish questions of boundary-adequacy relative to a particular purpose from questions of model purpose. (Forrester & Senge, 1980, p. 215).

In this argument, the selection of a proper aggregation way to group individuals is considered to be a problem of model purpose and not a possible technical problem. This perspective, in other words, implies an attitude that once the purpose of the model is defined, the selected aggregation scheme should be appropriate for analysis of the subject problem at the decided aggregation level.

In addition to ignoring the distinction between world components, as discussed above, the world-dynamics model was also criticized for oversimplifying the variables of model structures. In his description of the history of the world model, Hecox discusses a series of supporting projects that embodied these criticisms and "entailed expanded and disaggregated research on the 'predicament of mankind' and methods of assessing its future condition" (1976, p. 91). These projects produced the next generation of world models. These models tested their "disaggregated" world-aggregation schemes with expanded model variables.

Using one of the model efforts, ("Turning Point," by Mesarovic & Pestel, 1974) as an example, Hecox (1976, p. 93) says, "this work reflects a disaggregation of the world into ten sub-regions," but it also "introduces policy strategies into the model as ways to ameliorate world and regional problems." Since changing the aggregation scheme from a universal one to a subregional one was considered "disaggregation" of the aggregation level and modifications of the aggregate individuals and the variables are implemented in the models at the same time, these previous studies have never considered their study results, which

differ from those in the world model and differ among themselves, to be a matter of possible model sensitivity.

The Sample Model and Its General Applicability

A system-dynamic model of public school finance has been under development since 1979 at the Graduate School of Public Affairs at the State University of New York at Albany. The EDFIN model includes two types of aggregation: aggregation of variables and aggregation of individuals. The aggregated variables are such variables as the state tax base, the tax rate, and expenditures. The aggregated individuals include four types of aggregate school districts, which are hypothetically integrated according to the following school-district characteristics: wealth, size, and educational expenditures. They are called "local sectors."

The boundary of the model is set within a single state. The data used are from the State of New York. The sample model contains three major sectors: local taxation, state taxation, and education distribution. The aggregate school-district types contain identical structures of the local-taxation and education-distribution sectors but have different variable values to represent their different characteristics.

When using four kinds of local sectors in the model as the aggregation scheme, the EDFIN model may be questioned as to whether it is a fully aggregated or a semiaggregated simulation model and whether this study has general application to the other single-sector models.

If an aggregate simulation model can take only the universal disaggregation scheme as the scheme to represent the level of aggregation, the aggregation assumption may rely too much on the maintenance of average behaviors. It is argued here that selecting ways to represent the aggregation level should not be confused with the purpose of disaggregation. If the aggregate-simulation approach demands one universal aggregation scheme and any others are considered as semi- or disaggregate schemes, the aggregate approach is defective already. If changing aggregation schemes is regarded as a disaggregation level of analysis, then of course, changing the aggregation scheme can simply be regarded as a problem of model purpose instead of one of model sensitivity.

To illustrate, the EDFIN model decided to use four kinds of school districts to represent the local aggregation (school districts), because they are considered a proper scheme for an aggregate analysis such as EDFIN simulation. In the field of public-school finance, it may

be difficult to accept an assumption that all the school districts have similar characteristics and behave similarly in the system.

For another example, in a hypothetical case, a group of students are simulated in an aggregate model; and the model conclusions were found to be inaccurate. Two students in the class have been found with behavior very different from that of the others. The modeler aggregates the model sectors by looking at them as two kinds of students instead of one, and the accuracy of predicition has increased. In this case, the process of differentiating the students into two groups should not be regarded as disaggregating the population (from a full-aggregation to a semiaggregation level). Instead, it should be thought of as using a more proper aggregation scheme to represent the selected aggregation level; however, this study used several multisector aggregation schemes to evaluate the sensitivity of the model to the changes of aggregation schemes without raising validity questions of this sort.

