Money Supply and Creation of Deposits
– SD Macroeconomic Modeling (1) –

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Abstract

This is the first paper of a series of macroeconomic modeling that tries to model macroeconomic dynamics such as the determination of GDP (Gross Domestic Product) and money supply from system dynamics perspective. This paper tries to model money supply and creation of deposits on a basis of the principle of accounting system dynamics developed by the author. For this purpose, a simple model based on gold standard is constructed with the introduction of high-powered money and monetary base as two different stocks, contrary to the macroeconomic tradition that treat them identically. Then three different expressions of money supply based on these two stocks are presented, and it is shown how they differ each other. The model is further expanded to a complete money supply model that includes government securities so that central bank can exercise a discretionary control over money supply through open market operations.

1 Macroeconomic System Overview

Macroeconomics is one of the core economic subjects which has been widely taught, with the use of standard textbooks, all over the world by many macroeconomists. Under such circumstances, are there still something remaining to which system dynamics can add, I posed. An affirmative answer to this question has led me to work on this series of macroeconomic modeling. For instance, macroeconomic variables such as GDP, inventory, investment, price, money supply, interest rate, etc, could be more precisely presented by using a basic concept

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of stock and flow in system dynamics. Moreover, using SD modeling methods, determination of GDP and creation process of credits and money supply - two essential ingredients of macroeconomics- could be more precisely described as dynamic macroeconomic adjustment processes, compared with a traditional static approach.

System dynamics approach requires to capture macroeconomy as a wholistic system consisting of many parts that are interacting with one another. Specifically, macroeconomic system is viewed here as consisting of six sectors such as central bank, commercial banks, consumers (households), producers (firms), government and foreign sector. Figure 1 illustrates an overview of such macroeconomic system and shows how these macroeconomic sectors interact with one another and exchange goods and services for money.

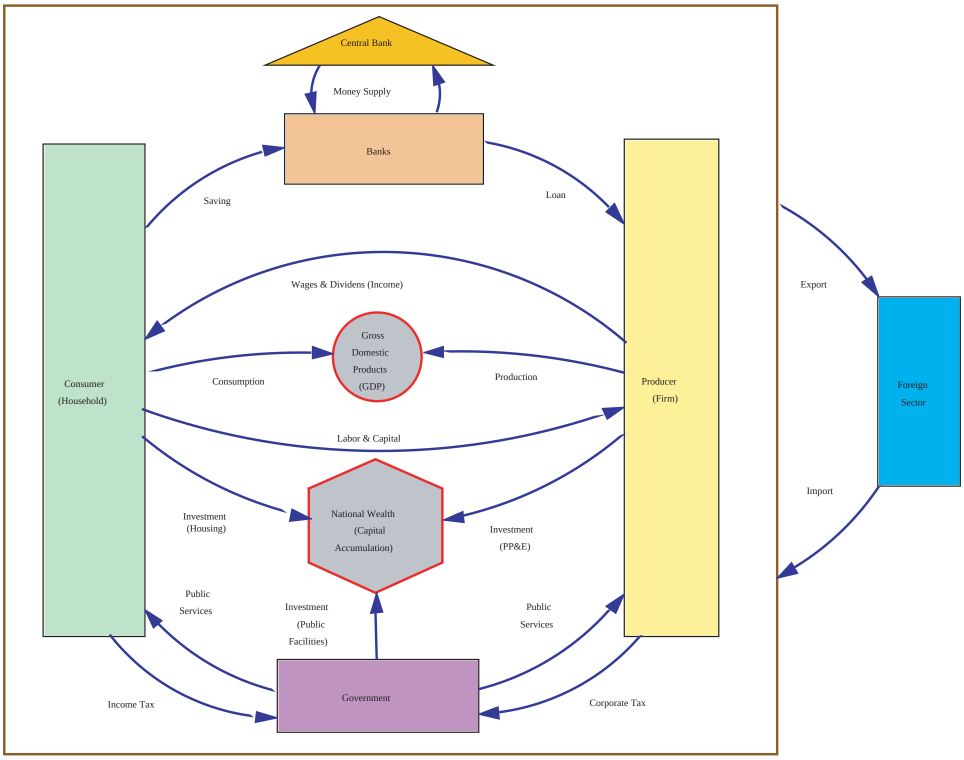

Figure 1: Macroeconomic System Overview

The objective of this research series on macroeconomics is to construct a complete system dynamics model of the above macroeconomic system. This paper is a first step towards such a complete macroeconomic system modeling, and only tries to model a portion of money supply.

2 Analytical Methods

Three Sectors of Money Supply

To model money supply and creation of deposits, six sectors in Figure 1 need to be reorganized into three sectors: central bank, commercial banks and non-financial sector (consisting of producers, consumers, and government). Foreign sector is excluded in this paper.

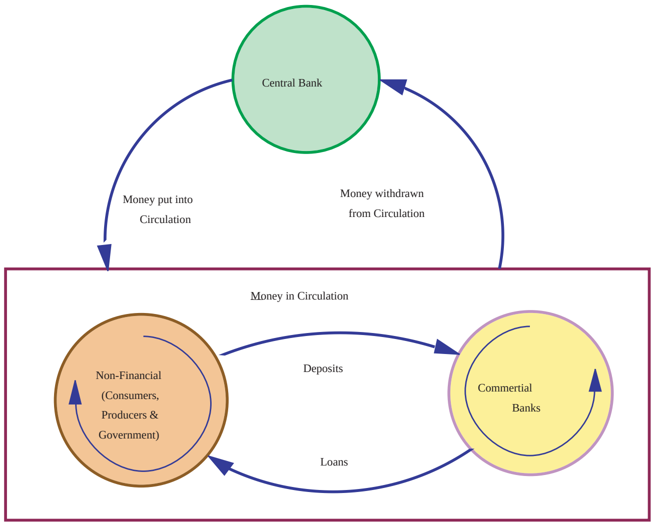

Figure 2: Three Sectors of Money Supply

Double Entry Rule

How can we describe transactions of money among three sectors? The method I employ here is the use of financial balance sheet. Balance sheet is an accounting method of keeping records of all transactions in both credit and debit sides so that they are kept in balance all the time as follows:

$$\text{Assets} = \text{Liabilities} + \text{Equity} \tag{1}$$

I have developed a modeling method of corporate balance sheet based on the following

> **Principle 5 (Double entry rule of bookkeeping).** All transactions in the accounting system are recorded as inflows and/or outflows of stocks in the balance sheet so that each transaction causes two corresponding stocks to change simultaneously in balance. For

this purpose, each transaction is booked twice on both debit and credit sides. Inflows of assets and outflows of liabilities and shareholders' equity are booked on the debit side, while outflows of assets and inflows of liabilities and shareholders' equity are booked on the credit side. [2]

In system dynamics modeling, this principle is illustrated as Figure 3.

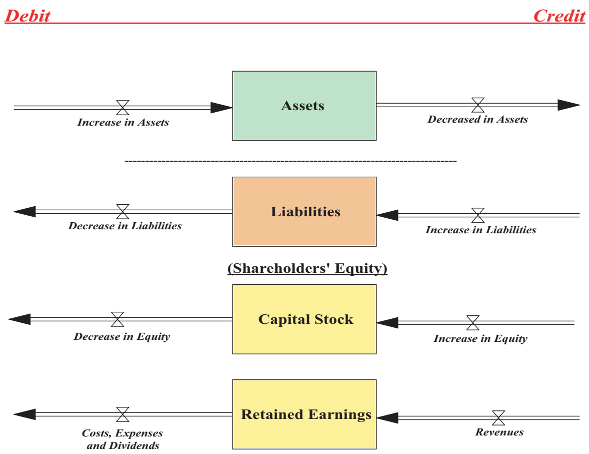

Figure 3: Double Entry Rule of Bookkeeping as Debit and Credit

Hence, all transactions of the central bank, commercial banks and non-financial sector are modeled respectively as inflows and outflows of money in their balance sheets. Moreover, macroeconomic transactions of money among three sectors not only influence their own balance sheets, but also other's balance sheets simultaneously. This makes our modeling a little bit more complicated compared with the case of corporate balance sheet in which we only need to focus on the balance of credit and debit sides of a specific company.

