MURCIA A/I, A MIXED SYSTEM DYNAMICS AND LINEAR PROGRAMMING MODEL FOR REGIONAL INVESTMENT PLANNING

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Abstract. When distribution of economic goods, equipments, ... ta kes place among different regions, it is expected to carry out 7 in an optimal way, considering "optimal" the way of distribution that assigns more to the neediest regions; thus, numerous fac- / tors such as economic conditions, actual equipments, social contors such as economic conditions, actual equipments, social con-
ditions, population, etc, should be taken into account. The presented model has a double aim: firstly to show the pre- /
sent behaviour of distribution system of investment in Comunidad Autonoma de Murcia, regionally, and secondly to get this distribution to optimize a linear function that represents the regio~ nal social welfare as a consequence of the social welfare in / each region of the Community and dependent on linear constraints To reach these objectives, a mixed model that combines both System Dynamics and Linear Programming techniques is constructed ;
a relation between both procedures is established in order to si
mulate both the natural behaviour of the distribution system and those decisions 'that make this distribution to be optimal.

Along this report the method carried out to handle mixed models/ as well as the particular model MURCIA A/I are described.

INTRODUCTION

In last years a centrifugal process is carrying in the structure of the Administration of Spain and the Regional governments ta- king actions to know the changing conditions and to improve the dynamics of the economical and social activities.

This paper takes part of an initial study of the Comunidad Autonoma de Murcia to establish an optimal way to assign the investment. The scope of the work is centered in the use of the System Dynamics methodology combined with the Linear Programming procedures in order to produce a simulation of the behaviour of a dis tribution system, in which the gap of the several factors determining the "necessity" is minimized.

In traditional dynamics systems simulation, various types of variables are handled in order to understand the behaviour of the systems by means of the behaviour of the variables along the ho rizon time. This behaviour is usually depicted trough mathematical functions which depend on either other system variables, pre vious values of the same variable or constants. -

As it is known, to build a system dynamics model it is necessary to find the mathematical relationships or functions able to describe the real behaviour of the system.

In this way, the "natural" behaviour can be modelled. However, I sometimes, when we are working with some specific kind of sys- $/$ tems, simulation or modelling of the natural behaviour of the / system may be not enough, because besides to the natural it may be interesting, or necessary, to simulate the decision-making of the managers, based on environmental factors, state of the sys- tem or policy constraints, in each timestep.

Several examples of this kind of systems have been already studied by the authors; those belong to areas so different as:

Farming policy, Toval,A. (1985), Labor-Market, or which we descri be below, about regional investment planning, MURCIA A/I.

Section II includes a formulation of System Dynamics and Linear/ Programming mixed models and section III relates the Assignement Investment Murcia model including numerical results for various/ hypothesis.

SYSTEM DYNAMICS AND LINEAR PROGRAMMING (SD-LP) MODELS

The aim of SD-LP models is to incorporate the simulation of deci sion-making, when this is carried out trough one or several $1i$ near programs, into the traditional system dynamics techniques . Thus, it is possible to obtain the optimal values of the system/ variables which participate as an objective function or as activities in some previously defined linear program, so making po- ssible the slmulation of decision-making at each time of the run

Althought the method is shown to use only linear programming, there is no problem to apply it to non-linear programming pro- / blems using the same SD-LP algorithm (fig.1) and the same interfaces to the sistem dynamics equations with minor changes.

To simplify, let us assume that we have defined an unique linear program together with the system dynamics equations, althought /
we could consider as many as we wish, with the only limitation / of the memory computer size.

The objective consists in simulating the optimal behaviour of a variable and, in consequence, the corresponding values for those variables which participate as activites in the definition of \sqrt{I} that variable.

With this purpose, let us fistly consider the usual linear pro- gramming form:

> (optimize) $z = c^r x$ subject to: A1 $x \le b1$ $A2 x = b2$ A3 $x \ge b3$ (1)

using matrix form.