Research Design

The research procedure can be divided into three major steps: (1) aggregation scheme identification, (2) model simulations, and (3) comparison of results and conclusion.

In step 1, five aggregation schemes of school districts were selected for EDFIN model testing. Two of them were selected from aggregation schemes presently used in the New York State school-finance system. Two others were identified, with the assistance of two methods of statistical clustering techniques. The last aggregation scheme was obtained by randomly assigning the school districts.

In step 2, models were replicated according to the identified aggregation schemes and simulated with test runs.

In step 3, there were two major objectives: first, to compare the testing results among the five models and second, to determine the results of the study.

In step 1, the identified schemes that are currently in use contain school districts that are grouped according to geographic and demographic criteria; these are: counties (by locations) and SMSA's vs. non-SMSA's. Statistical clustering was performed using two different clustering methods: an hierarchical and a nonhierarchical one, using the computer packages BMDP and BC TRY (Dixon & Brown, 1979; Tryon-Bailey). The statistical clustering schemes contain

school districts grouped by size, community type, property wealth, and income wealth. Random selection assigned the cases to different numbers and grouped the districts accordingly.

In step 2, data needed for the EDFIN-model simulations were extracted according to the aggregation schemes obtained. The data base is kept comparable among the aggregation schemes. Characteristics of different aggregation schemes can be observed, based on data obtained for local sectors. The geographic and demographic schemes evidence more similar local sectors than do the cluster-analyzed schemes. The randomly assigned aggregation scheme also shows similar local sectors, except for the New York City sector. These differences among aggregation schemes become the major determinant of model behaviors. This will be explained later in this paper.

Five aggregate EDFIN models were replicated, according to the extracted data sets. They are models with the aggregation schemes of counties by locations (COUNTY), SMSA's vs. non SMSA's (SMSA), hierarchical clustering result (HIER), nonhierarchical clustering result (NON-HIER), and random selection (RAN). The classification will be referred to in this study as COUNTY, SMSA, HIER, NON-HIER, and RAN.

Preliminary runs of all the models have been made, and all except HIER are in equilibrium. HIER is not in equilibrium because the input data cannot satisfy the modeled condition. Under this condition, in which only one of the five models was not in equilibrium, another version of HIER was built, by data modification. This new model, called HIERa, was tested along with the other models. The six models were simulated with 11 test runs, which are listed in Table 1.

(Table 1 here)

In step 3, the simulation results of the six models under the 11 test runs are compared according to the following criteria.

Base runs. This study cannot compare its outputs with the EDFIN simulation results, because the aggregate local sectors between the two studies are on different bases. Therefore, the simulation outputs of this study were compared with each other.

Simulation length. In this study, the model outputs were simulated within 20 years because, generally speaking, most of the dynamic behaviors of the models can be captured within this amount of time.

Sensitivity type. In this study evaluating the

sensitivity of an aggregate simulation model, the relevant type of model sensitivity was decided according to model usage. The EDFIN model is used to show the behavior of the modeled public school finance system. The policy implications of the variable behaviors are the central items of interest. In the EDFIN model, the policy conclusions are usually presented as increases or decreases of the major model variables, such as the variance of educational expenditures and the adequacy of educational expenditure.

Output measurements. Since the replicated models start from equilibrium at different variable values, the most important implications of variable behaviors were obtained by comparing them with the values prior to the testing implementations in each model. Therefore, the model behaviors were first compared with the original equilibrium values with respect to increase, decrease, or "no change." Then the implications were compared among the models to see whether they were different from one another. To facilitate analysis, overtime behaviors were analyzed at two points. A short-term change was defined as the condition at the second year after the change is simulated, and a long-term change was defined as the condition at the last year of simulation.

Observing variables. In order to preserve comparability among models with different local aggregation schemes, the model outputs were observed from three sectors: the state, New York City, and the remaining local sector. Within each of the three sectors, there are several major variables, indicating the essential model behaviors. The most important ones are selected and listed in Table 2.

