Business Dynamics Model for Market Acceptance Considering Individual Adoption Barriers

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Abstract:

The technological developments of our time provide the basis for a continuous flow of new applications and services. But even when allowing for significant improvements in everyday life, some of these innovations need an incredibly long time to be broadly – if at all – accepted in the market. Marketing and technical implementation are often only partially responsible. Consumers additionally need to overcome personal barriers to adopt an innovation. The height of this barrier is determined by manifold aspects like: Financials, required infrastructure, technical affinity, expected learning efforts, and safety concerns.

In this article we introduce a market diffusion model that explicitly takes individual adoption barriers into account, while maintaining the top down approach of Business Dynamics. An extendible method is used which bridges the gap between the level of individual traits and a macroscopic view. It enables system dynamics models to be designed on an individual level and to be simulated on an aggregated level.

The model includes a feedback loop between the number of existing adopters and the number of consumers prepared to adopt the innovation. It explicitly allows for the adoption by only a portion of the potential customers. Furthermore, the model can reproduce tipping points and visually explain slow market developments or unexpected late successes.

Keywords:

Business dynamics, market acceptance and diffusion, individual barriers, tipping points, mobile information society

1. Introduction

Two of the most influential technological developments of recent times are the launch of mobile communication networks for broad deployment and the development of the Internet/World Wide Web. GSM and its technical counterparts address very fundamental needs: Social contact, safety, business prospects and time saving. Its success is tremendous. Almost 50% of the world's population owns a mobile phone. Similarly impressive is the success of the World Wide Web/Internet. It is used as an information and communication platform by more than one billion users worldwide via over 500 million access units [BTK-r07].

With this basis of users, new applications are being developed that piggyback on one or both of the developments. Downloading of new computer software, Internet banking, payment via mobile phone, special information services, use of podcasts, purchasing of MP3, are but a few examples. The success of these applications is not obvious in terms of acceptance in general, and even less so

commercially. Despite true advantages and no direct costs (e.g. for Internet banking or podcast downloads) some services are not fully embraced.

The ultimate goal is therefore to fully understand these phenomena and to be able to apply well targeted measures that aid market success. Investigations show though that for markets of high involvement products (of which technically sophisticated mobile phone/Internet applications are part), a multitude of parameters influences the adoption process (see e.g. [GAS-b95]); the more stakeholders and interdependencies involved, the more complex it gets. Furthermore, end user behaviour is characterized by individual differences. A suitable model is thus required that can consider both: Complex market relations as well as individual behaviour. This study represents, as a first step, a new approach to such a model.

The basis of the model is given by the diffusion model of Bass [Bas-a69]. The Bass model is then extended by the possibility to make individual differences and to have a changing number of consumers who will adopt. The individual differences are accounted for with the help of distribution functions for perceived use and expected effort/overall barrier. We assume that when perceived use exceeds barriers and effort, a person (eventually) becomes an adopter. The changing number of customers is due to the inclusion of interdependencies. Other than e.g. Chatterjee and Eliasberg [ChE-a90], we explicitly consider that economies of scale, word of mouth effect, improved technical capabilities and general network effects impact the overall number of (possible) customers.

Furthermore, it is shown that – depending on the actual parameters – the feedback mechanisms can lead to typical tipping point behaviour. This again provides the basis for actively influencing market success. Despite the individualized approach we use Business Dynamics [Ste-b00] (and not Agent Based modelling), because we are interested in the top down view that potentially considers more stakeholders and very complex market relations [SM-a03] [BF-a04].

This study is structured as follows. Section 2 introduces the basic concept of acceptance and barriers deployed for our model. In Section 3 the model is explained (Subsection 3.2), based on the Bass diffusion model (see Subsection 3.1). Section 4 shows simulation results of "regular" and "tipping point" market situations. Section 5 closes the study with a summary.

2. Acceptance and Barriers

Following [Kol-b98], the acceptance of (high involvement) products happens in three steps: Choosing, installing and using. Choosing consists of the unconscious first contact and the conscious choice. In the installation phase the product is tested and made accessible, i.e. the product is bought and/or set-up for deployment. At first sight, this is sufficient for the product vendor; he gained a new customer. According to [Kol-b98] though, true acceptance is only reached when the customer also uses the innovation on a voluntary and problem solving basis. Only then will his/her experience initiate the acquisition of updates, the buying of additional services and a positive feedback on potential future customers.