3 Some Definitions of Money Supply

In this section, let us review some definitions of money supply widely used in macroeconomic textbook such as Frederic S. Mishkin [1].

What is money? Any commodity could be money if it can play such roles as a means of transactions and store of values. Currency in circulation can surely play such roles. Moreover, deposits held by commercial banks can also play similar roles. Hence, money supply is defined as follows[1]:

$$\text{Money Supply} = \text{Currency in Circulation} + \text{Deposits} \tag{2}$$

How is money held and used, then, between currency in circulation and deposits? Does the different use affect money supply itself? To see this, let us define currency ratio (α) as follows:

$$\text{Currency Ratio } (\alpha) = \frac{\text{Currency in Circulation}}{\text{Deposits}} \tag{3}$$

Then, one dollar put into circulation is divided into currency in circulation and deposits according to the following proportion:

$$1 \Rightarrow \begin{cases} \frac{\alpha}{\alpha+1} & : \text{Currency in Circulation} \\ \frac{1}{\alpha+1} & : \text{Deposits wit Banks} \end{cases} \tag{4}$$

Once commercial banks receive deposits, they have to make loans out of deposits to earn interest. However, they are required by law to have an account at the central bank and keep some portion of their deposits in it in order to meet unpredictable withdrawal by depositors. Deposits of commercial banks at the central bank are called reserves, which consist of legally required portion of reserves and excess reserves. Let us define reserve ratio as follows.

$$\begin{aligned} \text{Reserve Ratio } (\beta) &= \frac{\text{Reserves}}{\text{Deposits}} \\ &= \frac{\text{Required Reserves}}{\text{Deposits}} + \frac{\text{Excess Reserves}}{\text{Deposits}} \\ &= \beta_r + \beta_e. \end{aligned} \tag{5}$$

That is, reserve ratio β becomes the sum of required reserve ratio β_r and excess reserve ratio β_e.

With these two ratios defined above, let us now consider how one dollar put into circulation is transacted. From the equation (4), $1/(\alpha+1)$ dollars are deposited first, out of which commercial banks are allowed to make maximum loans of $(1-\beta)/(\alpha+1)$ dollars. This amount will be put into circulation again. In this way, one dollar put into circulation creates additional dollars to be put into circulation. Total sum of currency created by the original dollar and put into circulation is calculated as follows.

[1] In our simple model, it may not be needed to classify monetary aggregates further into M1, M2 and M3.

Total currency put into circulation

$$
\begin{aligned}
&= 1 + \frac{1-\beta}{\alpha+1} + \left(\frac{1-\beta}{\alpha+1}\right)^2 + \left(\frac{1-\beta}{\alpha+1}\right)^3 + \cdots \\
&= \frac{1}{1-\frac{1-\beta}{\alpha+1}} \\
&= \frac{\alpha+1}{\alpha+\beta} \qquad (6)
\end{aligned}
$$

This is a process of creating deposits by commercial banks, in which one dollar put into circulation is increased by its multiple amount. It is called money multiplier (m); that is,

$$\text{Money Multiplier } (m) = \frac{\alpha+1}{\alpha+\beta} = \frac{\alpha+1}{\alpha+\beta_r+\beta_e} \qquad (7)$$

Hence, money multiplier can be easily calculated if currency ratio and required reserve ratio as well as excess reserve ratio are given in a macroeconomy. Three sectors in Figure 2 play a role of determining these ratios. Depositors in the non-financial sector (consumers & producers) determine the currency ratio: how much money to keep at hand as cash and how much to deposit. Central bank sets a level of required reserve ratio as a part of its financial policies, while commercial banks decide excess reserve ratio: how much extra reserves to hold against the need for deposit withdrawals.

In this way, an additional dollar put into circulation will eventually create its multiple amount of money supply, which are being used as currency in circulation and deposits with banks as follows:

$$1 \Rightarrow \begin{cases} \frac{\alpha}{\alpha+1}\frac{\alpha+1}{\alpha+\beta} = \frac{\alpha}{\alpha+\beta} & : \text{Currency in Circulation} \\ \frac{1}{\alpha+1}\frac{\alpha+1}{\alpha+\beta} = \frac{1}{\alpha+\beta} & : \text{Deposits wit Banks} \end{cases} \qquad (8)$$

In a real economy, then, how much real currency or cash is actually being put into circulation? It is the sum of currency in current circulation and reserves that commercial banks withhold at the central bank. This sum indeed constitutes a real currency part of money supply issued by the central bank through which creation of deposits and money supply are made as shown above. In this sense, it is called high-powered money.

$$\text{High-Powered Money} = \text{Currency in Circulation} + \text{Reserves} \qquad (9)$$

To see the amount of money supply created by high-powered money, let us calculate a ratio between money supply and high-powered money as follows:

$$
\begin{aligned}
&\frac{\text{Money Supply}}{\text{High-Powered Money}}\\
&= \frac{\text{Currency in Circulation + Deposits}}{\text{Currency in Circulation + Reserves}}\\
&= \frac{\text{Currency in Circulation/Deposits + 1}}{\text{Currency in Circulation/Deposits + Reserves/Deposits}}\\
&= \frac{\alpha + 1}{\alpha + \beta} \qquad (10)
\end{aligned}
$$

This ratio becomes exactly the same as money multiplier calculated above in equation (7). Thus, money supply can be uniformly expressed as[2]

$$
\text{Money Supply} = m * \text{High-Powered Money} \qquad (11)
$$

As a special case of this money supply, let us consider a situation in which commercial banks hold no excess reserves; that is, excess reserve ratio $\beta_e = 0$. In this case, reserve ratio becomes required reserve ratio ($\beta = \beta_r$) and money multiplier attains its highest value $\bar{m} = (\alpha + 1)/(\alpha + \beta_r) \geq m$. Accordingly, money supply also attains its highest amount. This amount of money supply is here called

$$
\text{Money Supply (No Excess)} = \bar{m} * \text{High-Powered Money}. \qquad (12)
$$

In the above definitions, currency in circulation appears both in money supply and high-powered money. However, it is hard to obtain it in a real economy and in practice it is approximated by the amount of bank notes (currency) outstanding which is recorded in the balance sheet of the central bank. Accordingly, high-powered money is also approximated by the sum of currency outstanding and reserves. It is called

$$
\text{Monetary Base} = \text{Currency Outstanding} + \text{Reserves}, \qquad (13)
$$

because this is the amount of currency that the central bank can control. And most macroeconomic textbooks treat high-powered money equivalently as monetary base. For instance, a well-established textbook says: "This is why the monetary base is also called high-powered money" [1, p. 394]. However, SD modeling below strictly requires them to be treated differently.

If high-powered money is approximated by the monetary base, money supply could also be estimated similar to the equation (11) and it is called here money

[2]When money multiplier is calculated as the equation (7) and applied to the equation (11) to obtain money supply, it turns out that money suddenly jumps from the money supply defined in (2) as the currency ratio and reserve ratio are changed during a simulation. To avoid this problem, currency and reserve ratios need be constantly recalculated during the simulation. In the money supply model below, they are done as "actual currency and reserve ratios".

supply(base).

$$\text{Money Supply (Base)} = m * \text{Monetary Base} \tag{14}$$

It could be used as a reference amount of money supply with which true money supply is compared (or to which true money supply converges, as it turns out below).

In a real economy, however, money supply is calculated from the existing data as follows:

$$\text{Money Supply (Data)} = \text{Currency Outstanding} + \text{Deposits} \tag{15}$$

It is called money supply(data) here to distinguish it from the money supplies previously defined in equations (2) (or 11) and (14).

