A Contribution To Goodwin's Growth Cycle Model From A System Dynamics Perspective

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Abstract

The for economists well-known Goodwin model was one of the first models which tried to combine cyclical behavior and economic growth. The basis for this is the predator-prey model – a basic structure for every System Dynamicists. The economic literature about the Goodwin model is enormous, but so far, it was mostly concentrate on the mathematical behavior or on some extensions that could be implemented. In addition, there are only two papers from R. Solow and D. Harvie about an econometrical verification of the model and none from a System Dynamics' perspective. This article provides therefore two System Dynamics models of Goodwin's theory and tests the enhanced one on the German economic situation and on the data provide by Harvie 2000. Additionally there are some suggested modifications of the Goodwin model, tested from different authors, which reveal surprising outcomes for the understanding of Goodwin's theory.

Key Words

Goodwin; Predator-Prey-Model; Lotka-Volterra-Equation; Business Cycles; Phillips Curve

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1 Introduction

Goodwin proposed his simplified model about a new approach to the origin of the business cycles and growth in 1967; at that time he did not know that this model would influence so much the economic literature for the next 35 years.

Goodwin's model is about the relationship between employment rate and the workers' share of national income. From his point of view, it is "starkly schematized and hence quite unrealistic model of cycles in growth rates" (Goodwin 1967, 54). But the beauty of his model lies in its simplicity and it based on the classical Lotka-Volterra predator-prey model. Since then there have been many different extensions to it. Some of them focused on the assumptions, some on the variables and some on implementing more details. From a mathematical perspective, researches concentrated on the stability, the sensitivity or the relationship between the variables.

The Goodwin model is up to now one of the few examples of a simple, real dynamic model that is taught at the university all over the world. Nevertheless, the amount of papers that have proved the model with data from different countries is quite low. Besides the early work from Atkinson (Atkinson 1969), it took more than twenty years before Solow econometrically tested the model for the United States (Solow 1990). Ten years later Harvie did this for ten OECD countries (Harvie 2000) like UK, Norway, Canada, USA or Germany. He tested the long run relationship between the employment and the wage. Until today, there are no papers from a System Dynamics' perspective.

At the Chair of Macroeconomics from W. Cezanne in Cottbus, Germany (www.wiwi.tu-cottbus.de/vwl1-makro/) the Goodwin model is used to help students understand the income policy related to the business cycles and economic growth. However, this is more from a traditional perspective. The classical Lotka-Volterra predator-prey model is also often used as an introduction into non-linear behavior for new students in the field of System Dynamics, like in the courses from P. Davidson and E. Moxnes at the University of Bergen, Norway (http://www.ifi.uib.no/sd/). However, this is in most cases only on a much-unspecified level. Some universities are known to teach the Goodwin model from a System Dynamics' perspective, like M. Radzicki at Worcester Polytechnic Institute, USA (www.wpi.edu) or J. Kopf at the University of Würzburg, Germany (www.profkopf.de). Of course, there may be more, but it is clear that this is not a well-established practice until today.

This paper aims to present an approach to the Goodwin model from a System Dynamics' perspective. For this purpose, in section 2 we will first present the basics about the classical Lotka-Volterra predator-prey model and then recall the original model presented by Goodwin. In section 3 we explain a simple predator-prey model in System Dynamics based on Goodwin's two main equations. The impressive need for more Systems' Thinking at universities is explained in section 4, where we also try to transform the basic model into a more understandable System Dynamics model. For that reason, we will go back to the main structure behind Goodwin's equations. The tests of exemplary recommended enhancements from different authors are shown in section 5. Section 6 deals with the German business cycle between 1956 and 2004. Finally, section 7 summarizes the findings and ends with suggestions for further research in this field.

2 Origin And Equations Of Goodwin's Model

This section deals with the mathematics behind the model. The Goodwin equations will be deduced from the original model. We assume that the reader is not familiar with the origin of the predator-prey relationship and therefore, to understand the behaviors of the Goodwin model it is recommended we have a short summary.

2.1 Lotka-Volterra Equations

In the mid 1920's Lotka and Volterra (for further reading see Lotka 1925, Volterra 1926), independently from each other, figured out what was going to be one of the first mathematical model for biological systems based on fish in the Adriatic and its main predator.

The observed amount of predator in the Adriatic after the First World War was higher then expected. The fishery was destructed as a direct follow of the hostilities. We would anticipate that less prey would lead to fewer predators. However, the observations totally contradicted this belief.

Volterra assumed that the growth rate of the prey population x, under the restriction of absence of the predator, is constant given by a. In addition, it would be dependent on the density of the predator population y with a linear factor b. As a formula this leads to

$$(1) \qquad \frac{\dot{x}}{x} = +a - by \qquad \text{with a, b} > 0$$

On the other hand the predator population y dies if there is no prey fish x. This is given by the constant decay rate –c to the growth rate of the predator. Vice versa, the y population depends on the density of the prey population. This leads to

$$(2) \qquad \frac{\dot{y}}{y} = -c + dx \qquad \text{with c, d} > 0$$

From the above stated equations we come up with the basic differential equation of the Lotka-Volterra case (Hofbauer and Siegmund 1998, 11)

$$(3) \qquad \begin{aligned} \dot{x} &= x \cdot (+a - by) \\ \dot{y} &= y \cdot (-c + dx) \end{aligned} \qquad \text{where x=prey population and y=predator population}$$

These equations show us the change in predator and prey populations at a given time t. Therefore, x at the time t is $\dot{x}(t) = f(t, x)$ and y is $\dot{y}(t) = f(t, y)$. It is clear that this series needs a starting point or initial value. We will give the values $x(0)=x_0$ and $y(0)=y_0$.

The dependency can also be shown as the following causal loop diagram

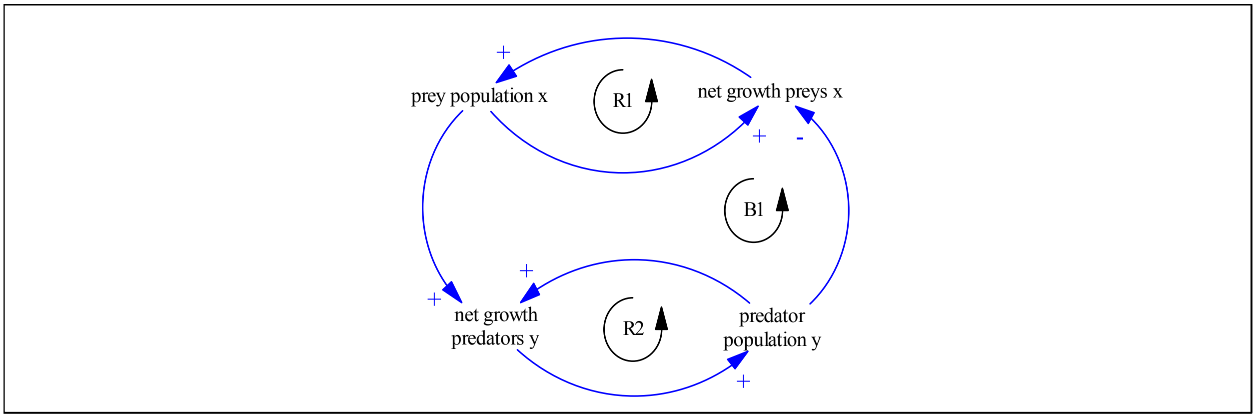

figure 1 - causal loop diagram of a predator-prey model

There are two main reinforcing loops. The stock of prey is dependent on the net growth of the prey. The stock of predator changes with their net growth. The balancing loop creates oscillations.

Figure 2 shows an exemplary phase portrait and a time graph. The orbits are created by different initial values and circulating around a critical point $(\bar{x}, \bar{y})$. This point is defined by a change of zero for $\dot{x}$ and $\dot{y}$. With transposing the equation (3) we get the critical points

$$\bar{x} = \frac{c}{d} \tag{4}$$

$$\bar{y} = \frac{a}{b} \tag{5}$$

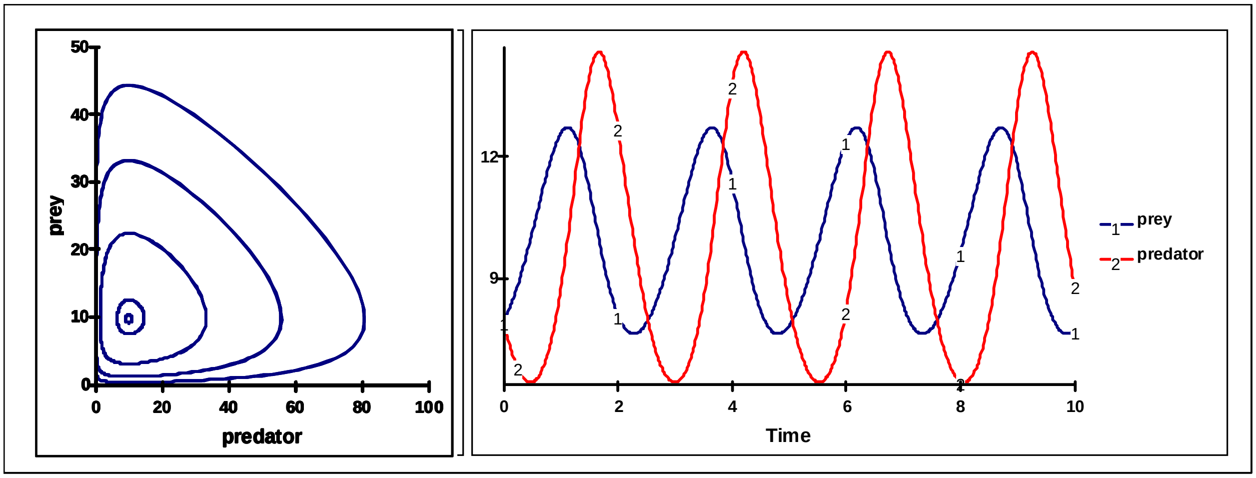

figure 2 - closed orbits and behavior of a predator-prey model

The average amplitude of the number of predator or prey is oscillating around this critical value for them. This paper does not aim to explain and proof the Lotka-Volterra model. But there are two more remarks.