We call SD-LP model to the system dynamics one embodying one or / several linear programs which have the following characteristics: the coefficients fathe vectors c, band matrix A, are not obli- ged to be constant, as it is usual in linear programming problems but they can be time-depending functions defined in relation to / the elements of the system to be modelled. Thus the usual SD-LP $/$ model is posed as:

> $(\text{optimize}) \quad z(t) = c^*(t) \quad x(t)$ subject to $A1(t)$ $x(t) \leq b1(t)$ A1(t) $x(t) \leq b1(t)$
A2(t) $x(t) = b2(t)$ $A3(t)$ $x(t) \geq b3(t)$ (2)

Some of the coefficients of A , b or c may be constant (constant / functions) if necessary. We can distinguish two kind of taking / part in SD_LP models:

Note that many of the LP-coefficients are included into SD one / although it may ocurrs that we use some LP-coefficients exclusive ly in the linear program, without any interest since the system 7 modelling vewpoint.

In this case, they can be performed as auxiliaries or merely as $/$ computer programming variables non essential for the model.

When we pose a linear program in this "dynamic" way, (2) , we can consider that during an iteration, t, coefficients of c(t), A(t), and $b(t)$, remain constant and so we can operate the linear pro- $\frac{1}{2}$ gram, in that moment, as an usual one (1), and we can to apply /
the simplex or revised simplex algorithm for the actual constant/ values. At the next iteration, t+1, the values of the coefficients $c(t)$, A(t) and b(t) will be changed, but during the current t+1 / iteration, they will again remain constant and we'll can again apply the same procedure to solve the linear program posed in this time and so on.

Before the run begins, we should give to the model the initial va lues of $c(0)$, $b(0)$ and $A(0)$, which will be either fictitious value es for those functions which are not constant, or the real values for the functions being really constant along the time horizon.

Thus, we have defined a dynamic linear program which is different each timestep of the run because its coefficients are changed. How ever, the structure of the linear program will remain unchanged except if at some iteration we add or substract activities (x) or constraints to it, although we seldom will do this.

As a consequence of the simplex run, we⁻¹¹ can obtaint, each time-

Step the optimal values of the variables which participate either as an objective function or as decision variables. This fact in-corporates to the system dynamics model the ability to simulate / the making decisions at each timestep as well as the results of applying these decisions, bearing in mind the environment chan-/ ging conditions.

Fig. 1 Algorithm to operate SD-LP models

Figure 1 shows the steps of the algorithm of operating SD-LP models.

To finish this section, we are going to give a warning: is necessary to bear in mind that as the linear program coefficients / values change, even randomly, it can be produced, at a certain / timestep, a linear program which cannot be solved because it may not exist either an optimal or a feasible solution. If some problem of this kind occurs, it could be a reason some of the following:

- The model (as a set of equations and linear programs) has inaccurately been posed.
- The chosen scenario is inadequate.
- The system will really reach a state where it will not be po-ssible to make an optimal decision according to the posed li- near program.

It is advisable to provide an alternative solution to prevent $/$ this event.

It is advisable too, the using of linear programming sensitivity methods to be able to use the optimal solutions reached at the / previous timestep, t-1, in order to make decrease the number of iterations of the simplex algorithm, to compute in a few time, $/$ the optimal solution at the present time, t , and so on. These / methods are well known in operations research and they can . methods are well known in operations research and they can applied when some of the linear program "data" vary. Changes can be made in:

- b vector (resources) values
- c vector (costs) values
- A matrix (technological coefficients) values
- x vector (activities or decision variables) values
- the number of activities or decision variables
- the number of constraints

Study of the variation of the solution with the variation of some coefficients is not difficult, the last two variations are a more intrincate matter.

Note that the last two kind of changes involve changes in the /
structure of the initial linear program, what will not be usual/ but provides more flexibility to these models. Procedures to pro gram these methods can be found in Prawda(1982), Sakarovitch (1983) and many other books about linear programming.

MURCIA A/I MODEL FORMULATION

Murcia A/I model has been designed to provide a tool to assist in decision-making to the "Consejeria de Políticia Territorial (CPT)" (Department of Territorial Policy) of the Comunidad Autonoma in Murcia in Spain, about the distribution of.investment in the community. This distribution will be carried out bearing in mind I the twelve regions wchich constitute the community of Murcia. (See Appendix 1) Appendix I)
Figure 2 shows the hierarchy of the organs of government over the community of Murcia including the Central spanish government.