In this way, we have now obtained three different expressions of money supply in the equations (2), (14), and (15) . It is one of the purposes of this paper to investigate how three expressions of money supply behave one another.

4 Money Supply Model under Gold Standard

To examine a dynamic process of money supply and creation of deposits, let us now construct a simple money supply model. It is built by assuming that the only currency available in our macroeconomic system is gold, or gold certificates (convertibles) issued by the central bank against the amount of gold. In short, it is constructed under gold standard. By doing so, we could avoid complicated transactions of government securities among three sectors, and focus on the essential feature of money supply. This assumption will be dropped later and government securities will be introduced to the model.

Figures 4 and 5 illustrate our simple money supply model under gold standard. For those who wish to reconstruct the model, a complete list of equations of the model is provided in the appendix.

In the model, currency outstanding in the central bank and currency in circulation in non-financial sector are illustrated as two different stocks. Thus, they need not be identically equal as most macroeconomics textbooks treat so. This is one of the features that system dynamics modeling can precisely differentiate itself from traditional macroeconomic modeling. It is interesting to observe how these two differentiated stocks of currency and three expressions of money supply derived from them will behave in the economy.

5 Simulations

High-Powered Money vs Monetary Base

Let us now run the model and see how it works. In the model, currency ratio and reserve ratios are set to be $(\alpha, \beta_r, \beta_e) = (0.2, 0.1, 0)$. Hence, money multiplier becomes $m = (0.2 + 1)/(0.2 + 0.1) = 4$. Meanwhile, from the balance sheet of

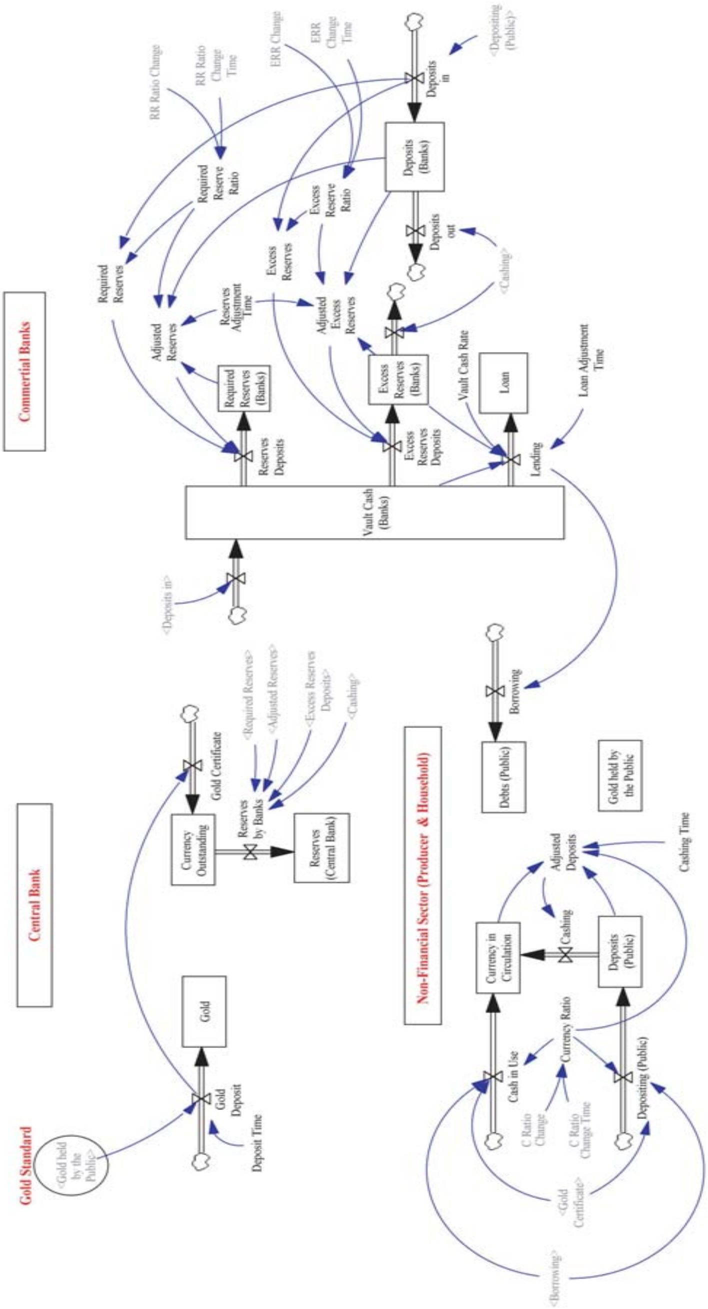

Figure 4: Money Supply Model under Gold Standard

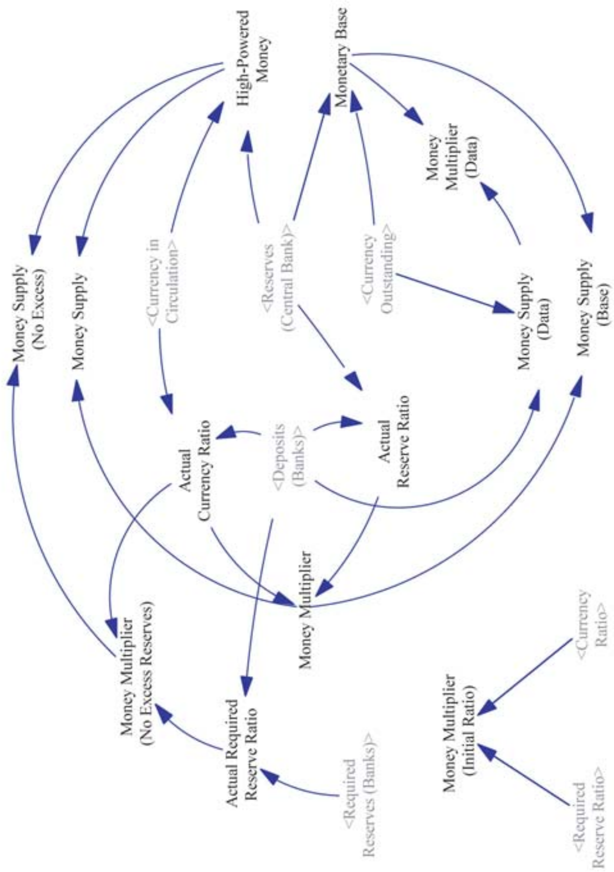

Figure 5: Money Supply Model under Gold Standard (Continued)

the central bank monetary base under gold standard is always equal to the fixed amount of gold, the only assets held by the central bank, which is here set to be equal to 200 dollars. This amount of gold is also equal to the gold held by the public. In other words, central bank is assumed to be trusted to start its business with the gold owned by the public and issue gold certificates against it.

From the equation (14), money supply(base) can be easily calculated as 800 (= 4 * 200) dollars without running the model. Meanwhile, true money supply based on high-powered money in equation (11) cannot be obtained without running a simulation.

Figure 6 illustrates our simulation result in which money supply(base), money supply, money supply(data), and money supply(no excess) are represented by

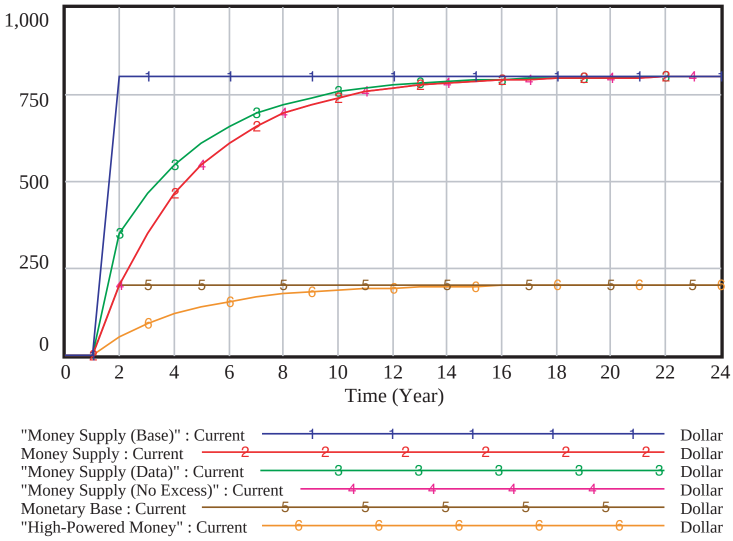

Figure 6: Simulation of Money Supply under Gold Standard

the lines numbered 1, 2, 3 and 4, respectively. Monetary base and high-powered money are illustrated by the lines 5 and 6.

