A System Dynamics Design of the US Economy Exit from the Stern Crisis

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Abstract

This paper re-defines two hypothetical laws of capital accumulation by including endogenous profit investment share and establishing an explicit inverse relation between this share and capital-output ratio. Other main state variables are labour productivity, employment ratio and unit value of labour force. A comprehensive Phillips equation, governing real labour compensation, is an element of a hypothetical law (HL-IR). A control law (CL-IR) determines a growth rate of surplus value by a gap between target and current employment ratios. A capital strive to higher profitability alters HL-IR and CL-IR.

After the recent neo-conservative defeat there is, likely, no place for stabilising policy with the same or similar aggressiveness as in 2002–2007. Based on the US macroeconomic data mainly for 1969–2007, computer simulation runs for a later period (through 2060) exhibit how a postponed non-aggressive application of altered CL-IR in 2012 and afterwards could smooth out long waves of capital accumulation and shorten a period of fluctuations from 24–30 to 14–16 years in the restructured US economy compared to evolution based on HL-IR altered in 2008. The present stern crisis of the capital accumulation, probably worst after the World War II, will last until 2018–2022 when the pre-crisis maximum of net output is restored and 2023–2026 when the pre-crisis maximum of employment is reached again.

Introduction

The previous papers (Ryzhenkov 2005a, 2005b, 2005c, 2007, 2008, 2009) unveil and explain the paradox that profit (money form of surplus value) is the decisive factor of big economic cycles under capitalism and could be the key for smoothing them. These papers demonstrate that a more efficient social control over the oscillatory macroeconomic system requires a substantial reshaping of primary income distribution that takes into explicit account this dual characteristic of profit (surplus value).

These previous papers define a hypothetical law (HL) of capital accumulation. The main state variables are the labour productivity, unit value of labour force, employment ratio, and capital-output ratio; a comprehensive Phillips equation governs a rate of growth of real labour compensation. An application of an extended Kalman filtering to the US macroeconomic data 1969–2002 and computer simulation runs demonstrate that long wave resulted from the socio-economic relations has been a viable pattern of capital accumulation. The characteristic of the inertia scenario based on HL is a strengthening of the secular tendency of the general profit rate to fall in 2002–2057. This was not accepted by the US state and business leadership that pursued an aggressive pro-growth stabilisation policy from 2002 until 2007 as understood now.

The transition to "new economy" in the XXI century is accompanied by decelerating growth or even by decline of labour force in the advanced capitalist economies. A skill gap (disparity between jobs' requirements and qualification of available workers) will likely aggravate in the coming years. A scientific treatment of these challenges requires models of capital accumulation with endogenous sup-

ply of labour force. HL includes the hypothetical partial law for the labour supply as a non-linear function of capital intensity. Although this partial law is not a necessary condition for long swings, it helps to portray them more accurately.

Table 1 lists the state and other variables of HL and CL. Time is viewed as a continuous variable. So the appropriate measure for the rate of change of a variable x is the derivative of x with respect to time ($\dot{x} = \frac{dx}{dt}$), while its fractional rate of change is $\hat{x} = \frac{\dot{x}}{x} = \frac{dx}{xdt}$. The same convention is appropriate for all variables. The main variables have the following units of measurement: a [millions of 2000 dollars per worker per year], u, v [dimensionless], s [years]. Calculations of u and s are done with the nominators and denominators measured in current prices. The employment ratio v is for the civil labour force (without accounting the latent and stagnant unemployment). The net fixed capital (K) is a sum of private and governmental produced non-residential fixed assets.

Table 1. The main variables of HL and CL

Variable	Notation
Real net output	P
Nominal net output	$P^*1 = P$
Employment	L
Labour force	N
Output per worker	$a = P/L$
Employment ratio	$v = L/N$
Fixed capital (net)	K
Real labour compensation (per worker)	w
Unit value of labour power (relative labour compensation)	u
Capital-output ratio	$s = K/P$
Surplus product	$(1 - u)P$
Profit	$M = (P - wL)^*1 = P - wL$
Surplus value	$S = (1 - u)L$
Profit investment share	k
Net accumulation of fixed capital	$\dot{K} = kM = k(1 - u)P$
Capital intensity	K/L
Profit rate (profitability)	$M/K = (1 - u)/s$
Rate of surplus value	$S/(L–S) = (1 - u)/u$

The inverse of output per worker ($1/a$) represents a total labour input embodied in a unit of net output, so it approximates a magnitude of labour value of this unit.[1] The value of a unit labour power is $u = w/a$, unit surplus value is $1 - u$; total surplus value is the labour value of surplus product, measured by surplus labour, $S = (1 - u)L$.

Total profit $M = Sa$ is the money form of surplus product. In HL and in CL, net output unit price is identically one whereas profit equals surplus product. Net output unit price (1) is omitted below for simplicity. A target employment ratio in CL only is denoted as $X = const$.

The previous papers uncovered long-term advantages of the closed loop control over total profit (or surplus value) in comparison with the open loop control. Based on examination of causal linkages, the supposed control law of primary distribution of income (CL) was derived as the more sophisticated

[1] Let Q is the total product, A is the direct material input per unit of total output, $l = L/Q$ is the direct labour input per unit of total output; $P = (1 - A)Q$ is the net output, while $Q = (1 - A)^{-1}P$. Then $L = lQ = l[(1 - A)^{-1}P] = P/a$ is the total labour input, and $1/a = l(1 - A)^{-1}$. The labour value of an output unit is approximated by the total labour embodied in this unit: $\omega = \omega A + l = l(1 - A)^{-1} = 1/a$.

modification of the initial HL. The new equation, representing feed-forward control, substitutes the comprehensive Phillips equation of the initial HL. A controlled transition to a non-trivial stationary state defined explicitly would alleviate the tendency of general profit rate to fall, maintain deliberately high employment ratio and uphold total profit.

HL and CL include positive feedback loops representing *reinforcing direct and roundabout economies of scale* (increasing returns) neglected in neoclassical models (Ryzhenkov 2009). Figure 1 reflects one of them.

Adopting a more pro-growth policy of primary distribution of income as demonstrated in (Ryzhenkov 2007) could be achieved through levy on excess income, *i.e.*, on excessive labour compensation or excessive profit. Figure 2 presents a particular additional 2nd order negative feedback loop in CL that includes the rate of compensation levy.

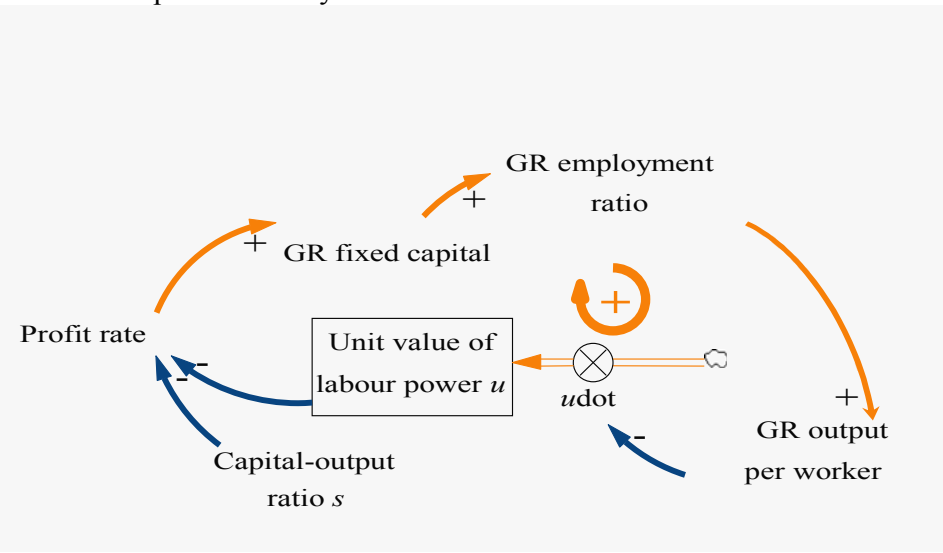

Figure 1. Increasing return in the 1st order positive feedback loop including unit value of labour power in HL and in CL (GR stands for a fractional growth rate)

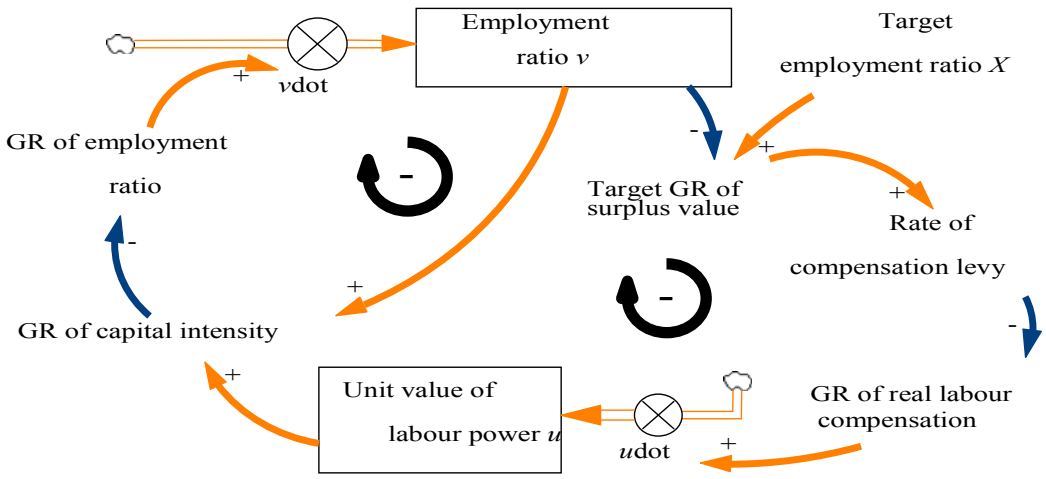

Figure 2. The 1st order negative feedback loop in HL and in CL, related to competition for jobs, and 2nd order negative feedback loop in CL only (GR is for a fractional growth rate)

The excess labour compensation levy is reduction in pre-levy primary labour compensation. The counter-part of excess labour compensation levy is subsidy (of the same quantity) on pre-levy primary profit. In the opposite case, excess profit levy equals subsidy on labour compensation receivable.

Workers' competition for jobs contributes to stabilising capital accumulation in HL and in CL. This competition is reflected as the 1st order negative feedback loop controlling the employment ratio (Figure 2). CL enforces pro-growth stabilisation of total surplus value, particularly, by creating an additional (anticipatory) negative 1st order feedback loop. It links positively growth rate of net fixed capital with growth rate of employment ratio that fosters increments of relative labour compensation detrimental for profit rate and – through it – for growth rate of fixed assets (Figure 3).[2]

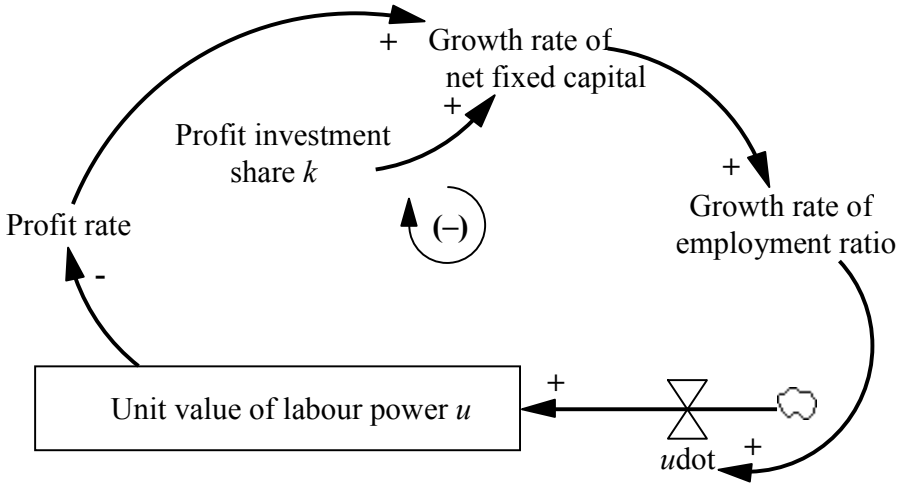

Figure 3. The 1st order negative feedback loop in CL

The former designs of HL and CL treat business cycles as exogenous influences in the basal period and abstract from them in prospective periods altogether. The present paper elaborates the hypothetic law (HL) and control law (CL) of capital accumulation by reflecting the endogenous nature of profit investment share.[3] This new endogenous variable belongs to the upgraded laws (HL-IR and CL-IR, respectively). The focus of the current research is again on pro-growth stabilisation policy that brings about shifts in distribution of net value added between the two main social classes. Still explicit financial and monetary factors are outside the models boundary in this paper.

The structure and content of this paper has been elaborated as a conscious attempt to satisfy standards outlined by Jay Forrester in the Special Issue of *SDR* devoted to system dynamics past and next great frontiers (Forrester 2007: 365–366). The rest of this text, besides mandatory references and conclusions, is organised in the following manner.

Chapter 2 re-formulates HL of capital accumulation for the modern US economy. A new partial dynamic law for the profit investment share reflects a pro-cyclical character of this variable in an extended law (HL-IR). There is a continuum of non-trivial non-stable stationary states depending on ini-

[2] "The labor compensation consists of wages and salaries (which are taxable), non-wage compensation (employer contributions to employee pension and insurance funds – which are not taxable), and employer contributions for social insurance (which are not taxable)… economists generally believe the employer tax is ultimately paid by workers in the form of relatively lower wages" (Economic Report of the President 2007. Washington. D.C. GPO, 2007: 43, 90).

[3] The net financial inflow from abroad is an import additional source for capital accumulation in the American economy. The external economic and financial relations need an additional research beyond the scope of this paper.

tial conditions. The trajectories repelled from an unstable stationary state are attracted by a limit cycle with a period of about 27 years.