(Table 2 here)

Study Results

Based on the research design, behaviors of the 20 variables in the six models under the 11 test runs were all recorded. The sensitive results are summarized in Table 3 according to the test runs and the variables. These sensitivities are reexamined in the light of model equilibrium situations. No systematic relationship has been found in this study between the final model equilibrium and the model sensitivities.

In the summary comparison table, the most apparent sensitivities appear under four test-run categories: implementing a cost-of-education index, increasing floor aid to $600 and $800, and increasing ceiling aid. Secondarily, even under the test runs where most model

behaviors are not sensitive, there are sensitive behaviors in several variables used to measure overall performance in the local sectors.

Sensitivity to test runs. The coded model behaviors of the four most sensitive testings are listed in Table 4.

Testing the implementation of a cost-of-education index has no impact on the models of COUNTY and SMSA but does have impact on the other four models. Among the affected models, the influenced behavior of model RAN is different from that of the others, HIER, HIERa, and NON-HIER. The testing of increasing floor aid to $600 has impact on HIER, HIERa, and NON-HIER. Testing of increasing floor aid to $800 has impact on all six models; however, influenced behaviors are different in the state sector between RAN and the other models. Testing of increasing ceiling aid also has impact on all of the models. The influenced behaviors show differences in the state sector between RAN and the other models and in the other sectors between COUNTY and SMSA and the other models.

The results have been found sensitive for six reasons:

1. Test runs have different kinds of influence on different models.

2. Test runs have different degrees of influence on

different models.

3. Models contain local sectors at various detail levels.

4. The number of positively or negatively influenced local sectors differs from model to model.

5. The characteristics of the most-influenced local sectors are different from model to model.

6. Competition among the local sectors is different from model to model.

Sensitivity to measuring variables. The second type of sensitivity that the comparison results show is that of the measurement of variables. From the summary comparison table, it is evident that there are occasions in which only partial model variables are sensitive, while the others behave similarly. In the seven test runs where the model output results are, for the most part, similar, the variables, using standard-deviation measurements, show sensitive results under all the test runs, except for the test run ofincreasing needed educational expenditures.

This type of model sensitivity should be considered different from the first type: test run sensitivity. The behaviors of these standard-deviation variables seem to be dominated by their initial data-distribution patterns.

When the variable distribution patterns of such factors as local tax rates and educational expenditures are very different among models, test runs can make the standard deviations change while the rest of the model variables behave similarly.

Sensitivity among model types. Behaviors are found to be different among certain kinds of models. Generally speaking, COUNTY and SMSA are usually more like one another when HIER, HIERa, and NON-HIER are similar to one another as well. RAN is similar to either of the two sets of models or unique unto itself, depending on the occasion; however, the models are not always sensitive according to these patterns. A few incidences of differences can also be found between similar models.

Characteristics of the four sensitive test runs. The model behaviors that are found to be sensitive under 4 out of 11 test runs are all under policy implementations rather than scenario changes. The major difference between these two kinds of test runs in this study is in their impact on the local sectors.

In this study, since all the local sectors are given identical taxation capacities, scenario test runs, more or less, have the same impact on all local sectors in all of the models. For instance, the scenario test run of decreasing the local taxation range decreases tax income in all local sectors and results in decreased spending levels and increased state aid in all of the local sectors. That is, under the scenario test runs, all local sectors are affected in a similar way; and only the degree of impact may differ. Under these kinds of test runs, unless the model types are extremely different, the general model behaviors will not be sensitive. (The measurements using standard deviations have shown sensitive behaviors in these situations, because the variables are influenced more by their original data distribution patterns than by model dynamics.)

Under the policy test runs, the local sectors are affected differently. Part of the local sectors may benefit while another part of them suffers, under one test run. For example, under the policy test run of implementing a COEI in the distribution formula, only the local sectors with COEIs higher than the fractions of the last year's aids over the general state aids will benefit and the other local sectors will suffer. Under the policy test runs of increasing floor-aid amount, the local sectors that are qualified for the aid increase will benefit at the expense of the unqualified local sectors. This is also true of the policy test run of increasing ceiling aid.

