

# The Ford Method: A Sensitivity Analysis Approach

Jinjing Huang, Enda Howley, Jim Duggan

Department of Information Technology, National University of Ireland, Galway

Email: colorfulginger@hotmail.com, enda.howley@nuigalway.ie, jim.duggan@nuigalway.ie

## Abstract

*In dynamic models, a system behavior is determined by the interaction of its feedback loops. The challenge for system dynamics modellers is to identify these loops, and also understand, over the runtime of a model, which loops dominate system behavior. The Ford method is a procedure that identifies changes in atomic behavior patterns in the presence, and absence, of feedback loops, in order to identify loop dominance for a specific time interval. For a candidate feedback loop, dominance is calculated based on setting the loops control variable to a constant value. Our approach proposes a variation on this method. Using sensitivity analysis, we explore a wider search space around a range of possible values for the control variable, and thereby the value of the loop gain. The outcome of this analysis is a richer set of loop dominance analysis for each atomic behavior pattern in the model. The sensitivity of various feedback loops is measured as an indicator of dominance. This approach has the potential to be selected for modellers and policy to analyze the structure-drive-behavior dynamic systems. The approach is illustrated through an analysis of the Yeast Model.*

## 1 Introduction

A core premise of system dynamics is that the behavior of a system is determined by its feedback structure. This feedback structure arises through the interactions of the physical and institutional structure of the system with the decision-making processes of the agents acting within it (Sterman, 2000). For a given problem of interest, the link between feedback structure and system behavior are captured early in the modeling process, through the formulation of a dynamic hypothesis. In the later stages of model development, once a simulation model is constructed, the resulting mathematical model can be formally analyzed in order to provide insights into the strengths and influences of the distinct feedback structures. Such formal analysis remains an important and challenging area in system dynamics and can be classified into two types, behavior-based methods and structure-based methods.

A prominent behavior-based method was proposed by Ford (1999), who writes that the purpose of feedback loop dominance analysis is to identify feedback structures that dominate behavior. Ford also argues for a rigorous approach to formal analysis, and presents a procedure, based on changes in the three atomic behavior patterns, to signal loop dominance for a given time interval. Our approach proposes an extension of Ford's method. Using sensitivity analysis, we generate a broader set of results around a range of possible values for the control variable, and thereby assess the impact of these changes to the behavior of the variable of interest. This approach suggests it could be used to select the candidate unique edge for the Ford method when more than one unique edge exist in the a feedback loop. It also provides an ideal basis to identify the dominant loop by evaluating the sensitivity over various loops. Two criteria are addressed in the assessment of

sensitivity. Furthermore, we also suggest a new criteria to quantify the loop dominance to the traditional Ford method.

The structure of the paper is as follows. In section 2, we summarize the existing approaches to formal analysis and provide a description to the Ford method. In section 3, we will present a detailed description of a variance of the Ford method relating to the use of sensitivity analysis. We also suggest a potential contribution that the sensitivity analysis could afford to make the Ford method more robust. Subsequently, we will illustrate the benefits of this novel approach through showing an example using the well known Yeast model in section 4. Finally, we will outline our conclusions and future research.

## 2 Related Research

In the area of formal analysis, there are three popular methods currently: the Ford method, pathway participation metrics (PPM) and eigenvalue elasticity analysis (EEA). As we said above, the former one is behavior-based while the latter ones are structure-based.

EEA method is a mathematic rigorous approach in system dynamics. Graham (1977) studied the relationship between system structure and system behavior. Forrester (1982) established the foundation for EEA. He made attempt to identify the important causal structures that underlie the cyclic behavior. He proposed a method using eigenvalue elasticity for identifying the dominant compact link (an imaginary link connects two states). However, this concept existed only in theory since researchers could not formally establish a complete feedback loop set for a system. Kampmann (1996) developed a solution to this from graph theory and suggested the independent loop set (ILS). It significantly reduced the number of feedback loops and offered a mathematical means to calculate the loop elasticity from causal link elasticity (In contrast to compact link, causal link is a normal link in a model). The way to obtain the causal link elasticity from the compact link elasticity is explained in details in (Gawad et al., 2005). Later, Oliva (2004) proposed a strategy to identify ILS based on the shortest loops, shortest independent loop set (SILS). This SILS algorithm does generate a unique loop set. Saleh (2000) linearized non-linear model and allowed EEA to apply on non-linear models. Another important contribution that he made is to allow EEA to produce assessments to the level of a particular variable rather than the level at the entire model. Guneralp (2005) improved the traditional EEA based on Saleh's work by introducing a new measure that allows consideration of the combination of all behavioral modes in determining the dominant feedback loops.

Mojtahedzadeh (1997) has proposed the PPM as a formal analysis tool distinguished itself by detecting any influential substructure besides the loop. These structures involve links of causal structure between two states, pathways. It uses seven behavior patterns to classify the system behavior. A single phase of behavior for a selected variable is a time interval where the variable retains its behavior pattern. The basic idea behind PPM is to calculate how much the net flow could change given a small change in the selected variable. It constructs the most influential pathway by adding the causal structure who contributes most to the total changes in the net flow. An existing software: Digest is used to implement and display the PPM result (Mojtahedzadeh, 2001; Mojtahedzadeh et al., 2004). One shortage of this method is that it is unable to deal effectively with oscillations (Kampmann and Oliva, 2005).

The Ford method is a prominent approach to shed light on the structure-drive-behavior dynamic

system. One contribution Ford made was that he improved the definition of atomic behavior patterns, classified them into three categories: exponential, logarithmic and linear. The formulation of the atomic behavior pattern is called behavior pattern indicator (BPI). For more details refer to (Ford, 1999). Richardson (1995) pointed out “The shifts in loop dominance that occur in nonlinear system arise naturally as changes in the sign of dominant polarity”. Thereby, dominance interval could be exactly determined. It is a time interval within which the atomic behavior pattern remains consistent. Ford’s another contribution is to isolate a candidate loop from other loops to see the impact on the variable of interest. He isolated the loop by deactivating its control variable. The control variable is not a variable in other feedback loops. After identifying the control variable, set it to be constant only in this loop, then simulate the changed model in each time interval.

The key steps of the Ford method are listed as follows:

1. Identify the variable of interest.
2. Divide the simulation length into time intervals in which different feedback loop may dominate. This division is conducted on the change of atomic behavior pattern. The time interval is also called phase.
3. Find a control variable for each loop. Control variable is a variable that is not in any other loops.
4. Deactivate the feedback loop by setting the control variable to be a constant. The constant is the same value as it is at the deactivating point. We also refer this value to default value.
5. Simulate the behavior of the variable of interest over the time interval with the candidate feedback loop deactivated. Repeat this process for every feedback loop in each period.
6. Identify dominant loops by checking if the atomic behavior pattern changes in each time interval.
7. If no dominant loops are identified, proceed the shadow feedback loops analysis.