We agree that the first contact with an innovation generally happens by chance. As a very first step, the principle use for the innovation is assessed. Only someone with a bank account and Internet access is going to be interested in home banking and only someone with an MP3 rendering device can be expected to buy MP3 encoded music. Those persons with potential use for the innovation become part of the potential customer basis P (see also Figure 1).

In the next step of our model, like in the model of Kolberg, the potential customer (and only the potential customer) evaluates the properties of the product and rates them against his/her use barriers. This is a very individual process, in which product related as well as personal parameters

play a role [AHS-p03]. Examples of product related factors are: Functionality in terms of relative advantage to existing solutions (time saving, more comfort/fun, increased safety, and more economic use), ease of use, compatibility, trial possibilities, risks of use, learning effort. Personal factors can be: Affinity to technology, prior user experience with technology, need to participate in societal fads (prestige), change related scepticism, social and demographic background, and attitude to safety [Gra-e06]¹. If the perceived use U is larger than the individual hurdle h, the potential customer will become a "future adopter FA" (i.e. a consumer prepared to adopt) and eventually – with a yet to be defined time delay – an actual adopter A and part of the installed base (see also Section 3.2).

In the first approach, we do not distinguish between installation and use, but assume that they are synonymous (see also [Rog-b95]). As soon as a person becomes an adopter the respective positive feedback is initiated (see also Figure 1): The more users there are the cheaper, better and more useful (especially in the case of network effects) the product will be, and the more users there are the more pronounced is the word of mouth effect. The latter as well as the actual using are not yet considered in our model, but seen as essential for later extensions.



Figure 1: Individual acceptance process behind the described innovation acceptance model

3. The Market Model

3.1. The Diffusion Model of Bass [Bas-a69]

In the diffusion model of Bass the number of existing adopters A (also denoted as "installed base") is put in relation to the possible customers P_t (also described as "market saturation limit") by following time dependent equations:

(1)
$$\frac{d}{dt}A(t) = g(t)(P_t - A(t)) \quad with \quad g(t) = a + bA(t).$$

g(t) represents the diffusion coefficient with *a* the innovation coefficient (referring to persons accepting an innovation based on impersonal communication like advertising) and *b* the imitation coefficient (referring to persons requiring personal communication within the social system to become customers). The remaining possible market P(t) declines as the number actual customers A(t) increases, because the number of possible customers P_t is assumed constant

(2)
$$P_t = const = P(t) + A(t)$$
.

¹ [Rog-b95] additionally adds environmental factors, like political and judicial aspects. In our model though, we focus on acceptance on an individual basis. Innovations that are enforced by regulation follow different patterns and are generally not subject to individual choice.

Figure 2 shows examples of how the customer base can develop over time in case of the two extremes: On the left side the consumers consist of innovators only (b=0), on the right side, the consumers consist of imitators only (a=0).



Figure 2: Examples for the development of installed adopter bases A(t) over time according to Bass; left side: pure innovation or exponential model (b=0), right side: pure imitation or logistic model (a=0)

The diffusion model of Bass represents the basis of a significant amount of research and its merits and limits have been discussed extensively (see e.g. [MMB-a90], to the best of the authors' knowledge individual adoption barriers have not been investigated in this context). For our investigation the following aspects are relevant (a detailed description follows in Section 3.2):

