Statistical Estimation and System Dynamics Models

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ABSTRACT

Much of the work done in system dynamics has been criticized for making insufficient use of statistical estimation techniques. There have been various responses to this criticism concentrating on the other sources of information available to the model builder. One of the major hurdles to the use of statistical estimation techniques is an understanding of when they are likely to be useful in system dynamics modeling. In this paper we consider different estimation techniques and how useful they can be in system dynamics modeling. The work is meant to be a pratical guide that will allow the modeler interested in statistical estimation to gain some understandng of the different approaches available. We concentrate or attention on the special problems that the system dynamics modeler is likely to encounter in estimation.

INTRODUCTION

In this paper we consider the problem of determining some or all of the parameters in a model through the use of statistical techniques. The paper is meant to be a practical introduction to the techniques available and the merits of these techniques relative to system dynamics models. The paper should be valuable as a guide for the system dynamics modeler who is either considering the use of statistical techniques, or simply trying to assess the value of existing statistically based estimates of parameters of concern. In order to give a good overview we have kept the notation to a minimum and sacrificed rigor. The reader is referred to the appropriate literature for a more complete discussion of the topics considered.

The paper is divided into four sections. The first section considers briefly the place of statistical parameter estimation and model validation techniques relative to the alternatives available. The second section discusses the modifications of a model that are required in order to apply statistical techniques. The third section gives a review of the commonly employed single equation estimation techniques as they apply to system dynamics models. The techniques considered in this section are widely used by economists and there are a plethora of efficient programs available to implement them. The final section considers slightly more involved estimation techniques that take greater advantage of the dynamic structure of system dynamics models.

This paper is meant to give an overview of the different techniques available for statistial estimation as they relate to system dynamics models. It is not meant to substitute for the study of statistical estimation techniques. To apply any statistical technique one should have a fairly good understanding of the assumptions underlying that technique and the properties of the resulting estimator. The purpose of this paper is essentially to help the reader decide where to look for estimators that might be applicable.

MODELS AND DATA

There have been strong criticisms leveled against system dynamics models because the models have been alleged to have no basis in existing data (Nordhaus 1973). The response of researchers in system dynamics to this criticism has been that recorded numerical data represent only a tiny fraction of the information upon which system dynamics models are based (Forrester, Mass and Senge 1974, Forrester 1980). The real disagreement comes about not because there are two definitions, but simply in the emphasis placed on the different stages of the model validation process. These deserve a brief review here.

The process of model testing and validation is extremely important and requires a very broadly based scrutiny of a model and its results. Forrester and Senge (1980) and Richardson (1981 chapter 5) discuss in some detail the problem of model validataion. There are a number of different issues that must be addressed in testing and assessing the validity of a model. These include: the sensitivity of the model to different parametric and structural changes, the adequacy of the model boundry, tests of extreme conditions and policies, the plausability of assumptions, and the ability of the model to accord with data. Of these we concentrate on the last.

The ability of the model to accord with data is a very broad concept and includes such things as: The ability of the model to predict changes in behavior modes, the ability of the model to predict the results of changes in policies and the ability of the model to produce behavior consistent with historical behavior. Relevant to the consideration of the last of these are the evaluation of periodicites, relative variable phasings and different apparent modes and patterns of behavior. The methods for evaluating the ability of a model to accord with the data include both visual evaluation of model output and the application of statistical tools.

Thus we see that the application of statistical tools is only one element in a variety of procedures aimed at assessing the validity of a model. In this paper we are concentrating on this one tool, and specifically, on how this one tool may be used in the estimation of parameters. The above discussion is meant to place the use of statistical tools in the overall context of model evaluation. In applying statistical estimation techniques to a model we are asserting that the ability of the model to accord with the data is an important aspect of the validity of a model. And though this assertion is in accord with the conventional practice of system dynamics it must be recognized that there are many other issues in assessing validity.

There are a number of different ways in which to estimate the parameters of a system dynamics model. Richardson and Pugh (1981 section 4.6), and Graham (1980) discuss the many available techniques for parameter estimation. Often it is possible to simply measure parameters. For example the number of toy cars that a machine will turn out in one hour could be measured. Other parameters often have managerial interpretations and simply asking the managers what they think is a reasonable value of the parameter will work. Introspection and guesswork are often required in setting parameters. Finally, statistical estimation techniques can be used in the determination of parameters. It is worth noting that some of the other estimation techniques may incorporate statistical estiamation. For example is a parameter can be measured, but only with noise, it may be necessary to take an average, a form of statistical estimation.