First, the period for a cycle is given by

$$T = \frac{2\pi}{\sqrt{ac}} \tag{6}$$

where a and c are the natural growth or decay rates as we already know from the equation (1). Note that this can only be assumed for initial values x and y near the critical point.

Second, the prey is going ahead with ¼ period T against the predator.

Having reviewed this we can go on to look at Goodwin's model.

2.2 Goodwin's Model

Goodwin presented his famous model in 1967. He tried to model economic growth and business cycles and he showed that the antagonist relationship between workers and capital owners could lead to cycles (Aquiar 2001, 2). Goodwin had a number of assumptions in his model, like (Goodwin 1965, 54):

(7) a steady technical progress,
(8) a steady growth in the labor force,
(9) only two homogenous and non specific factors of production: capital and labor,
(10) only real and net quantities,
(11) consumption of all wages,
(12) all profits are saved and invested.

Furthermore, he relied on a more empirical, but somewhat disputable view:

(13) a constant capital-output ratio,
(14) a real wage rate that rises in the neighborhood of full employment.

With these considerations in mind, we can develop the differential equations like in the Lotka-Volterra case (for further readings see Cezanne 2002, 483 or Neumann 1996, 259).

We start with the tautological equation

(15) $$q = \frac{y}{wf} \cdot \frac{wf}{n} \cdot n$$ where q=national income, wf=work force, n=labor supply

(16) $$q = a \cdot v \cdot n$$ where a=productivity, v=employment rate

A logarithmical derivation with respect to time gives us the growth rates:

(17) $$\frac{\dot{q}}{q} = \frac{\dot{a}}{a} + \frac{\dot{v}}{v} + \frac{\dot{n}}{n}$$

(18) $$\frac{\dot{q}}{q} = \alpha + \frac{\dot{v}}{v} + \beta$$ where α=increase in productivity and β=growth rate labor supply

Note that the "dot" on the variables in this paper means "differentiate with respect to time". The national income is 100% distributed between the profit rate and the workers' share. This leads to $1 = p + u$, where p is the profit rate and u the workers' share of the national income. If all profits are reinvested then follows

(19) $$\frac{\dot{k}}{q} = p$$ $$\frac{\dot{k}}{q} = 1 - u$$

A division with $\sigma = \frac{k}{q}$ leads to

$$(20) \qquad \frac{\dot{k}}{k} = \frac{1-u}{\sigma}$$

As we know that the growth rate of the capital depends from the workers' share, we can replace the growth rate with the growth rate of the national income. As we also know from the assumption (13) that we have a constant capital output ratio, the growth rate of the national income must be the same as the growth rate of the capital stock. In other words, a fluctuation in capital leads directly to a fluctuation in the national income.

$$(21) \qquad \frac{\dot{q}}{q} = \frac{1-u}{\sigma}$$

If we put now equation (18) and (21) together, we get the differential equation of the employment rate with

$$(22) \qquad \dot{v} = v\left[\left(\frac{1}{\sigma} - (\alpha + \beta)\right) - \left(\frac{1}{\sigma} \cdot u\right)\right]$$

Compared to the classical Lotka-Volterra model, the employment rate in equation (22) stands for the prey.

To define the equation for the predator population we should start with the Phillips curve. Phillips originally estimated a correlation between a change in wage and unemployment rate in United Kingdom for the period of 1861-1957 (Phillips 1958, 283). Goodwin seized this suggestion of a relationship, but for his purpose he linearized it. Figure 3 shows an example of simplified Phillips curve. To transform the unemployment rate into an employment rate we need to know that the labor supply is made up of employed and unemployed workers. This means that $1 = v + z$, where v=employment rate and z=unemployment rate. The figure 3 also shows that transformation.

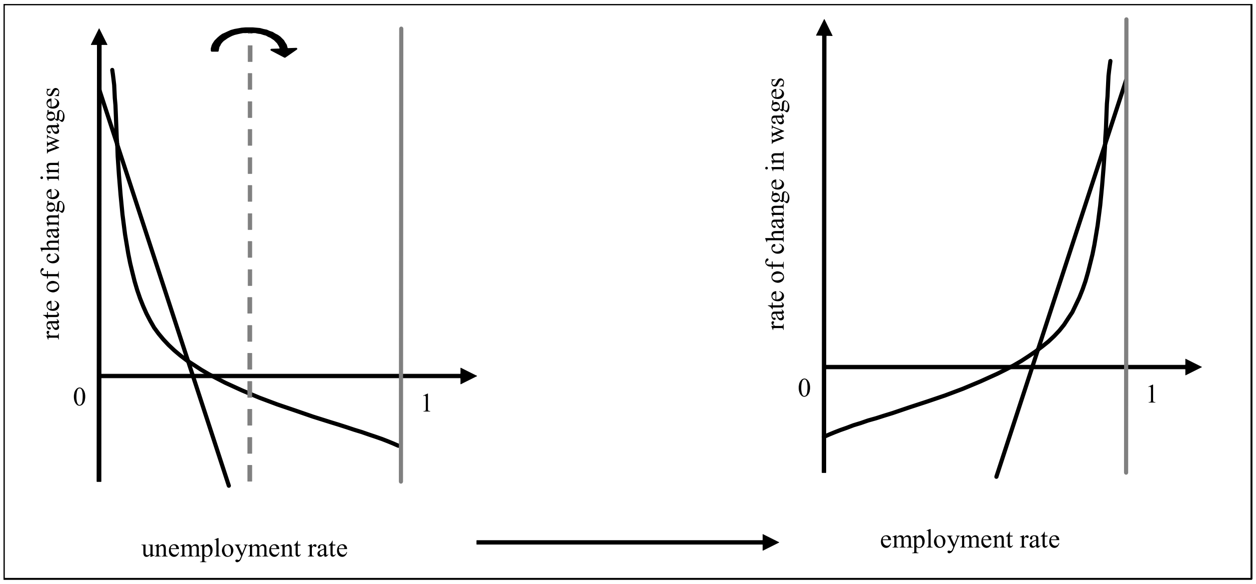

figure 3 - transformation of the Phillips curve

With the transformed and linearized Phillips curve we can write

(23) $$\frac{\dot{w}}{w} = -\gamma + \rho v$$ where w=wage, γ=intersection of the y-axis and ρ=increase

We also know that

(24) $$u = \frac{w \cdot l}{q} = \frac{w}{a}$$ where u=workers' share of the national income

A logarithmical derivation with respect to time gives us the growth rate of the workers' share

(25) $$\frac{\dot{u}}{u} = \frac{\dot{w}}{w} - \frac{\dot{a}}{a} = \frac{\dot{w}}{w} - \alpha$$

We get the final differential equation of the workers' share (26) if we make (23) and (25) equal.

(26) $$\dot{u} = u\left[-(\alpha + \gamma) + (\rho v)\right]$$

To summarize it, the pair of differential equations in the Goodwin model is as follows

(27) $$\dot{v} = v\left[\left(\frac{1}{\sigma} - (\alpha + \beta)\right) - \left(\frac{1}{\sigma} \cdot u\right)\right]$$ as for the prey population

(28) $$\dot{u} = u\left[-(\alpha + \gamma) + (\rho v)\right]$$ as for the predator population

The first summand in the square brackets is the 'natural growth rate' of the variables whereas the second summand gives us the density.

Next we need to define the critical points where all closed orbit circle around. For that, we must set growth rates of u and v to zero.

$$\bar{v} = \frac{\alpha + \gamma}{\rho} \tag{29}$$

$$\bar{u} = \frac{1}{\sigma} - (\alpha + \beta) \tag{30}$$

It is important to see that there is a difference between the critical points in the classical Lotka-Volterra model and those in the Goodwin model. The increase in productivity α is implemented into both differential equations. To achieve a closed orbit around a defined coordinate $(\bar{u}, \bar{v})$ in the Goodwin model, it is necessary that the values of the variables be in a specific ratio to each other.

The period of the cycles is like in the Lotka-Volterra model with respect to the variables

$$T = \frac{2\pi}{\sqrt{(\alpha + \gamma) \cdot \left(\frac{1}{\sigma} - (\alpha + \beta) \right)}} \tag{31}$$

The behavior is like in the general predator-prey model and is shown in figure 4. There are four quadrants regarding the central point. The small arrows indicate the behavior of the workers' share and the employment rate.

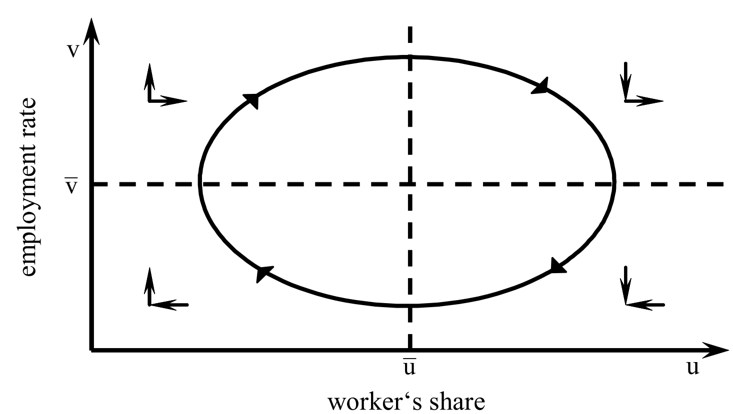

figure 4 - behavior of workers' share and employment rate in different sectors

To understand the behavior of the model later the figure 5 can help. Here we find a quick overview of the behavior of certain variables.

		critical point workers‘ share	critical point employment rate	period
		$\bar{u}$	$\bar{v}$	T
increase in productivity	α	-	+	+
increase in labor supply	β	-	none	+
capital output ratio	σ	-	none	+
intersection Phillips curve	γ	none	+	-
increase in Phillips curve	ρ	none	-	none

figure 5 - behavior of variables

3 A Simple Goodwin Model In System Dynamics

The engineer J.W. Forrester developed System Dynamics in the 1950s. It is a methodology for studying and managing of complex systems with feedback loops and time delays. (for further readings see Forrester 1961 or Sterman 2000). The main difference to System Thinking is that System Dynamics goes one-step further. It aims to analyze behavior of complex problems by constructing and testing models with the help of computer. Additionally we can easily test different policies (System Dynamics Society, 2005)

3.1 Problem Articulation

This section deals with a simple transformation of the differential equations of the Goodwin model that helps understand the relation between the variables. Most variables are exogenous. This has an advantage though, that we are able to test the model with different data.