Study Implications

This study has performed a kind of sensitivity test run that examines the model aggregation assumption and is assumed untestable through the other sensitivity test runs presently used in the field of system dynamics. This sensitivity is critical not only to the sample model conclusion but also to all of the system dynamics models using an aggregate modeling approach. The findings imply that the presumption of how the analyzed population should be presented has a definite influence on the model result. This identified model sensitivity questions the validity of the modeling approach and the applicability of the system-dynamics technique to policy analysis. Since sensitivity test runs with different aggregation schemes will increase the already vast amount of the modelers' work and delay the establishment of the model, systematic research to explore the patterns of and reasons for model sensitivity in general system-dynamics models is suggested before individual modelers can apply this type of sensitivity test run to regular modeling procedures.

As discussed in the beginning of the paper, in system dynamics modeling, the concepts of aggregation of individuals and of variables are mixed. Without a clear concept, common model sensitivity test runs on model structure cannot test the sensitivity of assumptions on the aggregation of individuals. The occasions that have proved to lead to model sensitivities cannot be discovered by test runs for model variables or structural expansion.

In the field of system dynamics, another common sensitivity test run for building confidence in models is parameter testing. System-dynamics models are generally considered to be less sensitive to parameter changes but are structurally dominated (Richardson & Pugh, 1981, p. 285). It is argued here that parameter sensitivity testing is similar to the scenario test runs in this study, in which, unless the model types are extremely different, the models are insensitive to the changes. The behaviors of the standard-deviation measurements are examples. The different impacts on the subpopulations and the possible competition with the aggregated sectors cannot be tested in this kind of sensitivity test runs.

This study raises an alert also to other modeling techniques using an aggregate simulation approach. System dynamics is known to be less sensitive to parameter changes than are the other modeling techniques. If such a technique should fail in a test using different aggregation schemes of a single data set, then other techniques, which may be more dominated by data values, are likely to have similar problems.

When reexamining the validity of aggregate simulation

models, a present trend in large-scale models is to aggregate or disaggregate model stuctural variables (Greenberger, et al., 1976, pp. 227-230); however, reexamination of the assumptions regarding the aggregation of individuals may reveal surprising insights into the problems that have been buried in the available model structure by selected aggregation assumptions.

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Table 1.

Eleven test runs applied in the study

1. Decreasing local taxation range
2. Decreasing state taxation range
3. Increasing needed educational expenditure per pupil
4. Increasing needed local noneducational expenditures
5. Increasing needed state noneducational expenditures
6. Decreasing pupil enrollment
7. Implementing a cost-of-education index in the operating aid formula
8. Eliminating floor aid
9. Increasing floor aid to $600
10. Increasing floor aid to $800
11. Increasing ceiling aid to $1800

Table 2.

Twenty behavioral indices (variables) for model-behavior observation

Sector	Variable
State	Tax rate
	Dollars to other expenditures
	Dollars to operating aid
	Adequacy of other expenditures
	Adequacy of operating aid
New York City	Educational expenditure per pupil
	Tax rate
	State aid fraction
	Adequacy of educational expenditure
	Adequacy of other expenditures
Remaining local sectors:	Average educational expenditure per pupil
	Average tax rate
	Standard deviation of tax rates
	Standard deviation of eduational expenditures
	Standard deviation of state aids
All local sectors (including NYC):	Average educational expenditure per pupil
	Average tax rate
	Standand deviation of tax rates
	Standard deviation of educational expenditures
	Standard deviation of state aids

Table 3.