Chapter 3 re-formulates CL for the modern US economy too. The same new partial dynamic law for the profit investment share reflects a pro-cyclical character of this variable in an extended law (CL-IR). There is also a continuum of non-trivial non-stable stationary states. The trajectories repelled from an unstable stationary state are attracted by a limit cycle with a period of about 8 years.

Chapter 4 transforms deterministic forms of HL-IR and of CL-IR into probabilistic. Their non-observable parameters are identified through application of a simplified version of the extended Kalman filtering (EKF) to macroeconomic data over a basal period 1969–2007. These two theoretic laws successfully pass behaviour reproduction tests in two distinguished sub-periods 1969–2001 and 2002–2007.

Chapter 5 explores inertia scenario I and two belated stabilising scenarios II and III based on the deterministic forms of HL-IR and CL-IR, respectively.

The inertia scenario I unfolds as a succession of long swings. In the first of them, a great recession is expected to be record deep and long for the whole period after the World War II. A step-wise decrease of a magnitude of a critical control parameter in an equation for the growth rate of capital intensity in altered HL-IR and altered CL-IR is a key assumption enabling explaining the current fast decline in the employment ratio.

In the stabilising scenarios II and III, altered HL-IR is operative in 2008–2011 only, whereas altered CL-IR governs capital accumulation afterwards.[4] The stabilising scenarios II and III smooth out long swing altogether. Aggressive adjustment of the growth rate of surplus value and of employment ratio to their stationary magnitudes in the stabilising scenario II produces business cycles with declining amplitude. Still overcoming the recession faster than in the inertia scenario I requires a substantial decline in labour compensation for several years. The less aggressive stabilising scenario III produces longer business cycles than those in the scenario II but shorter than long swings in the scenario I. A decline in labour compensation is not so drastic but the recession becomes longer than in stabilising scenario II.

The given analysis helps to validate HL-IR and CL-IR that could be useful in controlling structural crises. This study extends macroeconomic applications of system dynamics method. This investigation also uncovers and explains profound structural changes in the American economy.

1. Relative and absolute plethora of capital in the US economy

The American economy experienced sequentially the bursting of dot-com bubble, apparently shallow recession of 2001, economic expansion (up to November 2007), housing bubble (it began to unravel after house prices peaked and began to turn down in the middle of 2006), credit bubble and financial turmoil (since March–August 2007), production meltdown (since December 2007). Over-accumulation of capital has paved the way for the present acute economic and financial crisis. This crisis has symbolised the defeated and discredited mainstream neo-conservative economic policy. The powerful fractions of the ruling class have fallen in disarray, they try to win time for regrouping and next assault.

1.1. Capital as the barrier of capitalist production

Relative excess of capital

[4] See a supporting file on the stabilising scenario IV over 2002–2060 based on the unaltered CL-IR. Falling short to impose Scenario IV or improve upon it was labour's defeat!

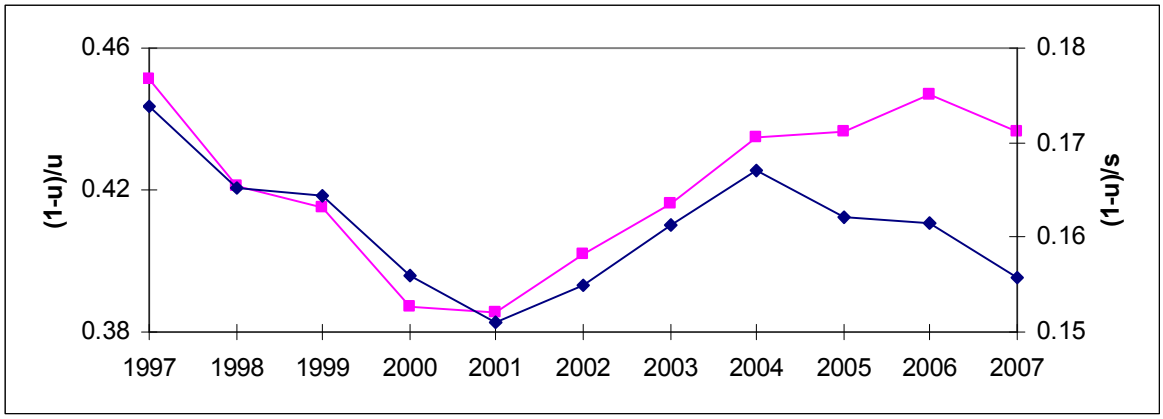

Figure 4. The profit rate (1–*u*)/*s* (diamond, the right scale) and rate of surplus value (1 – *u*)/*u* (square, the left scale), 1997–2007 (author's calculations based on BEA NIPA and BLS data)

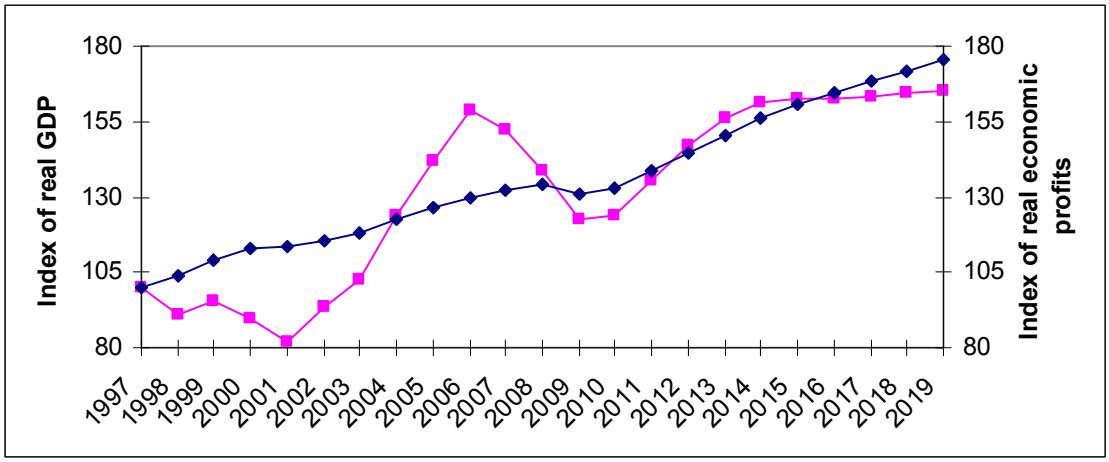

Figure 5. Indexes of real GDP (diamond, scale on the left) and of real economic profits (square, scale on the right), 1997–2019, 1997 = 100. Author's calculations based on CBO data (actual 1997–2007, estimated 2008, projected in January 2009 for 2009–2019)

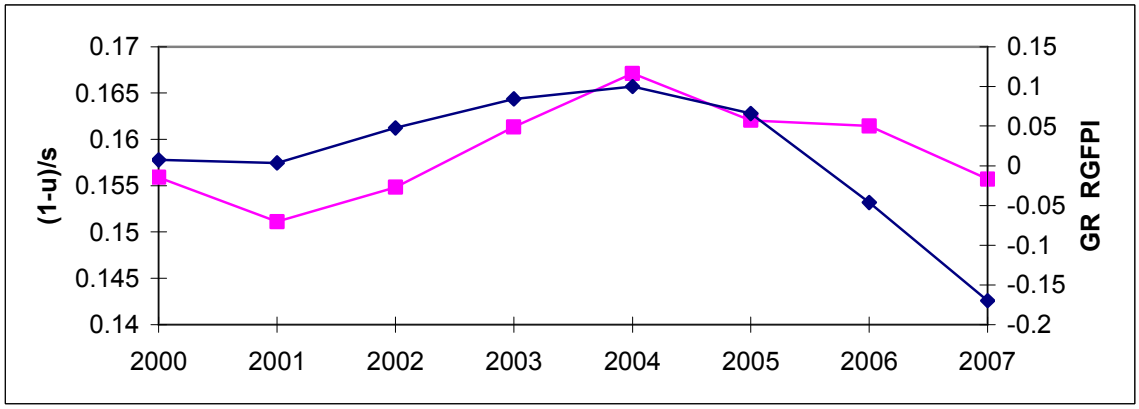

Figure 6. Profit rate (1 – *u*)/*s* (square, scale on the left) and growth rate of real ($ 2000) private residential investment GR RGFPI (diamond, scale on the right). Author's calculations based on BEA NIPA data and Statistical Abstract of the US, 2009

Owing to the mobilising policy carried out by the Bush Administration, the rate of profit grew over 2002–2004. Fall in the rate of profit due to a change in the composition of capital caused by the development of the productive forces at first occurred in 2005 (Figure 4). A declining rate of profit since

2005 and diminishing rate of surplus value since 2007 are evidences for relative excess of capital. Conflict between expansion of production and production of surplus value appears partly in periodical crises. Taking into account this fundamental conflict the paper (Ryzhenkov 2005a) predicted correctly a new crisis of over-production in the USA before 2010.

CBO has projected in January 2009 the conflict between expansion of production and creation of profit well into future (Elmendorf 2009). CBO has expected that while the pre-crisis maximum of GDP (2008) will be exceeded in 2011, the pre-crisis maximum of real economic profits (2006) will be surpassed in 2014, and the employment ratio will reach a limit of 0.949–0.952 in 2014–2019 lower than in 2007 (0.954). My original projections below are more cautious.

In the past and in the projection periods, profits vary more dramatically than labour compensation, falling sharply during recessions and then growing quickly during recoveries before capital is over-accumulated in relative or absolute terms (Figure 5). In my view, the prior movements of the profit rate have been decisive factor determining the overall economic activity including the residential sector (Figure 6).

1.2. Absolute excess of capital

Excess of capital arises from the same causes as those which call forth unemployment – a complementary phenomena, footing at the opposite poles. A fall in the rate of surplus value below a certain point in 2007 (Figure 4) has brought about the rise in unemployment. The unemployment rate has risen to 9.5 per cent, its highest level since 1983, the Labour Department announced on July 2 2009.[5] The US economy has apparently become increasingly trapped in a vicious circle of slumping consumer demand, falling business investment, rising unemployment and mounting losses in the banking system (OECD 2009).

A finer analysis distinguishes two forms of absolute excess of capital.

1) If the fall in the rate of profit is not compensated through the mass of profit, when the increased capital produced just as much, or even less, profit than it did before its increase: $\hat{M} = \hat{S} + \hat{a} = \hat{P} - \frac{\dot{u}}{1-u} \leq 0,$ therefore $\hat{S} \leq -\hat{a},$ or $\hat{P} \leq \frac{\dot{u}}{1-u}.$ When CL governs capital accumulation in agreement with the equation (19) below, this condition suits $X + \frac{\hat{a}}{c_2} \leq v$ for the increased capital (the target employment ratio plus the growth rate of output per worker divided by the positive adjustment coefficient is equal to or lower than the actual employment ratio).

2) Similarly, if the fall in the profit share (unit surplus value) is not compensated through the mass of surplus labour, when the increased capital produced just as much, or even less, surplus-value than it

[5] "Since the start of the recession in December 2007, the number of unemployed persons has increased by 7.2 million, and the unemployment rate has risen by 4.6 percentage points." See U.S. Bureau of Labor Statistics, Economic News Release, Employment Situation Summary, July 2 2009.

Unemployment surges, and there is worse to come. In February, the US authorities defined the "more adverse" scenario as one in which unemployment rose gradually to peak at 10.4 per cent in late 2010 (The Financial Times April 17, 2009). The "more adverse" scenario is in fact the new baseline.

Business fixed investment (or spending by businesses on structures, equipment, and software), which had plateaued during the first three quarters of 2008, plunged at a 21 per cent annual rate in the fourth quarter – a rate comparable to the worst declines observed in past post-war recessions (CBO March 2009: 26).

did before its increase: $\hat{S} \le 0 \equiv \hat{L} - \frac{\dot{u}}{1-u} \le 0$, or $\hat{L} \le \frac{\dot{u}}{1-u}$. When CL governs capital accumulation in agreement with the equation (19) below, this condition turns into $X \le v$ (the target employment ratio is equal to or lower than actual one) for the increased capital.