Where the line is drawn in the application of the different parameter estimation techniques is very much a matter of personal choice. There are no right or wrong methods for arriving at the parameters of a model. The tendency in the system dynamics literature has been to use statistical estimates only when the parameter in question was not easily enough interpretable to give a good idea of its appropriate value. The general position we take is that this is a justified procedure as long as a reasonable and replicatable verification of the model against available numerical data takes place. For more on this the reader is referred to Eberlein and Wang (1983).

In this paper we are concentrating on the statistical estimation of model parameters. This is not to advocate that all parameters be estimated in this manner. All of the techniques we will discuss can be implemented with some of the parameters in the model given predefined values. And this, in fact, is probably the preferable method for the estimation of parameters. It is most often the case that only a subset of the parameters in the model should be estimated through statistical techniques. It is also often true that it is not possible to estimate all the parameters from avaialble numerical data, and this will be touched on briefly.

To summarize, we see that the estimation techniques that we are concentrating on in this paper comprise only a small portion of the parameter estimation tecniques available to the system dynamics modeler. And we also see that the notion of statistical model validataion and statistical parameter estimation are closely linked, and that formal statistics are again only a small piece of the story.

STOCHASTIC DYNAMIC MODELS

System dynamics models are formulated in a continuous time setting but, for obvious reasons, must be estimated using discrete data. In addition, the general statistical model is based on the assumption that there is some amount of uncertainty in the system while this is not always the case in a system dynamics model. For these reasons it may be necessary to alter a system dynamics model slightly in order to apply statistical estimation techniques. Though these alterations will not always be of great importance in terms of the actual use of the model, they have important conceptual implications which need to be recognized. In this section we will discuss the principal characteristics of stochastic dynamic models and some of the ways that system dynamics models can be transformed to contain these characteristics.

One of the first issues that needs consideration is the question of continuous versus discrete time models. Most system dynamics models are conceptualized as continuous time models. However, they are both written and, for the most part, simulated as discrete time models. In general, this is not a serious problem as long as the integration interval (DT) is small enough to make the continuous time and discrete time solutions very close. We are going to take a similar approach in addressing the issues of estimation. However, in this case we are not free to choose the integration interval, but are constrained by the frequency with which data is available.

An important issue that arises in this is the problem of aliasing. As an example suppose that we were to observe the position of the sun in the sky every 36 hours starting at noon on the first day. Our observations of the suns position would alternate between completely up and completely set (noon and midnight). The sun is up, thirty six hours later it is set and thirty six hours later it is again up; we could conclude from this that the sun goes up and down with a 72 hour period. If it were not for the fact that we actually observe the sun more often than once every 36 hours we could not be sure that this was not the case. Any phenomena that has a period shorter than 72 hours will be subject to the same sorts of problems. Observing things only once every 36 hours we cannot distinguish phenomena with periods less than 72 hours from another phenomena with some period equal to or greater than 72 hours (Box and Jenkins 1976).

The problem that is described above is analagous to the problem of having a DT that is too large in a model. If this occurs then the model variables will behave erratically, taking on values far away from the last value at each successive DT. The solution to this problem is to simply shorten DT. And indeed the best solution to the problem of aliasing is to decrease the observation interval. However, this option is rarely open to us. The lack of frequently sampled numerical time series represents an obstacle for any modeler dealing with dynamic problems.

The general issue of using discrete time observations relative to a continuous time model will not be dealt with in detail in this paper. The aliasing problem described above is indicative of the kinds of problems that can be expected to arise. By assuming that the model is a discrete time model we are sidestepping these issues. The issues of estimating continuous time models are considered more fully in Bergstrom (1976) and Gandolfo (1981). For many practical problems assuming a discrete time model should not cause any real difficulties: The changes in the varaibles will often be slow enough relative to the observation interval to give reasonable results.

