

# The origins of behavior patterns

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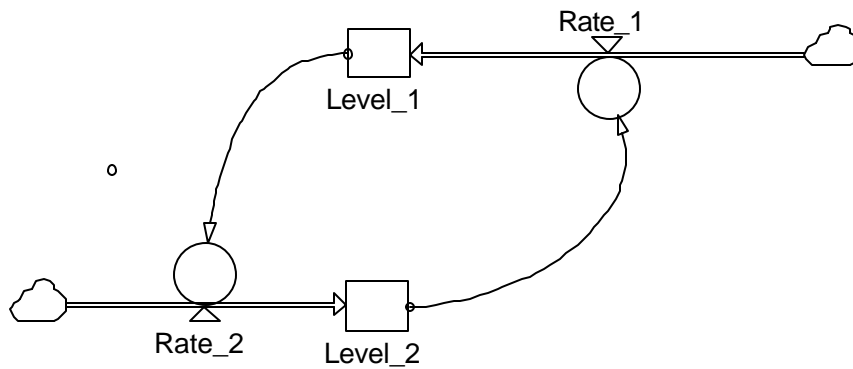
## Abstract

*Can a simple positive feedback loop generate a goal-seeking behavior? The answer is yes. Can the behavior generated from a loop with a constant gain shift from a goal-seeking one to an exponential growth one? The answer is also yes. From the above answers, one can conclude that link gains alone cannot explain the origin of behavior patterns. In this paper, we propose the “curvature-contribution” of a link as a more valid representation of the link’s role in generating behavior patterns. The “curvature-contribution” of link is the product of the link’s gain and the rate of change in the input (influencing) state. If a link has a very high gain but there is no change in the input state, then this link does not contribute at all to the behavior pattern of the output (influenced) state. Likewise, if the input state is changing rapidly but the link has zero gain, then this link does not contribute at all to the behavior pattern of the output state.*

**Keywords:** Model analysis, behavior patterns, dominant structures

## 1 The Challenge

As a demonstration of the issues stated in the abstract, we consider the model shown in figure 1. It is a simple, second order model that has a single positive feedback loop with a constant gain (determined by the model equations in table 1). This model can generate a goal-seeking behavior (as shown in figure 2), it can generate an exponential growth behavior (as shown in figure 3), and it can shift its behavior from a goal-seeking one to an exponential growth one (as shown in figure 4). In short, it can generate various sorts of behavior despite the fact that it is just contains a single loop with a constant gain. The implication is that knowing the structure of a model, more specifically the link gains characterizing that structure, is not sufficient to determine the current behavior of the model.



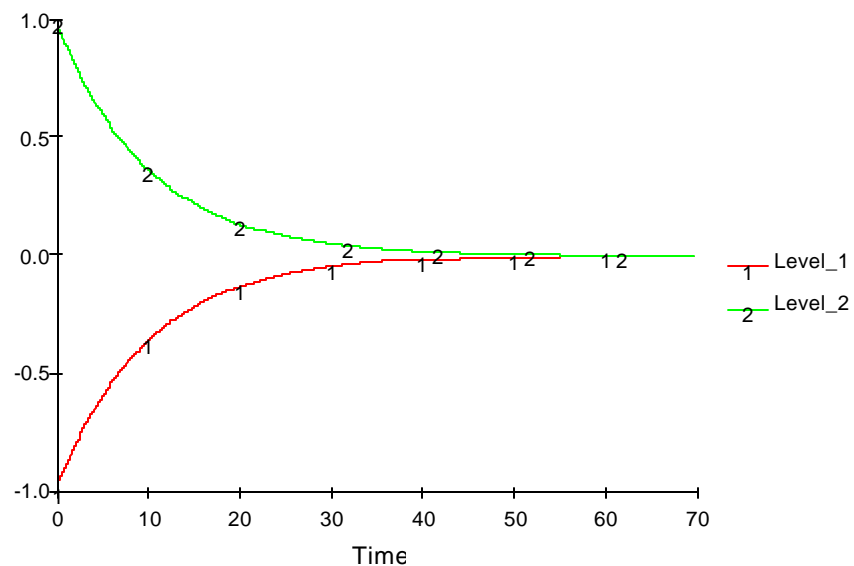
**Fig. 1: Example 1, stock and flow diagram**

$\text{flow Level}_1 = \text{Rate}_1$   
 $\text{flow Level}_2 = \text{Rate}_2$

$\text{Rate}_1 = 0.1 * \text{Level}_2$   
 $\text{Rate}_2 = 0.1 * \text{Level}_1$

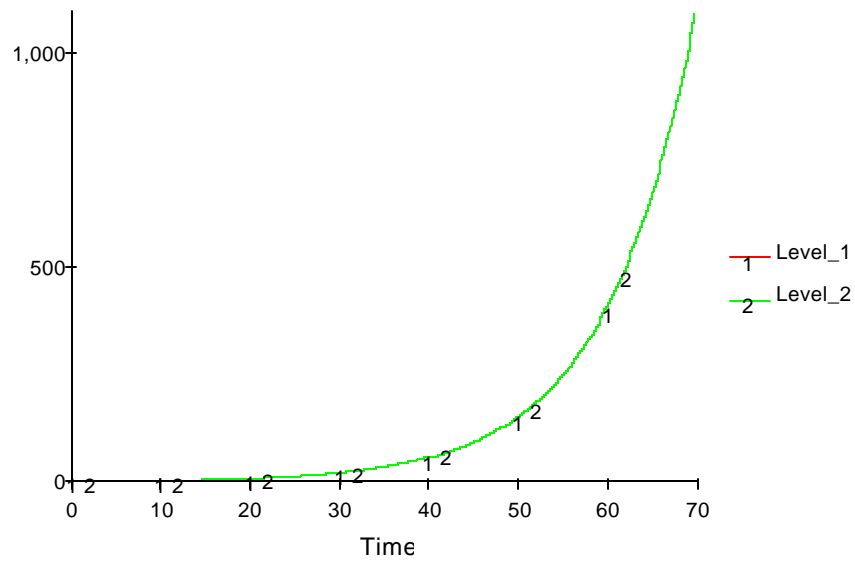
**Table 1: Example 1, equations**

Now, if we set the initial value of level\_1 to -1, and the initial value of level\_2 to 1, we get a goal-seeking behavior (convergent behavior), as shown in the figure below.