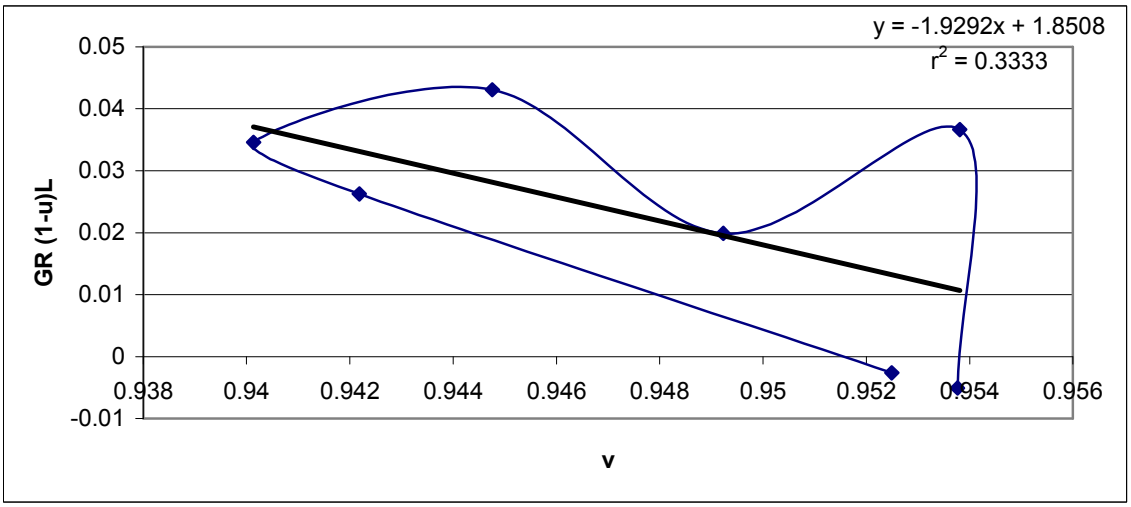

Figure 7. Employment ratio v and growth rate of surplus value GR $(1–u)L$, 2001–2007, clockwise (author's calculations based on BEA NIPA data and Statistical Abstract of the US, 2009)

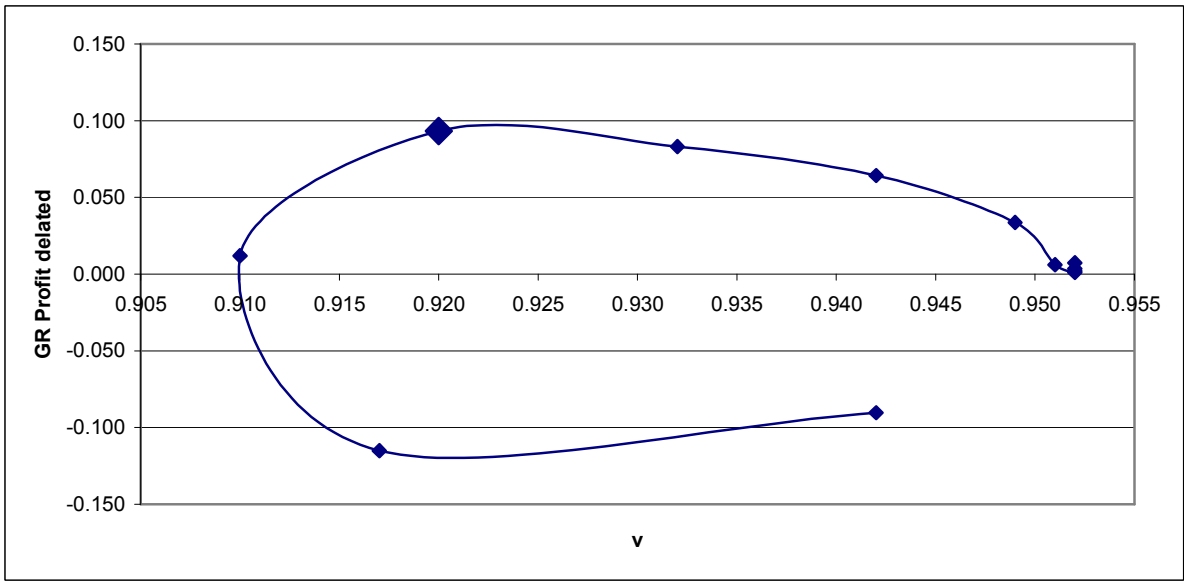

Figure 8. The employment ratio and growth rate of profit (the point of 2011 is emphasized): probable operation of HL in 2008–2011 and of CL in 2012–2019 (clock-wise) based on CBO data and projections (January 27, 2009)

CL has gotten a partial empirical support: a postulated negative association of the growth rate of surplus value with the actual employment ratio probably roughly characterised the real US economy in the period 2001–2007 taken as a whole (Figure 7). In the finished business cycle, the first condition $X + \frac{\hat{a}}{c_2} \le v$ and second one $X \le v$ were valid in 2005–2007 in agreement with the above presentation of capital over-accumulation. Yet this control law has been violated to some degree. The observed incre-

ment of the labour compensation in 2002–2007 over 2001 has been 5.6 per cent; the required one has to be 10.2 per cent. This disparity has strengthened acute capital over-accumulation since 2005.

The current global economic and financial crisis has apparently laid bare a lack of cohesion of economic interests *within* the capitalist class itself under the squeeze on total profits, when the loss in profit is very unevenly distributed, e.g. some sections of capitalists lose disproportionately while others even earn higher profits. The political difficulty of pursuing the stabilisation policy may then be positioned not so much in attaining cooperation between capital and labour, as between the capitalists themselves (cf. Bhaduri and Marglin 1990: 383).

Almost certainly, after the defeat of neo-conservative economic policy CL has not been operative even roughly in 2008 and in 2009 so far because the declining employment ratio has been accompanied by decreasing growth rate of real economic profits (in contradiction to CL). It is likely, that HL has been operating instead. The Congressional Budget Office (CBO) data and projections give additional tentative support for this reservation (Elmendorf 2009). There is no expectation of the negative association of the employment ratio and growth rate of profit, required by CL, in 2008–2011 and there is indeed such expectation for 2012–2019 (Figure 8). Therefore an opportunity of a probable rejection of CL in 2008–2011 and its return in the matter-of-fact economic policy beginning from 2012 is to be taken into account in elaborating scenarios of future evolution.

2. Re-formulating a Hypothetic Law of Capital Accumulation for the US Economy

The advanced capital does not include variable capital since workers are paid at the end of each completed circulation process. HL abstracts from capital of circulation. Natural capital and resource rent are not taken into explicit account; therefore magnitudes of general profit rate are biased.

2.1. An Extensive Deterministic Form of HL-IR

A deterministic model consists of the following equations:

$P = K/s,\ s > 0;$ (1)

$L = P/a;$ (2)

$u = w/a,\ 0 < u < 1;$ (3)

$\hat{a} = m_1 + m_2 \hat{K/L} + m_3 \psi(\hat{v}),$ (4)

$\psi(\hat{v}) = sgn(\hat{v}) |\hat{v}|^j,\ m_1 > 0,\ 1 > m_2 > 0,\ m_3 > 0,\ 1 > j > 0;$

$\hat{K/L} = n_1 + n_2 u + n_3 (v - v_c),$ (5)

$n_2 > 0,\ n_3 > 0,\ 1 > v_c > 0;$

$v = L/N,\ 1 > v > 0;$ (6)

$n = n_a + p_1 e_1^{-M_1 |K/L - K_c/L_c|^{i1}}$ for $0 < K/L < K_c/L_c,\ M_1 = 1;$ (7a)

$n = n_a + p_1 e_2^{-M_2 (K/L - K_c/L_c)^{i2}}$ for $K/L \geq K_c/L_c,\ M_2 = 1,\ p_1 > 0;$ (7b)

$\hat{w} = -g + rv + b\hat{K/L},\ g > 0,\ r > 0;$ (8)

$P = Q + \dot{K} = wL + (1 - k)M + \dot{K};$ (9)

$\dot{K} = k(1 - u)P = kM,\ 0 < k < 1;$ (10)

$\hat{k} = c_1 \hat{s},\ c_1 < 0;$ (11)

$$k = k_0\left(\frac{s}{s_0}\right)^{c_1},\ 0 < k_0 < 1,\ s_0 > 1. \qquad (11a)$$

Equation (1) postulates a technical-economic relation connecting the net fixed capital (K), net output (P) and capital-output ratio (s). Equation (2) relates labour productivity (a), net output (P) and labour input, or employment (L). Equation (3) describes the relative labour compensation (u), or unit labour value, as the ratio of real labour compensation (w) to labour productivity (a).[6]

Equation (4) is an extended technical progress function. It includes: the rate of change of capital intensity, K/L, and direct positive scale effect, $m_3\psi(\hat{v})$; $|x| \geq 0$ is an absolute value of x; $sgn(x) = -1$ for $x < 0$, $sgn(x) = 1$ for $x \geq 0$.

The non-linear continuous function $\psi(\hat{v})$ is analytical except at singular points with $\hat{v} = 0$ where its positive first derivative ($\psi'(\hat{v}) = j|\hat{v}|^{j-1} > 0$) becomes infinite. The derivatives of the function $\psi(\hat{v})$ of higher orders go to plus or minus infinity at the vicinity of $\hat{v} = 0$. This substantial singularity explains why the growth rate of labour productivity changes stepwise at local maximums and minimums of the employment ratio. Abruptness of economic crises follows from this essential singularity too if a closed loop control over total profit, total surplus value or profit rate is not enforced.

Equation (6) outlines the rate of employment (v) as a result of the buying and selling of labour–power. In the equation (8), the rate of change of the real labour compensation rate (w) depends on the employment rate (v), as in the usual Phillips relation, and on the rate of change of capital intensity (K/L) additionally. The capital intensity (K/L) is a proxy for qualification. A comprehensive Phillips equation gives the equivalent form of this equation:

$$\hat{w} = -g + rv + b(P\hat{/}L + K\hat{/}P)$$
$$= -g + rv + b(\hat{a} + \hat{s}),\ b > 0, g > 0, r > 0. \qquad (8a)$$

It will be compared to a new equation (19) of the growth rate of real labour compensation for closed loop control over total surplus value (Section 3).

Mechanisation (automation) manifests itself in a growing capital intensity. A high relative labour compensation and high employment ratio promote mechanization (automation) that shapes the labour supply.

The rate of change of capital intensity (K/L) in the equation (5) is a function of the relative labour compensation (u), difference between the real employment ratio (v) and some base magnitude (v_c).

A *key assumption* of all three scenarios below is a step-wise drop of a magnitude of this parameter in the year 2008 and its constancy afterwards as a means for stabilising capital accumulation under condition of a cease of the closed loop control over total surplus value that was roughly operative until the year 2007 as explained above. This drop immediately adversely affects the employment ratio and diminishes the relative labour compensation in the long term. A corresponding alteration of HL-IR and of CL-IR elevates the rate and amount of surplus value in the long run in prospective scenarios.

Following reasoning stays behind a hypothetical partial law for the labour supply. Before reaching a critical magnitude, mechanisation (automation) pushes new demographic groups (children, women, aged, immigrants from less developed countries) into a labouring population (as far as qualification really or potentially satisfies technological require-

[6] The equity $u = 1$ is not compatible with capitalist production relations as the use value of labour power ceases to exist for capitalists when they get no surplus value at all. The equity $u = 0$ would exclude the specific premise of capitalist production relations, namely, market supply of labour force. Therefore $0 < u < 1$. The existence of the labour market and necessity of an industrial reserve army for capital accumulation requires $0 < v < 1$.

ments) thus chiefly accelerating the growth of supply of labour force. Afterwards mechanisation (automation) becomes mainly a decelerating factor for the growth of supply of labour force because a substantial part of working-age population does not possess adequate qualification for being hired or self-employed.

Accordingly, the equations (7a) and (7b) determine the growth rate of supply of labour force (N) as a non-linear continuous function of capital intensity alone. Capital intensity, in turn, is a product of capital-output ratio and labour productivity ($K/L = sa$), it is implicitly applied in the Equation (14) below.

The growth rate of supply of labour force is monotonically increasing for $K/L \le K_c/L_c$, reaching an absolute maximum $n_{\max} = n_a + p_1$ at the point $K/L = K_c/L_c$; this rate is monotonically decreasing for $K/L \ge K_c/L_c$. Time evolution of supply of labour force (N) is typically S-shaped. A magnitude of the constant n_a is not determined a priory.

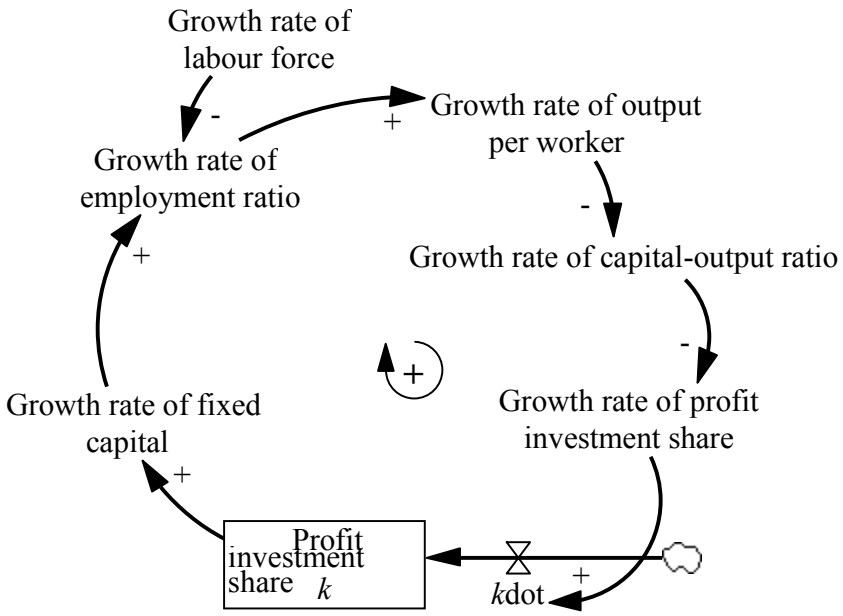

Figure 9. Endogenous profit investment share k reinforcing economy of scale in HL-IR and in CL-IR

In the equations (9) and (10), the net formation of fixed capital is $\dot{K}$, Q sums net export, final private and public consumption, $M = (1-u)P$ is the total profit in real terms.

An assumption on positive linear linkage between total profit and investment $\dot{K} = k(1-u)P$ is embodied in previous definitions of HL and CL. Their previous applications to the US economy have used the profit investment ratio (rate of capital accumulation) as a positive constant, $0 < k < 1$.

The papers (Ryzhenkov 2008, 2009) re-formulate HL for the modern Italian economy. Establishing an inverse relation between profit investment share and capital-output ratio not only smoothes long swings but slightly raises stationary profitability in stabilisation scenario II above stationary profitability in inertia scenario I. This paper uses positive experience in modelling the Italian economy for upgrading the system dynamics model of the US economy.

Following considerations support logically a working hypothesis on a bit lagging pro-cyclical nature of profit investment share. In the economic literature, output-capital ratio ($1/s$) represents typically a

proxy of utilization of the productive capacity. It is surmised that a growth rate of profit investment share ($\hat{k}$) depends strongly negatively on growth rate of capital-output ratio ($\hat{s}$).