If the variables move quickly relative to the observation interval, either in the sense that there is oscillatory behavior of high frequency, or that there is extremely fast adjustment to new conditions (for example a smooth with a very short time constant) then there can be severe problems. However, it will often be true that these problems can be localized. One of the most obvious examples of this type of localization would be the extraction of an auxiliary or rate equation from the model. Because such an equation defines the value of a variable in terms of other variables at the same time, the specific equation may be suitable for estimation with the sampled data. (There may be problems in doing this if some of the variables are observations on levels such as inventory while others are average of rates such as the production rate.) In addition it may be possible to estimate selected dynamics of the model while imposing certain others. One method of doing this is discussed in section IV with respect to simultaneous equation estimators.

We have said that it may be possible to isolate certain dynamics in order to allow partial estimation. This requires that certain dynamics be related to certain equations. This will generally not be the situation, and it is useful to look at the reason why this is. Consider a continuous time dynamic model of the form

$$\dot{\underline{x}} = \underline{Ax} ,$$

with $\underline{x}$ the set of states and $\underline{A}$ a square matrix. The discrete time approximation to this equation would be given by

$$\underline{x}_t = (\underline{I} + dt\underline{A})\underline{x}_{t-dt}$$

with dt the integration interval. Now suppose that we observe $\underline{x}$ every n(dt) units of time. Then we would be able to consider empirically an equation of the form

$$\underline{x}_t = (\underline{I} + dt\underline{A})^n \underline{x}_{t-(ndt)} \quad .$$

If $dt\underline{A}$ is small and n is not too large we can approximate fairly well the above expression by expanding the power of $(\underline{I} + dt\underline{A})$ and ignoring all but the first first term which is $n(dt)\underline{A}$. This is simply a statement of the notion of the dynamics not being too fast relative to the observation interval in terms of some more precise notation. But we want to use the notation to make the following point. It is the whole $\underline{A}$ matrix that must be small when raised to a power and multiplied by (n(dt)) raised to the same power. Thus even if an individual equation seems to imply reasonably slow movement, the estimation of that equation will yield good results only if the fast movement in the other equations does not influence the equation of interest, a strong requirement.

For the discussion that follows we will assume that the time between observations is equal to the integration time (DT) and for convenience choose time units so that DT is equal to 1. We will briefly touch on the issues raised above with respect to fast dynamics in section IV. For section III we will assume that the equations considered are reasonable in terms of the sampling interval.

The other aspect of a statistical model that may differ from a system dynamics model is that a statistical model inherently has a degree of uncertainty associated with it. For example, consider a difference equation of the form

$$\underline{x}_t = \underline{Ax}_{t-1} \quad ,$$

a stochastic version of this equation (there could be others) is given by

$$\underline{x}_t = \underline{Ax}_{t-1} + \underline{e}_{t-1} \quad ,$$

with $\underline{e}$ an error term.

The aspect of a system dynamics model that needs our special attention is the noise term $\underline{e}$. In the first model given above this term was taken to be zero. That is, only the deterministic behavior of the model was being considered. It is quite common in work in system dynamics to consider only deterministic model behavior. On the other hand, statistical models assume that there is noise entering the system at one or more points. Clearly it is possible to simply introduce noise into a system dynamics model at any number of points and then apply statistical techniques.

Though this method is often implicitly resorted to, some consideration of the sources of the noise is really called for.

There are a number of reasons why a relationship would not be expected to hold exactly in a model of interest. Perhaps the most important reason is the fact that the model is, by its nature, an abstraction, and can only be representative of the system under consideration. For example, consider a simple milk inventory model that assumes constant weekly milk output for each cow. It is clear that the quantity of milk a cow produces in a week will vary depending on the quality of the pasture the cows have been grazing in, water available, excercize and other factors. Yet, for a simple model it is not desirable to have many of these factors enter. Thus the equation determining the milk production rate could be written in the form

MP.K=DHS.K*MPC+E1.K

MP - Milk Production (litres per week)
DHS - Dairy Herd Size (cows)
MPC - Milk Per Cow (litres per cow per week)

with E1 some error term. The value that the error term takes on is determined by the numerous factors affecting a cows productivity that have not been included in the model. The type of argument that we have applied to productivity could easily be applied to a large number, if not all, of the equations in any given model. To do so explicitly would not necessarily be that informative for either the purposes of simulation or estimation. However, because in the process of estimation it is necessary to introduce error terms, it is desirable to consider the major source of the error and, given this source, the likely properties of the error.