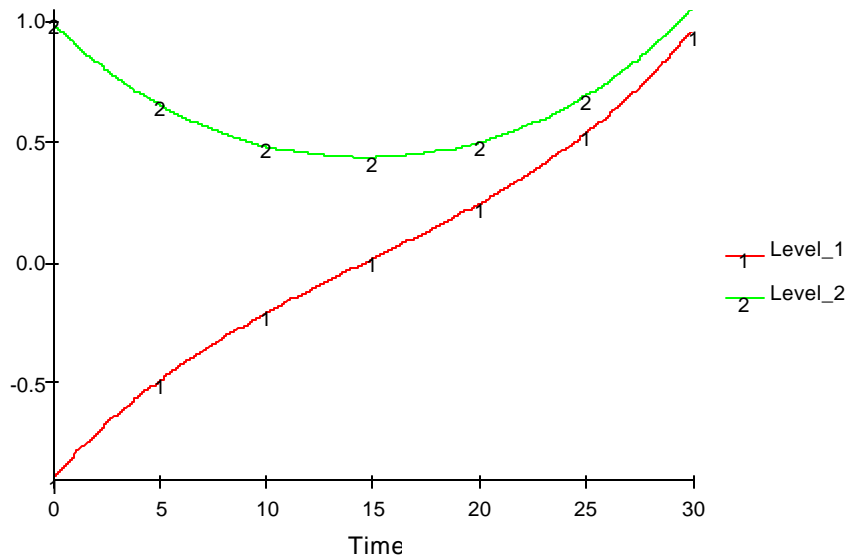
**Fig. 2: Example 1, behavior; init Level\_1= -1 & init Level\_2= 1**

Yet, if we change the initial value of level\_1 from -1 to 1, we get an exponential growth behavior (divergent behavior), as shown in the figure below.



**Fig. 3: Example 1, behavior; init Level\_1= 1 & init Level\_2= 1**

Moreover, if we set the initial value of level\_1 to  $-0.9$ , and the initial value of level\_2 to 1, the behavior shifts from a goal seeking one to an exponential growth one, as shown in the figure below.



**Fig. 4: Example 1, behavior; init Level\_1= -0.9 & init Level\_2= 1**

In the next sections, we will first develop quantitative measures for behavior patterns, and then we will identify the origins of behavior patterns. In this paper, we define behavior patterns as the types of behavior that can be observed over a tiny time interval. In any tiny time interval, one can only observe divergent behavior (i.e. exponential

growth or decline) and convergent behavior (i.e. goal-seeking behavior). Other types of behavior like oscillation and s-shaped behavior can only be observed over a large time interval, and are composed of a series of convergent/divergent patterns. This paper is based on our previous paper (Saleh & Davidsen, 2000), in which we developed two distinct approaches to characterize the current behavior, i.e. behavior pattern indexes and the current dominant modes of behavior -- where each mode of behavior is represented by an eigenvalue. Nathan Forrester (Forrester, 1982 & 1983) has established the theoretical foundation for identifying the origins of behavior modes (eigenvalues). In this paper we study the origin of behavior patterns, which is another approach to characterize the behavior. The curvature-contribution concept, which we mentioned in the abstract, is the core of our study as we are going to explain later.

Characterizing and analyzing the behavior of complex models is a sophisticated issue. Looking at this issue, from two distinct perspectives broadens our horizon, and enables us to capture the different aspects of this sophisticated issue. These two distinct perspectives are the empirical approach described in this paper, and an analytical approach based on eigenvalue analysis, which is described in (Saleh & Davidsen, 2001; Saleh & Davidsen, 2000; Forrester, 1982 & 1983).

Applying these two approaches to the same system dynamics models we may triangulate our results through thorough experimentations. Our ultimate goal (in both approaches) is to shed some light on the origin of dynamics in complex models, and apply the insights gained to the facilitation and enhancement of the management of the real systems represented by such complex models.

Note that the simple empirical interpretation method, described in this paper, is a localized perspective for determining the causes of the behavior of a certain state variable. It is a localized perspective as we only focus on the behavior of a particular variable, and for this specific state variable we search for the causes of its current behavior pattern in the near links, i.e. links that directly affect the state variable without any other intervening state variables. This only offers a fragmented view of the model as we assume that links far from the state variable (i.e. links that do not directly affect the state variable) have no effect at all. In contrast, the eigenvalue analysis gives us a global view of the whole model. I.e. we are able to determine the effect of any link on the behavior of any state variable (whether this link is a near or a far link). Moreover, the analytical approach based on eigenvalue analysis is more suited to studying models that exhibit oscillatory behavior, than the empirical approach, described in this paper. Recall that the analytical approach decomposes the total behavior of a model into monotonic and oscillatory modes of behavior. In contrast, in the empirical approach, we can only express monotonic behavior (that unfolds over a small time period). Hence an oscillatory behavior will be expressed as a series of alternating convergent and divergent fragments of behavior. Nevertheless, despite these reasons for applying the more potent analytical approach in linking behavior to structure – i.e. explaining the foundation of behavior – we apply the empirical approach in this paper to outline an interpretation to the origin of behavior. We think that a dual empirical/analytical approaches are vital to ensure that the analytical approach (based on eigenvalue analysis) is consistent with our empirical (intuitive) interpretation (described in this paper) of model behavior, model structure and the relationship between the two.

## 2 Theoretical Foundation

To investigate in depth the questions we raised in this paper and to understand the curvature-contribution concept, we begin by explaining the behavior pattern indexes.

### 2.1 Behavior pattern indexes

The properties of the behavior that we will be focusing on are the slope ( $s$ ), and the curvature ( $c$ ) of each state variable ( $x$ ). These are defined, in this paper, as follows:

- the slope,  $s$ , is defined as the first (time) derivative,  $\dot{x}$ , of that state variable,  $x$ ; and
- the curvature,  $c$ , is defined as the double (time) derivative,  $\ddot{x}$ , of that state variable.

The convergence/divergence of a state variable at any instant of time is defined as the rate of change of the absolute value of the slope,  $s$ , of this state variable, i.e.  $d|s|/dt$  (Ford, 1999). If the state variable is in a convergent behavior (goal-seeking behavior), then  $d|s|/dt$  will be negative, i.e. the absolute value of the slope,  $|s|$ , is decreasing with time. If the state variable is in a divergent behavior (exponential growth or decline), then  $d|s|/dt$  will be positive, i.e. the absolute value of the slope vector,  $|s|$ , is increasing with time. Note that  $|s|=0$  is a partial condition for equilibrium.

In the next paragraphs, we will develop a proxy measure for  $d|s|/dt$ . This proxy measure will serve as a normalized indicator for the convergence/divergence of a state variable at any instant of time. The definition of this proxy measure is the ratio of the curvature to the slope of the state variable. To understand the idea behind this proxy measure we begin by analyzing the following simple difference equation:

$$S_{x \text{ new}} = S_{x \text{ current}} + \Delta t * c_x$$

For a state variable  $x$ , we may express the new slope, i.e.  $S_{x \text{ new}}$ , in terms of the original slope,  $S_{x \text{ current}}$ , and the rate of change in the slope over the subsequent period of time, i.e. the curvature,  $c_x$ .