3.2 Formulation Of A Simulation Model

The model is build from the classical Lotka-Volterra model with the well-known differential equations as seen in (3)

(32) $\dot{x} = x \cdot (+a - by)$ for the prey population

(33) $\dot{y} = y \cdot (-c + dx)$ for the predator population

Figure 6 simply transforms these equations into a first System Dynamics model.

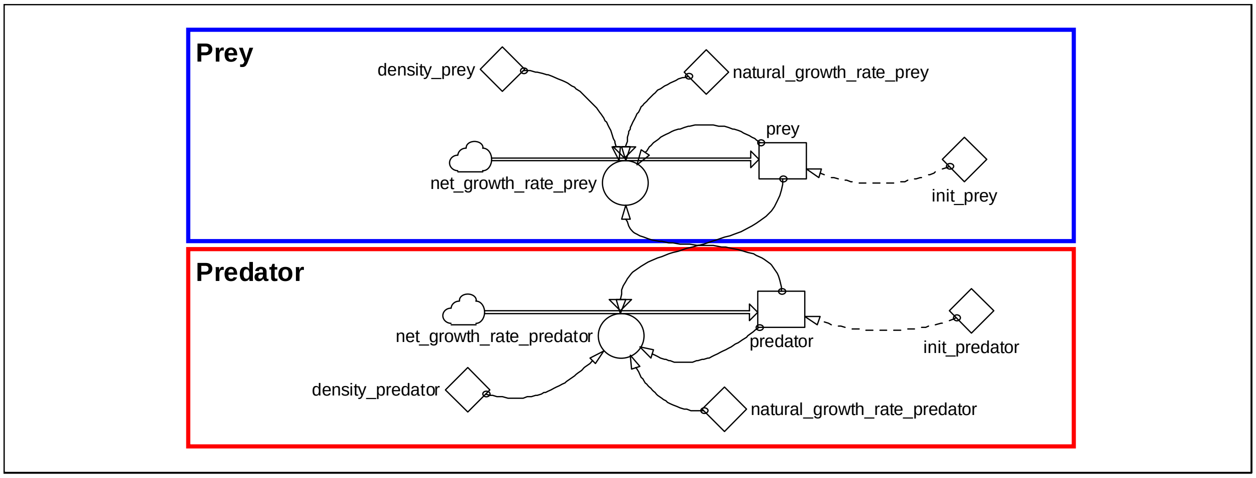

figure 6 - System Dynamics structure of a predator-prey model

For our purpose, we use stepwise modeling. To extend this basic structure into a simple Goodwin model we just add to the natural growth rates and the densities the equivalent Goodwin variables.

Hence, as it follows from (27) and (28)

(34) $a = \frac{1}{\sigma} - (\alpha + \beta)$ and $b = \frac{1}{\sigma}$ for the prey population and

(35) $c = \alpha + \gamma$ and $d = \rho$ for the predator population

With simple renaming of the x and y variables into v for the employment rate and u for the workers' share, we get the first model, which is shown in figure 7.

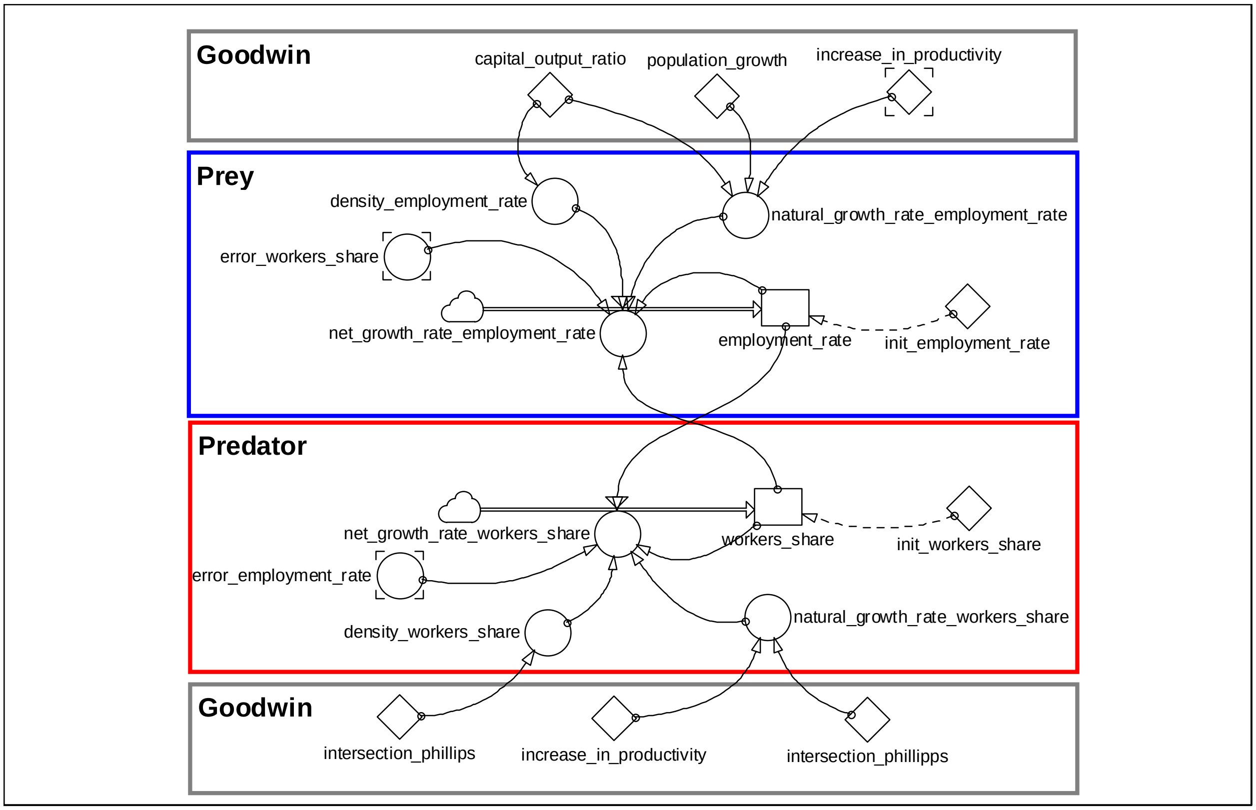

figure 7 - System Dynamics structure of a simple Goodwin model

The added parts to the normal Lotka-Volterra model are called Goodwin. As mentioned, the model is not stable with this structure, because the increase in productivity is implemented into both natural growth rates. This leads us to the problem that we only get closed orbits if the equation (34) and (35) fits for all variables. We do not have this problem with the classical Lotka-Volterra case, because the equivalent formulas are independent from each other regarding closed orbits. For that reason and to make the model independent from calculations of the correct ratios of the variables, we implemented error variables for both populations. With that, we are able to correct automatically the equations (29) and (30). We add automatically the needed value to achieve closed orbits. The correction is as follows, based on (4) and (5)

(36) $$\bar{v} = \frac{c + error\_workers\_share}{d}$$

where c=natural growth rate employment rate and d=density employment rate

(37) $$\bar{u} = \frac{a + error\_employment\_rate}{b}$$

where a=natural growth rate workers' share and b=density employment rate

The model structure for the error correction is shown in figure 8

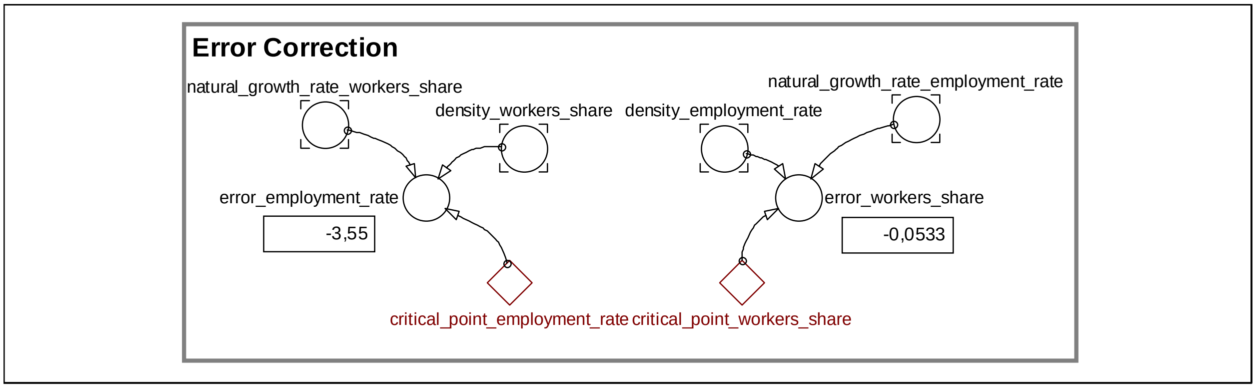

figure 8 - System Dynamics structure of the error correction in a simple Goodwin model

3.3 Error Testing

The challenge of all Lotka-Volterra models is that the models are sensitive to the variables. An equilibrium check was made: if the initial values are equal to the critical value, the closed orbit must have a range of zero. In addition, there should not be any cycles in the graph against time. The effect is shown in figure 9. The starting point for the workers' share and the employment rate is equal to the critical values calculated data. Furthermore, the error estimation was checked with the data from subsection 3.4. This leads an error correction of employment rate of -0.41 and to -0.58 for the workers' share.