There are five basic aspects to distribute the investment, in order to

- 1) Decrease the differences between the regions about the follo- wing eleven equipments goods and infrastructures: transporta- tion, housing, urban planning, supply, road network maintenance, road network improvements, ports, river edges and irriga- tion channels, public health, water depuration and environ- / ment.
- 2) Guarantee the minimum investment according to the weight of the regional population with respect to the total population / in the community.
- 3) Guarantee the minimum investment according to the region geo- graphic size.
- 4) Maximize a regional welfare index either using linear progra- mming, when possible, or unless, an equation as the European / Social Fund does.

Description of model components

Murcia A/I model, or simply A/I model, uses about 600 variables , 500 constants, 11 tables and 500 equations. This is the reason /
because we⁻il merely describe the most important relationships / because we'll merely describe the most important relationships and the general aspects of it. For further details about the mo- del, and software used to run it, the readers may look up Murcia/ A/I (1986).

From a geographical viewpoint, the model is thought of two le- /
vels: the upper one, or aggregate, which is constituted by the /
global community and the lower one, or disaggregate, which is / constituted by the twelve regions.

The advisable time horizon is the period of three years, making / corrections every month and every year in the data set. However , the model is run for the period 1983-1986 because of the available data.

Figure 3 shows a very simplified loop diagram of the model A/I The four major submodels are: Population, Labor, Aggregative economics and Distribution of CPT-investment.

Fig. 2 Hierarchy of the organs of government

Fig. 3 A/I Simplified causal diagram

Each one of these, includes variables which belong either to the aggregate or to the disaggregate level.

- A) Population model (regional and community)
- $-$ The vegetative growing depends on the difference between the $/$ birth and death rates
- Migrations are obtained by means of an equation describing the statistics behaviour of them, which includes three aspects: average familiy size, unemployment (with a certain delay structure) and the growth rate of the regional per capita income.
- B) Labo model (regional and community)
- Regional active population is obtained by multiplying the ac- tivity index by the corresponding population.
- Once we have got the community active population, by adding the regional one, active population per major sectors (fourth) are computed by fractions obtained from an Active Population / Sampling.
- The difference between active population and available jobs de termines the employment level per each major sector, respecti= vely.
- Employment net variations, per major sectors, depend on sto- / chastic equations, which are based on the following causes: / previous new jobs, Lq coefficient of population variation, to tal employment variation, irrigated land variation and indus-= trial land variation.
- Regional unemployment is determined by the difference between/ the active population and the total employment. The recorded / historical unemployment data, by extrapolating, determine the youthful unemployment.
- C) Aggregative economics model (regional and community)

At the upper level (community) the economic network is modeled. The essential relationships are:

- Foreign, Comunidad Autonoma, Business and Central Administra- tion investments are exogenous. Comunidad Autonoma investment/ should specially be simulated.
- CPT investment is computed as a fraction of Comunidad Aut6noma investment. This fraction will be one of the basic parameter /
to simulate the system.
- Inner private investment is assumed to be equal to the inner / saving. Also the inner saving depends both on the regional income and the per capita income.
- The distribution of the total investment per the fourth major sectors is one of the main aim of the model. The functions to distribute the investment are the following:
- $IA = FUNCTION(IA/II, EXA/EX, TEA/TE)$
- $IC = FUNCTION(IC/IT, TEC/TE)$

(3)

 $II = FUNCTION(II/IT, EXI/EX, TEI/TE)$

 $IS = IT - IA - IC - II$

the meaning of the variables is

IA: Agriculture investment IC: Building \mathbf{u} II: Industry \mathbf{u} IS: Services IT: Total EXA:Agricultural exports EXI:Industrial exports EX :Total exports TEA: Agrarian employment level
TEC: Building TEC:Building " \mathbf{u} TEI:Industry \mathbf{u} \mathbf{u} TES:Services α \mathbf{r} TE :Total

- The investment is converted to production by means of the frac tion ICOR(Incremental capital-output relation) using the rela= tions:

 $ICORA = DPA/IA$ $ICORC = DPC/IC$ $ICORI = DPI/II$ $ICORS = \frac{DPS}{IS}$

 $ICOR(X)$ means the fraction ICOR per each of the major sectors / and $DP(X)$ means the increment of production. Note that these increments may, jpossibly, be negative.