From the figure, three features are easily observed. First, money supply is equal to money supply(no excess); that is, line 2 = line 4, simply because $m = \bar{m}$ for $\beta_e = 0$.

Secondly, three expressions of money supply appear to have the following orderly relation.

$$\text{Money Supply (Base)} > \text{Money Supply (Data)} > \text{Money Supply} \tag{16}$$

Latter part of the inequality implies that actual money supply(data) overestimates true money supply. Since money supply(data) is the only figure actually obtained by using real data of the currency outstanding (liabilities of the central bank) and deposits (liabilities of commercial banks), the overestimation of true money supply might mislead economic activities in the real economy.

Thirdly, monetary base turns out to be greater than high-powered money.

$$\text{Monetary Base} > \text{High-Powered Money}, \tag{17}$$

which then leads to the following inequality from the definitions in equations (11) and (14):

$$\text{Money Supply (Base)} > \text{Money Supply}. \tag{18}$$

It also leads to

$$\text{Currency Outstanding} > \text{Currency in Circulation}, \tag{19}$$

which in turn implies

$$\text{Money Supply (Data)} > \text{Money Supply.} \tag{20}$$

In other words, actual money supply(data) (line 3) calculated by the central bank always overestimates true money supply(line 2) available in the economy, which, however, tends to approach to the money supply(data) eventually. On the other hand, it is impossible a priori to lead to the inequality relation between money supply(base) and money supply(data).

To understand the above features observed from the simulation, specifically the difference between currency outstanding and currency in circulation, let us consider the amount of currency that exists outside the central bank. From the money supply model, it may be referred to be the sum of cash in circulation in the non-financial sector (consumers and producers) and cash held in the vaults of all commercial banks. All other remaining currencies in the economy would be either held by the central bank as reserves or further put into circulation through the activities of making loans by commercial banks and borrowing debts by non-financial sector. Hence, the following relation may hold:

$$\text{Cash outside Central Bank} = \text{Currency in Circulation} + \text{Vault Cash(Banks)} \tag{21}$$

If this reasoning is correct, cash outside the central bank should be equal to

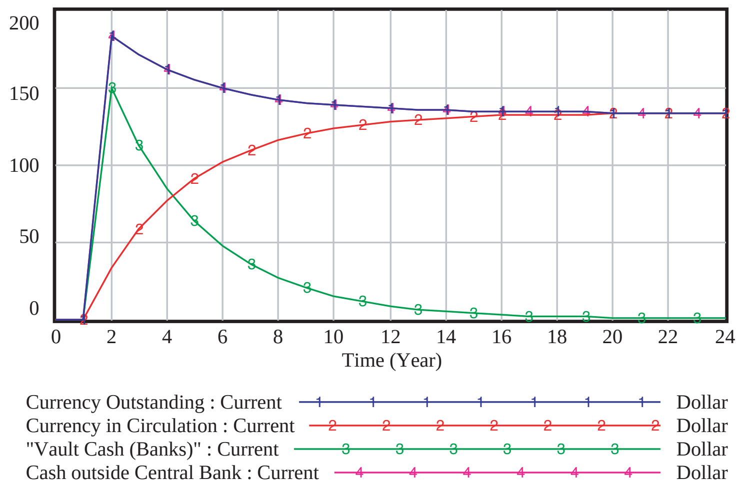

Figure 7: Simulation of Vault Cash and Cash outside Central Bank

cash outstanding in the balance sheet of the central bank; that is, the amount of cash that central bank owes to its outside world (non-financial sector and commercial banks).

Figure 7 shows that currency outstanding (line 1) is equal to the cash outside the central bank (line 4). Hence, we have correctly arrived at the equation:

$$\begin{aligned}\text{Vault Cash(Banks)} & \\ &= \text{Currency Outstanding} - \text{Currency in Circulation} \\ &= \text{Monetary Base} - \text{High-Powered Money}, \qquad (22)\end{aligned}$$

which in turn leads to the above inequality relations of equation (17) through (20) as long as vault cash is positive. In addition, all three expressions of money supply are shown to converge as long as vault cash tends to diminish, and overestimation of money supply will be eventually corrected.

Loan Adjustment Time

There is a case in which such a convergence becomes very slow and overestimation of money supply remains. In Figure 8 loan adjustment time is assumed to

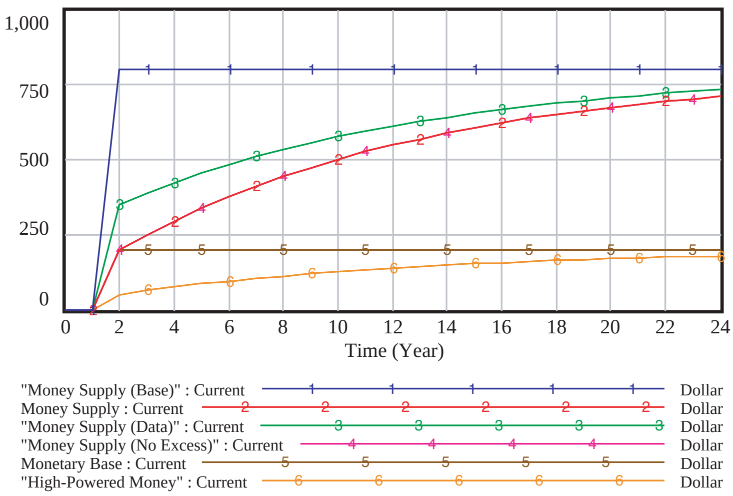

Figure 8: Money Supply when Loan Adjustment Time triples.

triple and become 3 periods. This is a situation in which a speed of bank loans becomes slower, or commercial banks become reluctant to make loans. Accordingly, money supply might converge to money supply (data), but extremely slow. In other words, money supply will not converge to the money supply(data) for a foreseeable future and overestimation of money supply remains. Specifically, money supply(data) (line 3) is always greater than money supply (line 2) during the simulation of 24 periods.

Excess Reserves

How can the amount of money supply be changed or controlled by the central bank? Under gold standard, monetary base is always fixed, and central bank can only influence money supply by changing a required reserve ratio. Even so, money supply may not be under the control of the central bank in a real economy. It could be affected by the following two situations. First, commercial banks may be forced to hold excess reserves in addition to the required reserves due to a reduced opportunity of making loans. Second, depositors in the non-financial sector may prefer to hold cash or liquidity due to a reduced attractiveness of financial market caused by lower interest rates. Money supply will be reduced under these situations.

Let us consider the situation of excess reserves first. To see how excess reserves affect money supply, let us increase the excess reserve ratio by 4%; that is, $\beta_e = 0.04$. Then, money multiplier is calculated as $m = (0.2 + 1)/(0.2 + 0.1 + 0.04) = 3.529$, and money supply(base) becomes 705.8 dollars (= 3.529 * 200). As Figure 9 illustrates, money supply (line 2) is lowered, due to the increase in excess reserves, from the original money supply(no excess) level of 800 dollars, and it approaches to 705.55 at t = 24. In this case, however, three expressions of money supply converges to the lowered level of money supply, and overestimation of money supply will be eventually corrected.

