Go Back

Simplified Translation of CLD's into SFD's

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Abstract--In this paper, I present constructs that enable simplified translation of CLD's (Causal Loop Diagrams) into SFD's (Stock and Flow Diagrams). As conventionally rendered, there are too many connection "possibilities" presented to immature, model builders who are just getting started. In this paper I will show that it is possible to limit some of these possibilities without any loss of robustness in the models that are developed. For model-builders who are unseasoned, the advantages are simplicity and prevention from creating a construct that is nonsensical. By implication, the automated translation of "fully developed" CLD's into SFD's is also possible. Algorithms implementing such automated translation are presented in the paper.

I. Introduction

In this paper, I shall describe rules, heuristics for translation of causal loop diagrams (CLD's) into stock and flow diagrams (SFD's). From these rules, one can conclude what is needed in order to have robust CLD's that are machine translatable into SFD's. Why should system dynamicists be interested in the translation of SFD's to CLD's? Because this is a crucial process step en route to a working simulation model of a problem. We start with verbal descriptions and people's understanding of the problem or process. From there CLD's are constructed. The CLD's are translated into S eventually into a running simulation within a computer modeling tool like Stella^{tm} or Vensim^{tm}.

Assume for the moment that each CLD is an assemblage consisting of a set Q of quantities and a set C of connectors amongst the quantities. Initially, the quantities and the connectors are not identified as to type. Doing so will effectively transition the CLD into an SFD. Thus, in this paper, the sets Q and C shall be decomposed into their respective subsets. Specifically, Q is decomposed into its stock, X, output Y, input U, auxiliary V, parameter P, and rate R subsets, while C shall be decomposed into its flow F and information I subsets. The translation of a CLD into a SFD can be thought of as a partition upon the sets Q and C into their respective subsets.

II. NOTATION

Let the causal diagram (CLD) or signed digraph D by which a system S is to be represented consist of the following assemblage:

$$D = \{Q,C\}, \tag{1}$$

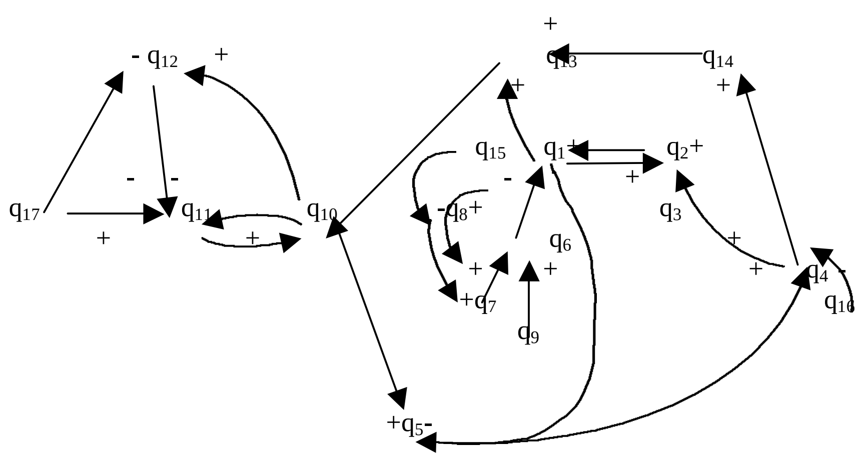


Fig.1. Causal diagram model D.

(all blank positions are zeros)

17	dimensions	1	2	3	4	5	6	7	8	9	10	11	12	13	4	15	16	
1	AA		1			-1			1									
2	AA/DD	1																
3	I/DD		1															
4	dimless			1											1			
5	CC/AA				1													
6	AA/DD	-1																
7	AA/(BB.DD)						1											
8	AA/ZZ							1										
9	BB						1											
10	CC					1					-1	1						
11	CC\DD									1								
12	DD										-1							
13	CC/DD									-1								
14	CC/(AA.DD)												1					
15	ZZ							-1										
16	CC/AA				-1													
17	CC										1	-1						

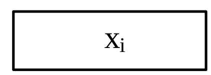
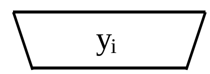
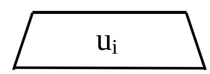
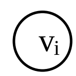
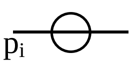
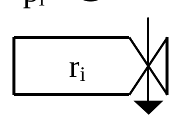
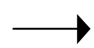
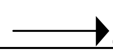
Fig 2. Square ternary matrix (STM) corresponding to causal diagram model D shown in Fig. 1.

where Q is the set of quantities used to represent the system and C is the set of signed connectors that exist between the quantities. The causal diagram is the graphic representation of D. The set Q shall be referred to as a finite space because the quantities it represents are constants and variables which are always finite in number. Let q be the vector of quantities contained in the set Q. A component of q shall be denoted by q_i. A system that can be represented by n quantities will possess a q vector of length n, whose associated quantity space Q is of dimension n. When the vector q(t) is an element of Q, this is denoted by $q \in Q$.