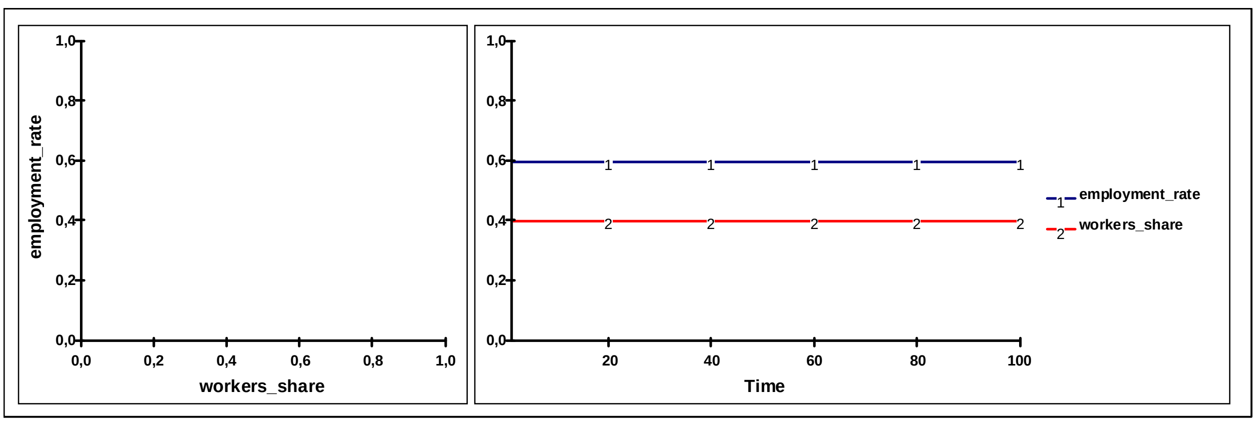

figure 9 - equilibrium check of a simple Goodwin model

3.4 Behavior

The initial randomly selected values for the cyclical behavior are

- capital output ratio σ=1.00
- population growth β=0.01
- increase in productivity α=0.01
- intersection Phillips γ=1.00
- increase Phillips ρ=1.00
- critical point employment rate $\bar{v}$=0.60
- critical point workers' share $\bar{u}$=0.40
- init employment rate v_0=0.50

- init workers' share u_0=0.50

This data leads to a period of T=12.83. Figure 10 shows the behavior. The left phase plot shows the expected cycle. The right graph against time shows the cyclical behavior.

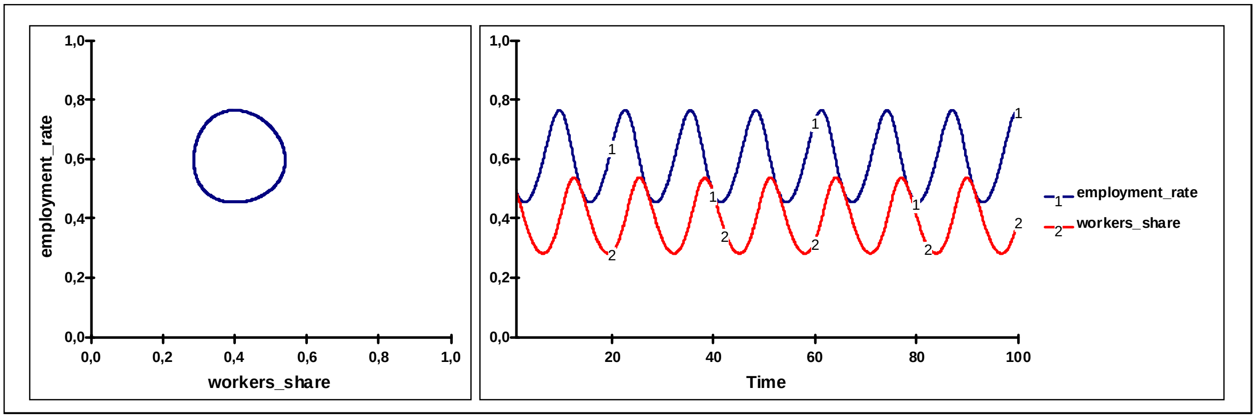

figure 10 - behavior of the simple Goodwin model

4 An Extended Goodwin Model In System Dynamics

4.1 Problem Articulation

The first model was introduced to show the possibility to transfer differential equations into System Dynamics. With this, we are able to show and to understand the consequences of various changes. The main problem of the reduced two differential equations is the lost of information. We cannot understand the sense of the following relaxations if we do not pay attention to the main stocks behind the simple model. The extended Goodwin model provides us this necessary details. By using self-correction mechanisms, the model gives us the possibility to "play" with the parameters and to understand the behavior behind the mathematics. But does this really help to understand the economic behavior? This can be doubted. This model leaves some space for improvement because it does not use the possibilities that System Dynamics really provides. Therefore, this section will go one-step back behind the economic structure of the Goodwin model to present an approach a classical economic perspective could never give.

4.2 Formulation Of A Simulation Model

The equation given by Goodwin (see subsection 2.2) is then defined by the following equations (Goodwin 1967, 54 and Harvie 2000, 352):

(38) $$a = a_0 e^{\alpha t}$$ where a=labor productivity,

(39) $$n = n_0 e^{\beta t}$$ where n=labor supply,

(40) $$q = \frac{1}{\sigma} k$$ where q=national income and k=capital,

(41) $$l = \frac{q}{a}$$ where l=employment,

(42) $u = \frac{w}{a}$ where u=workers' share and w=real wage,

(43) $\dot{k} = (1-u)q$ where $\dot{k}$ = investments,

(44) $\frac{\dot{k}}{k} = \frac{\dot{q}}{q} = \frac{1-{}^{w}\!/_{a}}{\sigma}$ where profit rate,

(45) $\frac{\dot{w}}{w} = -\gamma + \rho v$ where w=wage, γ=intersection of the y-axis and ρ=increase

The model will consists of four stocks – a stock of capital, a stock of productivity, a stock of labor supply and a stock of wages. The productivity stock and the labor supply stock grow exponentially with the rate of α and β. The investments influence the stock of capital and the Phillips curve estimation changes the stock of wages.

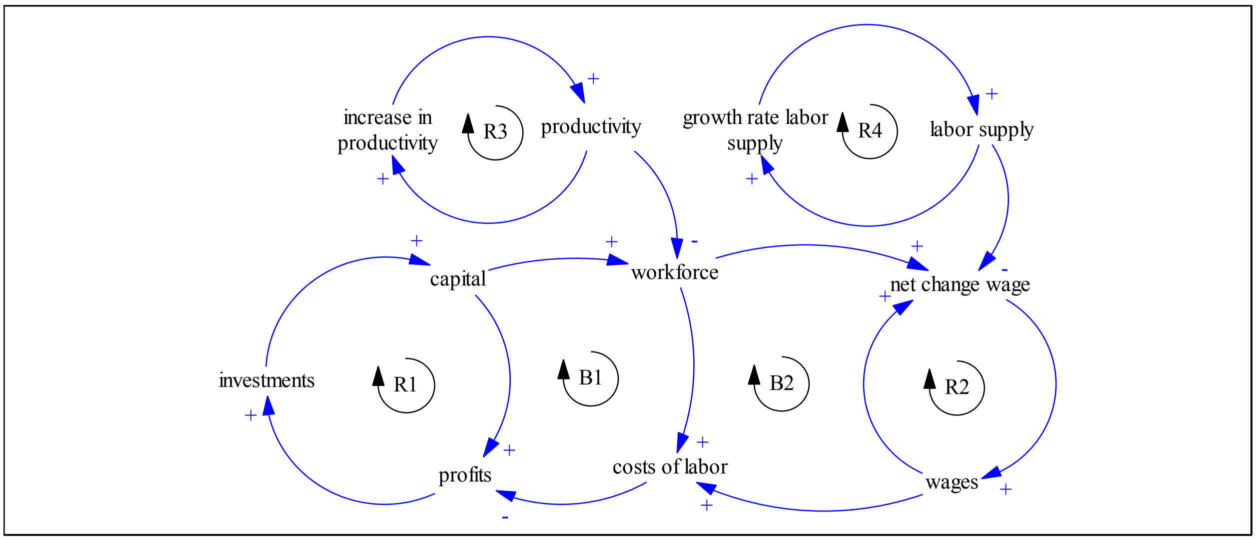

figure 11 - causal loop diagram of an extended Goodwin model

Figure 11 shows the causal loop diagram. This model has four reinforcing loops and two balancing loops. The loops R3 and R4 only support an exponential increase of the stocks of productivity and labor supply. The most important loops are the balancing loops B1 and B2. The loop B2 mainly causes the oscillations in the model.

For the initialization we must calculate the variables init productivity and init wage. Earlier we implemented a self-initializing process so that the correct values are automatically set. The initial values are calculated in reference to the initial start values of the employment rate and the workers' share. The exogenous variables are like in the previous simpler model – capital output ratio σ, increase in productivity α, growth rate labor supply β, increase of Phillips curve ρ and intersection of Phillips curve γ. Additionally there are two more variables to set. First, the initial value of the stock of capital and second the initial value of the stock of labor supply. With this, we can now adapt the model to specific country data and this will help us understand the behavior better. The figure 12 shows the full-extended model based on the original Goodwin assumptions.