In mathematical terms, the new proxy measure is defined by the following ratio:

$$c_x / S_{x \text{ current}}$$

Note that if  $c_x$  and  $S_{x \text{ current}}$  have the same sign, then this ratio will be positive, and the absolute value of the slope will increase (i.e.  $d|s_x|/dt > 0$ ), - an indicator of divergent behavior. If, on the other hand, the two have opposite signs, then the ratio will be negative, and the absolute value of the slope will decrease (i.e.  $d|s|/dt < 0$ ), - an indicator of convergent behavior. If  $c = 0$ , then the ratio will be 0, so that the absolute value of the slope will not change, i.e.  $d|s_x|/dt = 0$ . In such a case, if  $c_x$  changes its value from  $-\epsilon$  ( $\epsilon > 0$ ) to  $+\epsilon$ , then the ratio ( $c_x / S_{x \text{ current}}$ ) will change from negative to positive -- or the reverse, depending on the sign of  $S_{x \text{ current}}$ . As a consequence, the value of  $d|s_x|/dt$  will also change from negative to positive -- or the reverse, - an indicator of a transition in the pattern of behavior from convergence to divergence -- or the reverse. If  $S_{x \text{ current}}$  changes its value from  $-\epsilon$  to  $+\epsilon$ , then the ratio ( $c_x / S_{x \text{ current}}$ ) will change from  $-\infty$  to  $+\infty$  --

or the reverse, depending on the sign of  $c_x$ . That would be an indicator of a discontinuity in  $d|s_x|/dt$ , and thus, also in this case, a transition in the pattern of behavior from convergence to divergence – or the reverse.

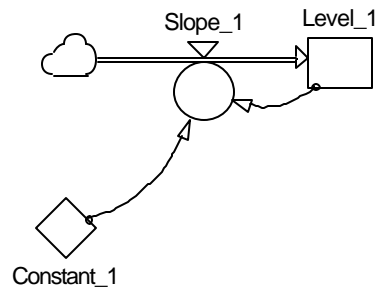
Since the ratio  $c_x / s_x$  current is a characterization of the pattern of behavior exhibited by the state  $x$ , we define an indicator of that behavior in the form of a **Behavior Pattern Index (BPI)**, associated with a particular state variable,  $x$ , as:

$$BPI_x = c_x / s_x$$

$BPI_x$  serves as a normalized proxy for  $d|s_x|/dt$ .

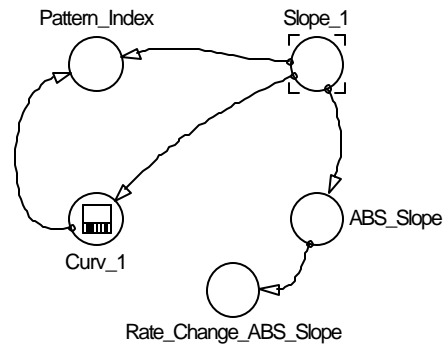
## 2.2 Behavior pattern indexes – example 2

Example 2 is a simple first order model with the state variable  $Level_1$ , governed by the  $Slope_1$ , - a model that can exhibit exponential growth or goal-seeking behavior, depending on the value of the parameter  $Constant_1$ . A positive value implies exponential growth, while a negative one implies a goal-seeking behavior.



**Fig. 5: Example 2, stock and flow diagram of the model**

In order to analyze the model in example 2, we need to add an auxiliary structure, see figure 4, that calculates the pattern index,  $Pattern\_Index$ , for the state variable based on the first derivative,  $Slope_1$ , and second derivative,  $Curv_1$ ; it also calculates the rate of change of the absolute value of the slope,  $Rate\_Change\_ABS\_Slope$ :



**Fig. 6: Example 2, stock and flow diagram of the calculation of the behavior pattern index, and the rate of change of the absolute value of the slope**

*Below is the table of equations for example 2.*

**init** Level\_1 = 1  
**flow** Level\_1 = Rate\_1

**Slope\_1** = Constant\_1\*Level\_1  
**Curv\_1** = DERIVN(Slope\_1)

*Note: DERIVN is the time derivative function.*

**Constant\_1** = 2 (or -2)

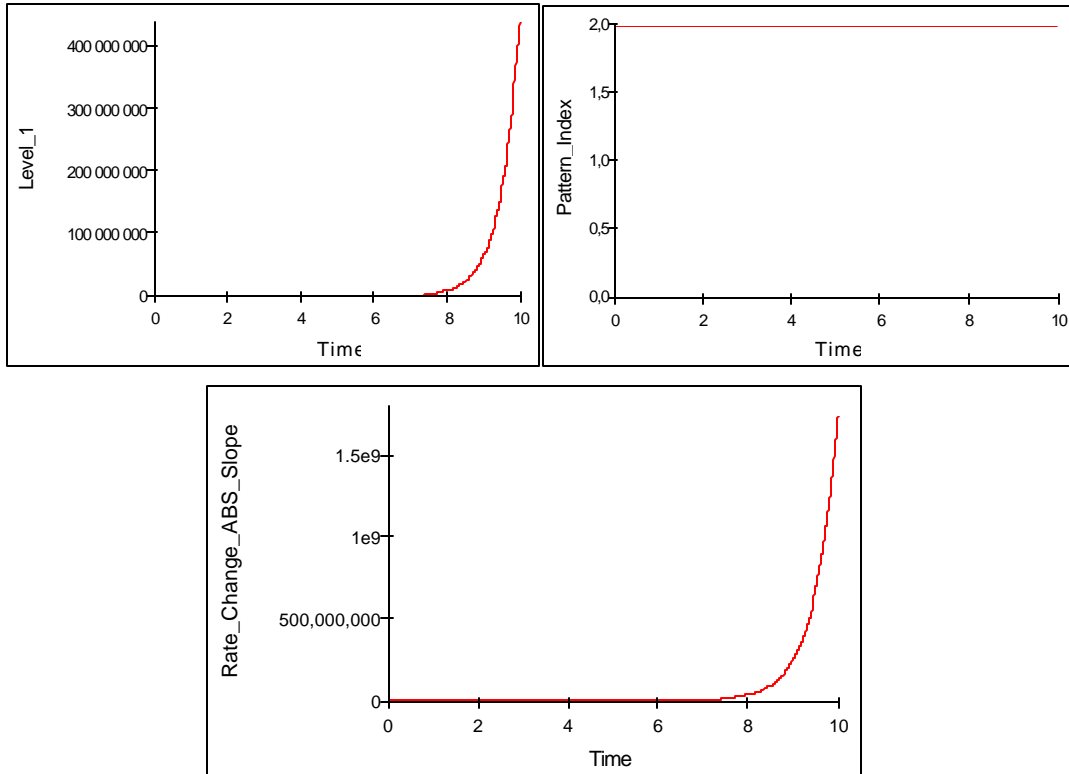
**Pattern\_Index** = Slope\_1/Curv\_1  
**ABS\_Slope** =ABS(Slope\_1)