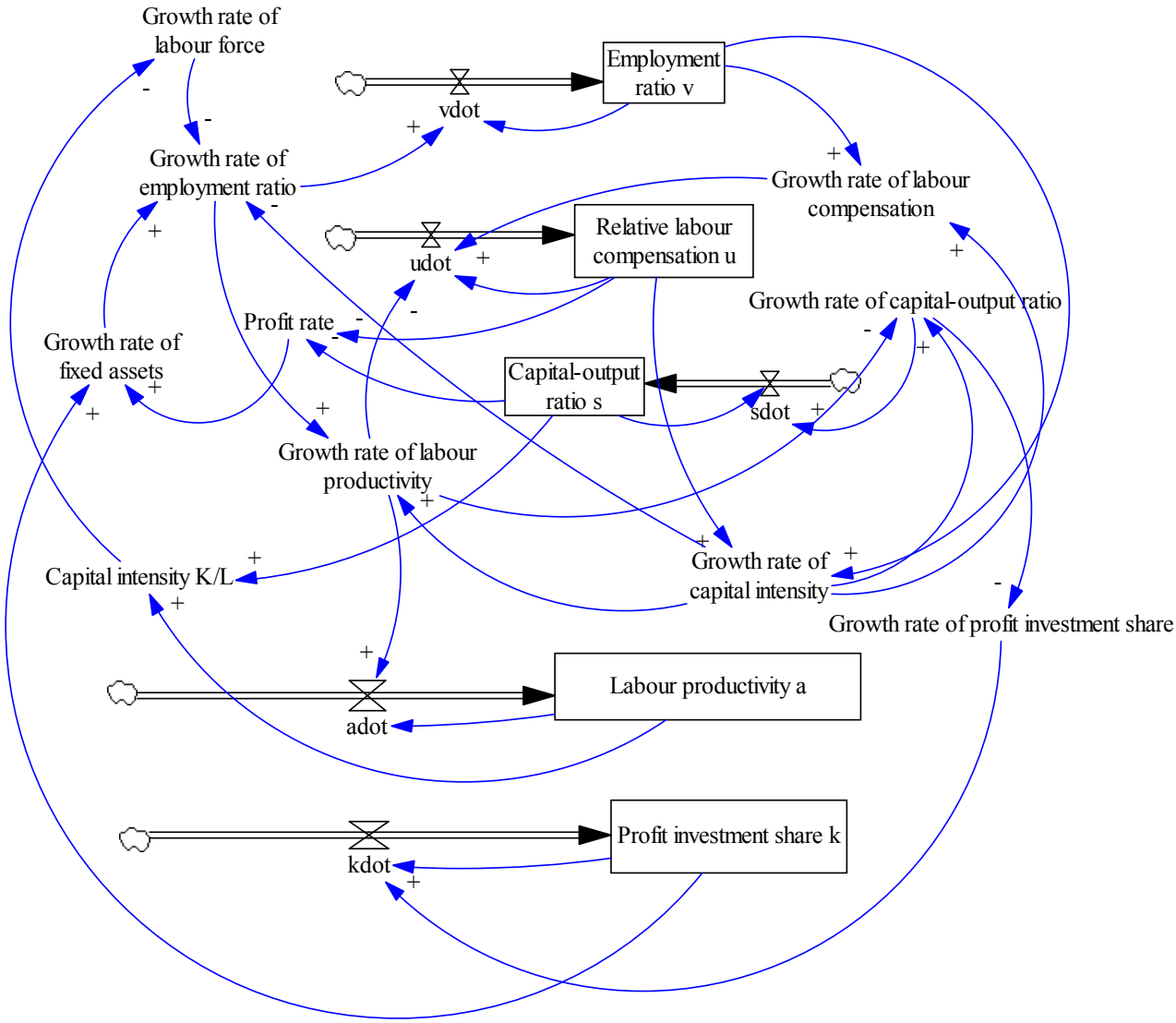

Figure 10. A condensed causal loop diagram of HL-IR (for $K / L \geq K_c / L_c$ in the equation (7b))

Increment in profit investment ratio facilitates growth of fixed capital and of employment ratio that, due to direct positive scale effect, fosters decline in capital-output ratio. The latter is, in turn, favourable for further extension of profit investment ratio (Figure 9). This positive feedback loop is an element of a greater structure of HL-IR (Figure 10).

The equation (11) defines a derivative control over the profit investment share, or the rate of capital accumulation, (k), whereby its fractional growth rate depends negatively and directly (for $c_1 < 0$) on a fractional growth rate of capital-output ratio.[7] The equivalent equation (11a) results from integration of the equation (11), where k_0 and s_0 are initial magnitudes of the respective variables (k and s).

[7] The endogenous profit investment share substantially neutralises the secular tendency of profit rate to fall emphasised in the previous publications on the US economy based on models with an exogenous profit investment share. The Italian experience prompts that the supposed functional form for

An extended Kalman filtering identifies c_1 for the basal period. The new endogenous variable k is a factor of business cycles in the structure of CL-IR but still not in the structure of HL-IR. CL-IR with this variable neutralises (for $c_1 < 0$) to a large extent the secular tendency of profit rate to fall.

2.2. An Intensive Deterministic Form of HL-IR

The deterministic model in an intensive form, derived from the equations (1) – (11), consists of five non-linear ordinary differential equations (11) and (12) – (15):

$$\dot{a} = \{m_1 + m_2 [n_1 + n_2 u + n_3(v - v_c)] + m_3 \psi(\hat{v})\} a, \tag{12}$$

$$\dot{s} = \{-m_1 + (1 - m_2)[n_1 + n_2 u + n_3(v - v_c)] - m_3 \psi(\hat{v})\} s, \tag{13}$$

$$\dot{v} = \left[k \frac{1-u}{s} - n_1 - n_2 u - n_3 (v - v_c) - n \right] v, \tag{14}$$

$$\dot{u} = \{-g + rv - m_1 + (b - m_2)[n_1 + n_2 u + n_3(v - v_c)] - m_3 \psi(\hat{v})\} u. \tag{15}$$

The reader may well notice the term $-n_3 v^2$ on the right hand side of the equation (14). It reflects labourers' competition for jobs that stabilises capitalist reproduction (Figure 2).

Analysing HL-IR with a help of the Lie derivative

Formally, properties of a system of ordinary non-linear differential equations can be examined with the help of the Lie derivative or the divergence defined in the present case as

$$\dot{V}/V = \operatorname{div}(f) = \frac{\partial \dot{a}}{\partial a} + \frac{\partial \dot{s}}{\partial s} + \frac{\partial \dot{v}}{\partial v} + \frac{\partial \dot{u}}{\partial u} + \frac{\partial \dot{k}}{\partial k}, \tag{16}$$

where V is the "volume" and $\operatorname{div}(f)$ is the divergence of the vector-function $f(a, s, v, u, k)$.

For the HL-IR intensive form (11) – (15) the Lie derivative is calculated as follows:

$$\dot{V}/V = \frac{k(1-u)}{s} - n + \hat{u} - n_3 v + (b - m_2) n_2 u + m_3 \psi'(\hat{v}) \left(\frac{k}{s} + n_2 u \right) + c_1 \hat{s} - c_1 m_3 \psi'(\hat{v}) \frac{k(1-u)}{s}. \tag{17}$$

In vicinity of critical (singular) points, the Lie derivative moves to positive infinity since the compound element $m_3 \left[\frac{k}{s} + n_2 u - c_1 \frac{k(1-u)}{s} \right]$ at $\psi'(\hat{v})$ is positive and $\psi'(\hat{v}) \to +\infty$ for $\hat{v} \to 0$. So induced technical progress, economy of scale and pro-cyclical character of profit investment share are at least locally destabilizing in vicinity of the critical points in the initial model.

A non-trivial stationary state in HL-IR

For finding a non-trivial stationary state of the system (11) – (15), it is necessary to equate each of the expressions on the right to zero. As $\dot{a} = 0$ is not true for a non-trivial stationary state, this system does not possess a non-trivial stationary state.

It is reasonable substitute the equation (7b) by equations (7b′) and (7c), keeping the equation (7a) intact

$$n = n_a + p_1 e_2^{-M_2 (K/L - K_c/L_c)^{i2}} \text{ for } K_m/L_m > K/L \geq K_c/L_c, \tag{7b′}$$

---

this variable (11) does not exclude other functional forms that may be applied in future research (Ryzhenkov 2008).

$$n = n_a \text{ for } K/L \geq K_m/L_m. \tag{7c}$$

Respectively, for $K/L \geq K_m/L_m$ the partial derivatives $\frac{\partial n}{\partial s} = 0$ and $\frac{\partial n}{\partial a} = 0$. This redefinition of the partial dynamic law of labour supply, being not harmful from the economic point of view, enables to have solutions with a steadily growing $(n_a > 0)$, declining $(n_a < 0)$ or constant labour force $(n_a = 0)$.

Assume that the system (11), (13) – (15) includes *n* defined by the equations (7a), (7b′) and (7c). For reducing the order of this system further, the equation (11a) will be used instead of the equation (11). Then the state variable *k* becomes an auxiliary. There is a non-trivial stationary state in the system (13) – (15) defined independently of the parameter m_3 as

$$E_a = (s_a, v_a, u_a), \tag{18}$$

where

$$s_a = k_a \frac{1-u_a}{i} = \left[\frac{i s_0^{c_1}}{k_0(1-u_a)}\right]^{\frac{1}{c_1-1}} = s_0\left[\frac{k_0(1-u_a)}{s_0}\frac{1}{i}\right]^{\frac{1}{1-c_1}},$$

$$v_a = \frac{g + (1-b)(i-n_a)}{r},\ u_a = \frac{i - n_a - n_1 - n_3(v_a - v_c)}{n_2}.$$

Additionally, a stationary profit investment share is determined as $1 > k_a = k_0\left(\frac{s_a}{s_0}\right)^{c_1} > 0$.

The stationary growth rate of real labour compensation, labour productivity and capital intensity is $\hat{w}_a = \hat{a}_a = \hat{K}_a / L_a = \frac{m_1}{1-m_2}$; the stationary growth rate of net fixed capital and net output is $\hat{K}_a = \hat{P}_a = i = n_a + \frac{m_1}{1-m_2}$. At this stationary state, the growth rate of the labour value of net fixed capital, employment and labour force is $\hat{K}_a / a_a = \hat{L}_a = n_a$. The stationary general profit rate is $\frac{1-u_a}{s_a}$. The stationary rate of surplus value is $\frac{1-u_a}{u_a} = \frac{-(i-n_a) + n_1 + n_2 + n_3(v_a - v_c)}{(i-n_a) - n_1 - n_3(v_a - v_c)}$. Table 6 contains the stationary magnitudes of the state variables.

It could be easily shown, that exogenous increases in a stationary labour productivity growth rate raise a stationary employment ratio, stationary profit investment ratio and stationary profitability but diminish stationary relative labour compensation and stationary capital-output ratio.

Whereas the social factors do influence on the long-run stationary ratio of profit to labour compensation (rate of surplus value) in HL-IR, in the neoclassical case the profit-labour compensation ratio is entirely determined by parameters of a production function quite independently of other substantial socio-economic parameters.

The system (13) – (15) cannot be linearised at the stationary state E_a. A trace of the Jacoby matrix for this system moves to positive infinity in the vicinity of the stationary state E_a since the product of

coefficients $m_3\left(\frac{k_a}{s_a}+n_2u_a\right)$ at $\psi'(\hat{v})$ is positive and $\psi'(\hat{v})\to+\infty$ for $\hat{v}\to 0$. This stationary state E_a is not asymptotically stable therefore. Computer simulations (skipped) show that it, being locally unstable in the sense of Liapunov too, repels trajectories to an attracting limit cycle with a period of about 27 years (for $v_c \approx 0.925$) that does not result from the Andronov – Hopf bifurcation.

3. Re-formulating a Control Law of Capital Accumulation for the Modern US Economy

Feed-forward control, as known, changes variables according to expected future states of the economy. It has been assumed that the decision-makers (the state officials, owners of capital, managers and, maybe, trade union leaders) set a desirable growth rate of total surplus value depending on a difference between a target (X) and current (v) employment ratios. An indicated growth rate of surplus value is

$$\hat{S}=c_3+c_2(X-v), \tag{19}$$

where $v<X$ is typical for recessions and depressions; it is assumed that $c_3=0$ for simplicity. When $c_2<0$ surplus value vanishes, and v sharply falls. The case $c_2=0$ would represent a tendency to greater equity in income distribution not observed in the studied historical period. So it is assumed realistically that the parameter c_2 is positive.

A new equation for relative labour compensation follows from the equation (19)

$$\begin{aligned}\dot{u}&=(\hat{L}-\hat{S})(1-u)\\&=[\hat{v}+n+c_2(v-X)](1-u).\end{aligned} \tag{20}$$

A new equation for a growth rate of real labour compensation follows from the equations (3) and (20):

$$\begin{aligned}\hat{w}&=\hat{a}+\hat{u}\\&=\hat{a}+\left[\hat{v}+n+c_2(v-X)\right]\frac{1-u}{u}.\end{aligned} \tag{21}$$

The growth rate of real labour compensation continues to depend positively on the employment ratio (v). Still the equation (19) is structurally different from the initial equation (8) and its equivalent form (8a). Therefore a structure of CL-IR (Figure 11) is different from the HL-IR structure only in this part (cf. Figures 10 and 11).