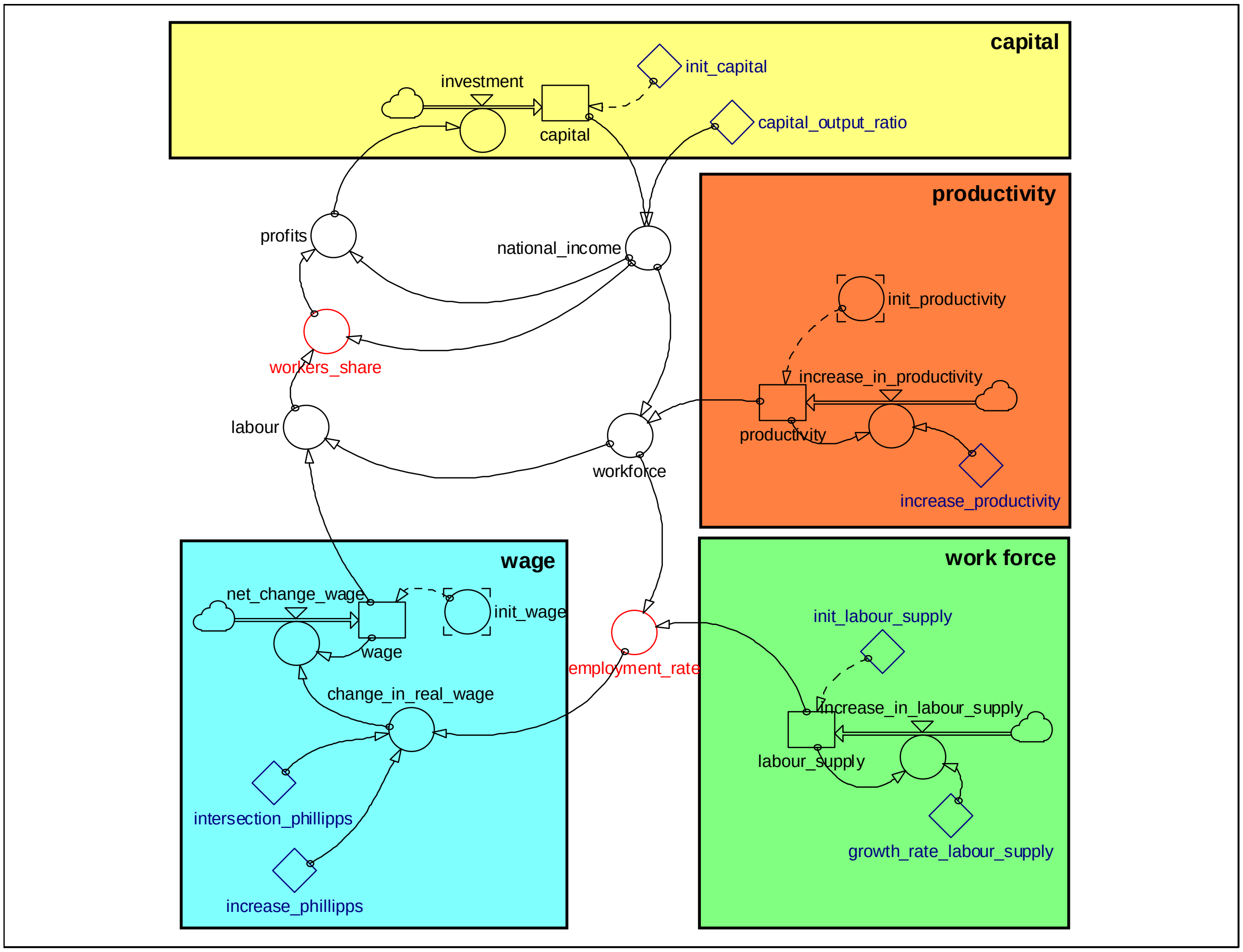

figure 12 - System Dynamics structure of an extended Goodwin model

4.3 Behavior

To compare the simple and this model we used the same initial values for the cyclical behavior like in section 3.4

- capital output ratio σ=1.00
- population growth β=0.01
- increase in productivity α=0.01
- intersection Phillips γ=1.00
- increase Phillips ρ=1.00
- critical point employment rate $\overline{v}$=0.60
- critical point workers' share $\overline{u}$=0.40
- init employment rate v_0=0.50
- init workers' share u_0=0.50

This data leads also to a period of T=12.83 and an error correction of employment rate of -0.41 and of workers' share of -0.58. The challenge was to implement the error variables into the model. With them we can compare the models in the behavior. It has to be the same. The error estimation is not necessary in later simulation, because of the endogenous character of the model. All further investigations are without the error variables.

5 Test of Modifications

This section is mainly about some released relaxations about the Goodwin model. Every modification is tested in the extended model.

5.1 A Non-linearized Phillips Curve

This section implements a suggestion offered by Desai et. al. (Desai 2003, 7). They proposed a critique to the linearized Phillips curve, because it does not reflect reality. Goodwin estimated the Phillips relationship in a linear equation (see section 2.2). He did this to simplify the model and meant that this would not change the relationship (Goodwin 1967, 54)

We changed the variable ‘change in wage’ compared to (23) into

(46) $$\frac{\dot{w}}{w} = -\gamma + \rho(\lambda - v)^{-\delta}$$ where λ=boundary and δ=auxiliary variable

This modified Phillips curve has the $\lim_{v\to\lambda} \frac{\dot{w}}{w} \longrightarrow +\infty$. Therefore, the boundary of the workers' share should be 1. We simulated several models with the data from Harvie (see also figure 16). The sense of this curve is obvious. Nevertheless, we had problems with the plotted behavior. The reason is that the bended Phillips curve leads to an increasing above average in the changing of the wages. This leads in the end to an enormous increase in the critical workers' share and additionally in a decreasing of the period.

With the enhanced model, it was simple to implement this policy. The outcome did not support this idea. By setting a higher level for the employment rate, we get the disadvantage of a tremendous increase in the span of workers' share, followed by an extreme change in the stock of wages. In figure 13 are some examples of graphs shown for a boundary λ=3.0 and an auxiliary variable δ=1.5. We could reduce the effects by changing the increase in Phillips curve and the intersection of the Phillips curve, but with this, we are going back to a nearly linearized Phillips curve.

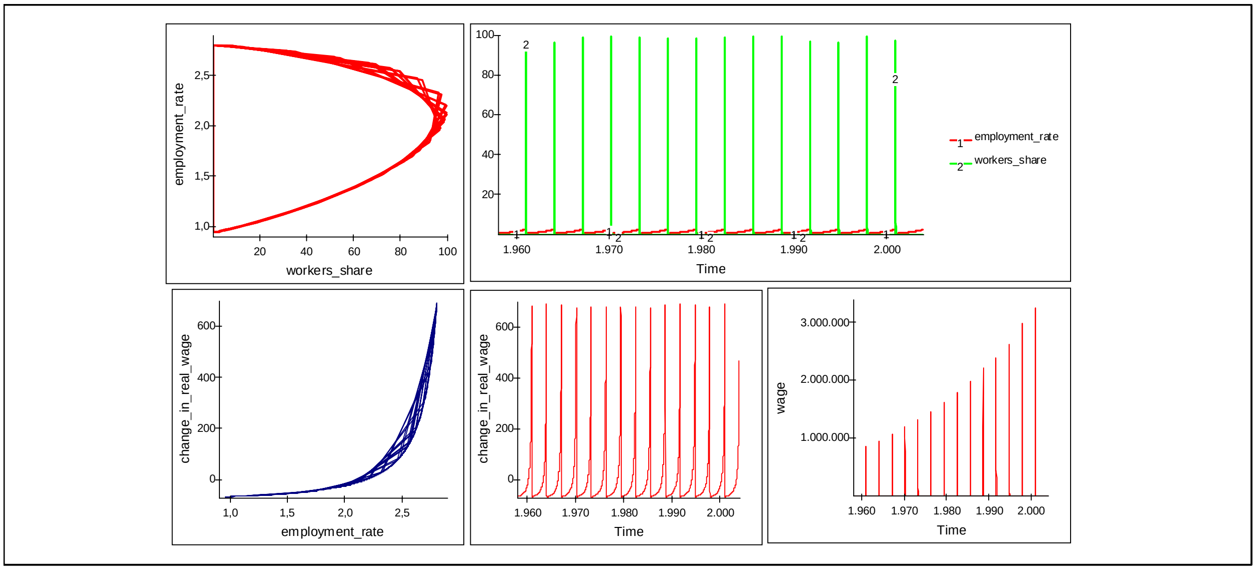

figure 13 - behavior of a non-linarized Phillips curve in an extended Goodwin model

With the modification, we implemented a discontinuity. The main balancing loop can not level the behavior any longer, and the result is that employment rate moves towards zero and the stock of capital as well. Therefore, we could not support this suggestion.

To summarize we have to say that this modification does not make sense even it is intuitive.

5.2 A Non-linearized Investment Function

In the previous subsection, we mentioned an article from Desai et. al. where a second change to the Goodwin model is strictly related to investment function $\dot{k} = (1-u)q$ where all profits are invested. Desai et. al. wanted to make the equations more flexible as long as they could within the limits of workers' share the capitalists allow (Desai 2003, 9). The rate of investment is written as a logarithmic function dependent on u and $\tilde{u}$ =maximum of accepted workers' share. The equation is as follows:

$$(47) \qquad \frac{\dot{k}}{k} = -\ln(1-\tilde{u}) + \ln(\tilde{u} - u)$$

The implementation is, as in the first case, done in the enhanced Goodwin model. Again, the outcome is surprising. The stock of capital is almost going zero and this leads to the behavior we could see in figure 14. A possible explanation for that could be that, from a mathematical point of view, this equation works for the aggregated Lotka-Volterra. Nevertheless, in our model we also have to take into account the stocks behind this equation. With that knowledge in mind, we have to admit that this investment function does not work properly.