The second source of error in a model is measurement error. The variables in a model are idealized concepts of variables which can often only be measured approximately, if at all. As a consequence it is ofen true that the empirical counterpart to a model equation can be expected to have different characteristics then the idealized equation. In the material that follows we will consider the implications that the source of the error has for the characteristics of estimated parameters.

SINGLE EQUATION ESTIMATION TECHNIQUES

In this section we will consider the most commonly applied estimation tools. These tools are very easy to apply and there are a very large number of statistical and econometric software packages available to implement them. Because the tools are so readily available and cheap to use they are potentially very useful, but their apparent simplicity has its pitfalls and these will be discussed.

The simplest case of a single equation is the linear equation of the form

$$z_t = (\underline{w}_t)\underline{b} + e_t$$

where $\underline{w}$ is a (column) vector of predetermined variables and e is an error term that is not correlated with any element of $\underline{w}$. It is further supposed that the expected value of e is zero and the variance of e is constant over time. Given these assumptions the value of $\underline{b}$ that minimizes the sum of squared errors is given by the usual formula

$$\underline{b}_{ols} = (\underline{W}^T\underline{W})^{-1}\underline{W}^T\underline{Z} ,$$

with the t'th row of $\underline{W}$ given by $(\underline{w}_t)^T$ and the t'th element of $\underline{Z}$ given by z_t. The estimate $\underline{b}_{ols}$ has the property that it is a consistent estimator of $\underline{b}$ in the sense that as the number of observations gets large the value of the estimate goes to the actual value of $\underline{b}$ (Johnston 1972, Pindyck and Rubinfeld 1976, Theil 1971).

The question that the modeler must face in applying ordinary least squares is how near to being satisfied are the assumptions likely to be. The major problem is, of course, the determination of whether the error term $\underline{e}$ is likely to be uncorrelated with the explanatory variables $\underline{w}$. The assumption that the error term e and the predetermined variables $\underline{w}$ are not correlated has traditionally gone untested, though such tests are possible and becoming much more common (Hausman 1978). However, in a great many cases in system dynamics modeling there are strong a priori reasons to expect that this assumption will fail. We consider some of these reasons below and then point out the kinds of modifications necessary to make the estimation techniques applicable.

Consider first the correlation over time (autocorrelation) of the error term e. By itself the existence of autocorrelation does not hurt the consistency of the ordinary least squares estimator. However, in system dynamics models all of the variables have a great deal of correlation over time as well. It is well known that if the predetermined variables contain a lagged value of z then the existence of autocorrelation in the errors yields an inconsistent estimate (Theil 1976 section 8.7). The reason for this is that the lagged value of z was partly determined by the lagged error, and the lagged error influences the current error, thus the lagged z and current error can be expected to be correlated.

While the above argument was given for a lagged dependent variable, it will also apply if any of the explanatory variables are in part determined through a lagged value of the dependent variable. Given the rich feedback structure of most system dynamics models it is likely that more than one element of the right hand side variables $\underline{w}$ will be partly determined by the past values of z. It is for this reason that the existence of autocorrelated errors will almost always lead to inconsistent estimates in a model with a great deal of feedback (related to this is the work of Engle, Hendry and Richard 1983). It is common practice in system dynamics models to assume that noise entering a model has positive autocorrelation (Richardson 1981 pp. 371-373, Britting 1973). The reason for this is that sources of errors are likely to have some amount of momentum behind them. This alone casts into doubt the usefulness of the standard model as it was given.

The problems associated with autocorrelated errors are relatively easy to overcome. Correction for first and second order autocorrelation is available in most existing software packages. Under certain circumstances the equation may not permit estimation, but normally this will not be a problem (see for example Theil 1971 section 8.2). There is another important point that must be made on the autocorrelation of the error terms. And this is that the degree of autocorrelation that exists over an observation interval will, in many cases, be small. Thus autocorrelation may not cause a great deal of problem, but it should be tested for. Because $\underline{w}$ is likely to have properties similar to a lagged value of z, the standard Durbin-Watson statistic is never a valid statistic. For this reason something such as the Durbin H-test (Durbin 1970, Pindyck and Rubinfeld 1976) should be employed.