Let c_{ij} represent the signed connector directed from q_i toward q_j. The connector shall, upon occasion, also be denoted by (q_i, q_j); thus, $c_{ij} = (q_i, q_j)$. The set C consists of all connectors c_{ij} as depicted in the causal diagram, is defined by a causal relation R on Q x Q, and can be sorted in the memory of any computer in the form of a square ternary matrix. A causal diagram D and its associated square ternary matrix (STM) are shown in Figs. 1 and 2, respectively. Corresponding to each D is a unique STM, and conversely. Each contains exactly the same and as much information about the components and interconnections within a system. If the ijth elements of the STM is denoted by m_{ij}, then c_{ij} is said to "exist" when $m_{ij} = \pm 1$, and c_{ij} is nonexistent otherwise.

TABLE I

SYMBOLIC BLOCKS USED IN Stock and Flow DIAGRAM

NAME	SUBSET MEMBER	SYMBOLIC BLOCK
Stock (level) variable	$x \varepsilon X$	 x_i
Output variable	$y \varepsilon Y$	 y_i
Input variable	$u \varepsilon U$	 u_i
Auxiliary variable	$v \varepsilon V$	 v_i
Parameter (constant)	$p \varepsilon P$	 p_i
Rate variable	$r \varepsilon R$	 r_i
Flow connector	$c_{ij} \varepsilon F$	i  j
Information connector	$c_{ij} \varepsilon I$	i  j

Once Q and C have been decided upon, the causal loop diagram can be delineated, and the next step is to determine the stock-and-flow diagram from the causal diagram. This step can be thought of as a problem involving a partition upon the sets Q and C into their respective subsets. In the sequel, the following quantity categories are distinguished – stocks (levels), outputs, inputs, auxiliaries, parameters, and rates; in addition, two connector categories are distinguished –flow connectors and information connectors. These categories are listed in Table I, where their set and symbolic representations are also indicated. Thus, x is the vector of stock variables whose associated space is X, and similarly for the other quantity categories. The space X can also be thought of as the set of quantities q_i that are stocks (its use shall be clear from context). To designate a particular quantity q_i as a stock, the notation $q_i \in X$, meaning q_i is an element of the set X, is used. If a quantity q_i is known to be a member of either of two quantity types, let us say the set of parameters P or inputs U, this is denoted by $q_i \in PU$.

Each of the subsets, X, Y, U, V, P, and R is mutually exclusive, and Q is the union of these sets. The objective is to use information provided in the STM to partition the element set Q into each of these subsets $\Pi_Q = \{X; Y; U; V; P; R\}$, and to partition the set of connectors C into its two subsets $\Pi_c = \{F;I\}$, where F and I are the set of flow connectors and the set of information connectors, respectively.

The capability to represent symbolically the connectors and quantities adjacent to or associated with a quantity q_j is needed in order to rigorously express concepts to follow. Let $A_c(q_j)$ represent the set of signed connector directed toward q_j, and let $E_c(q_j)$ represent the set of signed connectors directed away from q_j. Similarly, let $A_q(q_j)$ represent the set of quantities which have connectors directed toward q_j and therefore are adjacent to q_j, and let $E_q(q_j)$ represent the set of quantities which have connectors directed away from q_j and therefore are adjacent from q_j. The set $A_q(q_j)$ may be determined by picking those q_i which have $\pm$ 1 entries in the STM along the column associated with q_j. The set $E_q(q_j)$ are simply those q_i which have $\pm$ 1 entries in the STM along the row associated with q_j. The sets $A_c(q_j)$, $E_q(q_j)$ shall be referred to as the antecedent or affector subsets of q_j, whereas the sets $E_c(q_j)$, $E_q(q_j)$ shall be referred to as the subsequent or effector subsets of q_j.