## 2.1 Improved Ford method

To gain a deeper understanding of the Ford method and our variance method, we elaborate some issues in the application. One challenge to use the Ford method is to identify the feedback loops. No explicit method has been developed to select the feedback loops for the Ford method. A huge number of feedback loops could be detected in a system even in a small size model (Kampmann, 1996). In a model with  $n$  state variables and  $p$  auxiliary variables under the condition that it is maximally connected, the number of loops grows to  $2^n p * (n - 1)!$ . Even if in realistic models, which have far fewer links, it is still too time-consuming to analyze all of them. Actually, the selection of feedback loops in a complex system is already discussed by Oliva and Mojtahedzadeh (2004). The ideal loop set should represent the core structure of the system with an appropriate number of loops. Phaff (2008) proposed an automated version of the Ford method. He adopted the SILS as the loop set. This is a loop set originally developed for EEA. Whereas it is also applicable to the Ford method.

Another potential problem is the “control variable”. The first step of deactivating a candidate loop is to find the control variable. So once the loop set has been determined, we have to ensure that every loop has a control variable. This is a major constraint and is difficult to satisfy. Phaff (2008) proposed to use a unique edge instead of a control variable: deactivating a unique edge in a candidate feedback loop to deactivate that loop. The unique edge is an edge not shared by other loops. It is a similar concept of the control variable. The unique edge constrain is easier to satisfy. If a control variable exists, there must be at least two unique edges in this loop, but conversely a unique edge does not imply the existence of a control variable.

Finally, Ford use the change of atomic behavior pattern to determine if the candidate loop dominates. It only provides a binary answer to the analyst: yes or no. There is a need to visually demonstrate the degree of the dominance and its fluctuation over time. An alternative measure of dominance proposed here to show the relative dominance of various loops.

In section 3, we adopt the automated version of the Ford method.

### **3 The Sensitivity Analysis Approach**

Sensitivity analysis is used to determine how “sensitive” a model is to changes in the value of the parameters of the model and to changes in the structure of the model (Breierova and Choudhari, 1996). In this paper, we focus on the parameter sensitivity. Parameter sensitivity is usually performed with setting different values to the parameter to examine changes of a model’s behavior. By showing how the model responds to the change of the parameter, we could find the sensitive parameter to the model under the given set of conditions.

Before we present the proposed method, we still have to solve another problem in sensitivity analysis. We need to compare the sensitivity among different control variables, but it is easily to observe that variables in a model would be of different scale. How to fairly design a boundary? This boundary is used to determine the range of possible values for our sensitivity analysis on a specific control variable. We give up the idea that multiply the control variable’s default value by a factor as its upper bound and divide it by a factor as its lower bound. First, any selected factor would be considered arbitrary. There would be argues over this factor. Second, it causes difficulties to explain the analysis outcome in a specific model. In our sensitivity analysis of the Ford method, all the variables are evaluated in the simulation of the reference model, and phases are divided. We determined the control variable’s boundary by looking at its range in the relevant phase. Thus, the boundary region is identified by the behavior of the variable of interest and is not an arbitrary range. We believe it is a fair representation of a variable’s boundary for sensitivity analysis to a specified phase and model.

The procedure of sensitivity approach is as follows:

1. Select variable of interest in the model.
2. Set up SILS and identify all unique edges for each feedback loop in SILS. Identify the unique edges and the control variables.
3. Simulate the behavior of the variable of interest, use the Ford method to partition the phases.
4. Identify the boundary for all control variables from the simulation of the reference model.

5. Apply the sensitivity analysis. Simulate the behavior of the variable of interest over the divided phase with loop deactivated replacing the default value with a number of values in its boundary. Repeat this step for a number of values for individual control variable in each phase.
6. Display the results from the sensitivity analysis and integrate these findings into a single graphical representation for a clearer understanding of their implications.
7. If there are more than one unique edges in a feedback loop, choose the most sensitive one as the candidate edge to deactivate the feedback loop.
8. Using the default value of the selected control variable to simulate the behavior of the variable of interest. Repeat this step for every control variable in each phase.
9. Demonstrate the outcome from the Ford method by a new assessment of dominant and validate the conclusion from sensitivity analysis.

We propose a series of measurement of dominance and sensitivity for our approach. We first introduce a new assessment to determine a dominant loop, then, we will unfold a method for evaluating the sensitivity under different benchmarks. These extensions help to better demonstrate the analysis result.

The Ford method judges a dominant loop by checking the behavior pattern of the variable of interest after deactivating the candidate loop. If the behavior pattern is consistent with it in the reference model in the relevant phase, it would not be considered as a dominant loop. This criteria only indicates a binary answer. We have poor information on how dominant that loop is and how the other less significant loops perform throughout the time interval.

Our measurement of the dominance shows more descriptive information and affords to reveal the fluctuation of loop dominance over time. We use the behavior to simply the saying of the behavior of the variable of interest in the following explanation. First, we have the variable of interest  $\mathbf{v}$  in the reference model evaluated throughout the time  $\mathbf{v}_0^t$ , the subscript 0 represents the reference model. We also record its values in the simulation each time a feedback loop is deactivated. We assume there are  $n$  feedback loops. For a specific loop  $i$ , the value is given by  $\mathbf{v}_i^t$ . The difference between the behavior in the deactivated model with a specified loop  $i$  and the reference model is estimated by

$$\Delta \mathbf{v}_i^t = \mathbf{v}_i^t - \mathbf{v}_0^t \quad (1)$$

Its variance  $S$  is computed

$$S_i^t = (\Delta \mathbf{v}_i^t)^2 \quad (2)$$

The comparison of the influences to the behavior of the variable of interest between different feedback loops becomes easier when the difference is rescaled. The portion an individual feedback loop devotes to the behavior could be obtained by a series of steps. We calculate the relative contribution  $c$  that loop  $i$  devoted to the overall behavior of variable of interest at time  $t$  in Equation 3. The rescaled relative contribution  $r$  from loop  $i$  at time  $t$  is evaluated by dividing the sum relative contribution in Equation 4. The relative contribution of an individual loop varies between 0 and 1. It also could be expressed by the word “influence factor”. Rescaling the contribution of an individual feedback loop is a more straightforward way to present the analysis result.

$$c_i^t = \sqrt{\frac{S_i^t}{\sum_{m=1}^n S_m^t}} \quad (i = 1 \dots n) \quad (3)$$

$$r_i^t = \frac{c_i^t}{\sum_{m=1}^n c_m^t} \quad (i = 1 \dots n) \quad (4)$$

Our measurement shows that loop dominance can be assessed and compared on a scale evolving with time, thereby avoiding any binary classifications which can sometimes mask the underlying behaviors. A loop which is considered not dominant under the traditional criteria can now be shown to have some degree of dominance relative to the other loops in a model. Conversely a loop classified as dominant can be shown on a scale and compared with its peers. We can make use of the visualized result to identify the shadow loops as well. The loops with relatively larger dominance would be selected as the dominant loops.