Summary of Sensitive Model Result

Variable	Testing										
	Decr State Tax Range	Decr Local Tax Range	Incr Needed Edu Exp	Incr Needed Local Oth Exp	Incr Needed State Oth Exp	Decr Local Enroll	Imple- ment a COEI	Elimi- nate Floor Aid	Incr Floor Aid to $600	Incr Floor Aid to 800	Incr Ceiling Aid
State Sector:											
Tax Rate							X		X	X	X
$ to Other Exp							X		X	X	X
$ to Operating Aid							X		X	X	X
Adeqaucy of Oth Exp										X	
Adequcy of Op Aid							X		X		X
New York City:											
Edu Exp Per Pup							X		X		X
Tax Rate							X		X		X
State Aid Fraction							X		X		X
Adequcy of Edu Exp							X		X		X
Adequcy of Oth Exp							X				X
Local Sector: (excl. NYC)											
Average Edu Exp							X		X	X	X
Average Tax Rate							X		X	X	X
Stan Dev Tax Rate					X		X		X		X
Stan Dev Edu Exp		X			X		X	X	X		X
Stan Dev Sta Aid		X					X	X	X		X
Local Sector: (incl NYC)											
Average Edu Exp							X		X	X	X
Average Tax Rate							X		X	X	X
Stan Dev Tax Rate		X			X	X	X		X	X	X
Stan Dev Edu Exp	X	X		X	X	X	X		X	X	X
Stan Dev Sta Aid							X	X	X		X

Table 4.
Coded model behaviors under four testings

Testing: Implementing a Cost of Education Index

Model Variable		COUNTY	SMSA	HIER	HIERa	NON-HIER	RAN	
1. State Sector								
Tax Rate	S*	0	0	+	+	+	0	SEN
	L*	0	0	+	+	+	+	
Dollars to Other Exp	S	0	0	+	+	+	0	SEN
	L	0	0	0	0	0	0	
Dollars to Oper Aid	S	0	0	+	+	+	0	SEN
	L	0	0	+	+	+	+	
Adequacy of Oth Exp	S	0	0	0	0	0	0	
	L	0	0	0	0	0	0	
Adequacy of Oper Aid	S	0	0	-	-	-	0	SEN
	L	0	0	-	-	-	0	
2. New-York-City Sector								
Edu Exp Per Pupil	S	0	0	+	+	+	+	SEN
	L	0	0	+	+	+	0	
Tax Rate	S	0	0	-	-	-	-	SEN
	L	0	0	-	-	-	-	
State Aid Fraction	S	0	0	+	+	+	+	SEN
	L	0	0	+	+	+	+	
Adequacy of Edu Exp	S	0	0	+	+	+	+	SEN
	L	0	0	+	0	+	0	
Adequacy of Oth Exp	S	0	0	-	-	-	-	SEN
	L	0	0	+	0	0	0	
3. Remining Local Sectors								
Average Edu Exp Per Pup	S	0	0	-	-	-	-	SEN
	L	0	0	-	-	-	0	
Average Tax Rate	S	0	0	+	+	+	+	SEN
	L	0	0	+	+	+	+	
Stan Dev of Tax Rates	S	0	0	+	+	+	+	SEN
	L	0	0	+	+	-	+	
Stan Dev of Edu Exp	S	0	0	+	+	+	+	SEN
	L	0	0	+	+	+	0	
Stan Dev of State Aids	S	0	0	-	-	-	-	SEN
	L	0	0	+	-	-	-	
4. All Local Sectors (including NYC)								
Average Edu Exp Per Pup	S	0	0	-	-	-	-	SEN
	L	0	0	-	0	-	0	
Average Tax Rate	S	0	0	+	+	+	+	SEN
	L	0	0	+	+	+	+	
Stan Dev of Tax Rates	S	0	0	-	-	-	-	SEN
	L	0	0	-	-	-	-	
Stan Dev of Edu Exp	S	0	0	-	-	-	-	SEN
	L	0	0	+	+	+	0	
Stan Dev of State Aids	S	0	0	-	-	-	-	SEN
	L	0	0	+	+	-	-	

* S stands for short-term behaviors; L stands for long-term behaviors.