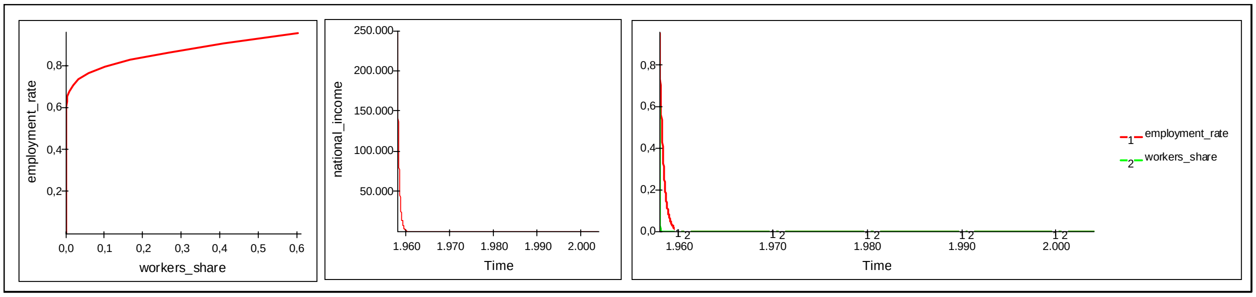

figure 14 - behavior of a non-linearized investment function in an extended Goodwin model

5.3 A Limit In The Wage Setting

Aguiar modifies the Goodwin model to build explosive or contracting cycles. He assumes it could be useful to impose a ceiling to one of the variables in order to avoid the explosion. A simple way is if the workers' share is above a certain limit, the worker would not try to increase the wages any longer. This leads to a change in wage of zero (Aguiar 2001, 12).

We studied this policy with the outcome that we could not support his suggestion. The main reason is that the employment rate is going to zero if the ceiling is reached. Additionally, the stock of capital decreases rapidly because of the high workers' share. The reinforcing loop from the national stock of capital to the national income to the profits empties the capital stock very fast. Again, the simulation gives a clear result that we cannot confirm Aguiar's idea.

5.4 Growth Process of Zero

Aghion and Howitt argue that this model creates cycles because of the growth process (Aghion and Howitt 1998, 234). We get a first suggestion by looking at the causal loop diagram in figure 11. The productivity and the labor supply are separated loops but there involvement in the fluctuation process is a constant. Therefore, the behavior is not different if we set them to zero. Verification by policy testing in the enhanced model clearly supports this, as we can see in figure 15.

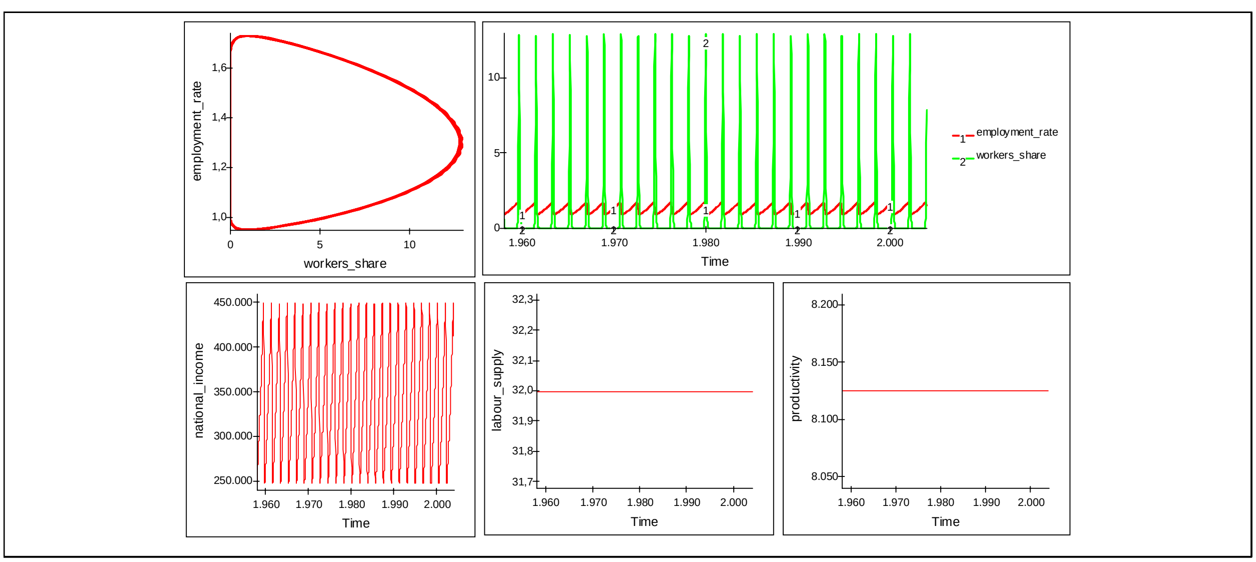

figure 15 - behavior of zero growth in productivity in an extended Goodwin model

5.5 Summary

In this section, we tested some examples of changes to the Goodwin model made by different authors. Using System Dynamics, we showed that the classical approach does not give the correct evidence about the assumed behavior. Furthermore, we demonstrated that taking care of the stocks behind the rates leads to failure behavior for most of the equations.

6 Verifying Economic Data of Germany

This section mainly refers to the paper of D. Harvie, who tested growth cycles econometrically in ten OECD countries (Harvie 2000). We reflect only the German data. The Goodwin equations are unmodified so that we can use both the simple and the extended model to compare and verify.

6.1 Harvi's Econometric Data: A Long-Term Perspective

Harvie did a log-linear parameter estimation of the productivity growth α, the labor force growth β, the capital-output ratio σ and the Phillips curve variables γ and ρ (Harvie 2000, 355). The time span is from 1956 until 1994. Figure 17 shows the employment rate and the workers' share for these years. Here we can see the behavior we try to rebuild with the model.

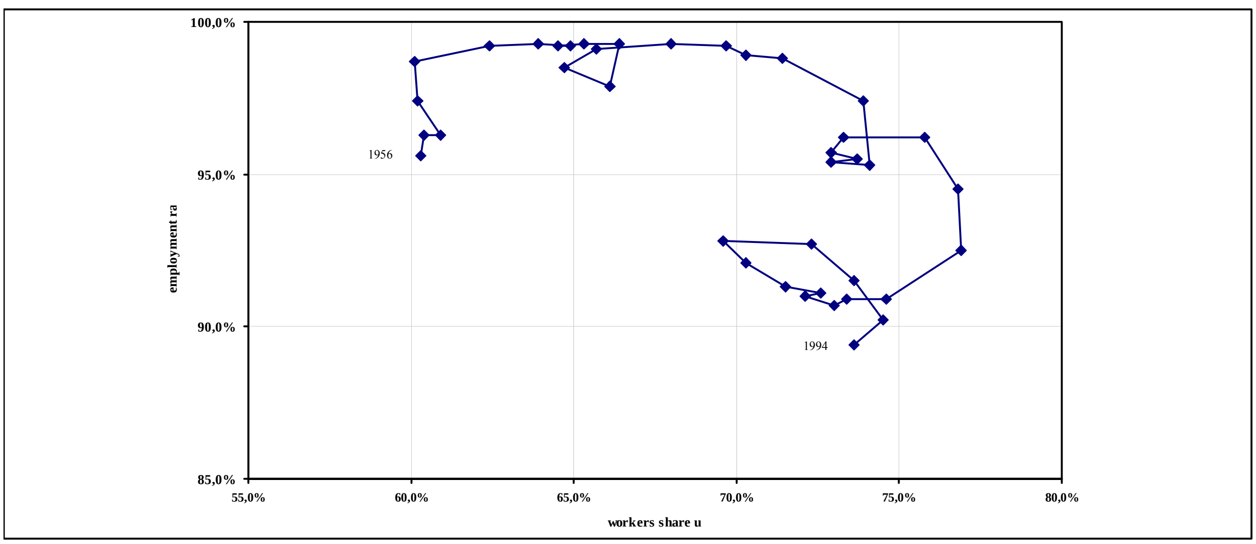

figure 16 - Goodwin's model with German data in a long-term perspective

Harvie estimated the following values (Harvie 2000, 362):

- capital output ratio σ=2.4941
- population growth β=0.004142
- increase in productivity α=0.0329
- intersection Phillips γ=85.49
- increase Phillips ρ=65.55
- critical point employment rate $\bar{v}$=1.30
- critical point workers' share $\bar{u}$=0.90

We start the dynamic model with initial values for the employment rate and the workers' share. We use the data from 1956 with init employment rate v_0=0.956 and init workers' share u_0=0.603. For the enhanced model it is also necessary to initialize the stock of capital and the stock of labor supply with init capital = 620 000 Mill. and init labor supply = 32 Mill units. We estimated the amount to achieve in the end of the simulation nearly current values. This gives a good approximation of the development of the stocks. But the behavior is totally independent from these two stocks.

The calculated period by the models are like in the data from D. Harvie with T=1.13 years, which supports the correctness of the model. To achieve closed orbits in the simple model there is an error employment rate=-0.308 and error workers' share=-0.00305 necessary. The other model is self-correcting, without error values. The behaviors are the same as we can see in figure 17.