*Note: ABS is the absolute function.*

**Rate\_Change\_ABS\_Slope** = DERIVN(ABS\_Slope)

**Table 2: Example 2, equations**

For Constant\_1 = 2, this model exhibits the following behavior with respect to the state variable (Level\_1),  $BPI_{Level_1}$ , and  $d|s|/dt$ :

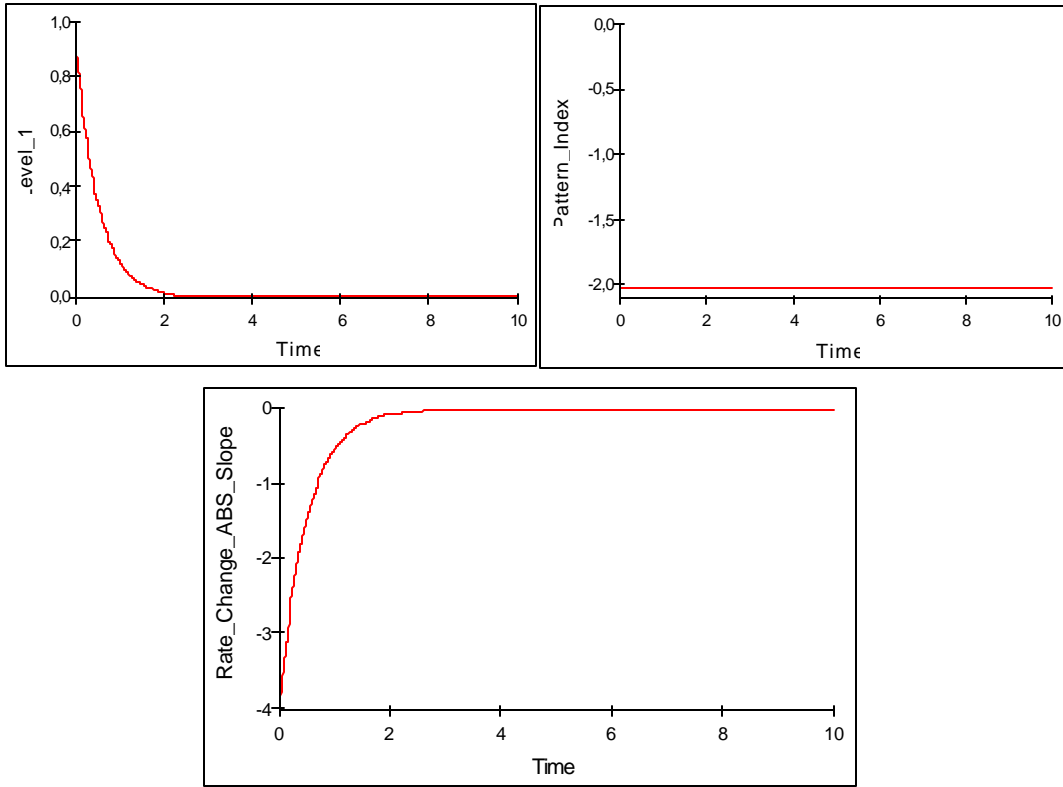


**Fig. 7: Example 2, behavior, Constant\_1= 2**

Note that the graphs for  $BPI_{Level_1}$  and  $d|s|/dt$  qualitatively provide the same information about the state variable  $Level_1$  -- that the state variable is divergent -- since both of them take positive values.  $BPI_{Level_1}$  is constant because it is a normalized expression of the divergence, i.e. of curvature,  $c$  with respect to slope,  $s$ .  $BPI_{Level_1}$  is thus a compact characterization of the mode of behavior exhibited by the state variable.

Now, if, in this model, we change the value of  $Constant_1$  from '2' to '-2', then we will obtain the following behavior with respect to the state variable ( $Level_1$ ),  $BPI_{Level_1}$ , and  $d|s|/dt$ :





**Fig. 8: Example 2, behavior, Constant\_1= -2**

Again note that the graphs for  $BPI_{Level_1}$  and  $d|s|/dt$  qualitatively provide the same information about the state variable  $Level_1$  -- as they both take negative values -- indicating that the state variable is convergent.  $BPI_{Level_1}$  is constant as it is a normalized value.

### 2.3 The curvature -contribution of a link

We first begin by defining the gain of a link in the model. The gain is defined as the change in the net rate (slope) of a state variable (say state i) in response to a change in the level (value) of another state variable in the model (say state j).

In mathematical terms the gain,  $g_j$ , is defined as  $(\frac{\partial \dot{x}_i}{\partial x_j})$

Now, as at any instant of time:

$$\dot{x}_i = F(x_1, x_2, \dots, x_n)$$

Then, by the chain rule, at any instant of time:

$$\ddot{x}_i = \frac{\partial \dot{x}_i}{\partial x_1} * \dot{x}_1 + \frac{\partial \dot{x}_i}{\partial x_2} * \dot{x}_2 + \dots + \frac{\partial \dot{x}_i}{\partial x_n} * \dot{x}_n$$

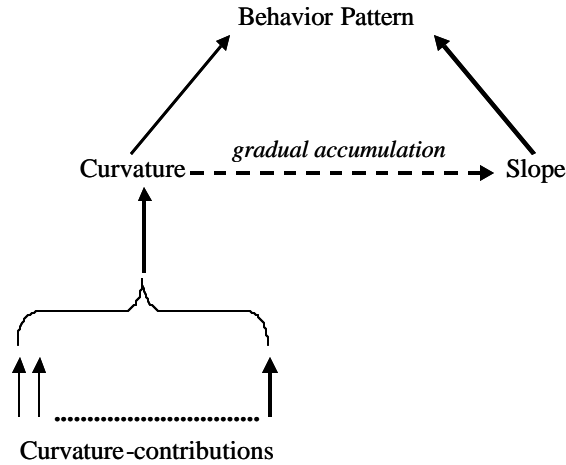
That is:

$$c_i = g_1 * s_1 + g_2 * s_2 + \dots + g_n * s_n \quad [1]$$

Equation 1 is valid for any nonlinear or linear model, as long as the model is a differentiable continuous one.