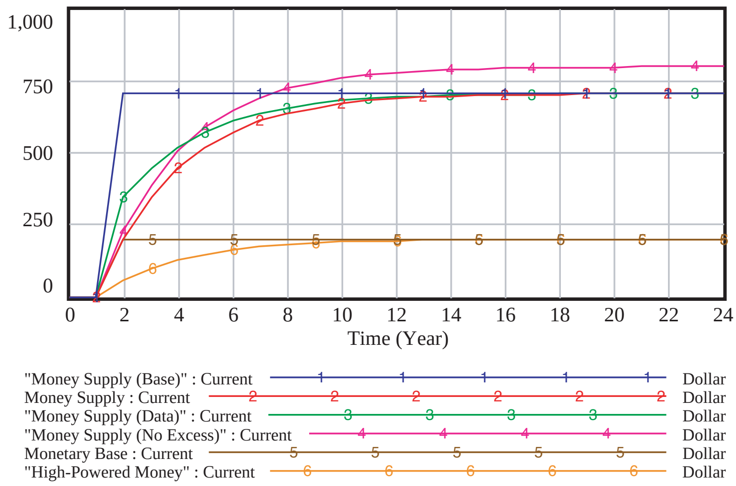

Figure 9: Money Supply when Excess Reserve Ratio is 0.04.

Vault Cash

Excess reserves may be withheld due to an imminent need for liquidity as mentioned above. To meet such demand for cash, commercial banks may additionally need to keep some portion of excess reserves as vault cash in their vaults. Let us consider how such behaviors further affect money supply by assuming that 80 % of excess reserves is withheld as vault cash; that is, vault cash rate = 0.8.

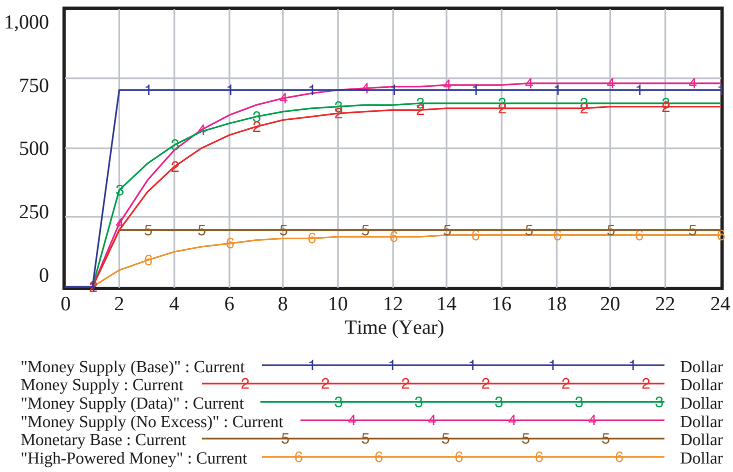

Figure 10: Money Supply when Vault Cash is held by Commercial Banks.

As Figure 10 illustrates, three expressions of money supply never converge each other and money supply continues to be overestimated. Specifically, at the period t = 24, money supply(data) (= 662.27) becomes larger than true money supply (= 645.03) by the amount of vault cash (=17.24).

Currency Ratio

Let us now consider the second situation in which non-financial sector prefers to hold more liquidity. To analyze its effect on money supply, let us assume that at t = 6 consumers suddenly wish to withhold cash by doubling currency rate from 0.2 to 0.4. Money multiplier is now calculated as $m = (0.4+1)/(0.4+0.1) = 2.8$ and money supply(base) becomes 560 dollars (= 2.8 * 200).

Figure 11 illustrates how money supply is reduced due to a sudden increase in liquidity preference in the non-financial sector. Three expressions of money supply, however, tend to converge this time since no excess reserves are withheld and accordingly no vault cash remains at the commercial banks.

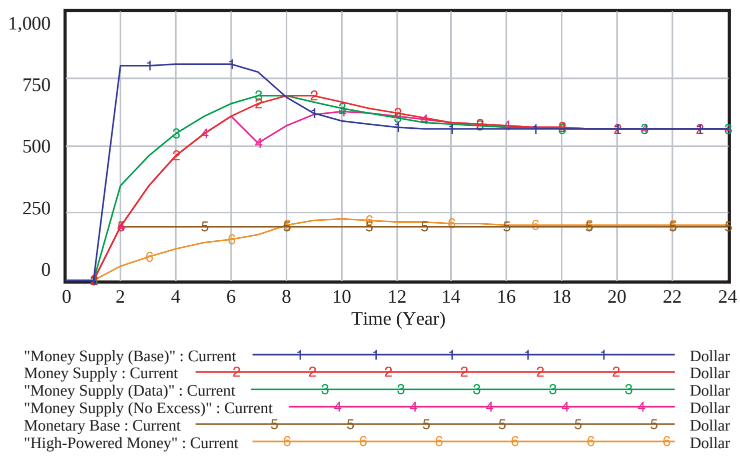

Figure 11: Money Supply when Currency Ratio doubles at t = 6.

6 A Complete Money Supply Model

Limit to Gold Standard System

As already mentioned above, monetary base is fixed under gold standard. How can we increase money supply under such circumstances to meet the need for increasing transactions as our economy continues to grow? Let us ask differently. What's the maximum amount of money supply the gold standard system can provide?

From equation 14, under the fixed amount of monetary base, only money multiplier can change the money supply(base), and money supply accordingly. Since currency ratio is not under the direct control of the central bank, the only discretionary policy the central bank can exercise is a change in required reserve ratio, as already shown above. Hence, money multiplier could be maximized if required reserve ratio is set to be zero (!), and commercial banks are allowed to fully make loans of all deposits. In this case, money multiplier becomes $m = (0.2 + 1)/0.2 = 6$ and the maximum money supply(base) increases from 800 to 1,200.

If our growing macroeconomy needs more money supply, gold standard system has to be eventually abandoned as a monetary system. This is what has historically happened. It was replaced with a paper money (legal tender) system in which central bank has a free hand of issuing bank notes mainly against government securities.

Non-Financial Sector

Let us now expand our money supply model under gold standard to a complete money supply model in which government securities are introduced and central bank can hold them as assets and issue its bank notes against them as liabilities. Under the introduction of government securities, our expanded money supply model becomes pretty complicated. To avoid further complication the model is now split into three parts based on three macroeconomic sectors.

Let us consider the balance sheet of non-financial sector first. Due to the introduction of government securities, this sector is subdivided further between non-financial public sector consisting of consumers and producers, and government sector.

Figure 12 illustrates the non-financial public sector, in which stock of government securities held by the public is newly added to the assets side of its balance sheet and stock of equity is added to its liabilities side.

On the other hand, Figure 13 illustrates a newly added government sector, in which stock of cash held by the government is added to the assets side of its balance sheet, and stocks of debts by the government and its equity are added to its liabilities side.

Apparently money supply is not affected by the introduction of government securities as long as they are purchased by consumers and producers, and government spends the money it borrowed within the non-financial sector.

Commercial Banks

Stock of government securities held by commercial banks has to be newly added to the assets side of the balance sheet of commercial banks. Now commercial banks have a portfolio choice of investment between loans and investment on government securities. Figure 14 illustrates a portion of commercial banks.

Money supply is not affected by the introduction of government securities as long as they are purchased by commercial banks and government spends the money it borrowed within the non-financial sector.

Central Bank

Stock of government securities held by the central bank has to be newly added to the assets side of the balance sheet of the central bank. It can now purchase government securities with its newly issued bank notes[3], which increases the same amount of its liabilities as currency outstanding, and accordingly monetary base. If it sells government securities to commercial banks or non-financial sector, currency in circulation is withdrawn back to the central bank, which decreases its currency outstanding, and hence monetary base. Purchase and sale of government securities by the central bank are known as open market operations.

[3] Direct purchase of government securities, or direct loan to government by the central bank is prohibited in Japan. Therefore, such purchases has to be indirectly done through market.