- The development of the installed base A(t) is purely dependent on time. All possible customers P_t eventually become part of the installed base A(t). We introduce an additional group: The potential customers P, who are all those customers for whom the application in principle makes sense. Those, for whom the individual barrier has been overcome, become part of the future adopters *FA*. The actual adoption then is, like in the model of Bass, purely a matter of time. In our model not all potential customers necessarily become adopters.
- For Bass, the number of users who eventually become adopters is constant; just the duration and form of the incidence changes. In our model, it is essential that the size of the group of future adopters *FA* can change. Owing to the feedback loop the use value *U* can change (e.g. because of a price decrease [RL-a75]) and with it the share of potential customers for whom the barrier is overcome by the product offer.
- In the Bass model, all customers behave identically. Our model is explicitly based on the assumption that adoption is an individually varying process and that it depends on psychological effects. We nevertheless think that for the overall perspective we intend to obtain, the aggregated approach we are using is appropriate and sufficient [EHGG-a03].
- The diffusion coefficient g(t) is elementary with *a* and *b* constant. This we keep.
- Furthermore, the overall number of potential customers *P* can also increase (see e.g. [MP-a78]). All the time more people buy for example an MP3 player or get an Internet connection for their homes. Nevertheless we will leave this effect for later extensions.
- Several other impacts like those of competitive products, network effects, renewal and replacement are not reflected in the Bass model. This we will also leave for later studies.

3.2. Diffusion Model Considering Individual Barriers

In Section 2 several aspects have been listed that show why the acceptance of a product can be so individually different. We will therefore describe every potential customer with an individual barrier (hurdle, h). We assume that only those persons become adopters for who the personal advantage U exceeds the barrier h. For the user the size of U is not an absolute value but one relative to existing solutions. Nevertheless we assume that the user behaves more or less rationally

in determining U as some kind of price equivalent. The following two examples illustrate the approach:

• **Mobile phone parking payment:** We assume a situation in which customers, who pay their parking via mobile phone, experience a cost advantage *c* compared with conventional payment. The frequency of use is *fr*. Those customers of the car park become adopters of the mobile phone payment application, when their frequency of use combined with the cost advantage exceeds the personal hurdle (e.g. the effort to register on a specific website)

 $(3) \qquad U = fr \cdot c > h.$

• Online banking: Persons who do online banking can save time τ . We assume that, depending on the customers' specific situations, a certain virtual earning *E* can be attached to their private time. Again those customers, whose frequency of use combined with the time advantage and earnings exceed the personal hurdle (e.g. security concerns), become adopters

(4)
$$U = fr \cdot \tau \cdot E > h.$$

For any application there are manifold parameters that can influence the calculation. For a start, we simplify the modelling by using an objective aggregated use parameter u and a personal factor p. All potential customers for whom

$$(5) \qquad U = p \cdot u > h$$

are ready for adoption and the actual process is only a matter of time. The individual values h and p are random variables. Because the decisive criteria is whether

$$(6) \qquad u > \frac{h}{p},$$

it is sufficient to use the probability density function for u, $n(\tilde{u},t)$ (which we normalized to the number of potential customers P, see also Figure 3). Figure 3 shows that the group of potential customers P is split into those for whom the barrier is smaller than the personal use (on the left side, the existing adopters A and the future adopters FA) and those for whom the hurdle is larger (on the right side, remaining potential customers PR). The choice of a Gaussian shape for the probability density function $n(\tilde{u},0)$ has been made for simplicity reasons. The very charm of this approach is that it can be changed to a log normal or other distribution if desired and suitable.



Figure 3: Probability density function considering personal use and hurdles in relation to the objective use

The number of potential customers having a threshold of about \tilde{u} is given by $n(\tilde{u},t) \cdot \Delta \tilde{u}$, where $\Delta \tilde{u}$ describes the width of a small but finite interval. The basic assumption of our model is that for such a group of potential customers the diffusion model of Bass is valid for the adoption process:

(7)
$$\frac{\partial}{\partial t}n(\widetilde{u},t)\cdot\Delta\widetilde{u} = -g(\widetilde{u})\cdot n(\widetilde{u},t)\cdot\Delta\widetilde{u} = -(a(\widetilde{u})+b(\widetilde{u})\cdot A(t))\cdot n(\widetilde{u},t)\cdot\Delta\widetilde{u}$$

Equation (7) describes the dynamics of the adoption behaviour on a "microscopic" level, as the diffusion coefficients $a(\tilde{u})$ and $b(\tilde{u})$ of every group of potential customers with the same \tilde{u} can be different depending on \tilde{u} . Mapping each of these groups of potential customers belonging to an interval of width $\Delta \tilde{u}$ to an indexed stock, Equation 7 becomes equivalent to a finite set of equations.