The impact of the growth rate of labour productivity $(\hat{a})$ on $\hat{w}$ has become unmitigated. The rate of surplus value $\left(\frac{1-u}{u}\right)$ is the correction factor for impact by the other variables on $\hat{w}$: the former constant g has been transformed into a product of the two new constants (c_2, X) and of rate of surplus value $\left(\frac{1-u}{u}\right)$; non-linear positive dependence of $\hat{w}$ on the rate of change of the employment ratio $(\hat{v})$ multiplied by $\left(\frac{1-u}{u}\right)$ has substituted its former positive linear dependence on the rate of change of capital-output ratio $(\hat{s})$; the former constant r has been transformed into a product of the new constant (c_2) and of the rate of surplus value $\left(\frac{1-u}{u}\right)$. As both $\frac{\partial\hat{w}}{\partial\hat{v}}>0$ and $\frac{\partial\hat{w}}{\partial n}>0$, decelerating growth of employment ratio and declining growth rate of labour supply are detrimental for growth of real labour compensation if the all other conditions remain the same.

Analysing CL-IR with a help of the Lie derivative

For CL-IR defined by the equations (11) – (14) and (20), the Lie derivative is given by:

$$\dot{V}/V = -n - n_3 v + m_3 \psi'(\hat{v}) \frac{k(1-u)}{s} - n_2(1-u) - \frac{\dot{u}}{1-u} + c_1 \hat{s} - c_1 m_3 \psi'(\hat{v}) \frac{k(1-u)}{s}. \qquad (22)$$

The reader may notice that the compound coefficient $m_3(1-c_1)\frac{k(1-u)}{s}$ at $\psi'(\hat{v})$ is positive and $\psi'(\hat{v}) \to +\infty$ for $\hat{v} \to 0$. Therefore the Lie derivative moves to positive infinity in vicinity of such critical (singular) points. Thus in this case economy of scale and pro-cyclical character of profit investment share are at least locally destabilizing in vicinity of these critical points.

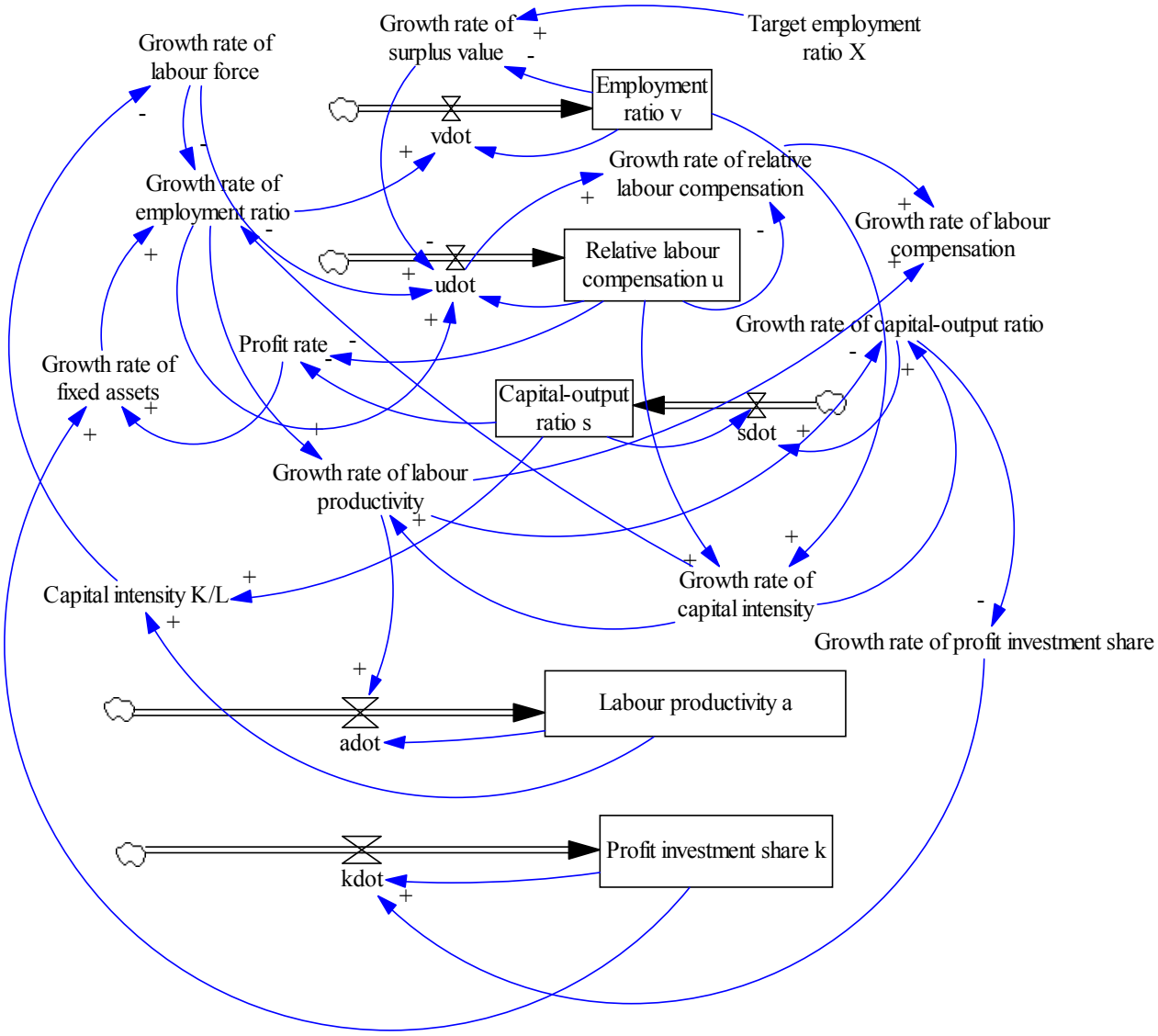

Figure 11. A condensed causal loop diagram of CL-IR (for $K/L \geq K_c/L_c$ in the equation (7b))

A non-trivial stationary state

The initial equations (11) – (14) and the new equation (20) that substitutes the initial equation (15) comprise the intensive deterministic form of CL-IR. If the equations (7a), (7b′) and (7c) for the growth rate of labour force and (11a) for the profit investment share are applied again, then the lower order system of the equations (13), (14) and (20) has a non-trivial stationary state defined independently of the parameter m_3 as

$$E_b = (s_b, v_b, u_b), \tag{23}$$

where

$$s_b = \left[\frac{i s_0^{c_1}}{k_0(1-u_b)}\right]^{\frac{1}{c_1-1}} = s_0\left[\frac{k_0(1-u_b)}{s_0}\frac{1}{i}\right]^{\frac{1}{1-c_1}},$$

$$v_b = X - \frac{n_a}{c_2},\ u_b = \frac{i - n_a - n_1 - n_3(v_b - v_c)}{n_2},\ i = \frac{m_1}{1-m_2} + n_a.$$

Additionally, a stationary profit investment share is determined as $1 > k_b = k_0\left(\frac{s_b}{s_0}\right)^{c_1} > 0$. At this stationary state, the rates of change for the value of net fixed capital, employment, labour force and surplus value are the same and equal $\hat{K}_b = \hat{L}_b = \hat{S}_b = n_a$. The stationary general profit rate is $(1 - u_b)/s_b = i/k_b$. Table 6 contains the stationary magnitudes of the state variables.

It could be shown, that confronted with exogenous increases in a stationary labour productivity growth rate the stationary employment ratio remains the same. These increases raise a stationary relative labour compensation and stationary profit investment ratio but diminish stationary profitability and stationary capital-output ratio. These consequences, especially the falling stationary profitability and declining stationary rate of surplus value weaken the capital interest in the CL-IR practical application.

A system (13), (14) and (20) cannot be linearised at the stationary state E_b. A trace of the Jacoby matrix for this system moves to positive infinity in the vicinity of the stationary state E_b since the product of coefficients $m_3 i$ at $\psi'(\hat{v})$ is positive and $\psi'(\hat{v}) \to +\infty$ for $\hat{v} \to 0$. Therefore this stationary state E_b, as E_a, is not asymptotically stable. Computer simulations (skipped) show that it, being locally unstable in the sense of Liapunov too, repels trajectories to an attracting limit cycle with a period of about 9 years (for $v_c \approx 0.9253$) that does not result from the Andronov – Hopf bifurcation.

4. A Historical Fit of HL-IR and CL-IR for the US Economy in 1969–2007

4.1. Probabilistic Forms of HL-IR and CL-IR

For estimating probable states of the economy and for identifying unobserved parameters in the basal period the deterministic models *HL-IR* and *CL-IR* have been transformed in two respective stochastic models, taking into account measurement errors and an impact of factors neglected in the model assumptions.[8] This makes implicit allowances for short-term and middle-term economic fluctuations by

[8] It is not possible to check whether the given deterministic model is able to replicate behaviour and create understanding of the observable economic behaviour without estimating parameters that usually

specification of the random components. The latter models include state equations and measurement equations for discrete moments of time

$$\mathbf{x}(\tau) = \mathbf{f_i}[\mathbf{x}(\tau - 1)] + \mathbf{w}(\tau),$$
$$\mathbf{z}(\tau) = \mathbf{H}\mathbf{x}(\tau) + \mathbf{v}(\tau),$$

where $\tau = 1, 2, \ldots, T$ is an index of data samples, $\mathbf{x}(0)$ – a vector of an initial state of the system, $\mathbf{w}(\tau)$ – a vector of equations errors (driving noise), $\mathbf{v}(\tau)$ – a vector of measurement errors. The deterministic parts $\mathbf{x}(\tau) = \mathbf{f_i}[\mathbf{x}(\tau - 1)]$, $i = 1, 2$ corresponds to the systems (11) – (15) for $i = 1$ and (11) – (14) and (20) for $i = 2$. The symbol **H** is for a rectangular matrix. The residuals are not due entirely, or largely, to pure random influences. On the contrary, these residuals contain highly systematic, non-random components.

Table 2. Initial and average observable magnitudes for US economic development in 1969–2007

	Profit investment share (k)	Capital-output ratio (s)	Employment ratio (v)	Relative labour compensation (u)	Profit rate $((1 - u)/s)$
Initial for the year 1969	0.237	1.767	0.965	0.709	0.165
Average for 1969–2001	0.184	1.932	0.938	0.710	0.151
Average for 1969–2007	0.174	1.923	0.939	0.708	0.152

A simplified version of an extended Kalman filtering (EKF), realised in the Vensim software developed by Ventana Systems, Inc., has been applied. This software enables to estimate the unobservable components of the both systems by a procedure of maximum likelihood.

Table 3. Comparison of HL-IR realizations for the USA and Italy

Condition or equation	USA, 1969–2001, 2009–2011	Italy, 1980–2004
technical progress function (4)	$\psi(\hat{v}) = sign(\hat{v})\lvert\hat{v}\rvert^j, 0 < j < 1$	$\psi(\hat{v}) = \hat{v}$
	$m_3 > 0$	$m_3 < -1$
mechanisation function (5)	$n_3 > 0$	$n_3 < 0$
labour force equation (7)	$n \geq 0$ is a function of capital intensity	$n = const > 0$
generalised Phillips equation (8)	$1 > b > m_2 > 0$	$b = 0$
profit investment share equation (11)	$\hat{k}$ is a function of growth rate of capital-output ratio	$\dot{k}$ is a function of profitability
inequality (27b)	not valid	valid

The value of one parameter was chosen a priory: $n_a = 0$. An application of the EKF to the US macroeconomic data for the basal period 1969–2007 has identified the other unobservable components of the above probabilistic forms of HL-IR and of CL-IR: $b \approx 0.563$, $c_1 = -20$, $c_2 = 4.678$, $e_1 \approx 2.5$, e_2

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requires construction of a stochastic model. A direct measurement of parameters' values, rarely achievable in macroeconomic modelling, is not for this particular study.

≈ 582.665, $i_1 \approx 0.336$, $i_2 \approx 0.337$, $g \approx 0.043$, $j \approx 0.35$, $K_c / L_c \approx 0.098$, $m_1 \approx 0.0068$, $m_2 \approx 0.257$, $m_3 \approx 0.04$, $n_1 \approx -0.24$, $n_2 \approx 0.340$, $n_3 \approx 0.588$, $p_1 \approx 0.030$, $r \approx 0.052$, $v_c \approx 0.925$, $X = 0.945$, $i \approx 0.0091$. Parameters b, g and r from the comprehensive Phillips equation (8a) are not applicable for CL-IR; parameters X and c_2 from the control equation (19) are not applicable for HL-IR.

Simulation runs have used the observed magnitudes for the initial year (1969) posted in Table 2 (additionally $a_0 \approx 0.04521$ millions 2000 dollars per person a year, $N_0 \approx 80705.1$ thousands persons, $P_0 \approx 3520.7$ billions 2000 dollars). They calculated the most probable (still sub-optimal) magnitudes of these four and other variables in the subsequent years.

For the stationary states E_a (18) and E_b (23) the following properties are satisfied

$$\frac{\partial u_a}{\partial v_c} = \frac{\partial u_b}{\partial v_c} = \frac{n_3}{n_2} > 0 \qquad (24)$$

and

$$\frac{\partial v_a}{\partial v_c} = \frac{\partial v_b}{\partial v_c} = 0. \qquad (25)$$

The reader sees that the probable plummeting of the magnitude of the parameter v_c in 2008 brings about the drop of the stationary magnitudes of the relative labour compensation in the inertia and stabilising scenarios (Table 6).

An exogenous growth in the stationary growth rate of labour productivity $\hat{a}_a$ benefits the stationary employment ratio v_a:

$$\frac{\partial v_a}{\partial \hat{a}_a} = \frac{1-b}{r} > 0. \qquad (26)$$

An exogenous growth in the stationary growth rate of labour productivity $\hat{a}_a$ enhances the stationary relative labour compensation if

$$\frac{\partial u_a}{\partial \hat{a}_a} = \frac{r-(1-b)n_3}{rn_2} > 0. \qquad (27a)$$

We see that exogenous growth in the stationary growth rate of labour productivity $\hat{a}_a$ raises the stationary relative labour compensation u_a if the following condition is satisfied:

$$n_3 < \frac{r}{1-b}. \qquad (27b)$$

As $\frac{\partial v_b}{\partial \hat{a}_b} = 0$, a following condition guarantees a positive derivative of the stationary relative labour compensation u_b in relation to the stationary growth rate of labour productivity $\hat{a}_b$

$$\frac{\partial u_b}{\partial \hat{a}_b} = \frac{1}{n_2} > 0. \qquad (28)$$

There are important qualitative differences with the identified parameters for the US and for Italy presented in Table 3. The condition (27b) was probably not valid for the US, unlike Italy, in particular. So an exogenous increase of labour productivity would exacerbate income inequality in the American economy in an inertia scenario based on HL-IR unlike conventional wisdom. Still an exogenous increase of labour productivity would diminish income inequality in the scenario(s) based on CL-IR when the condition (28) is valid.