- The total production per major sectors (defined as the gross / added value) is obtained by adding the increments per year to/ the previous production. In this way, the productions per sector are rate variables, but they work as level variables and in this manner is as we are going to operate with them (from a system dynamics viewpoint).
- The total employment per sector is computed by dividing the / sector productions by the corresponding average productivity.
- Community income depends on the sum of the sector gross added/ values, computed as the net added value and it is added to the remaining income originated by the CPT investment. Per capita/ income is computed from the regional income previously defi- / ned.

D) Consejeria de Politica Territorial (CPT) investments

This model is the main subject and the justification of the present work. The current release accurately formulates the main questions.However, a completely satisfactory solution is not yet given. According to the CPT-criteria, the expected objective to
be reached by the distribution is a double one, firstly, to a be reached by the distribution is a double one: firstly, to a-
ttend to the regional deficits due to the absolute shortages in ttend to the regional deficits due to the absolute shortages equipment and infrastructure; and secondly, to close the gap due to the differences between the twelve regions.

In case of having information enough about both potential needs/ and actual equipments, we could find out both the relative ,and overall deficits. However, this is not the case, and we'll be / forced to operate, in this first release, with the inter-regio--
nal relative deficits which have been subjectively estimate by nal relative deficits which have been subjectively estimate the experts and political in charge of different areas in rela- tion to equipments or infrastructure.

On the other hand, inter-regional priorities to decrease the deficits in equipments and infrastructures will be determinated by political criteria and they will be established by the CPT.

There are two main formular investing for the investment direc-tly depending on CPT:

- a) By using an economic equation which distributes the invest--/ ment in a proportional way, as the European Social Fund does.
- b) By using a SD-LP model to obtain the optimal distribution taking into account some linear constraints.

In both cases, a regional discomfort index is used, which is defined as

 $IZj = 0.7$ (0.8 PAROJj + 0.2 PAROAj) + $\frac{0.30}{T R P C T i}$, j=1..12 (4)

Note that variations in IZj are dynamics, and they depend, among other causes, on the quantity of investment assigned to the re- gion.

In fact, this investment contributes to decrease the unemployment and to increase the income.

The variables in (4) mean:

IZj PAROJj Regional youthful unemployment PAROAj Regional adult unemployment IRPCj Regional per capita income index : Regional discomfort index

Now, we'll superficially describe both possibilities.

a) Economic equation

In this case, the criteria to distribute the investment are to.

- 1) Assign an investment ratio to be proportional to the popula- tion size.
- 2) Assign an investment ratio to be proportional to the economic size, which is measured by.the total employment.
- 3) Assign an investment ratio to be proportional to the regional geographic size.
- 4) Compensate the differences between the regions due to the relative deficits in equipment and infrastructure.
- 5) Distribute the remaining investment -once we have substracted from the total investment the first three ratios- to be pro-portional to the regional discomfort index.

The'corresponding equations to these criteria are:

 $ICPTj = ICPT*$ ($FPOBLA*Pj/P + FEMP*Ej/E + FSUG*SGj/SG + FEO*Cj$) $i = 1...12$ $ICPTP = \Sigma ICPT$ j $IIIZ = \Sigma IIZj$ $ICPTj = ICPTj + (ICPT - ICPTP) *IZj/IIZ$ (5) (6)

The variables not described yet mean:

Let us see now the other option

b) SD-LP model

Now a linear program is formulated where the objective function/ which we wish to maximize is the total welfare, IB,(7), defined/
as the sum of the regional welfare coefficients,(1/IZj)*ICPTj , which can be understood as the profit resulting from the investment ICPTj in the region number j, which has a discomfort (wel-- fare) index IZj (1/IZj). Decision or activities variables are ICPTj and cost coefficients or profitabilities per invested unit value are IZj.

The interpretation is not difficult: let us assume the next single case, where T is the time

 $-$ T=k and IZ3(k) < IZ5(k)

In this time it is more profitable to invest more in the re- gion number three better than in number five, because number/ three is more depressed, in this period.

 $-$ T=k+1 and IZ3(k+1) > IZ5(k+1)

Now it is more profitable to invest more in the region number five because in this timestep it is more depressed.