The other source of error that was mentioned was measurement error. Measurement error in the elements of the $\underline{w}$ vector will cause the resulting estimator to be inconsistent. The reason for this can be intuitively explained as follows. Suppose that the correct model is

$$z_t = (\underline{w}'_t)\underline{b} + e_t$$

but that we have measurements on $\underline{w} = \underline{w}'+\underline{e}'$ where $\underline{e}'$ is some error term. If $\underline{e}'$ is uncorrelated with $\underline{w}'$ the measured variables $\underline{w}$ will show substantially more movement (have a higher variance) than the actual variables $\underline{w}'$ will. As a rule of thumb we say in this case that the elements of the $\underline{b}_{ols}$ vector will be biased toward zero. Senge (1977) using the market growth model gives some indication of how great the bias can be.

The measurement error problem in isolation is not that serious in most system dynamics models. The reason for this is that it is possible to derive a consistent estimator of $\underline{b}$ by the method of instrumental varaibles. The idea behind instrumental variables is quite simple and ingeneous. Find some variables ($\underline{V}$) that are correlated with the actual variables $\underline{w}'$ but not correlated with the measurement error $\underline{e}'$. Call these variables the instruments. Define the instumental variables estimator

$$\underline{b}_{iv} = (\underline{V}^T\underline{V})^{-1}\,\underline{V}^T\underline{Z}\;;$$

this estimator is consistent for $\underline{b}$. The problem with the instrumental variables estimator thus defined is in the existence of the instruments $\underline{V}$. However, for most system dynamics models the lagged values of the right hand side variables will work very well (that is $\underline{v}_t = \underline{w}_{t-1}$).

Because of the dynamics of the model lagged and unlagged variables will generally be highly correlated, at the same time as long as the measurement noise is not highly autocorrelated the correlation with the measurement errors will be minimal. Morecroft (1977) employs this method on some synthetic data generated by a market growth model with quite good results.

In the above discussion we have assumed that the equation to be estimated is essentially a model equation. This will not always be the case. It is often necessary to manipulate model equations so that they can be estimated statistically. For example, if an information or material delay exists in the model then we may replace the delayed value by an approximate formula in terms of lagged values of the actual variables (Hamilton 1980). Alternatively it may be necessary to amalgamate a number of equations by solving out for values of computed but unobserved variables. Deriving such transformations is something of an art and it is difficult to lay out any simple rules. One is referred to Hamilton (1980) and the references therein for a number of examples.

The discussion so far has been with respect to a linear equation. In the case of nonlinear equations the same general points still remain true. Nonlinear least squares is a readily available tool for the estimation of nonlinear equations, and autocorrelation correction is usually available as well. Nonlinear instrumental variable estimators exist but are somewhat less common.

FULL MODEL ESTIMATION

We have considered the estimation of a single equation in a model. In many cases it may be desirable to consider more than one equation or all of a model. This is clearly more difficult than the single equation case and often the effort required to obtain estimates can be susbstantial. In this section we consider three basic classes of system estimators. The first is a relatively simple extension of the single estimation techniques considered above and is usually fairly straightforward to implement. The second estimation technique to be discussed is designed to work with simultaneous equations and is of only limited applicability in system dynamics modeling. The third technique to be considered is a method developed primarily in the control engineering fields that allows for estimation when only some of the variables are observed.

The first system estimation method we consider is generalized least squares. Generalized least squares is an estimation technique that is designed to take account of the fact that the errors entering different equations may be closely related (Johnston 1972 chapter 7, Theil 1971 chapter 6). For example an equation for agricultural production and an equation for ice cream sales may both have an error partly due to the weather. In this case the two equations are seemingly unrelated but, by virtue of the fact that the weather is an excluded influence on both, combining the two equations can improve our estimator. In this case improve simply means to decrease the variance in the estimated coefficients.

It is not difficult to implement the generalized least squares estimator. However, the gains from implementation are normally rather small. The real usefulness of the generalized east squares estimator is that it allows the same parameter to appear in more than one equation. In this manner we may use two or more equations in order to get a value of a parameter that appears in the equations. The alternative to doing this is to estimate the equations separately and take some average value of the estimated parameter.

The second estimation method we consider is simultaneous equations estimation (Johnston 1972 chapters 12 & 13, Theil 1971 chapters 9 & 10). A great deal of effort has been devoted in econometric research to the development of simultaneous equation estimators. System dynamic models are generally built without any simultaneities. Strictly speaking, simultaneous equations are non-causal in the sense that a number of variables determine each other without any mechanism by which the influence can be transmitted. Nonetheless there are situations in which the use of simultaneous equations can be quite helpful. For example in the model used in N. Forrester (1982) the interest rate is determined by equating the demand and supply of liquid assets. This is a particular case of what might be refered to as fast dynamic simultaneities.