0 stands for no change, + stands for increase, and - stands for decrease.

(Table 4-2)

Testing: Increasing Floor Aid to $600

Model Variable		COUNTY	SMSA	HIER	HIERa	NON-HIER	RAN	
1. State Sector								
Tax Rate	S	0	0	+	+	+	0	SEN
	L	0	0	+	+	+	0	
Dollars to Other Exp	S	0	0	+	+	+	0	SEN
	L	0	0	0	0	0	0	
Dollars to Oper Aid	S	0	0	+	+	+	0	SEN
	L	0	0	+	+	+	0	
Adequacy of Oth Exp	S	0	0	0	0	0	0	
	L	0	0	0	0	0	0	
Adequacy of Oper Aid	S	0	0	-	-	-	0	SEN
	L	0	0	0	0	0	0	
2. New-York-City Sector								
Edu Exp Per Pupil	S	0	0	-	-	-	0	SEN
	L	0	0	+	0	0	0	
Tax Rate	S	0	0	+	+	+	0	SEN
	L	0	0	+	+	+	0	
State Aid Fraction	S	0	0	-	-	-	0	SEN
	L	0	0	-	-	-	0	
Adequacy of Edu Exp	S	0	0	-	-	-	0	SEN
	L	0	0	0	0	0	0	
Adequacy of Oth Exp	S	0	0	0	+	0	0	
	L	0	0	0	0	0	0	
3. Remining Local Sectors								
Average Edu Exp Per Pup	S	0	0	+	+	+	0	SEN
	L	0	0	-	0	0	0	
Average Tax Rate	S	0	0	-	-	-	0	SEN
	L	0	0	-	-	-	0	
Stan Dev of Tax Rates	S	0	0	+	+	+	0	SEN
	L	0	0	+	+	+	0	
Stan Dev of Edu Exp	S	0	0	+	+	+	0	SEN
	L	0	0	+	0	0	0	
Stan Dev of State Aids	S	0	0	-	-	-	0	SEN
	L	0	0	-	-	-	0	
4. All Local Sectors (including NYC)								
Average Edu Exp Per Pup	S	0	0	+	+	+	0	SEN
	L	0	0	-	0	0	0	
Average Tax Rate	S	0	0	-	-	-	0	SEN
	L	0	0	-	-	-	0	
Stan Dev of Tax Rates	S	0	0	+	+	+	0	SEN
	L	0	0	+	+	+	0	
Stan Dev of Edu Exp	S	0	0	+	+	+	0	SEN
	L	0	0	+	0	0	0	
Stan Dev of State Aids	S	0	0	-	-	-	0	SEN
	L	0	0	-	-	-	0	