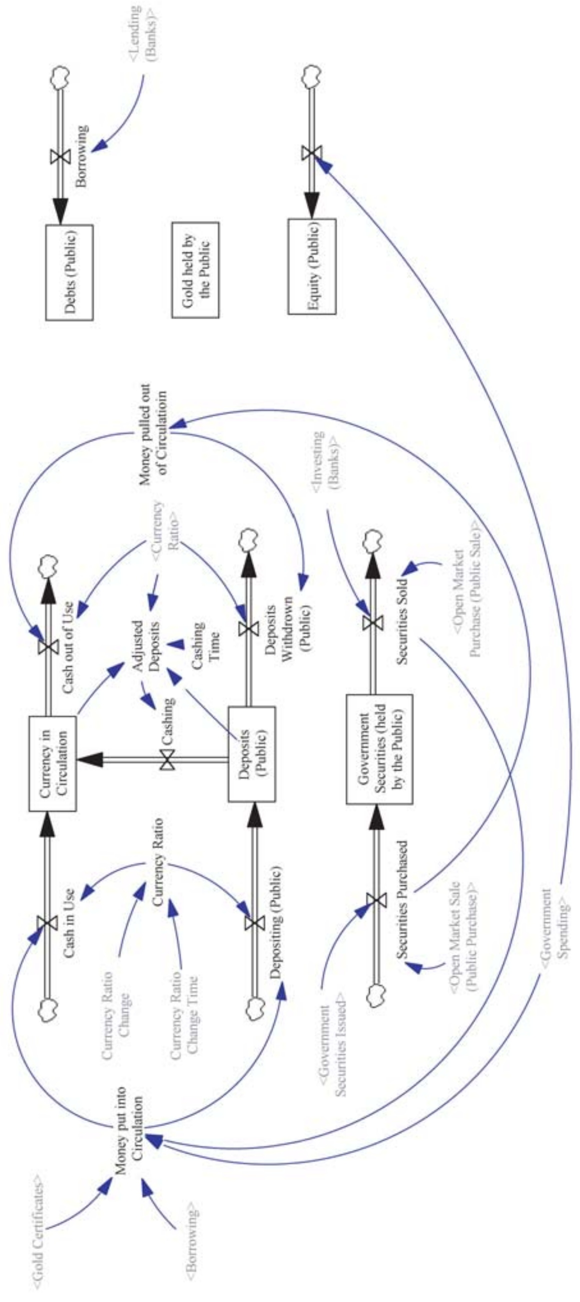

Figure 12: Money Supply Model of Non-Financial Public Sector

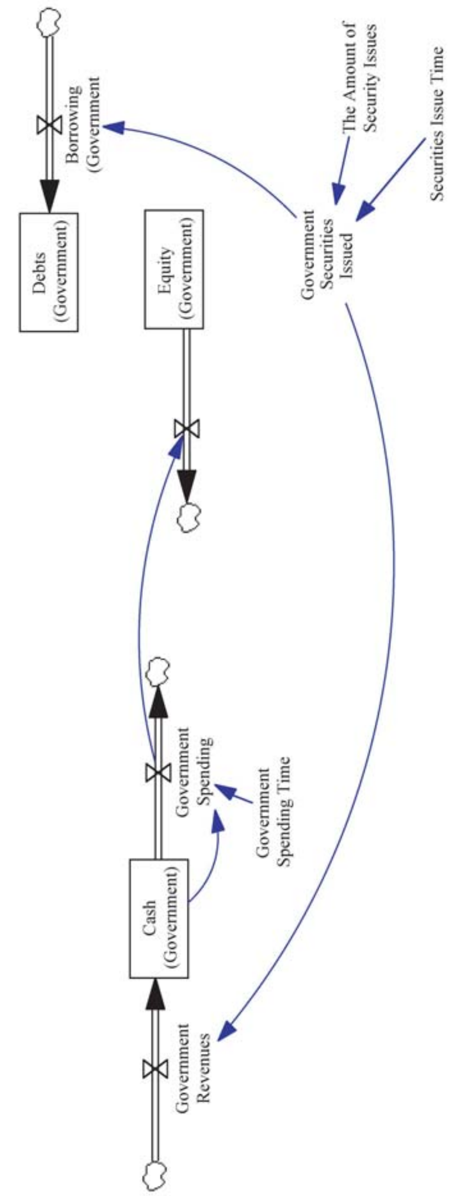

Figure 13: Money Supply Model of Non-Financial Sector (Government)