4.2. Behaviour reproduction tests of HL-IR and CL-IR

HL-IR and CR-IR probabilistic forms are to pass behaviour reproduction tests. In particular, the Theil inequality statistics (Table 4) are used for estimating historical fit (Theil 1966).

Rather small root-mean-square errors as the percentage of the means (RMSE as percentage of the mean) and prevailing non-systematic errors of incomplete co-variation (U^C) over bias (U^M) and over difference in variation (U^S) show that these probabilistic forms track observations of the major variables in the basal period agreeably (Table 4). Figures 12–18, demonstrating a certain likeness between simulated and realised (observed) magnitudes in the basal period 1969–2007, support this conclusion.

Table 4. Decomposition of errors of the retrospective forecast for 1969–2007

Variable	*MSE* (units^2)	U^M	U^S	U^C	$\frac{\sqrt{MSE}}{mean}$,per cent
a	1.09E-06	0.278	0.023	0.699	1.915
s	1.30E-05	0.011	0.120	0.870	0.188
v	2.63E-06	0.024	0.198	0.778	0.173
u	0.0001	0.190	0.028	0.781	1.506
k	0.0004	0.031	0.006	0.964	11.234
$(1-u)/s$	3.20E-05	0.183	0.008	0.809	3.709

Two peak magnitudes of the profit rate, $(1-u)/s$, were observed in 1966 and 1997, a trough – in 1982; the simulated peaks refer to 1969 and 1999, simulated trough – to 1982 (Figure 18). The maximal magnitudes (observed and simulated) of the both variables are lower than their magnitudes (observed and simulated) in 1969. The uncovered tendency of the profit rate to fall is unfavourable for the employment ratio in the long-term.

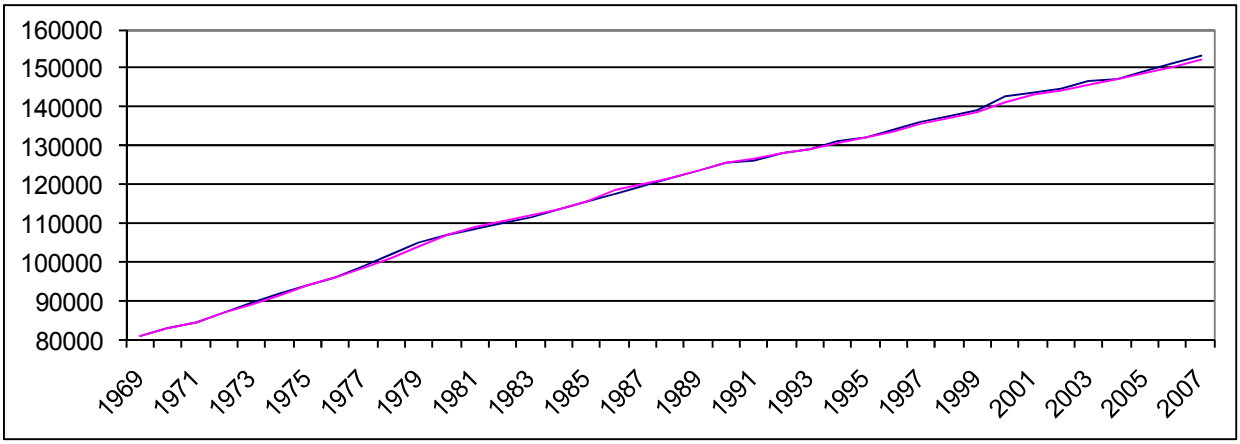

Figure 12. The observed (blue) and simulated (violet) civil labour force *N* (thousands of persons), 1969–2007

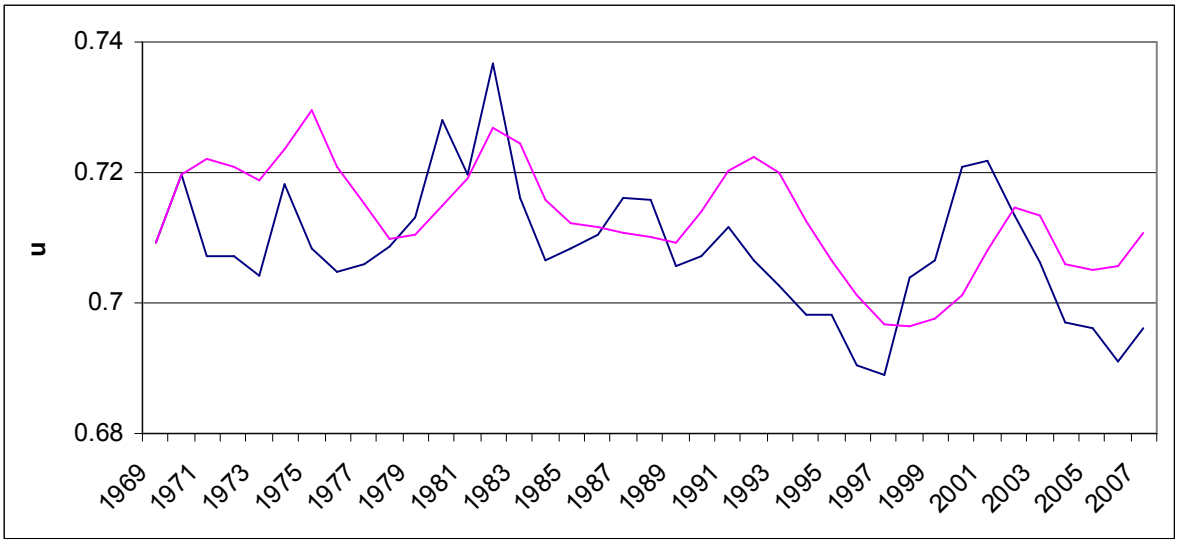

Figure 13. The relative labour compensation (u): observed (blue), 1948–2007, and simulated (violet), 1969–2007

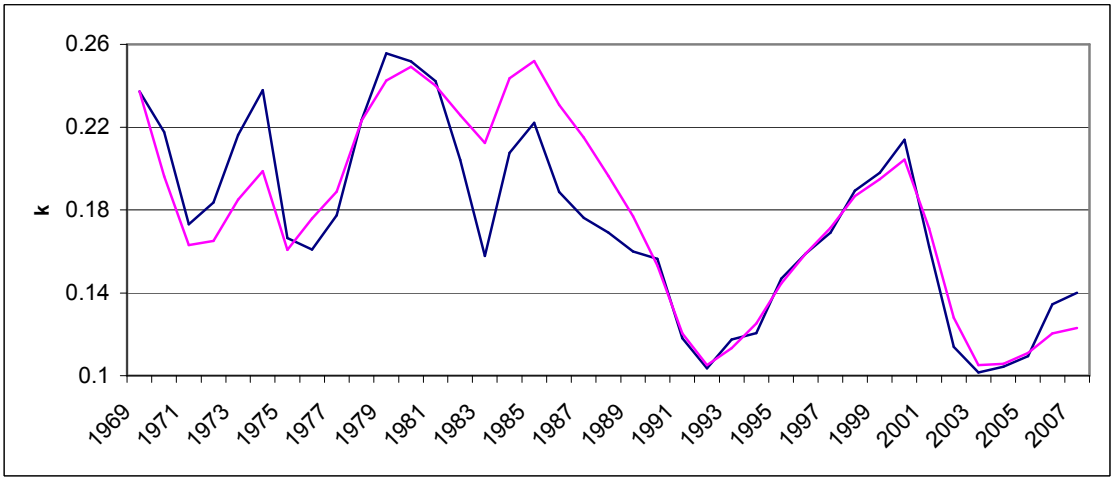

Figure 14. The profit investment share (k): observed (blue) and simulated (violet), 1969–2007

5. Prospective scenarios of US Economic Development

5.1. Inertia Scenario

An extrapolation of the retrospective forecast for the year 2008 and beyond, based on the deterministic model HL-IR (1) – (11) is called the inertia scenario I. The simulated magnitudes of the variables based on the probabilistic forms of HL-IR (for 1969–2001) and of CL-IR (for 2002–2007) are also included in the inertia scenario for the basal period 1969–2007 as a whole.

The previous long wave has manifested itself in 1969–2000 when two peak magnitudes of the employment ratio, v, were observed and simulated, whereas a trough in that long wave was observed in 1982, whereas the simulated trough – in 1983 (Figure 17).

The parameters values are given above (section 4.1). The new magnitude of the critical parameter v_c in altered HL-IR is 0.88 that is a qualitatively plausible expert guess for 2008–2060 and later. It is lower than the former magnitude of this parameter (0.925) identified by EKF for the basal period as a

whole (1969–2007). Without such step-wise change unaltered HL-IR and unaltered CL-IR do not generate steep decreases in the employment ratio and in net output observed in 2008–2009.

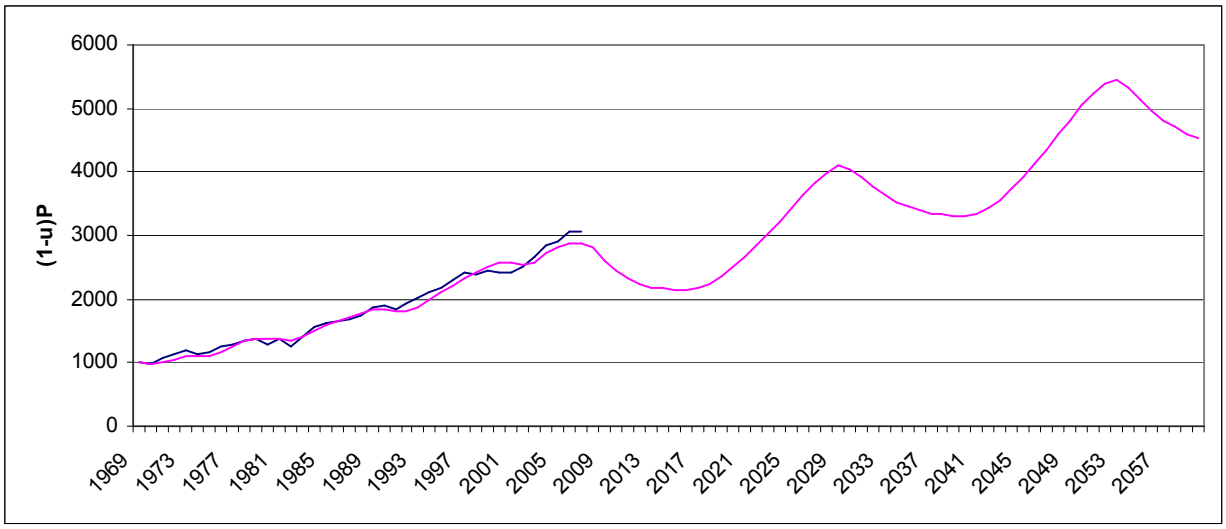

Figure 15. The profit $M = (1 - u)P$ (billions 2000 dollars a year) realised (blue), 1969–2007, and simulated (violet) in the inertia scenario, 1969–2060

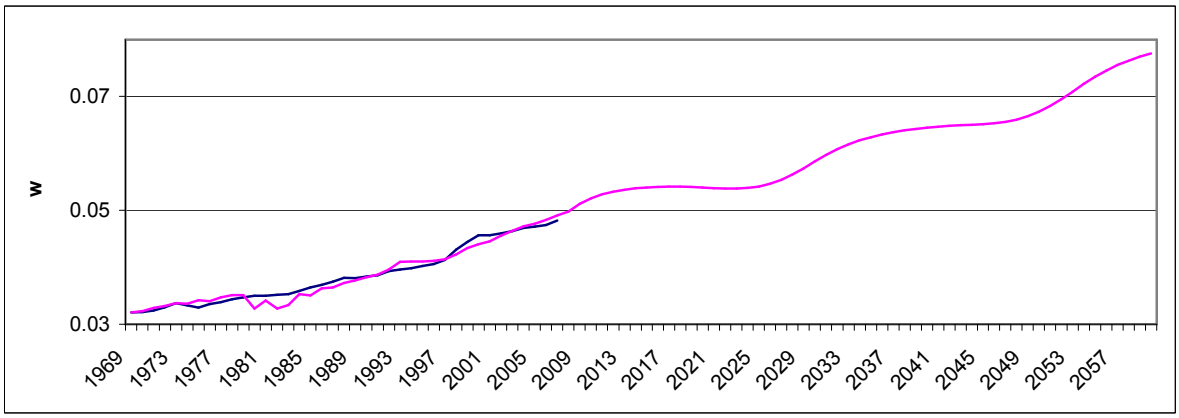

Figure 16. The labour compensation w (millions 2000 dollars per person a year) realised (blue), 1969–2007, and simulated (violet) in the inertia scenario, 1969–2060