We now describe two alternative measurements of the sensitivity. For a certain model, we assume there are  $M$  control variables,  $T$  unique edges and  $T \geq M$ . The amount of the possible values for a control variable is  $N$ . The behavior of the variable of interest  $v$  in the model deactivated by unique edge  $e$  with  $i^{\text{th}}$  value at time step  $t$  is expressed in notation  $v_{e,i}^t$ . Here, we provide two criteria to measure the sensitivity of a variable to an edge. They compare with different benchmarks. Measurement 1 makes use of the average behavior from  $N$  different behaviors in sensitivity analysis while measurement 2 takes the behavior in the deactivated model using the default value as its benchmark. Note that these behaviors are from the simulation with a same edge deactivated. The average behavior of  $N$  behaviors is described in Equation 5. The notation of the behavior with default value is:  $v_{e,0}^t$ . The subscript 0 represents the default value.

$$\bar{v}_e^t = \frac{\sum_{i=1}^N v_{e,i}^t}{N} \quad (5)$$

The difference from a specified behavior to the default value behavior and the average value behavior are calculated in Equation 6 and Equation 7 respectively.  $v_{e,i}^t$  is the behavior with the unique edge  $e$  deactivated using test value  $i$  at time step  $t$ . The subscript  $i$  represents any value that assigned to do the sensitivity analysis within the corresponding control variable's boundary.

$$\Delta v_{e,i}^t = v_{e,i}^t - v_{e,0}^t \quad (6)$$

$$\Delta v_{e,i}^t = v_{e,i}^t - \bar{v}_e^t \quad (7)$$

The calculation of the overall variance  $S$  for  $N$  sensitivity analyse with a unique edge  $e$  deactivated is shown in Equation 8 and the average value of it is then obtained by dividing the overall variance with  $N$  in Equation 9. This represent the average variance of edge  $e$  sensitivity with respect to the variable of interest  $v$ .

$$S_e^t = \sum_{i=1}^N (\Delta v_{e,i}^t)^2 \quad (8)$$

$$\bar{S}_e^t = \frac{S_e^t}{N} \quad (9)$$

Finally, we adopt the standard average variance  $\sigma$  presented in Equation 10 as our criteria to compare the sensitivity between different edges throughout all the phases.

$$\sigma_e^t = \sqrt{\bar{S}_e^t} \quad (10)$$

## 4 Illustration of Approach

We choose Yeast model to illustrate this method. The Yeast model is a well known model used in system dynamics analysis. It is a second order non-linear model and an classical example of overshoot-and-collapse dynamics. The stock-flow diagram is presented in Figure 1. The equation of Yeast model is in appendix. It has four feedback loops in SILS and only L1 (Cells-births-Cells) is positive, the polarity of each loop is marked in Figure 1. The definition and the calculation of the loop polarity is elaborated in (Richardson, 1995; Sterman, 2000). The polarity of the loop is a typical symbol that shows the relationship between the structure and the behaviors.

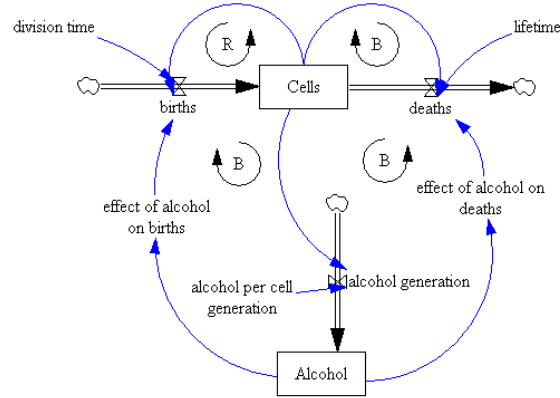


Figure 1: stock-flow diagram for Yeast model

We carry out our approach step by step as follows:

1. Choose the variable of interest: Cells.
2. Set up SILS as its loop set. There are four loops in SILS in Yeast model (Figure 1).

**L1** Cells-births-Cells

**L2** Cells-deaths-Cells

**L3** Cells-Alcohol generation-Alcohol- effect alcohol on birth (*eab*)-births-Cells

**L4** Cells-alcohol generation-Alcohol- effect alcohol on death (*ead*)-deaths-Cells

Identify all the unique edges in SILS. We identify two unique edges in *L3* and *L4* respectively. The variables *eab* is exclusively belonging to *L3*, thereby deactivating either unique

edge associated with it them makes no difference to assess the loop's dominance. It is identical to variable *ead*. The candidate unique edges are selected and presented in Table 2.

I	II	III	IV
exponential growth	logarithmic growth	exponential decay	logarithmic decay
0, 50.84	50.84, 65.49	65.49, 74.52	74.52, 90

Table 1: Time intervals over simulation length and behavioral patterns

L1	L2	L3	L4
Cells→births	Cells→deaths	eab→births	ead→deaths

Table 2: Candidate unique edges in each loop

3. Simulate the behavior of the variable of interest in the reference model, identify the time intervals. Figure 3 plotted the behavior of cell over 90 units. Table 1 list the time intervals and its corresponding behavior patterns. The equations for the Yeast model are listed in Table 3.