While Equation 7 allows for different diffusion coefficients, the probability to get in contact with existing adopters (and therefore the innovation as such) depends solely on A(t) and is the same for all potential customers. Individuals are assumed to have a spatially uniform distribution and influences as those opinion leaders can have are not considered (perfect mixing). The number of adopters A(t) can be regarded as a global variable. This is the same for the objective use value of the innovation u (which is the same for all individuals). When u changes all potential customers will notice this at the same time.

Our model assumes the diffusion coefficient $g(\tilde{u})$ to be a step function:

(8)
$$g(\tilde{u}) = \begin{cases} (a+b\cdot A); & \tilde{u} \le u \\ 0 & ; & \tilde{u} > u \end{cases}$$

If $\tilde{u}>u$ then the coefficients $a(\tilde{u})$ and $b(\tilde{u})$ disappear and the respective potential customers do not become adopters. Furthermore a and b are constant in case $\tilde{u} \le u$. A more accurate model would use strictly monotonic decreasing diffusion coefficients because, as is easily imaginable, the actual value of \tilde{u} -u can influence the adoption behaviour. Someone, who uses a specific parking lot very often will more readily adopt new payment methods than someone for whom the new payment model would only result in a small advantage. We leave this distinction for later extensions to the model.

By inserting Equation (8) into Equation (7) and cancelling $\Delta \tilde{u}$ we obtain:

(9)
$$\frac{\partial}{\partial t}n(\tilde{u},t) = \begin{cases} -g(t) \cdot n(\tilde{u},t) = -(a+b \cdot A(t)) \cdot n(\tilde{u},t); & \tilde{u} \le u \\ 0 & ; & \tilde{u} > u \end{cases}$$

For those customers with too large a barrier (u p < h) the probability density function does not change over time, while for those whose barrier is overcome, the actual adoption follows Bass' approach.

In the scenario shown in Figure 3 *u* is constant. In this case, the actual number of adopters is (like in Bass' model) the difference between the original number of future adopters $FA_0=FA(0)=const$ and the remaining number of future adopters FA(t)

(10)
$$A(t) = \int_{0}^{\infty} (n(\widetilde{u}, 0) - n(\widetilde{u}, t)) d\widetilde{u} = \int_{0}^{u} n(\widetilde{u}, 0) d\widetilde{u} - \int_{0}^{u} n(\widetilde{u}, t) d\widetilde{u} \int_{FA_{0}(t)}^{u} n(\widetilde{u}, t) d\widetilde{u}.$$

It has been said before though, that the aggregated use parameter *u* can be expected to change over time (and with it $FA_0 = FA_0(t) \neq const$): Production costs decrease [BC-e72], the product becomes

more convenient, usable in more situations etc. As a first approach we assume a simple dependency between use value u and number of adopters A

(11)
$$u = u_0 + \beta \cdot A(t),$$

in which u_0 and β are constants and A depends on time. As we do not (yet) know the exact relationship between u and A(t), Equation (11) can be seen as the Taylor series of the correct function up to the linear term. The dependency of u on A(t) generates the only feedback mechanism in the system. If $\beta=0$ then u(A)=const. and the model is reduced to the Bass model. We are using this linear function for numerical calculations but it can be replaced by any other function if desired.

Equation (11) completes the model. Overall, our model depends on three essential functions:

- $n(\tilde{u},0)$, the probability density function (in the given example a Gaussian distribution)
- $g(\tilde{u})$, the diffusion coefficients (here a step function)
- *u*(*A*), the use value (here a linear approximation)

The exact quantitative solution of any simulations performed using these functions will depend on exactly this choice of functions and parameter values. However, the qualitative characteristics do not depend on them so much. We expect a similar behaviour of the solutions for a wide range of meaningful functions. With our choice of functions, only few parameters are needed, which makes it easier to explore the parameter space. For practical applications, validated functions have to be chosen which may depend on the product under consideration, level of information, country etc. To be able to use the model with different functions to represent the individual differences is the very benefit of the model.

With the increase of u the number of customers whose barrier has been overcome (i.e. the number of future adopters) will increase as well. Figure 4 visualises the resulting change in the probability density function.