From the equation above, it is clear that, at any time, the curvature of state i is the sum of “contributions” from all state variables. Each contribution is the product of the link’s gain and the rate of change in the input state. As we have stated before, if a link has a very high gain but there is no change in the input state, then this link does not contribute at all to the curvature of the output state, and thus this link does not contribute at all to the behavior pattern of the output state. Likewise, if the input state is changing rapidly but the link has zero gain, then this link does not contribute at all to the curvature of the output state, and thus this link does not contribute at all to the behavior pattern of the output state

As we mentioned before, the behavior pattern of any state variable is determined by its slope and curvature. Moreover, the slope is the integration of the curvature; i.e. the slope results from the gradual accumulation of the curvature. This is illustrated in figure 9. As shown in that figure, the curvature contribution of a link determines the role of that link in shaping the current curvature, which in turn constitutes the rate of change of the current slope. Moreover, both the current curvature and slope determine the current behavior pattern. Thus, we can conclude that the behavior pattern “originally” comes from the curvature contributions of links. For this reason, we define, in this paper, the significance of a link with respect to the behavior pattern of a certain state variable by the contribution of this link to the curvature of the state variable. Note that as mentioned before, this is not -- by any means -- the only way to define the significance of a link with respect to behavior. It is just another empirical localized perspective on the relationship between structure and behavior.



**Fig. 9: The origin of behavior pattern**

From equation 1, one can directly compute the proportional curvature-contribution (or, in short, proportional contribution) of each link. In this paper, we will denote the proportional contribution by  $p'_{ij}$

$$p'_{ij} = 100 * (g_j * s_j) / c_i$$

Now there is a tricky part left. Recall that whenever the curvature has the same sign as the slope (i.e. positive BPI) then the state variable exhibits a divergent behavior; and when the slope and curvature have opposite signs the behavior is convergent. We will benefit from this fact and expand on it in the following way: A link is acknowledged to have a positive curvature-contribution when it pushes the curvature in the same direction as the slope (i.e. the effect of the link is to increase the absolute value of the slope), and is acknowledged to have a negative curvature-contribution when it pushes the curvature in a direction opposite to that of the slope (i.e. the effect of the link is to decrease the absolute value of the slope). To incorporate this in our computations, we introduce a slight modification for the sign of the proportional contribution to obtain the relative contribution. In this paper, we will denote the relative contribution by  $p_j$

$$p_j = \text{sign}(c_i / s_i) * p'_{ij}$$

Where the sign function returns 1 if its argument is greater than zero, 0 if its argument equals zero and -1 if its argument is less than zero.

I.e.

$$p_j = \text{sign}(BPI_i) * p'_{ij}$$

Usually in system dynamics models, the relative contributions for model links will change with time and may even change their signs. By tracking the changes in the relative contributions of model links one can develop valuable insights about the complicated and concealed dynamics of the model. Hitherto we have offered theoretical material, and maybe it is time now for practical examples.

To demonstrate the curvature-contribution concept, will use it to analyze two models. We first begin by analyzing example 1 (please revise fig 1 & table 1) which we presented in the introductory section.

### 3 Examples

#### 3.1 Back to example 1

We will study the model both in the case when it exhibits a goal-seeking behavior, and when it exhibits an exponential growth behavior.

- *Goal seeking behavior (as shown in fig. 2):*

For simplicity we will study the model at time = 0, where Level\_1 = -1; Level\_2 = 1

Thus Slope\_1 = Rate\_1 =  $g_{12} * Level_2$ ;

Where  $g_{12}$  is the gain of the link from State\_2 to State\_1.  $g_{12} = 0.1$

And Slope\_2 = Rate\_2 =  $g_{21} * Level_1$ ;

Where  $g_{21}$  is the gain of the link from State\_1 to State\_2.  $g_{21} = 0.1$

By substituting the initial values of the state variables we obtain;

Slope\_1 =  $0.1 * 1 = 0.1$ ; Slope\_2 =  $0.1 * -1 = -0.1$

By differentiating the rate equations with respect to time we obtain;

Curvature\_1 =  $g_{12} * Slope_2 = 0.1 * -0.1 = -0.01$

Curvature\_2 =  $g_{21} * Slope_1 = 0.1 * 0.1 = 0.01$

Hence we can compute the behavior pattern indexes as follows;

BPI\_1 = Curvature\_1 / Slope\_1 =  $-0.01 / 0.1 = -0.1$

BPI\_2 = Curvature\_2 / Slope\_2 =  $0.01 / -0.1 = -0.1$

As we stated before, negative behavior pattern indexes indicate convergent behavior for the two state variables.

Now we can compute the proportional contributions ( $P'_{ij}$ );

$P'_{12} = 100 * (g_{12} * Slope_2) / Curvature_1 = 100\%$

$P'_{21} = 100 * (g_{21} * Slope_1) / Curvature_2 = 100\%$

As there is only one link that influences the rate of each state variable, then the proportional contributions of links are 100%. However, we will concentrate on the signs of the relative contributions. Here is the formula and computations for the relative contributions ( $P_{ij}$ ):

$$P_{12} = \text{sign}(\text{BPI}_1) * P'_{12} = -100\%$$

$$P_{21} = \text{sign}(\text{BPI}_2) * P'_{21} = -100\%$$

Now we will trace the dynamics of the model in order to explain the reasons behind negative values for the relative contributions. We start from the link from variable State\_2 to State\_1, i.e. Link\_12. As Slope\_2 is negative and as the gain of Link\_12,  $g_{12}$ , is positive, then the Link\_12 will push Curvature\_1 in the negative direction. Yet as Slope\_1 is positive, then, in reality, Link\_12 is causing BPI\_1 to be negative (i.e. the effect of the Link\_12 is to decrease the absolute value of Slope\_1). The same line of reasoning applies for the link from State\_1 to State\_2, i.e. Link\_21. As Slope\_1 is positive and as the gain of Link\_21,  $g_{21}$ , is positive, then Link\_21 will push Curvature\_1 in the positive direction. Yet as Slope\_2 is negative, then, in reality, Link\_21 is causing BPI\_2 to be negative (i.e. the effect of the Link\_21 is to decrease the absolute value of Slope\_2).

- *Exponential growth behavior (as shown in fig. 4):*

For simplicity we will study the model at time = 0, where Level\_1 = 1; Level\_2 = 1

$$\text{Thus Slope}_1 = \text{Rate}_1 = g_{12} * \text{Level}_2;$$

$$\text{Where } g_{12} \text{ is the gain of the link from State}_2 \text{ to State}_1. g_{12} = 0.1$$

$$\text{And Slope}_2 = \text{Rate}_2 = g_{21} * \text{Level}_1;$$

$$\text{Where } g_{21} \text{ is the gain of the link from State}_1 \text{ to State}_2. g_{21} = 0.1$$

By substituting the initial values of the state variables we obtain;

$$\text{Slope}_1 = 0.1 * 1 = 0.1; \quad \text{Slope}_2 = 0.1 * 1 = 0.1$$

By differentiating the rate equations with respect to time we obtain;

$$\text{Curvature}_1 = g_{12} * \text{Slope}_2 = 0.1 * 0.1 = 0.01$$

$$\text{Curvature}_2 = g_{21} * \text{Slope}_1 = 0.1 * 0.1 = 0.01$$

Hence we can compute the behavior pattern indexes as follows;

$$\text{BPI}_1 = \text{Curvature}_1 / \text{Slope}_1 = 0.01 / 0.1 = 0.1$$

$$\text{BPI}_2 = \text{Curvature}_2 / \text{Slope}_2 = 0.01 / 0.1 = 0.1$$

As we stated before, positive behavior pattern indexes indicate divergent behavior for the two state variables.