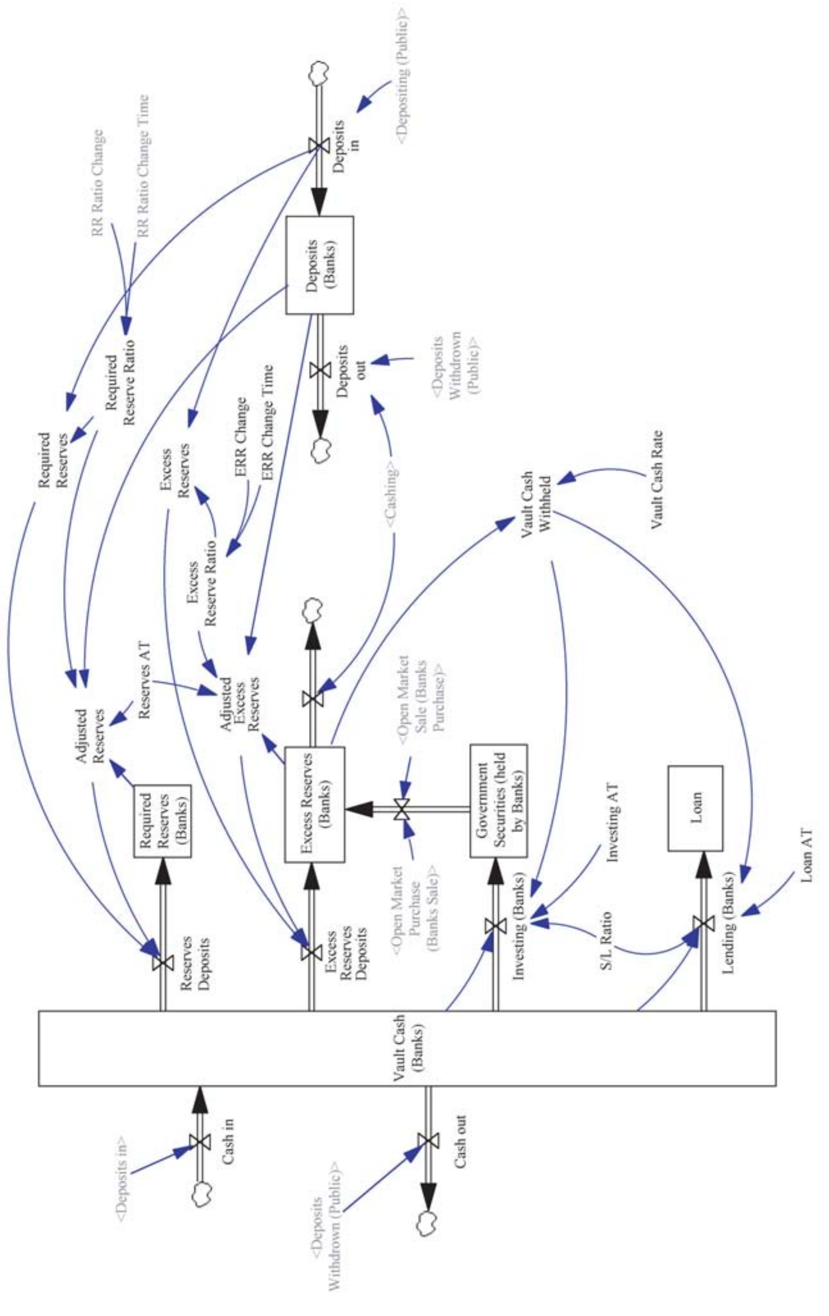

Figure 14: Money Supply Model of Commercial Banks

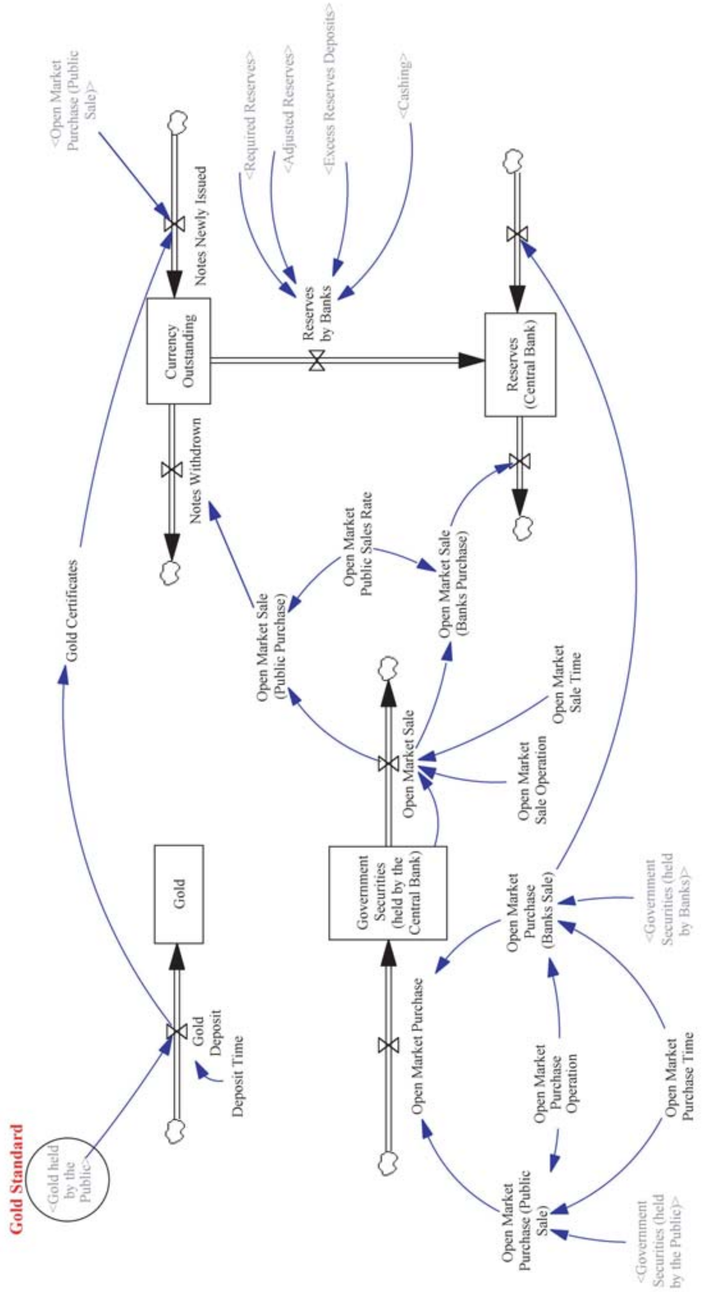

Figure 15: Money Supply Model of Central Bank

In this way, with the introduction of government securities, the central bank has a discretionary control of monetary base and money supply. Figure 15 illustrates a portion of the central bank.

7 Open Market Operations

Let us now see how our complete money supply model works. Figure 16 illustrates how open market operations affect the behavior of money supply. It is assumed that government issues securities (and makes loans) of 100 dollars at the period t = 3, which are purchased by the public (consumers and producers) and commercial banks. At the period t = 6 central bank purchases 50% of government securities held by the public and commercial banks through open market purchase operation. Accordingly, monetary base (line 5) is now increased from the original 200 dollars to 250 dollars. Since money multiplier is obtained as $m = (0.2 + 1)/(0.2 + 0.1) = 4$, money supply(base) (line 1) now starts to increase to 1,000 dollars (= 4 * 250), while money supply(data) and money supply also continue to grow.

At the period t = 10 the central bank sells 50% of the government securities it holds, and monetary base decreases to 225 dollars. Money supply(base) (line 1) is now reduced to converge to 900 dollars (= 4 * 225). Money supply(data) (line 3) slightly reduces at that period, but then continues to grow, while money supply (line 2) only continues to grow. Eventually three expressions of money supply converge to 900 dollars.

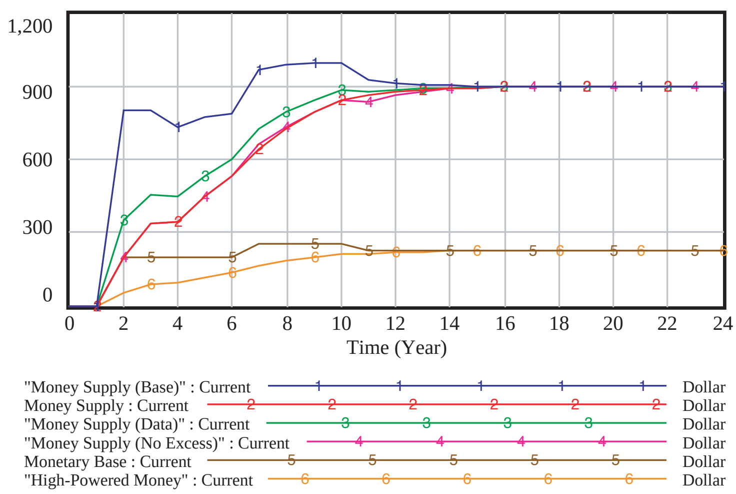

Figure 16: Simulation of Open Market Operation

In this way, the central bank can increase money supply by its discretionary monetary policy of open market operations, and theoretically there exists no ceiling or upper boundary of money supply.

Even so, there is a case in which the central bank cannot control money supply. Figure 17 illustrates the case in which a currency ratio is additionally doubled from 0.2 to 0.4 at t = 12. Money multiplier is now calculated as $m = (0.4 + 1)/(0.4 + 0.1) = 2.8$, and money supply(base) becomes 630 dollars (= 2.8 * 225); a reduction of money supply by 270 dollars. Three expressions of money supply all converge to this reduced amount.

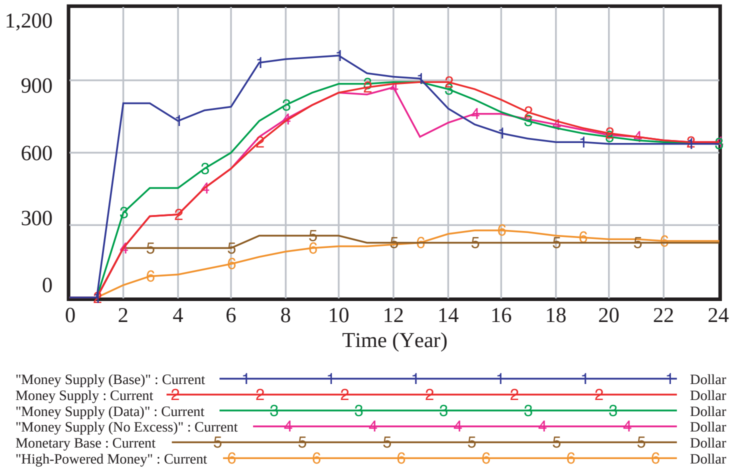

Figure 17: Simulation of Open Market Operation with Currency Ratio doubled

Conclusion

From the overview of macroeconomic system, six sectors are rearranged to three sectors: central bank, commercial banks and non-financial sector. Then a simple model of money supply under gold standard is constructed to examine some essential features of money supply and creation of deposits. This modeling process inevitably requires the distinction between currency in circulation and currency outstanding, and accordingly high-powered money and monetary base that are traditionally treated equivalently in macroeconomics.

Three expressions of money supply are derived from the distinction, and it is shown that money supply(data) obtained from actual economic data tends to overestimate true money supply based on high-powered money. It is also shown that three expressions of money supply tend to converge as long as vault cash held by commercial banks diminishes.