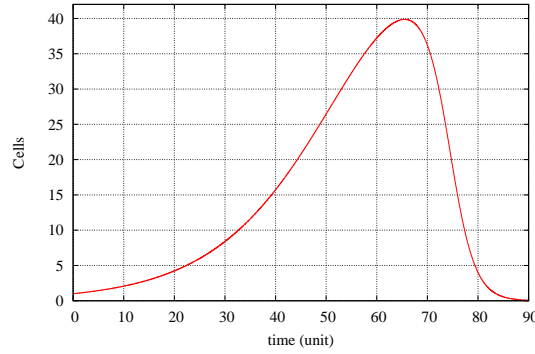


Figure 2: Behavior of Cells over the simulation time

4. Apply sensitivity analysis on the control variable in each phase. So we display the behavior of all the control variables in the reference model in Figure 3 and 3. Then the boundary for individual control variable is determined and given in Table 4. We mark the boundary as “*B*”, *N* represents the number of test values. A series number of values of the control variable is calculated by Equation 11. We use 100 test values across its boundary. These test values are used to substitute for the default value in the deactivated model in turns. Therefore, we apply sensitivity analysis on each unique edge 100 times across all the phase.

$$\text{Min}(B) + \frac{\text{Max}(B) - \text{Min}(B)}{N} \times k \quad (k = 0 \dots N - 1) \quad (11)$$



timestep=0.01 ; simulation length=90  
 Stocks:  
 Cells=INTEG(births-deaths, 1)  
 Alcohol=INTEG(alcohol generation, 0)  
 Flows:  
 births=Cells/division time\* effect of alcohol on birth  
 deaths=Cells/life time\* effect of alcohol on death  
 alcoholgeneration=Cells\*alcohol per cell generation  
 Auxiliaries:  
 effectofalcoholonbirths=(-0.1\*Alcohol)+1.1  
 effectofalcoholondeaths=EXP(Alcohol-11);  
 life time=30 ; division time=15  
 alcohol per cell generation=0.01

Table 3: Equations for Yeast model

	Cells	eab	ead
I	(1, 27.45)	(1.1, 0.646)	(0, 1.575E-3)
II	(27.45, 39.87)	(0.646, 0.133)	(1.575E-3, 0.1846)
III	(39.87, 21.12)	(0.133, -0.175)	(0.185, 5.768)
IV	(21.12, 0)	(-0.175, -0.25)	(5.768, 11.78)

Table 4: Boundary for control variables in the Yeast model

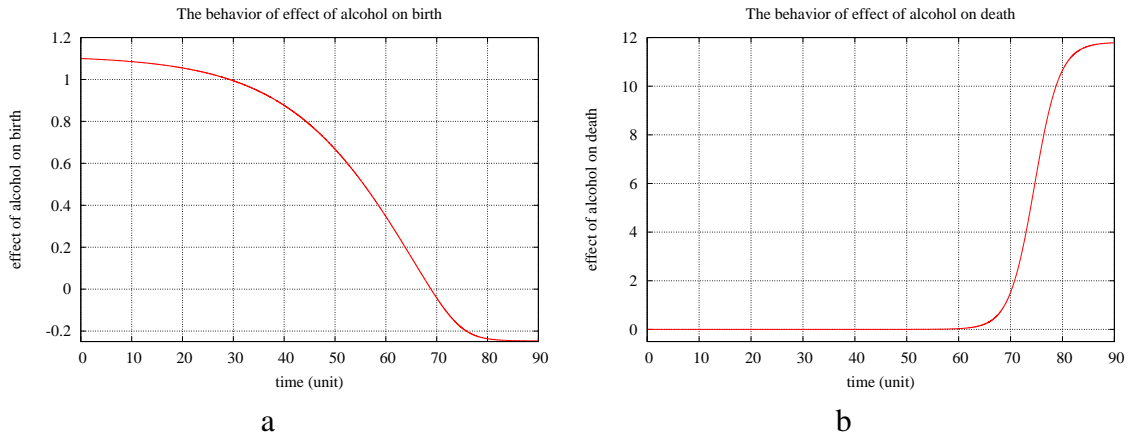


Figure 3: Behavior of variables in reference

5. Analysis of the experiment result. There are four feedback loops in the yeast model in SILS. In each phase we plot a set of four figures to show the behavior of Cells under the sequential deactivation of four unique edges. Figure 4(a) depicts the behavior of Cells under the deactivation of *edge* Cells $\rightarrow$ births with a certain of different values on the control variable Cells in phase *I*. We plot only 10 curves for the clarity. Similarly, in Figure 4(b), we can identify an almost identical experiment involving the loop *L2*. Subsequently, results are presented from a sensitivity analysis involving *L3* and *L4*, in Figure 4(c) and Figure 4(d) respectively. The “default” in the legend is short for the “default value”.

After generally describing the displayed graphs, let us focus on Figure 4(a), these curves demonstrate how the variable of interest responds to the change of values on the control variable throughout this phase. Compared it with other three peers in the Figure 4(b) and Figure 4, it is easy to observe that Cells is the most sensitive to the change of the control variable via the *edge* Cells $\rightarrow$ birth. We can not simply address the variable of interest is sensitive to the control variable, because the *L1* and the *L2* both share the same control variable (Cells), but they exhibit completely different sensitivity. To be more precise, the impact results from change of the control variable is propagated via the unique edge and the relevant loop. There is almost no change on the behavior of Cells no matter how the control variable varies across their boundaries in *L2* and *L4*. As to the control variable *eab* in *L3*, a number of variance varies the behavior of the Cells but it is not as significant as the *L1*.

Figure 5(a) is plotted under the proposed measurement *I* of the sensitivity. The standard variance of edge sensitivity is an indicator of the degree of sensitivity. This quantitative indicator presents how significant the edge influences the variable of interest. In a conclusion, Cells is most sensitive to *L1* and insensitive to *L2* and *L4* while Cells gradually increased its sensitivity to *L3*. Based on this knowledge, we infer that the *L2* and *L4* can not be a dominant loop and *L1* has a high possibly to be the dominant loop in the current phase.

The outcome of the the behavior with edges deactivated in phase *II* is illustrated in Figure 6, and the overview of different edges sensitivity is in 5(b). Edge Cells $\rightarrow$ death in *L2* remains almost no impact on Cells as in phase *I*. No matter how the control variable changes across its boundary, the variable of interest exhibits almost the same trajectory. Hence, the variable