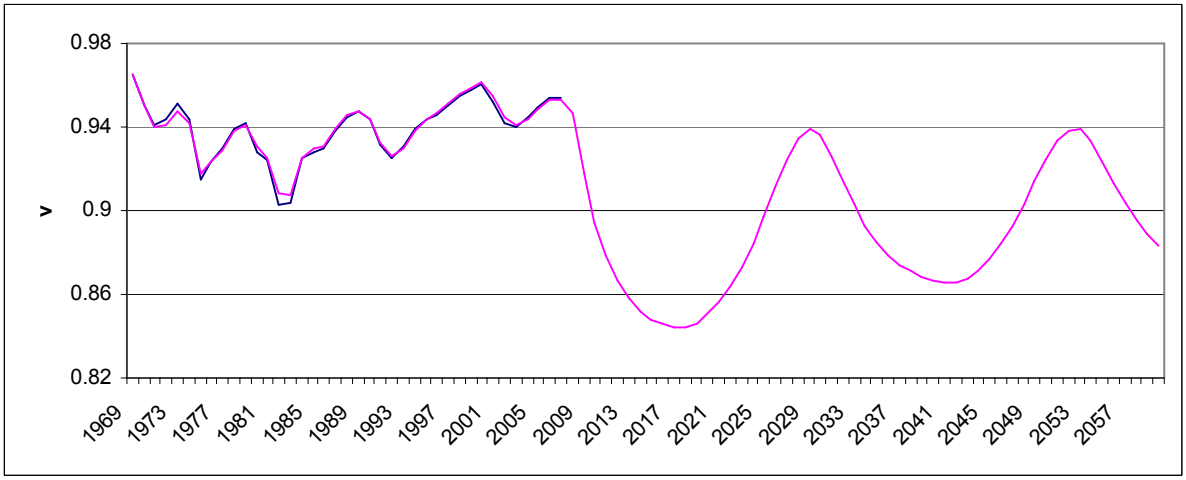

Figure 17. The employment ratio v realised (blue), 1969–2007, and simulated (violet) in the inertia scenario, 1969–2060

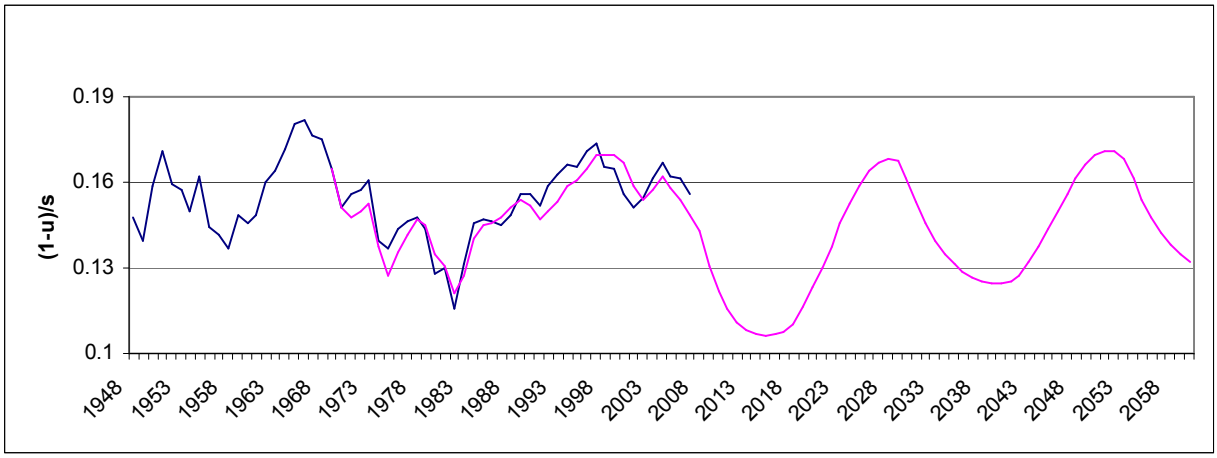

Figure 18. Profit rate $(1 - u)/s$ observed (blue), 1948–2007, and simulated (violet) in the inertia scenario, 1969–2060

Computer simulations reveal that phase variables (s, v, u, k), gross profit rate, growth rates of labour productivity and real labour compensation as well as some other variables fluctuate. These long-term fluctuations are anharmonic. The first distinguished complete quasi-cycle of the profit rate encompasses 1999–2028. The decrease in the magnitude of the parameter v_c gives support for profitability that is projected to be higher on the average during the second prospective long swing (2028–2052) than in the first (Figure 18).

Profit in real terms and profit rate will fall uninterruptedly in 2008–2015 (Figure 15 and Figure 18). The decline during the first quasi-cycle of the 21st century of surplus value lasts until 2015, of the employment ratio – until 2017 mainly because the growth rate of the real labour compensation exceeds the growth rate of labour productivity until 2015. Only when the latter surpasses the former the long wave starts to move upwards. For the employment ratio, a remaining period of the first prospective long swing started in 2000 is about 20 years (until 2029 inclusive); for the same variable, a total period of the second long swing is about 24 years (Figure 17). The real labour compensation w expected to stagnate for twelve years 2012–2023 (Figure 16).

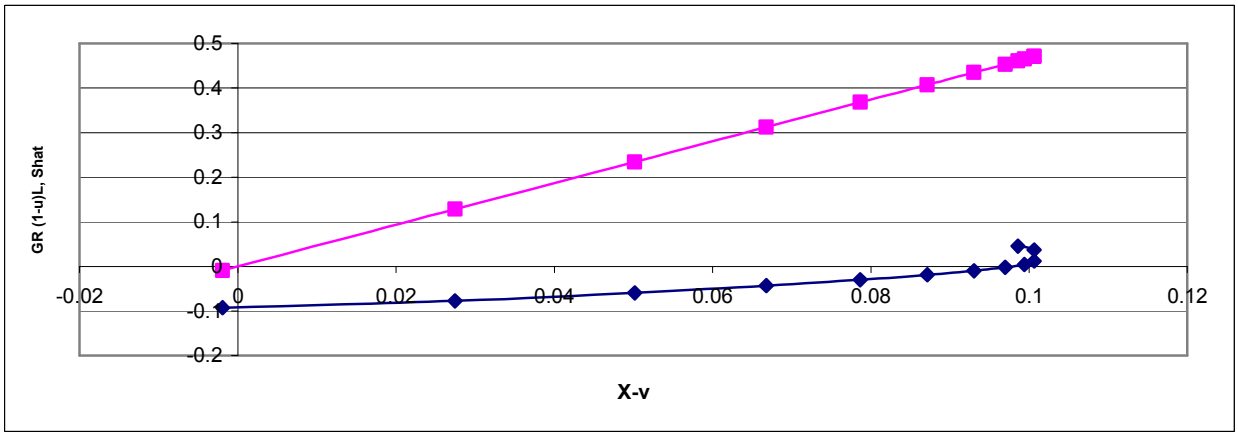

Figure 19. GR$(1–u)L$ – the inertia explicit growth rate of surplus value (blue), Shat – the implicit target growth rate of surplus value (violet) for the inertia scenario I depending on shortage employment ratio $(X – v)$ for $c_2 \approx 4.678$, 2008–2019 (from the left to the right)

The scatter graphs on Figure 19 demonstrate that the inertia growth rate of surplus value in HL-IR is much lower than the target growth rate of surplus value required by CL-IR that is not operative in

this inertia scenario for different magnitudes of the employment ratio. These 'scissors' make obvious that the inertia scenario rejects mobilising policy implied by CL-IR. The higher is the gap $X - v$, or the shortage employment ratio, the stronger is a labour resistance for mobilising policy since the higher is a target growth rate of surplus value the deeper is a required plunge in labour compensation as wee see below.

5.2. Two Stabilising Scenarios

It is assumed in stabilising scenarios II and III that a belated application of aggressive and non-aggressive mobilising policies based on altered CL-IR take place beginning from 2012 whereas altered HL-IR governs capital accumulation in 2008–2011. The both policy variants use the same initial magnitudes of state variables for 2008 (Table 5) and the same relevant parameters' values identified for the probabilistic forms of unaltered HL-IR and unaltered CL-IR by EKF for 1969–2007 except the updated magnitude of the parameter v_c (0.88 instead of 0.925) from the mechanisation function (5).

Table 5. Initial magnitudes of main variables in three different scenarios of US economic development for the year 2008

Profit investment share (k)	Capital-output ratio (s)	Employment ratio (v)	Relative labour compensation (u)	Profit rate $((1-u)/s)$
0.094	1.977	0.947	0.718	0.143

Table 6. Parameters of the simulation runs and stationary magnitudes in two basal periods and in three prospective scenarios for 2008 and beyond

Parameter	1969-2001	2002-2007	Scenario I	Scenario II	Scenario III
i	0.009	0.009	0.009	0.009	0.009
v_c	0.925	0.925	0.88	0.88	0.88
c_2	…	4.678	…	4.678	0.4678
X	…	0.945	…	0.945	0.945
v_a	0.909	…	0.909	…	…
v_b	…	0.945	…	0.945	0.945
u_a	0.761	…	0.682	…	…
u_b	…	0.699	…	0.620	0.620
s_a	1.918	…	2.025	…	…
s_b	…	1.940	…	2.042	2.042
k_a	0.073	…	0.058	…	…
k_b	…	0.059	…	0.049	0.049
$(1-u_a)/s_a$	0.125	…	0.157	…	…
$(1-u_b)/s_b$	…	0.155	…	0.186	0.186

The magnitude of c_2 identified for 2002–2007 is used in the aggressive stabilising scenario II, whereas the non-aggressive stabilising scenario III applies this initial magnitude divided by 10. The higher is c_2, the deeper is initial fall in $\hat{w}$ and the higher is the employment ratio overshooting a few years later, the higher are both amplitude and peak-to-trough ratios of converging middle-term economic oscillations. A period of oscillations is longer for lower c_2 than for initial one.

Table 6 presents relations of some important parameters with the stationary magnitudes of the main variables. The reader sees that capital accumulation tends by different means to decrease a stationary relative labour compensation (from 0.761 to 0.620) and increase a stationary profit rate (from 0.125 to 0.186). A maximal profitability after the Second World War was about 0.182 observed in 1966.

In the CBO forecast of March 2009, a recovery begins to take hold late in 2009 and quickens in 2010. OECD (March 2009) projects a decelerating decline of US GDP in 2009 and accelerating resumption of its growth in the 1st quarter 2010. The outlook of this paper is darker: in the inertia scenario I, a recovery begins in 2016, in the stabilizing scenario II – in 2013, in the stabilizing scenario III – in 2014.

The Outlooks Through 2019

In the CBO forecast, the unemployment rate declines to 8.5 per cent by the end of 2010. In the three scenarios, the unemployment rate increases to 10.5 per cent by the end of 2010 (0.1 percentage point higher than in the official "more adverse" scenario) and grows further (in the inertia scenario I – until 2017, in the stabilizing scenario II – until 2012, in the stabilizing scenario III – until 2014). In 2009–2019 the employment ratio is lower on average in the three scenarios than in the CBO projection (Table 8, Figure 20, panel 1).

Table 7a. Summary statistics of main labour variables in three scenarios for 2008–2019

Scenario	Mean			Normalised standard deviation (variation)		
	Employment ratio (v)	Relative labour compensation (u)	Labour compensation (w)	Employment ratio (v)	Relative labour compensation (u)	Labour compensation (w)
Inertia I	0.870	0.755	0.053	0.037	0.020	0.025
Stabilising II (aggressive)	0.922	0.671	0.048	0.041	0.108	0.096
Stabilising III (non-aggressive)	0.881	0.731	0.052	0.029	0.025	0.016

Table 7b. Summary statistics of the main capital variables in three scenarios for 2008–2019

Scenario	Mean			Normalised standard deviation		
	Surplus value ($(1-u)L$)	Profit ($(1-u)P$)	Profit rate ($(1-u)/s$)	Surplus value ($(1-u)L$)	Profit ($(1-u)P$)	Profit rate ($(1-u)/s$)
Inertia I	33036.38	2322.1	0.115	0.091	0.086	0.094
Stabilising II (aggressive)	47138.18	3403.6	0.163	0.245	0.262	0.245
Stabilising III (non-aggressive)	36670.7	2613.6	0.129	0.083	0.101	0.094

Table 8. Averaging economic indicators in three scenarios and in the CBO projection for 2009–2019

Scenario	Employment ratio	Growth rate of				
		total labour compensation	profit	net output	labour force	net fixed capital
Inertia I	0.863	0.000	-0.015	-0.004	0.003	0.003
Stabilising II (aggressive)	0.919	0.006	0.039	0.010	0.003	0.013
Stabilising III (non-aggressive)	0.875	0.001	0.010	0.004	0.003	0.004
CBO projection (March 2009)	0.939	0.025	0.015	0.025	0.006	0.029