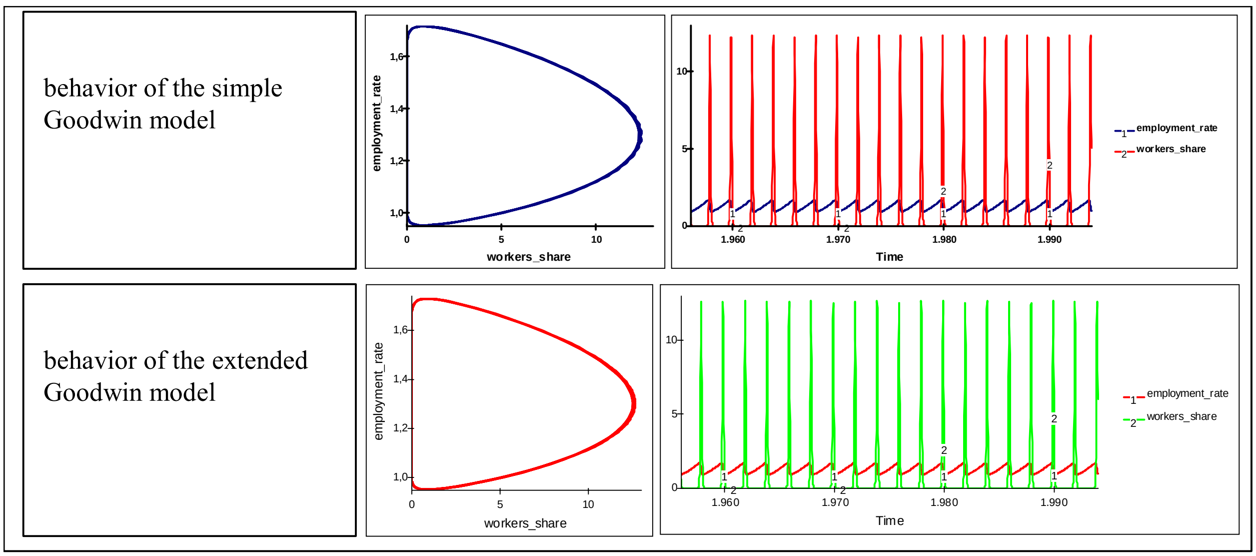

figure 17 - behavior of the System Dynamics models with data from D. Harvie

The cycles are as predicted. Harvie too mentioned that a cycle outside the (0;0)-(1;1) boundary does not make sense from an economic point of view. Harvie argues that the Goodwin model provides a qualitative support but not a quantitative one (Harvie 2000, 363).

So why then extend a model, if the behavior is the same? We can answer this question by looking at the additional information the second model provides in figure 18.

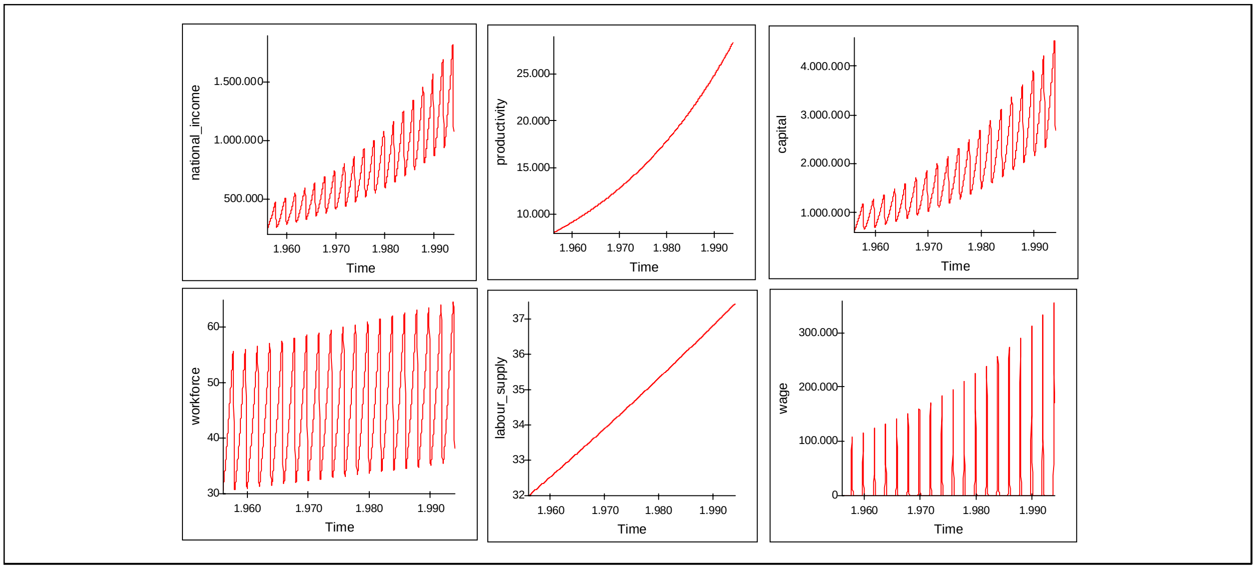

figure 18 - behavior of certain variables based on data from D. Harvie

As we assumed, the national income grows exponentially, so do the labor supply and the productivity. The wages oscillate between nearly zero and the upper level. But, the workforce and the employed people are changing dramatically. It starts with a low fluctuation and increases with the time. If the simulated time is to long then the model creates huge amplitudes of certain variables. Harvie did a very good parameter estimation for 10 OECD countries to prove the validity of the Goodwin model. But as we demonstrated, the Goodwin model can only describe short-term or medium-term business cycles. These two facts could never be shown with the simple model or with econometrical parameter estimation and is clearly a huge advantage of System Dynamics modeling. Veneziani supported this suggestion when he assumed that the long-term change is more often interpreted as a product of structural change (Veneziani 2001, 16)

6.2 A mid-term perspective of Germany

With the gained information, the next step is clear. Is the behavior in a mid-term perspective better? Figure 20 shows the reference data from Germany from 1984-2004. For that, we model in the second version with a timeframe of twenty years from 1984 until 2004.

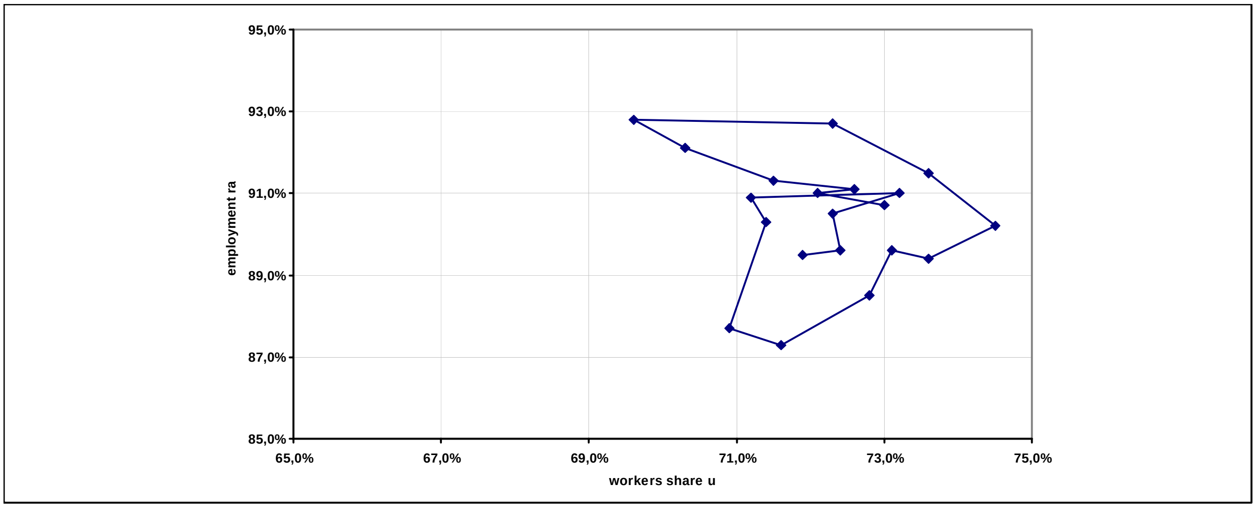

figure 19 - Goodwin‘s model with German data from 1984-2004

We accept most of the given data by Harvie

- capital output ratio,
- population growth β=0.004142,
- increase in productivity α=0.0329,
- intersection Phillips γ=85.49.

Of course, the Goodwin model does not predict the perfect outcome. So we adjust the model with changing the critical point which we estimated by the graph in figure 20

- critical point employment rate $\overline{v}$=0.91
- critical point workers' share $\overline{u}$=0.72

For that reason, we change the capital output ration to σ=7.2 and the increase Phillips ρ=95.00. The critical point is now inside the (0;0)-(1;1) boundary. With that, we transfer a possible error of the model to the input variables. In other words, the inputs may not be correct and differ to the real data; we can use it as a good approximation so that the output is close to the expected outcome.

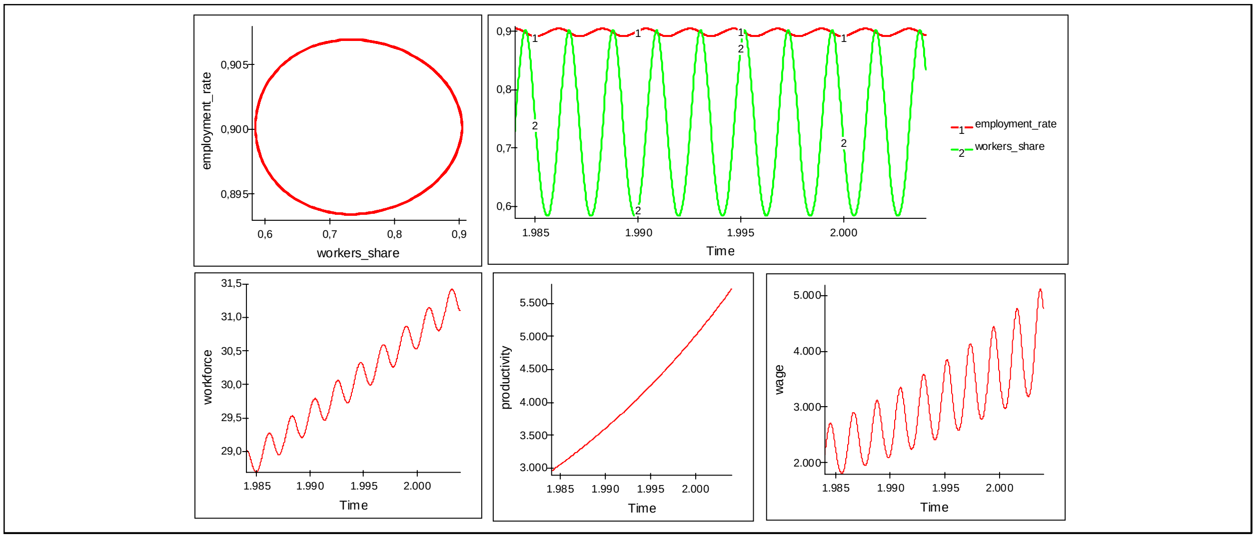

figure 20 - behavior of the extended Goodwin model with German data in a mid-term perspective

The output is still not extremely satisfactory, because the period is too short and it is not spiraling downwards. However, this could be changed with different variables like productivity or intersection Phillips curve. With these, we would accept in all variables slight error values.