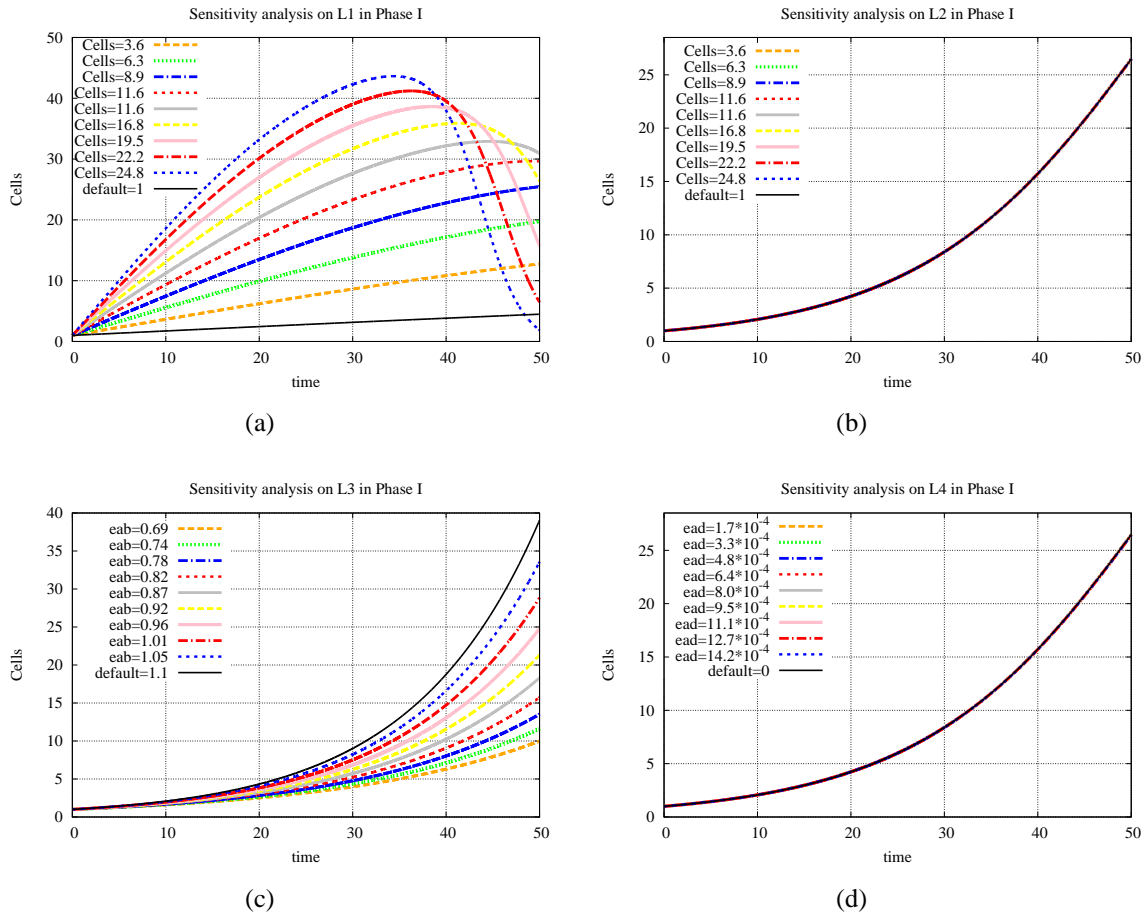
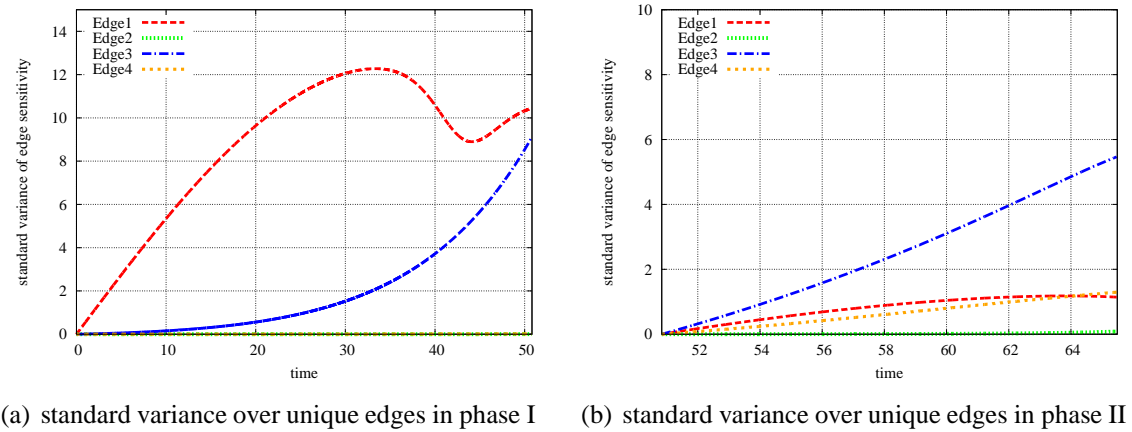
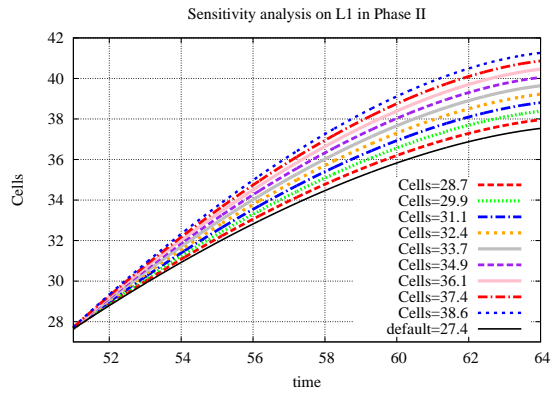


Figure 4: Sensitivity analysis on edges in phase I

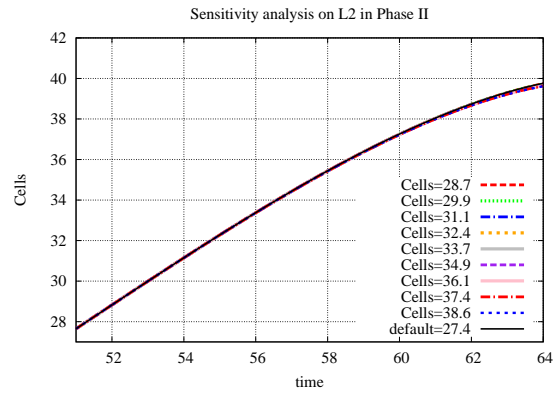


(a) standard variance over unique edges in phase I (b) standard variance over unique edges in phase II

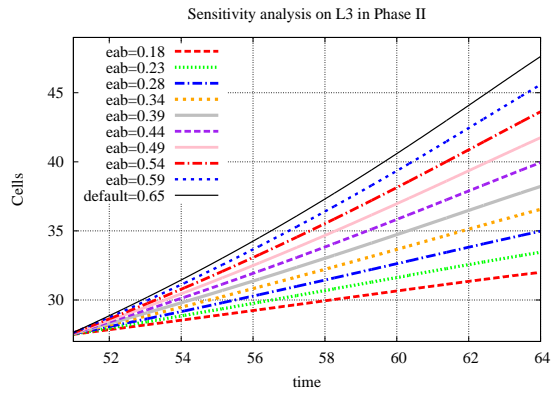
Figure 5: Sensitivity analysis—standard variance