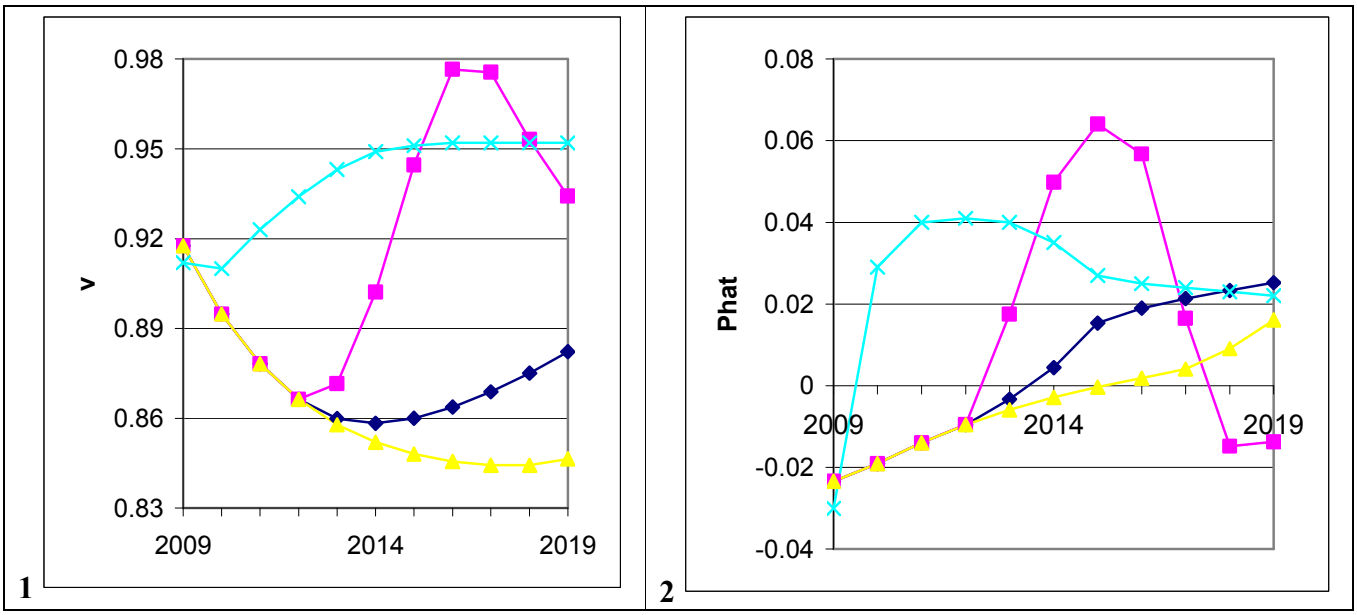

Figure 20. Evolution of the employment ratio (v) on panel 1 and of growth rate of net output (Phat) on panel 2 in three scenarios compared with CBO's projections 2009–2019 (yellow – inertia I, violet – aggressive stabilising II, blue – non-aggressive stabilising III, aqua – CBO March 2009)

CBO expects that net output will grow at an annual rate of 2.3 per cent, profit at an annual rate of 1.5, wages and salaries at an annual rate of 2.5 per cent on average during the 2009–2019 period (CBO March 2009). Net output, profit and labour compensation will grow at an annual rate of -0.4, -1.5 and 0.0 per cent on average in the inertia scenario I, of 1.0, 3.9 and 0.6 per cent – in the stabilizing scenario II, of 0.4, 1.0 and 0.1 per cent – in the stabilizing scenario III (Table 8). The gaps between growth rates of net output in the CBO projection and in scenarios I and III will be narrow to the end of this period (Figure 20, panel 2).

Whereas CBO projects the growth of potential hours worked average 0.6 per cent annually from 2009 to 2019, scenarios I, II and III offer an annual growth rate of labour force 0.3 per cent on average. CBO anticipates the pace of capital accumulation averaging 2.9 per cent annually during the period, whereas the rate of growth of net fixed capital will be annually 0.3 per cent in the inertia scenario I, of 1.3 per cent – in the stabilizing scenario II, of 0.4 per cent – in the stabilizing scenario III.

The Outlooks Through 2060

Both stabilising scenarios smooth out long swings; in the inertia scenario I only long swings are present with a period of oscillations for 1999–2060 diminishing from about 30 to 24 years; a period of fluctuations in the aggressive stabilising scenario II (8) is typical for business cycles; a period of fluctuations in the non-aggressive stabilising III (14–16) approximately equals the mean of the two periods in the scenarios I and II. Scenario III has the highest profit investment share and growth rates of net output and labour productivity, on the average. It may represent a new social compromise, especially taking into account international competition of national economies.

Table 9. Projecting years of the first match with maximal magnitudes of economic variables achieved in 1995–2008 in three scenarios, 2008–2060

Variable	Year of previous maximum	Year of the first exceeding the previous maximum		
		Inertia I	Stabilising II (aggressive)	Stabilising III (non-aggressive)
Net output (P)	2008	2022	2015	2018
Profit ($(1 – u)P$)	2007	2023	2013	2018
Surplus value ($(1 – u)L$)	2006	2024	2013	2020
Rate of surplus value ($(1 – u)/u$)	1998	2025	2013	2020
Profit rate ($(1 – u)/s$)	1999	2051	2014	2034
Employment (L)	2007	2026	2015	2023
Employment ratio (v)	2000	outside reach	2016	outside reach

Table 10a. Summary statistics of three main labour variables in three main scenarios for 2012–2060

	Mean			Normalised standard deviation (variation)		
Scenario	Employment ratio (v)	Relative labour compensation (u)	Labour compensation (w)	Employment ratio (v)	Relative labour compensation (u)	Labour compensation (w)
Inertia I	0.889	0.715	0.062	0.033	0.041	0.118
Stabilising II (aggressive)	0.941	0.624	0.056	0.019	0.046	0.132
Stabilising III (non-aggressive)	0.918	0.669	0.062	0.027	0.048	0.121

Table 10b. Summary statistics of three main capital variables in three main scenarios for 2012–2060

	Mean			Normalised standard deviation (variation)		
Scenario	Profit investment ratio (k)	Capital-output ratio (s)	Profit rate ($(1 – u)/s$)	Profit investment ratio (k)	Capital-output ratio (s)	Profit rate ($(1 – u)/s$)
Inertia I	0.070	2.045	0.140	0.833	0.044	0.143
Stabilising II (aggressive)	0.063	2.023	0.186	0.399	0.018	0.086
Stabilising III (non-aggressive)	0.079	2.007	0.165	0.446	0.028	0.109

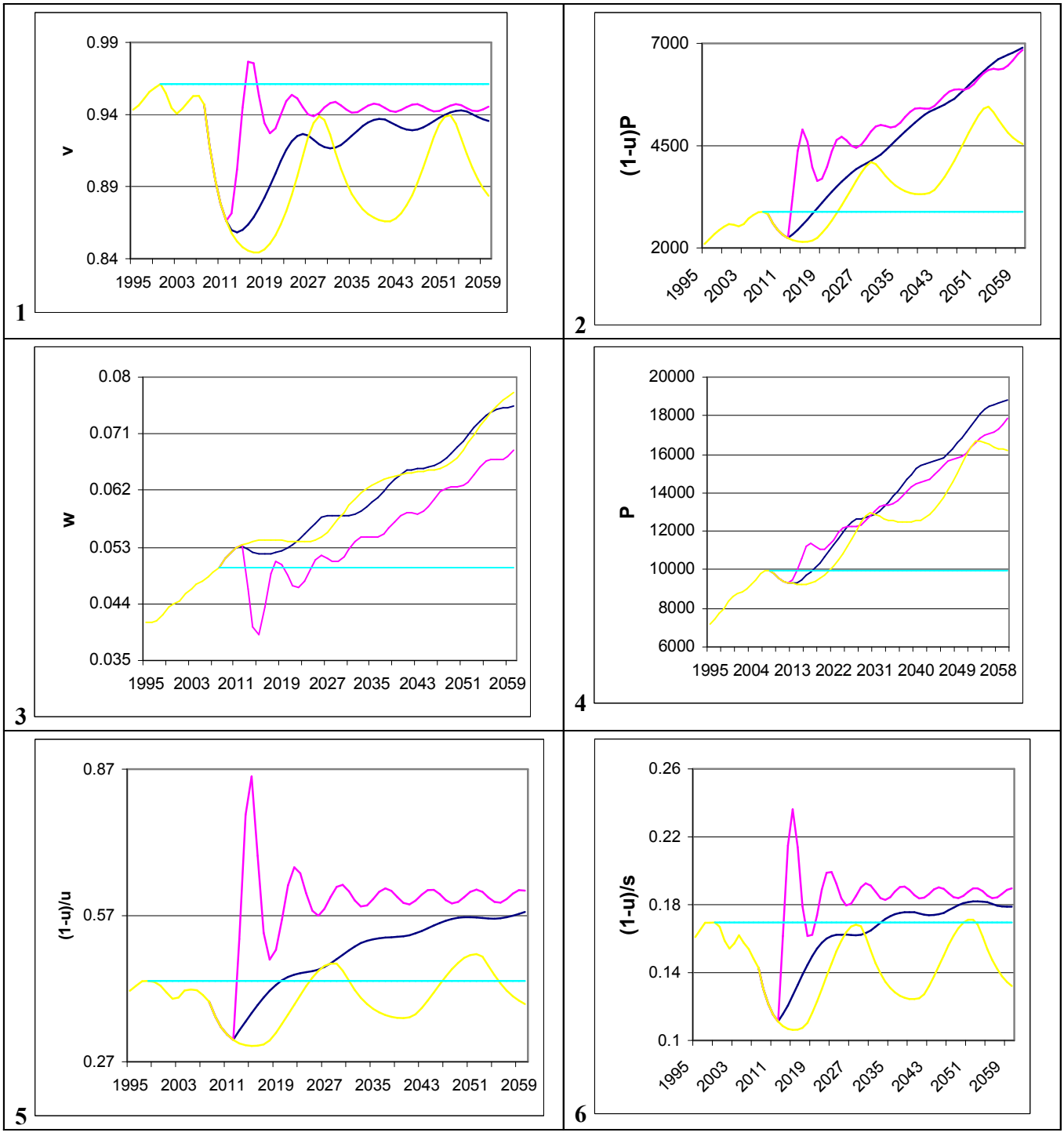

Figure 21. Evolution in three scenarios (yellow – inertia I, violet – aggressive stabilising II, blue – non-aggressive stabilising III, aqua – frame matching maximum for 1995–2008; 1 – employment ratio, 2 – profit, 3 – labour compensation, 4 – net output, 5 – rate of surplus value, 6 – rate of profit)

The longer projections confirm that the aggressive stabilising scenario II is best for capital judging by the rates of profit and surplus value and by the total profit and surplus value (Tables 9, 10a and 10b,

Figure 21). Still as the experience of 2002–2007 teaches this stabilising policy contains seeds of its own destruction. Moreover it requires a dramatic plunge of labourers' living standard for a protracted period with returning to the level of 2007 only in 2024. Mostly probably this scenario could be not realised on practice (at least without engaging in a grand war – to be avoided at all costs!).

The non-aggressive stabilising scenario III is superior for capital than the inertia scenario I. It has some advantages for labour too. Still labourers face an employment ratio – relative labour compensation trade-off. The scenario III will extend the secular tendency of relative labour compensation to fall that was a characteristic of the basal period 1969–2007 taken as a whole (Figure 13). Besides that the previous local maximum of the employment ratio of 2007 will be outside reach.

The reader sees that the supposed structural change of the law of capital accumulation in the non-aggressive stabilising scenario III may help the US economy to overcome the declining phase in the long swing faster than in the inertia scenario I without this change. Still compared with the inertia scenario I the real labour compensation (w) will be lower in 2012–2021, total real labour income (wL) in 2012–2018, relative labour compensation in an almost all period 2012–60 with few exceptions.

Conclusion

This paper offers a synthesis of theoretical laws of capital accumulation grasped at the definite level of abstraction and the historical contingency thus strengthening analytical foundations of economic theory. It advances, in particular, a system dynamics understanding of the role of the variable profit investment share in capital accumulation by establishing the explicit inverse relation between growth rate of this share and growth rate of capital-output ratio. This new endogenous variable is incorporated into HL-IR and into CL-IR. Capital endeavours by different means explained to decrease a stationary relative labour compensation (unit value of labour power) and increase a stationary profit rate. A secular decline in profit investment share substantially neutralises the secular tendency of profit rate to fall.

This paper tests the probabilistic forms of these laws statistically for the basal period of the US economic evolution, 1969–2007. Most likely, these laws roughly operated in the historical periods under study (1969–2001 and 2002–2007, respectively). The lower observed growth rate of real labour compensation in 2002–2007 than required by unaltered CL-IR has contributed to sharp capital over-accumulation in 2005–2007. Moreover, capital rejects unaltered CL-IR as a trap: it does not provide the maximal profit rate as in 1966. A strive of capital dominated by its relentless financial arm to higher profit and higher profitability hides behind the *explosive nature* of capitalist reproduction. Still this paper argues that after the defeat of the neo-conservative mobilising policy in 2007–2008 there is hardly place for stabilising policy with the same or similar aggressiveness as in 2002–2007.

In the inertia scenario I, HL-IR (altered in 2008) shapes long swings with a period of about 24–30 years. Non-trivial stationary states of a deterministic form of HL-IR are unstable; a limit cycle is simulated with a period of fluctuations of about 27 years. Other characteristic of this scenario is reinforcement of the tendencies of the general profit rate to fall until 2015 and of employment ratio to decline until 2017.

Stabilising scenarios II (aggressive) and III (non-aggressive) based on altered HL-IR (2008–2011) and on altered CL-IR (2012 –2060) describe transients to vicinity of new stationary states. Altered CL-IR smoothes long swings and shortens their quasi periods in these scenarios to 8 and 14–16 years, respectively, compared with the inertia scenario I. The new stationary profitability is higher than stationary profitability in inertia scenario I. There may be substantial increases in the total profit, profitability and employment ratio in comparison with the inertia scenario I.

Apart from scenario II, the present stern crisis of the capital accumulation, probably worst after the World War II, will last in scenarios III and I until 2018–2022 when the pre-crisis maximum of net output is restored and 2023–2026 when the pre-crisis maximum of employment is reached again. These

projections are more realistic, in my view, than the recent official scenarios (prepared by the US government and by CBO in the 1st quarter of 2009) and than the OECD projection (March 2009).

The American labourers have been, are and will be carrying the main burden of the expected economic revival. This absolute and relative worsening of the labourers' living standard may be interwoven with substantial dollar devaluation, considerable inflation and growing national debt. This motion picture is gloomy but not strange for the economic theory named dismal science!

A task of the future research is figuring out whether HL-IR and CL-IR, presented in this paper, really govern capital accumulation in the modern US economy. The efficiency and robustness of the supposed stabilisation policy is to be checked repeatedly using a permanent data stream and system dynamics parameters optimisation tools.

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