Now we can compute the proportional contributions ( $P'_{ij}$ );

$$P'_{12} = 100 * (g_{12} * \text{Slope}_2) / \text{Curvature}_1 = 100\%$$

$$P'_{21} = 100 * (g_{21} * \text{Slope}_1) / \text{Curvature}_2 = 100\%$$

Like the goal-seeking case, as there is only one link that influences the rate of each state variable, then the proportional contributions of links are 100%. However, we will concentrate on the signs of the relative contributions. Here is the formula and computations for the relative contributions ( $P_{ij}$ ):

$$P_{12} = \text{sign}(\text{BPI}_1) * P'_{12} = 100\%$$

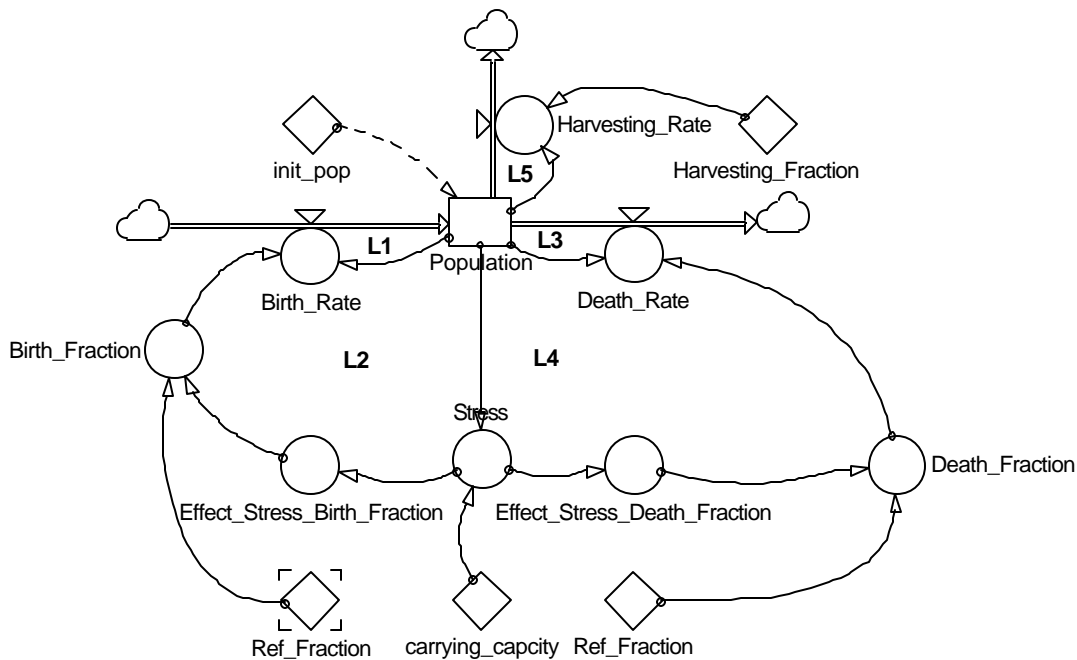
$$P_{21} = \text{sign}(\text{BPI}_2) * P'_{21} = 100\%$$

As we did before in the goal-seeking case, we will trace the dynamics of the model in order to explain the reasons behind positive values for the relative contributions. We start from the link from variable State\_2 to State\_1, i.e. Link\_12. As Slope\_2 is positive and as the gain of Link\_12,  $g_{21}$ , is positive, then the Link\_12 will push Curvature\_1 in the positive direction. Yet as Slope\_1 is positive, then, in reality, Link\_12 is causing BPI\_1 to be positive (i.e. the effect of the Link\_12 is to increase the absolute value of Slope\_1). The same line of reasoning applies for the link from State\_1 to State\_2, i.e. Link\_21. As Slope\_1 is positive and as the gain of Link\_21,  $g_{12}$ , is positive, then Link\_21 will push Curvature\_2 in the positive direction. Yet as Slope\_2 is positive, then, in reality, Link\_21 is causing BPI\_2 to be positive (i.e. the effect of the Link\_21 is to increase the absolute value of Slope\_2).

As we may have observed, this example is very simple as the relative contributions are either 100% or -100%. However, it sets the stage for analyzing more interesting models, and extending the analysis to models with a large number and a variety of links is straightforward, as we will demonstrate in the following example.

### 3.2 The population model

This model, example 3, is a typical simple population model as shown in the figure below.



**Fig. 10: Example 3, stock and flow diagram**

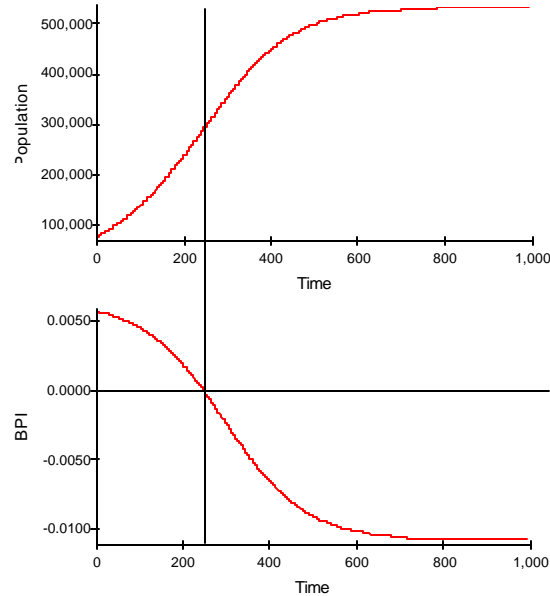
This model consists of only one level, the population level, that has one inflow, i.e. the Birth\_Rate, and two outflow, i.e. the Death\_Rate and Harvesting\_Rate. So the Net\_Rate is equal to the Birth\_Rate – Death\_Rate – Harvesting\_Rate.