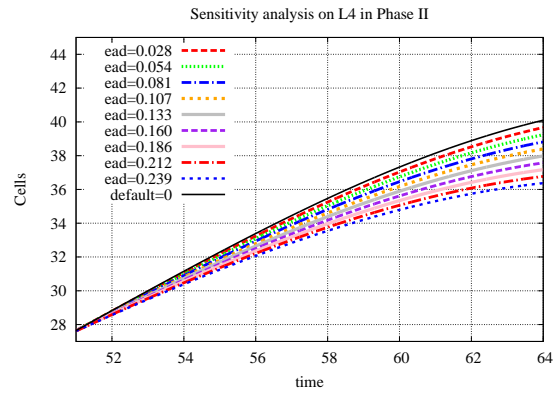
(a)



(b)



(c)



(d)

Figure 6: Sensitivity analysis on edges in phase II

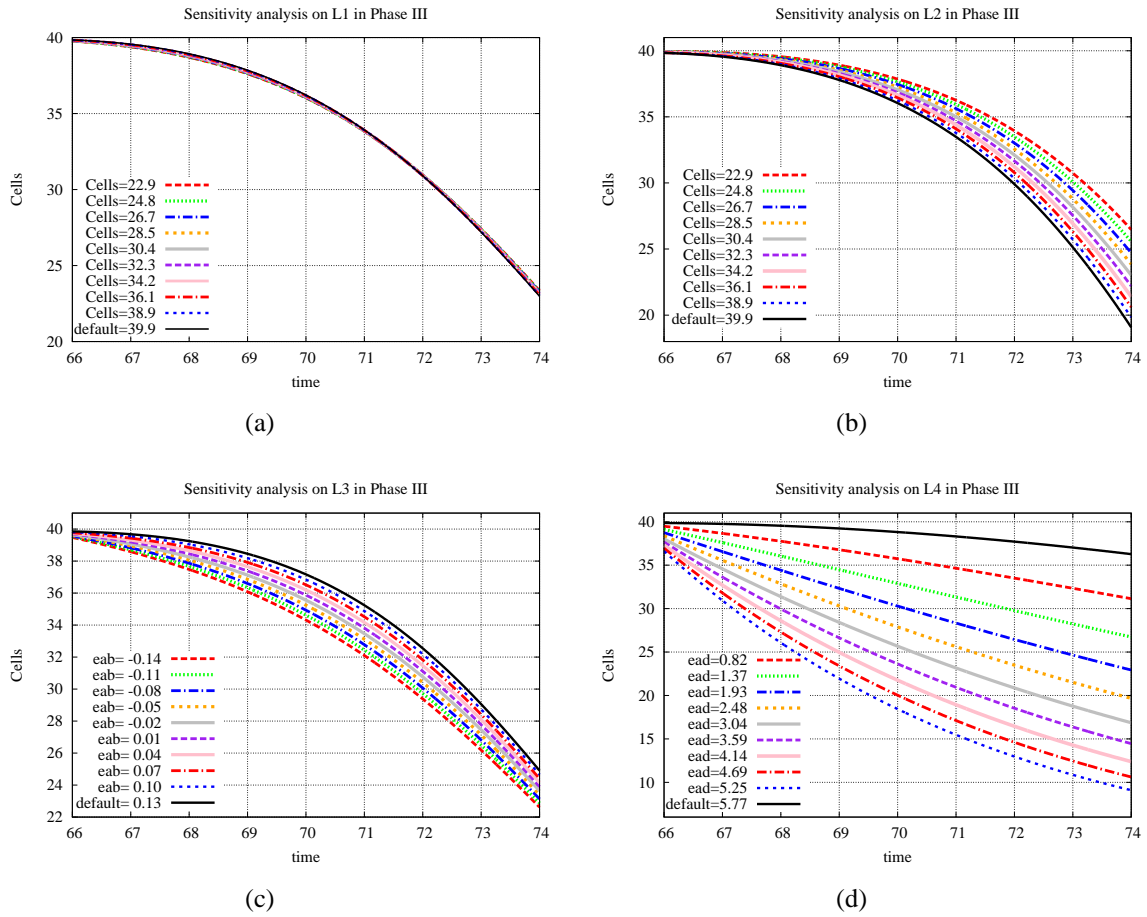
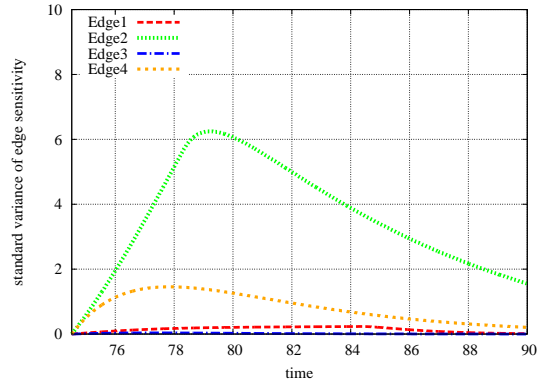
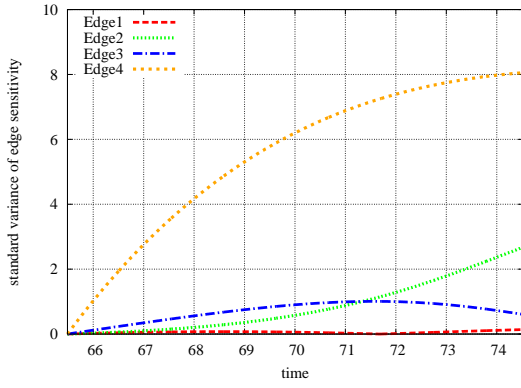


Figure 7: Sensitivity analysis on edges in phase III

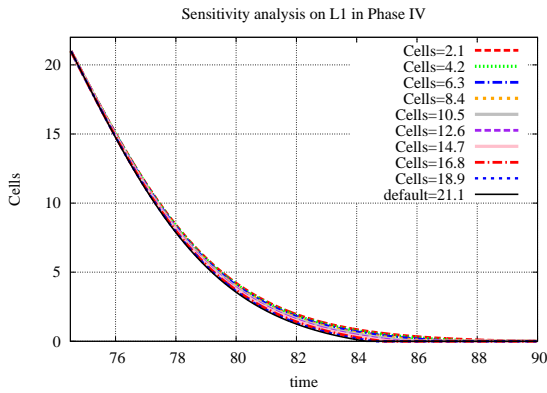
is not sensitive to  $L2$  at all throughout this phase and implies that  $L2$  would not be a dominant loop. In contrast,  $L3$  keeps its growing sensitivity and becomes the most sensitive factor to the variable of interest. At the same time,  $L1$  now does not greatly affect the variable of interest as in phase  $I$ . We can also identify  $L2$  and  $L4$  has the similar influence in the relevant graphs. However, their impact is much less than  $L3$ . Recall corresponding graph in phase  $I$ , the standard variance reflects that the difference of sensitivity from different loops is much smaller than in phase  $I$ .