In this way, taking into account that each timestep the system environment is different, the linear program computes the opti mal distribution for each timestep of the run.

The constraints beared in mind, (8), are similar to the reported above, for the option (a). Thus, the linear program would/ be:

IB = *L* (1/IZj)*ICPTj *)* **maximize** (7) subject to ICPTj = ICPT ICPTj FPOBLA*(Pj/P)*ICPT ICPTj \geq FEMP*(EMj/E)*ICPT ICPTj FSUG*(SGj/SG)*ICPT ICPTj FEQ*Cj*ICPT

Note that the optimal solution, whether exists, should be on /
the hyperplane Σ ICPT; = ICPT. the hyperplane

Note also that all the inequations are in the same sense (\geq); to avoia redundancy we-ll consider the inequation which has the highest value on the right part.

Perhaps the best way of formulating this model would be using / the SD-LP way, but the option a) is not lost of sight to cover/ the iterations where the linear program doesn-t give a right so lution, because either unfeasibility or unboundedness or other7 problem.

MURCIA A/I **Scenarios**

In order to show the possible uses of the model, various different scenarios are formulated. Suggestions are made in order to manage to the future users and specially to the CPT-technical / staff.

Firstly, the constants and tables which are more adequate to manipulate the model behaviour are suggested. These constants and tables are in relation to exogenous economic policy variables,equipment and infrastructure (to be modified when information will be available) and CPT-investments variables.

Five different scenarios are used to run the model.

MURCIA A/I Images

An image is the set of results corresponding to the model run with a scenario given. In order to compare the results of the /

run, some tables have been included in this paper, which show / I run, some tables have been included in this paper, which show / the most important consequences of the different policies to $_{\text{dis}}$ tribute the investment: tables 1 and 2 show the values correspon ding to both the relative per capita income indices (IRPC) and 7 the discomfort index (IZ) generated by the chosen scenarios. Outs tanding remarks could be:

- 1) IRPC~83 fluctuates between 1.126 and 0.764 with a standard deviation 0.105
- 2) For any image the:disparities are minimized
- 3) The discomfort indices on 1983 fluctuate between 0.440 and $/$ 0.310 with a standard deviation 0.041
- 4) Unlike it happens with IRPC, in I-4 (1986), the differences / are maximum, with limits between 0.332 and 0.457 and a stan-dard deviation 0.0400
- 5) It is noted that the minimum IZ corresponding to any image on 1986 is higher than the 1983 minimum one. However, changes are not measurable.

Finally, tables 4(a) and 4(b) show the percentage distribution of CPT-investments, per region, depending on the policy (or scenario) chosen. It is obvious to note how remarkable differences exist. / Such differences involve very different impacts on both the gene- rated remaining incomme and the generated employment. To facilitate the comparisons the reader could see the bars diagram on appendix II .

Table 1 Regional per capita **income**

Table 2 Statistical analysis of the per capita **income**

Table 3(a) Regional discomfort indices

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፲ 2 \subseteq IT c *"T* I IT u: $\tilde{ }$ u: IT 5 c 2)> <u>∑</u> c= u: u: š <u>اء</u> ;:;: :< u: ГT, ₿ <u>;</u> ~ *c* ϵ c 0: IT ::X

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Table 4(a) CPT- investment distribution

Table 4'(b) Statistical analysis of investment distribution

CONCLUSIONS

As we said above, this work is about a first releas of the model for the investments planning. To: improve the currentrelea se it will be necessary to dispose more statistic data ab0ut 7 the actual community of Murcia state. Moreover, some ideas and suggestions could be useful to

- a) Establish an average productivity of the regional³ employ-ment with actual regional income data
- b) Introd se an economic distance index associated with the (distribution percentage of the constant FDE (minimum fraction of ICPT due to economic distance) whose current value/ is 0
- c) Use welfare indices associated with known- equipments (te-lephones, roads, hospital beds, sewer system, schools, etc.)
- d) Recalibrate the model when 1983 information from Bilbao $\sqrt{2}$ Bank for our region will be available

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APPENDIX I. GEOGRAPHIC SITUATION

APPENDIX II. BARS DIAGRAMS