Below is the table of equations for example 3.

$$\begin{aligned}
 &\text{init Level}_1 = \text{init\_pop} \\
 &\text{flow Level}_1 = \text{Birth\_Rate} - \text{Death\_Rate} - \text{Harvesting} \\
 \\ 
 &\text{Stress} = \text{Population}/\text{carrying\_capacity} \\
 &\text{Effect\_Stress\_Birth\_Fraction} = 1 - (\text{Stress} - 0.75)^{2/5} \\
 &\text{Birth\_Fraction} = \text{Ref\_Fraction} * \text{Effect\_Stress\_Birth\_Fraction} \\
 &\text{Birth\_Rate} = \text{Birth\_Fraction} * \text{Population} \\
 &\text{Effect\_Stress\_Death\_Fraction} = 0.1 + \text{Stress}^{1.4} \\
 &\text{Death\_Fraction} = \text{Ref\_Fraction} * \text{Effect\_Stress\_Death\_Fraction} \\
 &\text{Death\_Rate} = \text{Population} * \text{Death\_Fraction} \\
 &\text{Harvesting\_Rate} = \text{Population} * \text{Harvesting\_Fraction} \\
 \\ 
 &\text{Ref\_Fraction} = 0.02 \\
 &\text{init\_pop} = 75000 \\
 &\text{carrying\_capacity} = 1000000 \\
 &\text{Harvesting\_Fraction} = 0.0095
 \end{aligned}$$

**Table 3: Example 3, equations**

Below are the graphs of the behavior of the population state, and the behavior pattern index of the population. BPI is the ratio of the Curvature to the Slope. The Slope is equal to the Net\_Rate of the population, and the Curvature is equal to the time derivative of the Slope (can be computed using the DERIVN function in Powersim software).



**Fig. 11: Example 3, behavior and BPI**

It is clear from figure 11 that the model exhibits a s-shaped behavior, where, initially, it exhibits a divergent behavior (as indicated by a positive BPI), thereafter, the behavior shifts to a convergent behavior (as indicated by a negative BPI).

Returning back to the model loops, as shown in figure 9 and table 3, the model also has five loops L1...L5, where:

L1: Population? Birth\_Rate—?? Population (—?? symbolizes integration)

We will denote its gain by  $g_{b1}$ , where  
 $g_{b1} = \text{Birth\_Fraction}$  (pos. gain)

L2: Population? Stress? Eff\_Stress\_Birth? Birth\_Fraction? Birth\_Rate—??  
 Population

We will denote its gain by  $g_{b2}$ , where  
 $g_{b2} = (1/\text{carrying\_capacity}) * (-0.4 * (\text{stress} - 0.75)) * \text{Ref\_Fraction} * \text{Population}$   
 (pos. gain)

L3: Population? Death\_Rate—?? Population

We will denote its gain by  $g_{d1}$ , where  
 $g_{d1} = -\text{Death\_Fraction}$  (neg. gain)

L4: Pop.? Stress? Eff\_Stress\_Death? Death\_Fraction? Death\_Rate—?? Pop.



We will denote its gain by  $g_{d2}$ , where

$$g_{d2} = -1 * (1 / \text{carrying\_capcity}) * (1.4 * \text{stress}^{0.4}) * \text{Ref\_Fraction} * \text{Population}$$

(neg. gain)

L5: Population? Harvesting—?? Population

We will denote its gain by  $g_h$ , where

$$g_h = -\text{Harvesting\_Fraction} \text{ (neg. gain)}$$

Now, the stage is set to compute the relative contributions. The point of departure is the following formula that relates Curvature to Slope.

$$\text{Curvature} = g_{\text{total}} * \text{Slope}$$

Where:

$$g_{\text{total}} = g_{b1} + g_{b2} + g_{d1} + g_{d2} + g_h$$

Thus:

$$\text{Curvature} = (g_{b1} + g_{b2} + g_{d1} + g_{d2} + g_h) * \text{Slope}$$

Hence the relative contribution due to loop L1 (denoted it by  $P_{b1}$ ) is given as;

$$P_{b1} = \text{sign}(\text{BPI}) * 100 * (g_{b1} * \text{Slope}) / \text{Curvature}$$

The relative contribution due to loop L2 (denoted it by  $P_{b2}$ ) is given as;

$$P_{b2} = \text{sign}(\text{BPI}) * 100 * (g_{b2} * \text{Slope}) / \text{Curvature}$$

The relative contribution due to loop L3 (denoted it by  $P_{d1}$ ) is given as;

$$P_{d1} = \text{sign}(\text{BPI}) * 100 * (g_{d1} * \text{Slope}) / \text{Curvature}$$

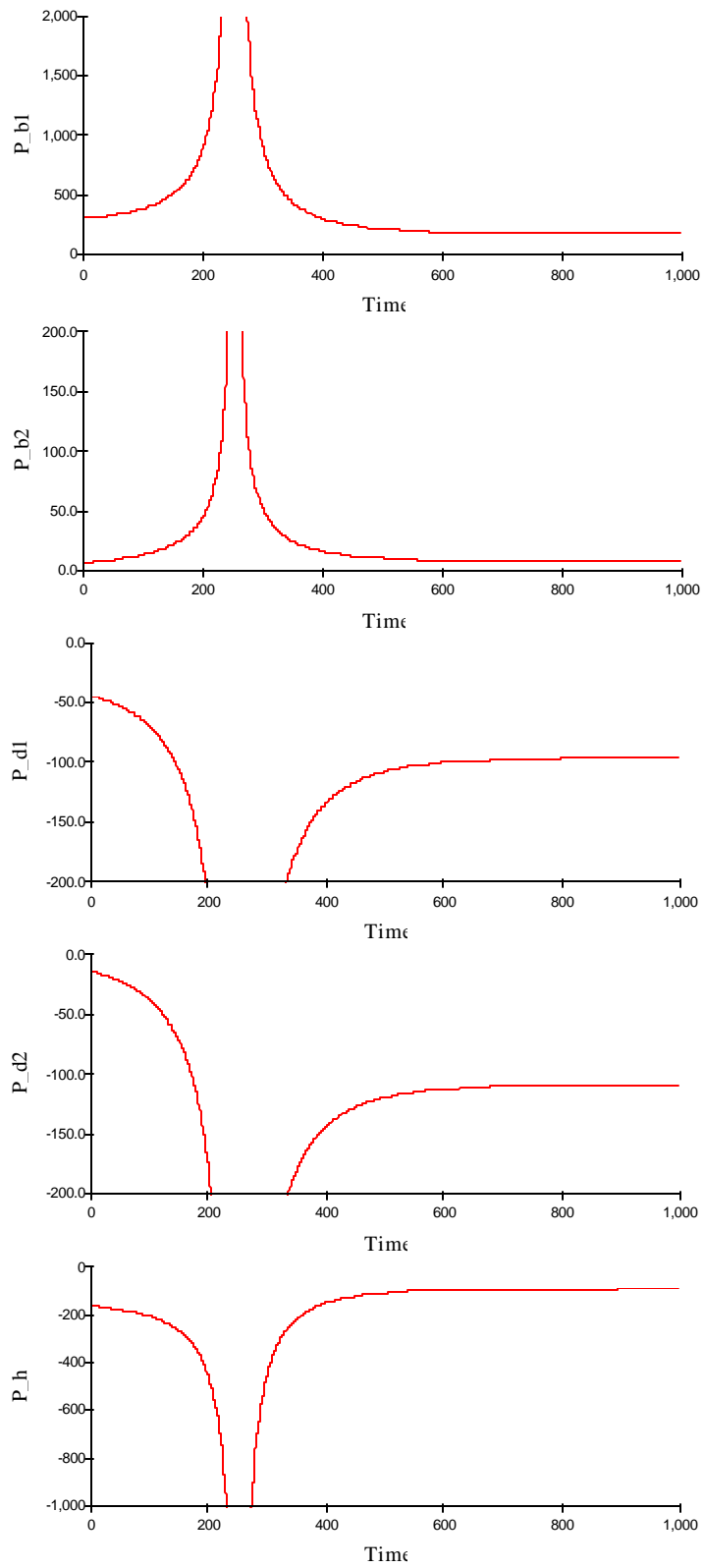
The relative contribution due to loop L4 (denoted it by  $P_{d2}$ ) is given as;

$$P_{d2} = \text{sign}(\text{BPI}) * 100 * (g_{d2} * \text{Slope}) / \text{Curvature}$$