We are all familiar with the multitude of behavior modes displayed by any given model. In many cases some of these behavior modes will be much faster than others. If we are not specifically intereseted in the fast behavior modes then we may choose to deliberately remove them from a model. The most obvious example of this is the removal of a smooth that adjusts to the input value so quickly as to be largely irrelevant to the purposes of the model. In this case we would set a perceived variable equal to the actual value even though it is known that there should be some sort of an information delay.

If the observation interval is long relative to the time constants of the faster processes in a model and those processes are stable then simultaneous equation based estimates can be useful. The basic approach is quite simple. The fast stable dynamics are assumed to have approximately reached an equilibrium state. The remaining dynamics are then estimated from the avialable data. This approach is considered in Eberlein (1984 Chapter 5, see also Fisher 1961, Senge 1979).

The final estimator we consider is the full information maximum likelihood via optimal filtering (or FIMLOF) estimator (Peterson 1975, 1980, Peterson and Schweppe 1974, Schweppe 1973). This approach to estimation was developed by workers in control engineering and has not yet been widely employed in econometrics work. The method is expensive to use both in terms of human and computer time and has a number of limitations. These will be discussed, but first the basic approach will be introduced.

The basic model for the discussion of filter based estimators is the following linear dynamic system.

$$\underline{x}_t = \underline{Ax}_{t-1} + \underline{Bu}_{t-1} + \underline{e}_{t-1} ,$$

$$\underline{y}_t = \underline{Cx}_t + \underline{c}_t$$

with $\underline{x}$ the state vector and $\underline{y}$ the set of variable that are observed. The two noise terms $\underline{e}$ and $\underline{c}$ are assumed to be uncorrelated with one another, and to display no autocorrelation.

The major difference between the above model and the equations discussed earlier is that it is no longer assumed that all the variables are observed. Typically, $\underline{y}$ is some subset of the variables contained in the vector $\underline{x}$. This is a strong and useful generalization of the more standard econometric models. Typically, system dynamics models will contain a large number of variables that cannot be observed. One can put a system dynamcis model into the above framework without having to do violence to the model.

The first problem that one is faced with when given a model in the form of equation 5 is that the value of the state $\underline{x}$ at any time t is not known. If the value of the state vector $\underline{x}$ were known at all times then it would be possible to apply a least squares criterion in order to arrive at parameter estimates. The first step in developing an estimator is therefore to get values of the state vector $\underline{x}$ at all times. Once this is done the minimization of the sum of squares can proceed much as in the single equation case.

The estimation of the value of the state variable at time t proceeds by an iterative process first proposed and justified by Kalman (1960) and Kalman and Bucy (1961). This process has become known as Kalman filtering. The basic idea behind the Kalman filter is very simple and it is the fact that it has certain optimality properties that is the real difficult part of its derivation. We will give here only a heuristic explanation of the workings of the Kalman filter. For a more detailed discussion the reader is referred to (Anderson and Moore 1979, Gelb 1974, Sage and Melsa 1971 or Schweppe 1973).

We suppose that we have some value for the different unknown parameters of the model we are considering. This is, of course, just a device that allows us to pick the best such set of parameters numerically. Given the parameters of the model suppose that we have a best guess of the value of $\underline{x}_{t-1}$. We want to use this to get a best estimate of $\underline{x}_t$. One obvious guess for this is that

$$\underline{x}_{t(\text{first try})} = \underline{Ax}_{t-1(\text{best})} + \underline{Bu}_{t-1}$$

since the error term is assumed to be zero. This estimate of $\underline{x}_t$ implies that a good estimate of $\underline{y}$ would be given by

$$\underline{y}_{t(\text{first try})} = C\underline{x}_{t(\text{first try})} .$$

However, we observe $\underline{y}$ and we can compare this observation to our first try estimate. If the value of the first try guess for $\underline{x}$ at time t is different from the actual value of $\underline{x}$ then we would expect the value of our guess for $\underline{y}$ to be different from the actual value. And we can also work backwards from the value of $\underline{y}$ to that of $\underline{x}$. This is done through what is called the Kalman gain and in this manner we can adjust the first guess for $\underline{x}$ in order to arrive at the best guess. Now that we have $\underline{x}_t$ we can go onto to t+1.