(Table 4-3) 29

Testing: Increasing Floor Aid to $800

Model / Variable		COUNTY	SMSA	HIER	HIERa	NON-HIER	RAN	
1. State Sector								
Tax Rate	S	+	+	+	+	+	0	SEN
	L	+	+	+	+	+	+	
Dollars to Other Exp	S	+	+	+	+	+	0	SEN
	L	0	0	0	0	0	0	
Dollars to Oper Aid	S	+	+	+	+	+	0	SEN
	L	+	+	+	+	+	+	
Adequacy of Oth Exp	S	+	+	+	+	+	0	SEN
	L	0	0	0	0	0	0	
Adequacy of Oper Aid	S	-	-	-	-	-	-	
	L	-	-	-	-	-	-	
2. New-York-City Sector								
Edu Exp Per Pupil	S	+	+	+	+	+	+	
	L	+	+	+	+	+	+	
Tax Rate	S	-	-	-	-	-	-	
	L	-	-	-	-	-	-	
State Aid Fraction	S	+	+	+	+	+	+	
	L	+	+	+	+	+	+	
Adequacy of Edu Exp	S	+	+	+	+	+	+	
	L	+	+	+	+	+	+	
Adequacy of Oth Exp	S	-	-	-	-	-	-	
	L	0	0	0	0	0	0	
3. Remining Local Sectors								
Average Edu Exp Per Pup	S	+	+	+	+	+	+	
	L	-	-	-	-	-	-	
Average Tax Rate	S	+	+	-	-	-	+	SEN
	L	+	+	-	-	+	+	
Stan Dev of Tax Rates	S	+	+	+	+	+	+	
	L	+	+	+	+	+	+	
Stan Dev of Edu Exp	S	+	+	+	+	+	+	
	L	+	+	+	+	+	+	
Stan Dev of State Aids	S	-	-	-	-	-	-	
	L	-	-	-	-	-	-	
4. All Local Sectors (including NYC)								
Average Edu Exp Per Pup	S	-	-	+	+	+	-	SEN
	L	-	-	-	-	-	-	
Average Tax Rate	S	+	+	-	-	-	+	SEN
	L	+	+	-	-	-	+	
Stan Dev of Tax Rates	S	-	-	+	+	+	-	SEN
	L	-	-	+	+	-	-	
Stan Dev of Edu Exp	S	+	+	+	+	+	-	SEN
	L	+	+	+	+	+	-	
Stan Dev of State Aids	S	-	-	-	-	-	-	
	L	-	-	-	-	-	-	

(Table 4-4) 30

Testing: Increasing Ceiling Aid

Models: / Variables:		COUNTY	SMSA	HIER	HIERa	NON-HIER	RAN	
1. State Sector								
Tax Rate	S	-	-	-	-	-	0	SEN
	L	-	-	-	-	-	+	
Dollars to Other Exp	S	0	0	-	-	-	0	SEN
	L	0	0	0	0	0	0	
Dollars to Oper Aid	S	-	-	-	-	-	-	SEN
	L	-	-	-	-	-	+	
Adequacy of Oth Exp	S	0	0	0	0	0	0	
	L	0	0	0	0	0	0	
Adequacy of Oper Aid	S	+	+	+	+	+	+	SEN
	L	0	0	-	-	-	-	
2. New-York-City Sector								
Edu Exp Per Pupil	S	-	-	+	+	+	+	SEN
	L	0	0	+	+	+	+	
Tax Rate	S	+	+	-	-	-	-	SEN
	L	+	+	-	-	-	-	
State Aid Fraction	S	-	-	+	+	+	+	SEN
	L	-	-	+	+	+	+	
Adequacy of Edu Exp	S	-	-	+	+	+	+	SEN
	L	0	0	+	+	+	+	
Adequacy of Oth Exp	S	+	+	-	-	-	-	SEN
	L	0	0	0	0	0	0	
3. Remining Local Sectors								
Average Edu Exp Per Pup	S	+	+	-	-	-	-	SEN
	L	0	0	-	-	-	-	
Average Tax Rate	S	-	-	+	+	+	+	SEN
	L	-	-	+	+	+	+	
Stan Dev of Tax Rates	S	-	+	-	-	-	-	SEN
	L	-	+	-	-	+	-	
Stan Dev of Edu Exp	S	+	+	-	-	-	-	SEN
	L	0	0	-	-	-	-	
Stan Dev of State Aids	S	-	-	+	+	+	+	SEN
	L	-	-	+	+	+	+	
4. All Local Sectors (including NYC)								
Average Edu Exp Per Pup	S	+	+	-	-	-	-	SEN
	L	0	0	-	-	-	-	
Average Tax Rate	S	-	-	+	+	+	+	SEN
	L	-	-	+	+	+	+	
Stan Dev of Tax Rates	S	+	+	-	-	-	-	SEN
	L	-	+	-	-	-	-	
Stan Dev of Edu Exp	S	+	+	-	-	-	-	SEN
	L	0	0	-	-	-	-	
Stan Dev of State Aids	S	-	-	+	+	+	-	SEN
	L	-	-	+	+	+	-	