And the relative contribution due to loop L5 (denoted it by  $P_h$ ) is given as;

$$P_h = \text{sign}(\text{BPI}) * 100 * (g_h * \text{Slope}) / \text{Curvature}$$

In figure 12, we plot of the various percentage contributions over time.



**Fig. 12: Example 3, relative contributions**

There are two basic insights from our empirical analysis, which can be observed in figure 12.

First, as a consequence of our normalization, although the values of some relative contributions at some point of time are very high (much bigger than 100), yet the sum of relative contributions is either equal to 100% or –100% at any time.

Second,  $P_{b1}$  and  $P_{b2}$  associated with loops 1 and 2, respectively, are always positive, i.e. have positive contributions that influence the population behavior divergently. Moreover, loops 1 and 2 dominate the behavior in the initial divergent phase. Note that the sum of all relative contributions, in the initial divergent phase, is 100%.

While  $P_{d1}$ ,  $P_{d2}$ , and  $P_h$  associated with loops 3, 4 and 5, respectively, are always negative, i.e. have negative contributions that influence the population behavior convergently. Moreover, loops 3, 4 and 5 dominate the behavior in the final convergent phase. Note that the sum of all relative contributions, in the final convergent phase, is -100%.

The same kind of analysis can effectively be applied to more complex and sophisticated system dynamics models.

## 4 Conclusion

The purpose of this paper has been to illustrate that link gains alone cannot explain the origin of behavior. Moreover, the goal of the paper was not only to present the challenge that most system dynamics practitioners face when they attempt to interpret behavior, but also to explain how such an analysis can be performed utilizing the curvature-contribution concept. This new concept may shed more light on the hidden and complicated dynamics that operate in the model unnoticed if one only considers the gains of the model. Furthermore, we proposed the proportional contribution as a normalized measure for the curvature-contribution, and in addition we proposed the relative contribution to incorporate the correlation relationship between the contribution and the behavior pattern index.

Our final mission, from such kind of analysis, is to tell the story of a system dynamics model based on a detailed mathematical analysis, rather than merely by intuition, which may sometimes be misleading. Here is how we propose to proceed: Partition the time horizon of the model into a set of disjoint time-intervals. For each such interval, select a time instant contained within that interval, and then construct a contribution-map that visually displays the relative contributions for all links (at that time instant). Additionally, one can identify the dominant modes of behavior (eigenvalues) at the current time; divergent, convergent, oscillatory, convergent oscillation, and divergent oscillation. In this paper, we did not dwell on the scheme to identify the dominant modes of behavior. The interested reader may refer to (Saleh & Davidsen, 2001). Finally, in order to complete the analysis, track the changes in the various contribution-maps generated at different time instants to tell the dynamic story of the model.

## References

Barlas, Y.; Kanar K. (2000): Structure-oriented Behavior Tests in Model Validation. Proceedings of the 2000 International System Dynamics Conference. Bergen.

Davidsen, P. (1991): The Structure-Behavior Graph. The System Dynamics Group, MIT. Cambridge.

Eberlein, R. (1984): Simplifying Dynamic Models by Retaining Selected Behavior Modes. Ph.D. Thesis, M.I.T., Cambridge, MA.

Forrester, N. (1982): A Dynamic Synthesis of Basic Macroeconomic Policy: Implications for Stabilization Policy Analysis. Ph.D. Thesis, M.I.T., Cambridge, MA.

Forrester, N. (1983): Eigenvalue Analysis of Dominant Feedback Loops. The 1983 International System Dynamics Conference, Plenary Session Papers, pp. 178-202.

Ford, D. (1999): A Behavioral Approach to Feedback Loop Dominance Analysis. System Dynamics Review. Volume 15, Issue 1, pp. 3-36.

Goncalves, P.; Lertpattaraong, C.; Hines, J. (2000): Implementing Formal Model Analysis. Proceedings of the 2000 International System Dynamics Conference. Bergen.

Guthrie, S. (1999): Mini-Model Presentations \_ A Tool for Teaching Dynamic Systems Thinking. Proceedings of the 1999 International System Dynamics Conference. Wellington.

Kampmann, C. (1996): Feedback Loop Gains and System Behavior. Proceedings of the 1996 International System Dynamics Conference. Boston.

Kreyszig, E. (1979): Advanced Engineering Mathematics. Fourth Edition. John Wiley & Sons.

Luenberger, D. (1979): Introduction to Dynamic Systems: Theory, Models and Applications. John Wiley & Sons.

Mojtahedzadeh, M. (1996): A Path Taken: Computer-Assisted Heuristics for Understanding Dynamic Systems. Ph.D. Thesis. Rockefeller College of Public Affairs and Policy. Albany NY.

Myrtveit, M.; Saleh, M. (2000): Superimposing Dynamic Behavior on Causal Loop Diagram of System Dynamics Models. Proceedings of the 2000 International System Dynamics Conference. Bergen.

Ogata, K. (1997): Modern Control Engineering. Third Edition. Prentice-Hall.

Press, W.; Teukolsky, S.; Vetterling, W.; Flannery, B. (1992): Numerical Recipes in C. Second Edition. Cambridge Univ. Press.

Reinschke, K. (1988): Multivariable Control: A Graph-theoretical Approach. Lecture Notes in Control and Information Sciences. Springer-Verlag.

Richardson, G. (1984): Loop polarity, loop dominance, and the concept of dominant polarity. System Dynamics Review Vol. 11; pp. 67-88.

Saleh, M.; Davidsen P. (2000): An eigenvalue approach to feedback loop dominance analysis in non-linear dynamic models. Proceedings of the 2000 International System Dynamics Conference. Bergen.

Saleh, M.; Davidsen P. (2001): The origins of business cycles. Proceedings of the 2001 International System Dynamics Conference. Atlanta.

Sterman, J. (2000): Business Dynamics: Systems Thinking and Modeling for a Complex World. McGraw-Hill.