The model is then expanded to include government securities so that the central bank can exercise a discretionary control over money supply through its open market operations. Under the complete model of money supply, the above results are shown to hold similarly.

This paper has only presented a basic framework of money supply, and its relations with demand for money, interest rate and a real part of macroeconomy, as well as the actual analysis of money supply based on real monetary data, are left for later research.

References

[1] Frederic S. Mishkin. *The Economics of Money, Banking, and Financial Markets*. Addison Wesley, New York, 6th edition, 2000.

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Appendix

List of Equations for the Money Supply Model under Gold Standard

```
(01) Actual Currency Ratio=XIDZ(Currency in Circulation,
        "Deposits (Banks)" , 0 )
        Units: Dmnl
(02) Actual Required Reserve Ratio=XIDZ("Required Reserves (Banks)",
        "Deposits (Banks)" , 0 )
        Units: Dmnl
(03) Actual Reserve Ratio=XIDZ("Reserves (Central Bank)",
        "Deposits (Banks)" , 0 )
        Units: Dmnl
(04) Adjusted Deposits=(Currency Ratio*"Deposits (Public)"-
        Currency in Circulation)/Cashing Time
        Units: Dollar/Year
(05) Adjusted Excess Reserves=("Deposits (Banks)" *
        Excess Reserve Ratio - "Excess Reserves (Banks)") /
        Reserves Adjustment Time
       Units: Dollar/Year
(06) Adjusted Reserves=(Required Reserve Ratio*"Deposits (Banks)" -
        "Required Reserves (Banks)")/Reserves Adjustment Time
        Units: Dollar/Year
(07) "B/S Check: Central Bank"=Gold-Currency Outstanding-
        "Reserves (Central Bank)"
        Units: Dollar
(08) "B/S Check: Commercial Bank"="Vault Cash (Banks)"+
        "Required Reserves (Banks)" + "Excess Reserves (Banks)" +
        Loan-"Deposits (Banks)"
         Units: Dollar
(09) "B/S Check: Non-Financial"=Currency in Circulation+
        "Deposits (Public)"-"Debts (Public)"-Gold held by the Public
          Units: Dollar
(10) Borrowing= Lending
        Units: Dollar/Year
(11) C Ratio Change=0
        Units: Dmnl [-0.2,0.8,0.01]
(12) C Ratio Change Time=0
        Units: Dmnl
(13) Cash in Use=(Gold Certificate+Borrowing)* (Currency Ratio/
        (1+Currency Ratio))
        Units: Dollar/Year
(14) "Cash outside Central Bank"=Currency in Circulation +
        "Vault Cash (Banks)"
        Units: Dollar
```

```
(15) Cashing=Adjusted Deposits
        Units: Dollar/Year
(16) Cashing Time=2
        Units: Year
(17) Currency in Circulation= INTEG (Cash in Use+Cashing,0)
        Units: Dollar
(18) Currency Outstanding= INTEG (Gold Certificate -
        Reserves by Banks,0)
        Units: Dollar
(19) Currency Ratio=0.2 + STEP(C Ratio Change, C Ratio Change Time )
        Units: Dmnl [0,1,0.005]
(20) "Debts (Public)"= INTEG (Borrowing,0)
        Units: Dollar
(21) Deposit Time=1
        Units: Year
(22) "Depositing (Public)"=(Gold Certificate+Borrowing)/
       (1+Currency Ratio)
        Units: Dollar/Year
(23) "Deposits (Banks)"= INTEG (Deposits in - Deposits out,0)
        Units: Dollar
(24) "Deposits (Public)"= INTEG ("Depositing (Public)" - Cashing,0)
        Units: Dollar
(25) Deposits in= "Depositing (Public)"
        Units: Dollar/Year
(26) Deposits out=Cashing
        Units: Dollar/Year
(27) ERR Change=0
        Units: Dmnl [0,0.1,0.01]
(28) ERR Change Time=0
        Units: Year [0,24,1]
(29) Excess Reserve Ratio=STEP(ERR Change, ERR Change Time )
        Units: Dmnl [0,0.5,0.01]
(30) Excess Reserves=Deposits in*Excess Reserve Ratio
        Units: Dollar/Year
(31) "Excess Reserves (Banks)"= INTEG (Excess Reserves Deposits -
         Cashing,0)
        Units: Dollar
(32) Excess Reserves Deposits=Excess Reserves +
        Adjusted Excess Reserves
        Units: Dollar/Year
(33) FINAL TIME  = 24
        Units: Year
(34) Gold= INTEG (Gold Deposit,0)
        Units: Dollar
(35) Gold Certificate=Gold Deposit
        Units: Dollar/Year
```

```
(36) Gold Deposit=PULSE(Deposit Time, 1) * Gold held by the Public
        Units: Dollar/Year
(37) Gold held by the Public=200
        Units: Dollar [0,500,100]
(38) "High-Powered Money"=Currency in Circulation+
        "Reserves (Central Bank)"
        Units: Dollar
(39) INITIAL TIME  = 0
        Units: Year
(40) Lending=("Vault Cash (Banks)" - "Excess Reserves (Banks)"*
        Vault Cash Rate)/Loan Adjustment Time
        Units: Dollar/Year
(41) Loan= INTEG (Lending,0)
        Units: Dollar
(42) Loan Adjustment Time=1
        Units: Year
(43) Monetary Base=Currency Outstanding+"Reserves (Central Bank)"
        Units: Dollar
(44) Money Multiplier=XIDZ( Actual Currency Ratio + 1,
        Actual Currency Ratio + Actual Reserve Ratio,0)
        Units: Dmnl
(45) "Money Multiplier (Data)"=XIDZ("Money Supply (Data)",
        Monetary Base , 0)
        Units: Dmnl
(46) "Money Multiplier (Initial Ratio)"=(Currency Ratio+1)/
       (Currency Ratio+Required Reserve Ratio)
        Units: Dmnl
(47) "Money Multiplier (No Excess Reserves)"=XIDZ(
         Actual Currency Ratio +1, Actual Currency Ratio +
         Actual Required Reserve Ratio,0)
        Units: Dmnl
(48) Money Supply=Money Multiplier * "High-Powered Money"
        Units: Dollar
(49) "Money Supply (Base)"= Money Multiplier * Monetary Base
        Units: Dollar
(50) "Money Supply (Data)"=Currency Outstanding+"Deposits (Banks)"
        Units: Dollar
(51) "Money Supply (No Excess)"="Money Multiplier (No Excess Reserves)"*
         "High-Powered Money"
        Units: Dollar
(52) Required Reserve Ratio=0.1 + STEP(RR Ratio Change,
        RR Ratio Change Time )
        Units: Dmnl [0,0.3,0.01]
(53) Required Reserves=Deposits in * Required Reserve Ratio
        Units: Dollar/Year
(54) "Required Reserves (Banks)"= INTEG (Reserves Deposits,0)
```

```
        Units: Dollar
(55) "Reserves (Central Bank)"= INTEG (Reserves by Banks,0)
        Units: Dollar
(56) Reserves Adjustment Time=2
        Units: Year
(57) Reserves by Banks=Required Reserves + Adjusted Reserves +
        Excess Reserves Deposits - Cashing
        Units: Dollar/Year
(58) Reserves Deposits=Required Reserves + Adjusted Reserves
        Units: Dollar/Year
(59) RR Ratio Change=0
        Units: Dmnl
(60) RR Ratio Change Time=0
        Units: Dmnl
(61) SAVEPER  = TIME STEP
        Units: Year
(62) TIME STEP  = 1
        Units: Year
(63) "Vault Cash (Banks)"= INTEG ( Deposits in - Reserves Deposits -
        Excess Reserves Deposits - Lending ,0)
        Units: Dollar
(64) Vault Cash Rate=0
        Units: Dmnl [0,1,0.05]
```