Figure 4: Probability density function in case of a changing objective use value u

To start with the overall potential market P (the saturation limit) remains constant in the simulation results shown here, even though this might be different in real cases

(12)
$$P = const = PR(t) + FA(t) + A(t)$$
.

Equations (9) and (10) describe the time and barrier dependency of the density function $n(\tilde{u},t)$ completely. Unfortunately, they are not easy to solve and cannot simply be interpreted in terms of stocks and flows. Instead of evaluating $n(\tilde{u},t)$ we thus restrict our investigation on the (integrated)

values A(t) and FA(t). In Appendix A it is shown that the differential equation describing the time development of existing adopters A(t) is given by:

(13)
$$\frac{d}{dt}A(t) = g(t)\big(FA_0(t) - A(t)\big).$$

The distribution $n(\tilde{u}, 0)$ has to be known in order to calculate the market increase (see also [Kal-a85] for the similar use of the same basic idea):

(14)
$$FA_0(t) = \int_0^u n(\widetilde{u}, 0) d\widetilde{u} = \int_0^{a+\beta \cdot A(t)} n(\widetilde{u}, 0) d\widetilde{u}$$

In contrast to the original microscopic picture Equations (13)) and (14) describe the model on a macroscopic level, which can be interpreted in terms of socks and flows even without the need to descretise the probability density function $n(\tilde{u},t)$ with help of small $\Delta(\tilde{u})$. For the solution of Equations (13) and (14) thus standard tools can be used. Notice that the transformation leading to the macroscopic model is exact. However, there is a loss of information regarding the distribution $n(\tilde{u},t)$.

The proposed model is extendible to include e.g. competition or opinion-forming. To be consistent, extensions to the model always should be done at microscopic level. For such models of higher complexity it seems to be a challenge either to simulate it directly or to transform it into a macroscopic standard system dynamics model.

A comparison between the equation (1) of Bass and Equation (13) shows that there is a difference in the number of possible customers. In the model presented P_t is not constant, but like $FA_0(t)$ a function of time that depends on A(t) as described by Expression (14). This again leads to a positive feedback on A.

4. Numeric Evaluation

This section presents examples of some of the effects that can be shown with the model. The examples are not (yet) validated, but represent effects that can be expected in real life situations.

4.1. Basic approach

A numerical analysis of Equations (13) and (14) can be performed by using standard simulation programs. For gaining a deeper insight into the structure of stocks and flows we reformulate these equations, such that existing adopters A(t), eventual adopters FA(t) and remaining potential customers PR(t) serve as stocks. In Appendix B it is proven that Equations (13) and (14) are equivalent to

(15)
$$\frac{d}{dt}A(t) = Sales$$
,

(16)
$$\frac{d}{dt}FA(t) = Market increase - Sales$$
, and

(17)
$$\frac{d}{dt}PR(t) = -Market \ increase$$
.

The flows Sales and Market increase are

(18)
$$Sales = g(t) \cdot FA(t)$$
 and

(19) Market increase = $FA(t) \cdot \beta \cdot n(u,0) \cdot g(t)$.

In our simulations we assume n(u,0) to be Gaussian distributed:

(20)
$$n(\tilde{u},0) = P \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-0.5 \cdot \left(\frac{\tilde{u}-mean}{\sigma}\right)}$$

where n(u,0) is normalized to the number of potential customers P.

Figure 5 shows the resulting Business Dynamics simulation model in graphical representation. It can be seen that the diffusion coefficient g(t) as well as the parameter that decides on the use increase β impact the Market increase.



Figure 5: Simulation model (the Bass model can be obtained with $\beta=0$)

Figure 6 shows results of typical product diffusion. The adoption behaviour is highly non-linear [War-b08]. Most important though, the final number of adopters $A(\infty)$ is larger than the potential customers, for whom the product was acceptable right at the beginning of market introduction $FA_0(0)$. The peak of sales and the peak of market increase are closely correlated, while for both only a medium increase of the use value *u* was required.

To determine the final number of adopter $A(\infty)$ we investigate the stationary state of Equation (13). The number of adopters A(t) grows as long as $FA_0(t) > A(t)$ and approaches a limit $A(\infty)$, when the derivative of A(t) disappears, i.e. when

(21)
$$FA_0(t) = A(t)$$
.