Figure 7 illustrates the experiment results in phase  $III$ . It is apparent from these results that many alternative test values of the control variable in  $L4$  drive the variable of interest to display the diverse behaviors. Thus, a significant effect is contributed to the variable of interest by  $L4$ . Contrarily, we can see there are no changes when control variable changes in  $L1$ . This indicates that the unique edge in  $L1$  has no impact over the variable of interest. By the observation on Figure 4(b) and Figure 4(c), the variable of interest shows similar sensitivity to  $L2$  and  $L3$ , but toward the end of this phase,  $L2$  slightly raises its influence on the variable of interest and overtakes  $L3$ . Figure 8(a) shows the standard variance of edge sensitivity, and also confirms the above analysis, it encapsulates the information from the other four graphs and express it in a succinct way.

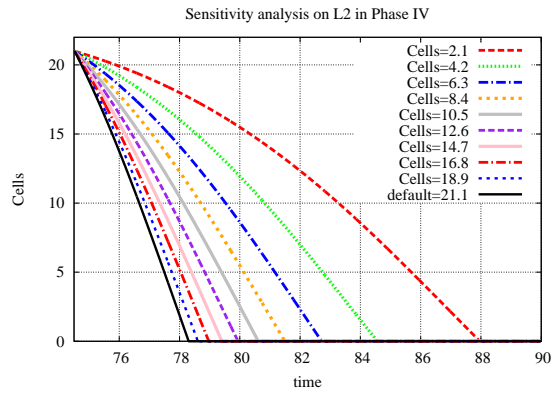


(a) standard variance over unique edges in phase III (b) standard variance over unique edges in phase IV

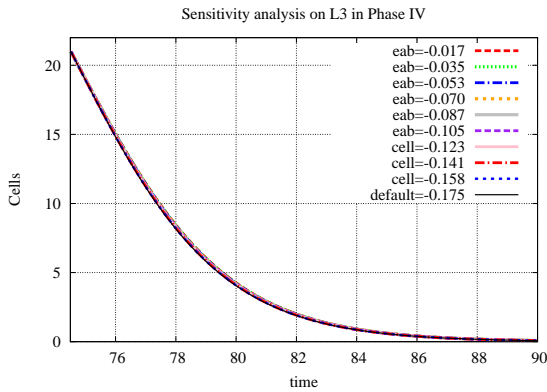
Figure 8: Sensitivity analysis—standard variance



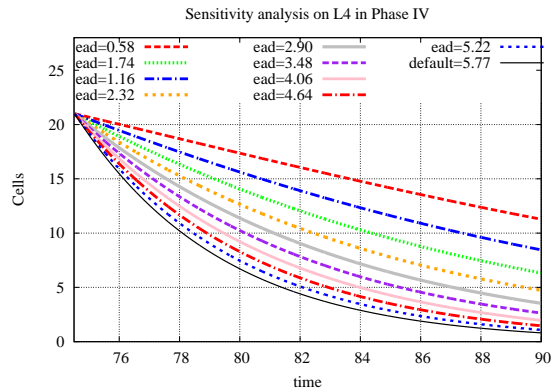
(a)



(b)



(c)



(d)

Figure 9: Sensitivity analysis on edges in phase IV

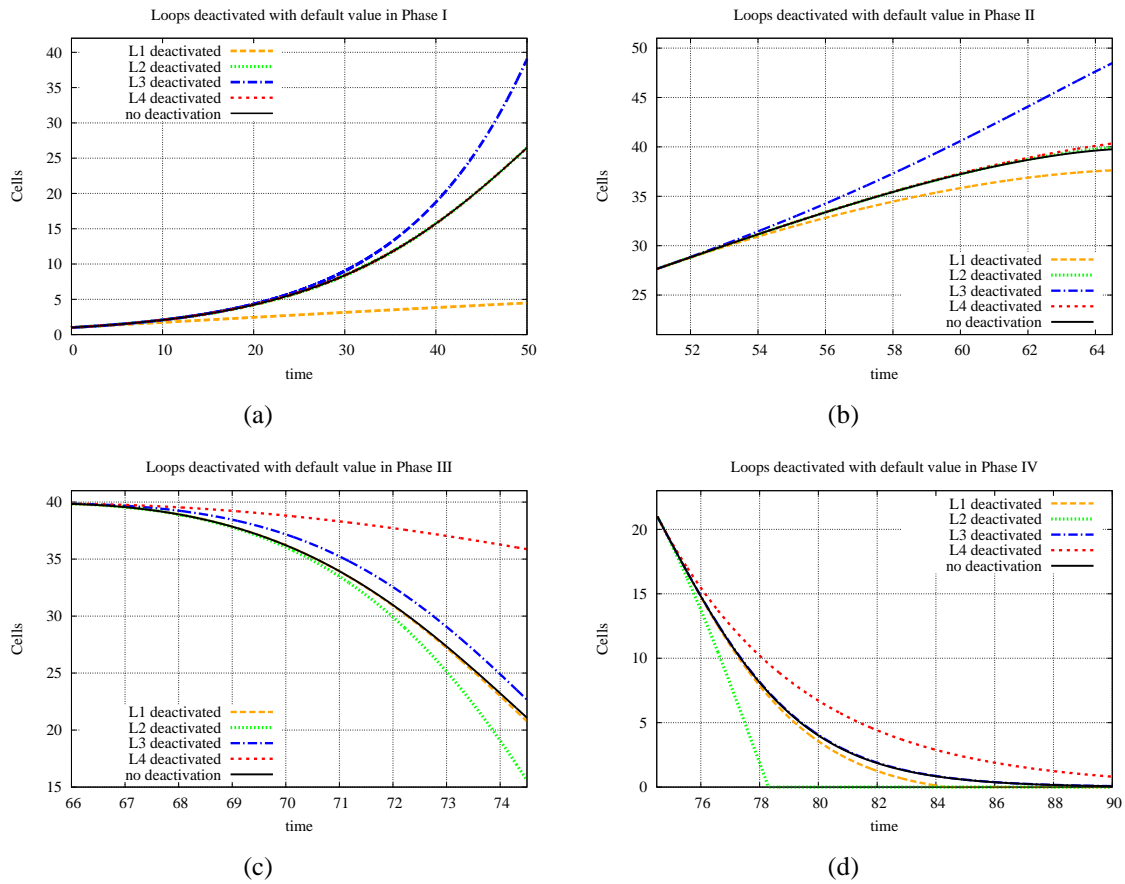
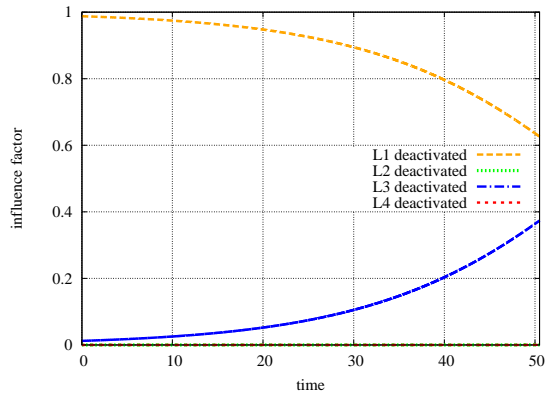