Given these best estimates of $\underline{x}$ it is possible to evaluate the sum of squared errors. The errors that are used are the errors made in predicting $\underline{y}$, since this is what is observed. It is important to note that the estimate of the values of $\underline{x}$ will depend on the parameters of the model. because of this for each different parameter choice considered the entire procedure outlined above has to be repeated. It is for this reason that the parameter estiamation technique FIMLOF is so expensive.

The above discussion has been for a linear system. It is possible to apply the techniques to nonlinear systems through successive linearizations in order to derive the Kalman gain. Such a strategy has been employed in Peterson (1975, 1980, see also Sage and Melsa 1971 and Schweppe 1973). The resulting estimator will not have the optimal properties associated with the Kalman filter for the linear model, but the application of the technique is often successful.

There are two very important issues that need to be addressed in the use of FIMLOF estimators. The first is the importance of noise to the resulting estimates and the second is the question of identification. In order to implement a FIMLOF estimator the way in which the noise enters the model must be carefully specified. Normally this means not only which equations the noise enters but also what the characteristics of the noise are. Changing the specified noise characteristics can have rather profound effects on the resulting estimate. This is because the Kalman gain that is used in the updating of the state estimates is strongly influenced by the characteristics of the noise entering the equations.

The other related issue in the use of a FIMLOF estimator is that of identification. Identification refers to the ability of the data to distinguish between different model parameters. (This is the econometric meaning of the term identification. In the control theory literature identification refers to the estimation of model parameters.) Suppose, for example, that two model parameters always appeared multiplying one another. Then doubling the first parameter and halving the second would alter nothing in the model. In such a situation the two parameters would not be identified, although their product might. There are potentially a very large number of parameters in a system dynamics model. These include the variances of the noise entering the model and the variances of the measurement noise, as well as all the regular model parameters that one might wish to estimate. If it is not possible to severely restrict the qualities of the noise entering the model it will not be possible to identify the model. This is because the approach considered is capable of estimating the covariance structure of at most the same number of errors as there are observed variables (Mehra 1974). In the authors' own experience and that of Peterson (1975) this problem of identification has proved quite serious.

We have discussed the Kalman filter in terms of its use in conjunction with parameter estimation. There are alternative uses for the Kalman filter that should also be pointed out. The first is that the Kalman filter can sometimes be used alone for parameter estimation. This is the case if the parameters can be treated as though the were states. If this is done then simply employing the Kalman filter one can sometimes arrive at good estimates (Sage and Melsa 1971). The other point is that the Kalman filter can be used in order to generate a sum of squares for a model. This is useful as a device for evaluating the performance of the model relative to the data. This is the approach used in Eberlein and Wang (1983).

SUMMARY

In this paper we have considered various techniques for the estimation of parameters in system dynamcis models. As is easily seen the parameter estimation techniques are all, strictly speaking, based on assumptions that are known to be false. This casts into doubt the strict interpretation of the results. However, it does not imply that using statistical estimation techniques will give any worse results than will making informed guesses. The real question here is one of expediency. Is the value to the user of a parameter obtained through statistical estimation sufficient to warrant the expense of such estimation. Again the answer to this will depend on both model purpose and the final consumer of the model. Certainly the value of statistical estimation (and model validation) in the academic and scientific community is quite large. And in such cases a good deal of effort is justified.

If one is interested in the use of statistical parameter estimation then one must still choose the best method to apply. In this area the best plan is likely to use the cheapest thing available that is likely to yield reasonable restults. This means that if the variables in an equation are all available in some historical time series, or it is possible to solve out for unavailable variables, then the estimation of that equation through least squares or instrumental variables is probably the best method. If the variables are not observed, then it is necessary to resort to a estimation technique based on the Kalman filter or a similar approach.

In applying the techniques considered (or other techniques) there is no substitute for a solid understanding of the tools being applied. In this paper we have tried to point out where such an understanding can be obtained. Many of the issues are quite subtle, and present the system dynamic modeler with some formidable barriers to statistical estimation. However, in time it can be hoped that improved software quality and availablility will make the task of evaluating and implementing a given estimation technique much simpler, and extremely practical.

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