According to Equation (14) FA(t) is a concatenation of the primitive function of $n(\tilde{u}, 0)$ and u(t).

(22)
$$FA_0(t) = \int_0^{u(t)} n(\widetilde{u}, 0) d\widetilde{u} = G_0(u(t)).$$

The condition for the stationary solution of A(t) can thus be obtained when substituting A(t) with help of Equation (11)

$$(23) \qquad G_0(u) = \frac{u - u_0}{\beta}$$

Figure 7 shows a typical example. Minor changes of β or u_0 may have a strong impact on the point of intersection, which represents the stationary solution.



Figure 6: Examples of typical simulation results (all examples are created with Vensim)²



Figure 7: $G_{\theta}(u)/P$ and $(u-u\theta)/\beta P$ plotted versus the aggregated use parameter u

$$\frac{d}{dt}\frac{A(t)}{P} = g(t)(P - A(t))\frac{1}{P} = g(t)(1 - A_p(t)) = \left(a + bP\frac{A(t)}{P}\right)(1 - A_p(t)) = \frac{d}{dt}A_p(t) = (a + pA_p(t))(1 - A_p(t)) = g(t)(1 - A_p(t)),$$

$$p = b \cdot P.$$

As can be seen, whether absolute or normalized values are used impacts, among other, the order of magnitude of the imitation coefficient (b, p). In its normalized value p, in the example of Figure 6, is in a range common in case studies (see e.g. [Art-a06]).

² Note that Equations (1), (7), (8), and (9) use absolute values instead of the commonly used relative values or shares. To derive the relative values from our approach the number of adopters A(t) in Equation (1) needs to be normalized by the number of potential customers *P*

4.2. Tipping Points

Tipping points refer to the moment of important change, when a supposedly stable situation suddenly evolves. This phenomena can be observed in social [Gla-b00], economic [War-b08] as well as engineering or other domains.

While generally caused by self-enforcing mechanisms, tipping points often have – depending on the properties of the actual good – irreversibility attached to them. When for example the diffusion of a technological standard reaches the critical mass, a technological lock-in can occur [FG-a85]. Because a certain number of consumers are already using the standard, more are attracted and fewer are likely to discard the choice for anticipatory economic reasons [BS-r99]. If consumers follow the path of many, the probability decreases that they are stranded on a system no longer supported and faced with high costs of a required system change [KS-a94] [FJ-a98]. Tipping points are thus very important to anticipate for innovation break-throughs.

Figure 8 shows an example for diffusion with a late break through. For a very long time the adopter base grows weakly. The decisive value change of u, causing sudden acceptance and reflecting the tipping point, is reached at a very late stage. The diagram on the left visualises $G_0(u)/P$ and $(u-u0)/\beta P$. Other than in the example shown in Figure 7 the graphs of both functions come very close a long time before they actually cross (i.e. the stationary state). At this point dA/dt is very small and A(t) is increasing only slowly. As the graphs diverge A(t) starts growing rapidly until it reaches its maximum value. FA(t) exhibits a sharp peak at the tipping point due to a rapid change of market increase.



Figure 8: Example for adopter development showing a late tipping point

5. Summary and Outlook

In this article we presented a market diffusion model that explicitly takes individual use barriers and the influence of existing adopters on the attractiveness of the product (and thus on the adoption behaviour of future customers) into account. Based on the probability density function of individual user barriers and the diffusion approach as introduced by Bass, we presented a business dynamics model with which the respective effects can be shown. Tipping points, late market break-throughs and saturation limits can also be modelled.

The approach shown describes the basis for more complex investigations. For future research extensions are foreseen with respect to word of mouth feedback (which will influence the individual barrier of potential customers), feedbacks owing to network effects (which will influence the use value of an innovation), markets with many stakeholders, influence and size of support by subsidising, renewals and replacements of products (e.g. services) and products in which the adoption itself consist of several steps (like registration, use, use of additional services etc.). The latter better reflects the distinction between installation and actual use as made by [Kol-b98]. Furthermore, the theoretical approach needs to be validated with case studies. The parameter estimation is decisive as well as the impact of estimation errors.