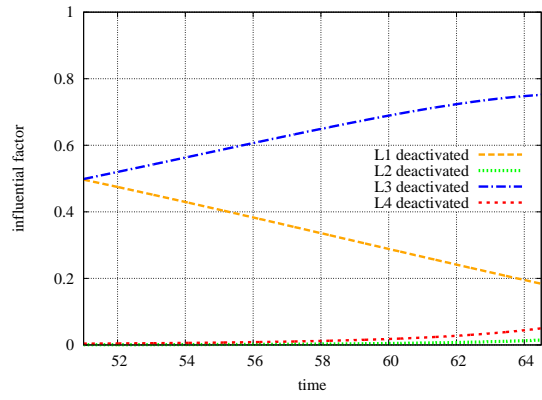
Figure 10: Feedback loop analysis with the default values

Finally, in phase *IV*, Look at the Figure 9(a) and Figure 9(c), the curves plotted in them are seemed to be one bold curve, this shows that the variable of interest is not sensitive to *L1* and *L3*. However, the curves for the peers in Figure 9(b) and Figure 9(d) are radiate. With the addition of Figure 8(b), we come to a conclusion that in phase *IV* *L2* is the most sensitive and *L4* is less sensitive while *L1* and *L2* are almost insensitive. The the significance of the loop dominance is in the same order as the sensitivity.

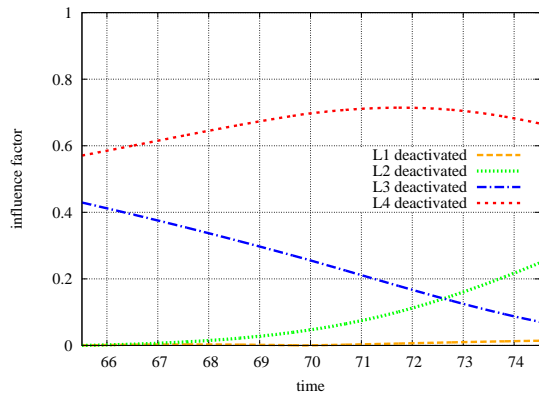
6. In the Yeast model, we select the candidate unique edge by observation, so there is no need to resort to sensitivity analysis to do the selection. In the sensitivity analysis, we have simulated the behavior of Cells with default values of different control variables already. They are plotted in relevant graphs in black lines. However, we display this result in Figure 10.
7. Use the proposed quantitative assessment to calculate their “influence factor”, thereby determine the dominant loops. Figure 11 present the normalized influence factor over time. In phase *I*, we can see the *L1* is definitely dominant for most of the time while toward the end of this phase, *L3* rises its impact and *L1* decreases its dominance. Meanwhile, *L2* and *L4* hardly have any impact on the variable of interest. In phase *II*, *L3* still gains its influence on “Cells” and becomes the dominant loop soon after entering the phase *II*. The influence of *L1* decreases by the time goes but it still retains some impact on the variable of interest. *L2*



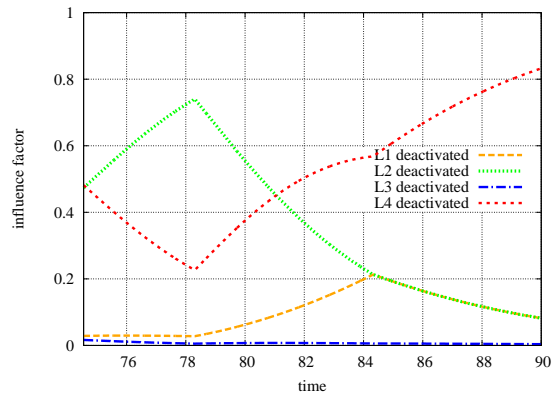
(a) phase I



(b) phase II



(c) phase III



(d) phase IV

Figure 11: Dominance measured for deactivated loops



and  $L4$  remain the same as in the previous phase—almost no effect, the impact of  $L4$  grows slightly toward the end of this phase. In phase *III*,  $L4$  is dominant, but  $L3$  also retains certain significance but becomes less and less important while  $L2$  increases its effect at the late phase.  $L1$  almost has no impact over “Cell”. In phase *IV*, from 74.5-78, the impact of  $L2$  grows swiftly and dominates during this period, but after that, it loses its dominant to  $L4$  quickly.  $L4$  is a dominant loop until the end.

Compared the analysis result from the original criteria in the Ford method (Phaff et al., 2006), they agree with each other most of the time but there is also discrepancy. In phase *III*, our assessment attributes  $L4$  to be the only dominant loop, whereas the original one determines both  $L3$  and  $L4$  to be dominant. In phase *IV*, our method indicates  $L2$  and  $L4$  each dominates for half phase while  $L2$  is the dominant loop under the original criteria.

	Dominant loops			
	I	II	III	IV
Sensitivity analysis	$L1$	$L3$	$L4$	$L2$
New assessment	$L1$	$L3$	$L4$	$L2$ & $L4$
Original assessment	$L1$	$L3$	$L3$ & $L4$	$L2$

Table 5: Comparison of dominant loops

Another comparison is conducted with the sensitivity analysis approach, these conclusions are consistent except in the last phase, sensitivity analysis owes to  $L2$  to be a dominant loop while it is together with  $L4$  to be dominant in the new assessment. To clarify the comparison between the three outcomes, we outline them in table 5 above.

## 5 Conclusions and Future Research

The Ford method is an important formal analysis method which offers a way to investigate the dynamic system. We propose an extension of this method by integrating a sensitivity analysis approach in order to enhance our understanding of the dominant loops in a model. We also propose that sensitivity analysis can be applied in order to select the unique edge if there are more than one unique edges in a feedback loop. For the future work, we believe the sensitivity analysis can be integrated added into an automated version of the Ford method to make it more powerful and robust, and become an practical and effective means of performing robust policy analysis.

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Jeffrey J. Shelstad.