6.3 The Advantage Of System Dynamics In Modeling Dynamic Structures

As seen in section 4.3, enhanced model still does not reflect reality. A big advantage of System Dynamics is the possibility of implementing dynamics into the model. An econometric estimation could hardly deal with real data for the productivity increase or a changing in the capital output ratio, because these are mostly point estimations (for further readings see Sterman 1991, 16). This section we base on the enhanced model with the mid-term data and simulate continuous changes in

a) capital output ratio σ=7.2,
b) growth rate labor supply β=0.004142,
c) increase productivity α=0.0329,
d) increase Phillips curve ρ=95.00,
e) intersection Phillips curve γ=85.49.

To understand the behavior we double the initial values over the timeframe with a RAMP function. Figure 21 shows the output. Some effects are bigger than others are. It is important to refer to the data of the Phillips Curve and to the mathematical equations. For example, the influence of productivity is much bigger than the growth rate of the labor supply.

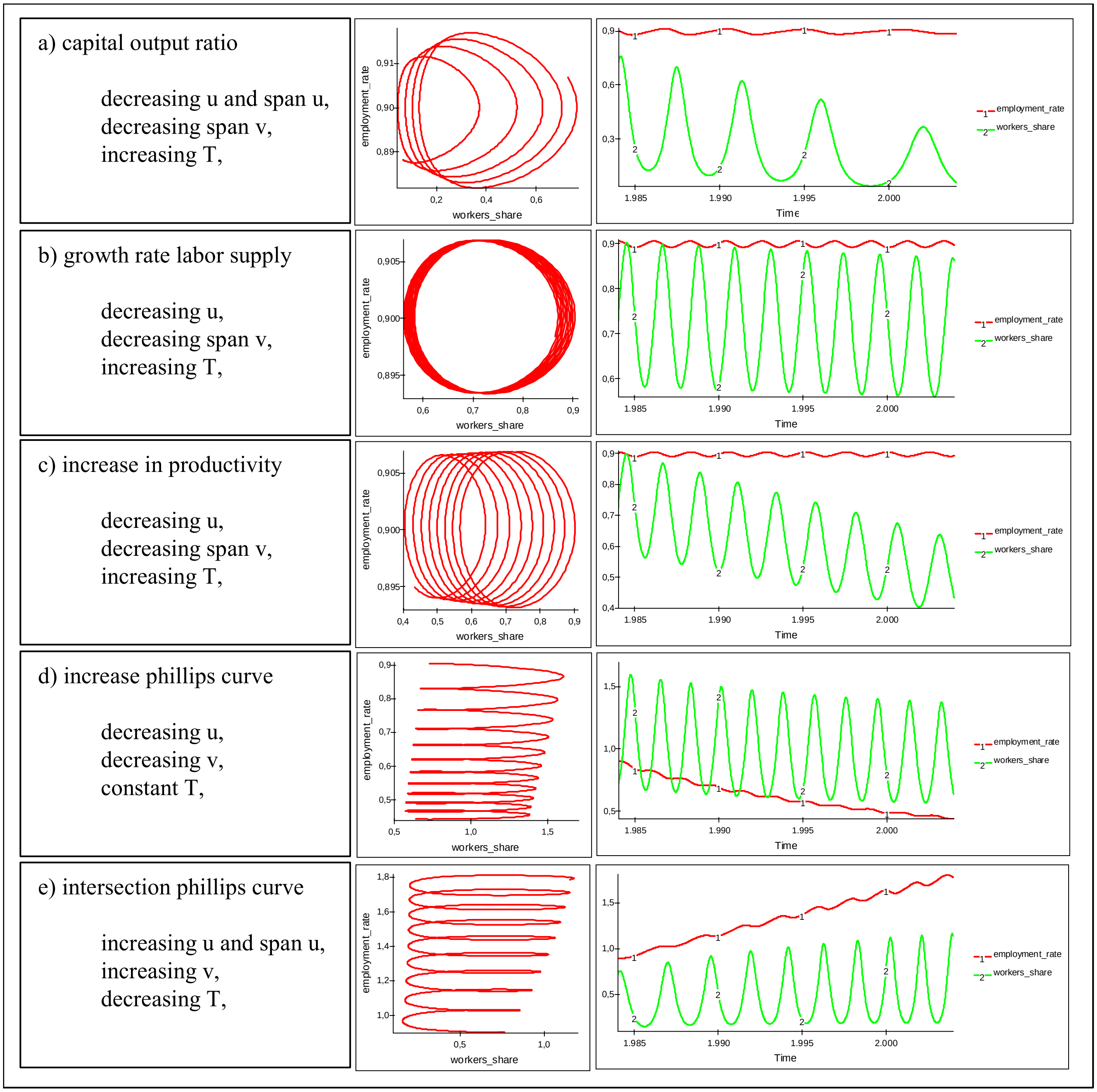

figure 21 - behavior of certain variables in the extended Goodwin model with increasing values

With this gained information, we could try to reconfigure the mid-term Goodwin model to adopt reality better. What if we let move the Phillips curve toward a higher unemployment rate? In the long-term perspective, the Phillips curve was moved several times because of changes in the structure of the production, like shown in figure 22.

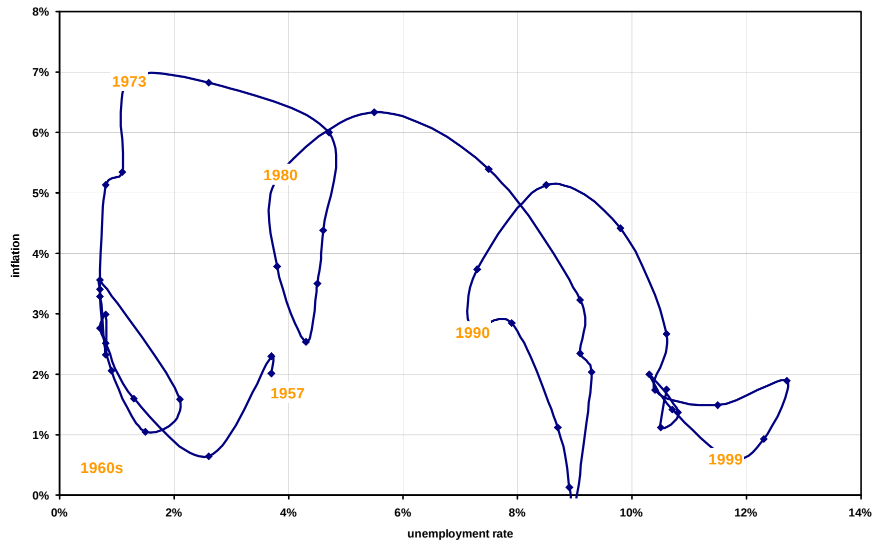

figure 22 - modified Phillips curve in a long-term perspective with German data

For modeling this, we us the RAMP function with a slight change in the intersection of the Phillips curve. The output in figure 23 proves that. We can see that the circle headed to a lower employment rate shows almost what happened in the reference mode (see figure 19).

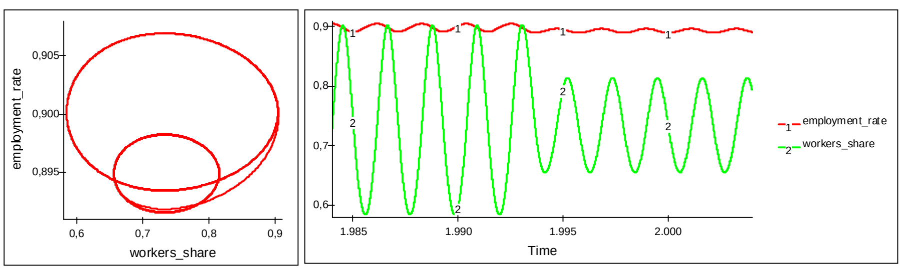

figure 23 - behavior of the extended Goodwin model with a change in the Phillips curve

To summarize this subsection, we conclude that System Dynamics provides a tool with which we can model the reality and evaluate the economic theories. The advantage of this approach is that with the classical tools we would never gain some additional information.

7 Conclusion

In this paper, we first reviewed basic information about the mathematical Lotka-Volterra differential equations, and then we developed two models to understand the economic behavior of the Goodwin model. We analyzed the behavior and identified the problems of a long-term econometric estimation. Looking at the stocks, we were able to understand the underlying structure and could show the problem of focusing only on the mathematics and aggregated equations. Furthermore, we showed, in section six, the advantage of System Dynamics in dealing with dynamic structures and that it provides a much better model in explaining the reality. With changes with respect to time, we were able to support the theory in a better way than the so-called “orthodox” approaches. This method can be used to improve teaching in universities and helps understand the theory better.

Further research could deal with a micro foundation of the aggregated behavior of the Phillips curve. The change in wage could be modeled individually for a single worker, which would lead to a more random distributed reaction. A possible basis could be an agent-based view (for further readings see Borshchev and Filippov 2004). There is also the problem of the non-linearized Phillips curve (see section 5.1). Further research is important. In addition, it could be interesting to use the real data from different countries with an implemented misperception variable to rate the Goodwin model. The main research from a System Dynamics point of view should be focused on how this can help to make up for the lack of explanation from the econometrical approaches.

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