6. Appendix A

By integration of Equation (7) we obtain

(24)
$$\int_{0}^{u} \frac{\partial n(\widetilde{u},t)}{\partial t} d\widetilde{u} = -(a+b \cdot A(t)) \cdot \int_{0}^{u} n(\widetilde{u},t) dt = -(a+b \cdot A(t)) \cdot FA(t).$$

Note that *u* is explicitly time dependent (u = u(t)) due to Equation (11). Using the definition of FA(t) and the relation

(25)
$$\frac{d}{dt}\int_{0}^{u} n(\widetilde{u},t)d\widetilde{u} = \int_{0}^{u} \frac{\partial n(\widetilde{u},t)}{\partial t}d\widetilde{u} + n(u,t)\cdot\frac{du}{dt}.$$

Equation (24) can be rewritten in the form

(26)
$$\frac{d}{dt}FA(t) = -g(t)FA(t) + n(u,t)\frac{d}{dt}u(t)$$

From Figure 4 we know that

(27)
$$FA(t) + A(t) = FA_0(t)$$

Equation (26) thus takes the form

(28)
$$\frac{d}{dt}FA_0(t) - \frac{d}{dt}A(t) = -g(t)\underbrace{\left(FA_0(t) - A(t)\right)}_{FA(t)} + n(u,t)\frac{d}{dt}u(t).$$

Figure 3 illustrates a situation where function $n(\tilde{u},t)$ is discontinuous at the value *u*. The term n(u,0) thus represents the limit from the left. In the next step we calculate the derivative of $FA_0(t)$

(29)
$$\frac{d}{dt}FA_0(t) = \frac{d}{dt}\int_0^u n(\widetilde{u},0)d\widetilde{u} = n(u,0)\frac{d}{dt}u(t).$$

By inserting Equation (29) into Equation (28) we obtain

(30)
$$\frac{d}{dt}A(t) = g(t)(FA_0(t) - A(t)) + (n(u,0) - n(u,t))\frac{d}{dt}u(t)$$

The second term on the right side of Equation (30) can be neglected for all practical purposes: If u is considered time independent then the time derivative is equal to 0. If, on the other hand, u is time dependent but grows continuously with the existing adopters A, then u exhibits the same behaviour and grows continuously, too. Like shown in Figure 4 the distribution $n(\tilde{u},t)$ remains continuous at u. The final equation is therefore

(31)
$$\frac{d}{dt}A(t) = g(t)(FA_0(t) - A(t)).$$

7. Appendix B

In order to understand the model more easily and to be able to perform simulations it is useful to identify stocks and flows. Because of the mechanisms explained (and visualized in Figure 4) the following variables are suitable stocks:

- Existing adopters *A*(*t*)
- Eventual adopters *FA*(*t*)
- Remaining potential customers PR(t)

From Equations (13) and (27) we know the inflow of A(t)

(32)
$$\frac{d}{dt}A(t) = \underbrace{g(t) \cdot FA(t)}_{Sales}$$

Equations (27), (29) and (11) yield inflow and outflow of FA(t)

(33)
$$\frac{d}{dt}FA(t) = \frac{d}{dt}FA_0(t) - \frac{d}{dt}A(t) = n(u,0)\frac{d}{dt}u(t) - \frac{d}{dt}A(t) = n(u,0)\cdot\beta\cdot\frac{d}{dt}A(t) - \frac{d}{dt}A(t).$$

With the help of Equation (32) we then receive

(34)
$$\frac{d}{dt}FA(t) = \underbrace{n(u,0) \cdot \beta \cdot g(t) \cdot FA(t)}_{Market increase} - \underbrace{g(t) \cdot FA(t)}_{Sales}.$$

By using Equations (12), (32) and (34) we finally get

(35)
$$\frac{d}{dt}PR(t) = -\frac{d}{dt}FA(t) - \frac{d}{dt}A(t) = -Market \ increase \ .$$

Equations (32), (34) and (35) form a complete set of ordinary differential equations which is equivalent to Equations (13) and (14). Apart from the intuitive interpretation of the stock and flows this set of equations has the advantage to use the density function n(u,0) directly instead of the primitive function of $n(\tilde{u},0)$ according to Equation (14).

8. References

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