In the ensuing discussion, set operators are used to denote the union, intersection, and subset of sets, using the symbols $\cup$, $\cap$, and $\subseteq$, respectively, whereas logical operators are used to denote the AND and OR operations between propositions, using the symbols $\wedge$ and $\vee$, respectively. The notation $A_c(q_j) \subseteq I$, for example, denotes the proposition, considered to be true, that $A_c(q_j)$ is a subset of the set I. When its occurrence is simultaneous with the proposition $E_c(q_j) \subseteq I$, the compound proposition is denoted $A_c(q_j) \subseteq I \wedge E_c(q_j) \subseteq I$. Using the suggested notation, the assumptions can be stated in the next section.

III. Assumptions

Evidently, the approach herein discussed assumes that an appropriate causal diagram or signed digraph and its associated STM have been arrived at. Such a signed digraph is specified by causal relations defined on Q x Q in which the concept of “causal n from Klir [23]. It is assumed that an inherent behavior or set of behaviors is prescribed by the interaction hypotheses contained in the signed digraph; the purpose of the process herein described is to extract that behavior or set of behaviors.

A causal diagram of the proper form is crucial to the success of this approach. Preliminary signed digraphs developed with policymaker assistance must be revised into the form appropriate for this method. The influence relation(s) required to facilitate specification of casual loop diagrams must be causal, deterministic, and time-invariant [23]. In addition, each connector in the relation(s) must be directed and signed. And, the causal relation by definition disallows any formulations that do not result in a clear separation of independent from dependent quantities at each node in the digraph [23]. In particular, the following formulations are disallowed:

1) self-loops involving a single quantity,
2) loops involving exclusively information paths or auxiliary variables,
3) more than one connector joining any two quantities (q_i,q_j), and
4) connectors which have more than one originating quantity q_i or more than one destination quantity q_j.

In general, these formulations are not permitted by causal relations; thus the specific causal relation(s) being considered here does not depart from these conventions.

Having considered the characteristics of the relations, their properties are discussed next. First, they do not necessarily possess any of the properties of symmetry, reflexivity, completeness, or transitivity [17]. Nevertheless, the set of connectors C that are in the causal relation(s) is assumed to be partitionable into two subsets: F and I. This assumption follows the Forrester contention that two distinct types of interactions exist between the elements in a set of quantities Q. In [22] it is shown that the two types of causality acknowledged by the Kane method [20], [21] are analogous to the flow and information connectors in the Forrester methodology. Specifically, Kane-style interactions of the form $\dot{q}_i = \alpha_{ij}q_j$ are shown to be analogous with the flow connector, whereas interactions of the form $\dot{q}_i = \beta_{ij}\dot{q}_j$ are analogous to the information connector. (Here $\dot{q}_i$ represents the time rate of change of q_i – the derivative of q_i with respect to time). In effect, both Forrester and Kane are hypothesizing that deterministic causality is comprised of just these two types and all connectors must belong to one of these two classes. Such precedent is maintained in this methodology (stated in axiom A1).

Other aspects of the relation(s) are worth noting. Flows (the transporting of substance) must be explicitly displayed as causal linkages which they ordinarily would be, provided rate quantities are explicitly included in Q. In addition, there should not appear any linkages that, properly interpreted, represent information paths directed from rates or to levels. Under such conditions all connectors directed toward a particular quantity are of the same type, either F or I, and similarly for connectors directed away from a particular quantity when considered in the context of the causal diagram or signed digraph. For example, all connectors directed both toward and away from an auxiliary are information connectors because of the nature of auxiliaries. In a similar vein, all connectors directed toward a stock are flow connectors; all connectors directed away from a parameter or an input and all connectors directed toward outputs are information connectors.

It is very infrequent that a mixture of inward-directed or outward-directed connectors is observed when the structure of the model is delineated by means of a causal diagram. These mixtures will be momentarily neglected in favor of the elegant simplicity that results from such benign neglect. This we state as supposition S1, the consistency supposition.

S1. *Consistency*: For any q_{ij},

$$[A_c(q_j) \cap I = \{\emptyset \vee A_c(q_j)\}] \wedge [\{A_c(q_j) \cap F = \{A_c(q_j) \vee \emptyset\}].$$

Also,

$$[E_c(q_j) \cap I = \{\emptyset \vee E_c(q_j)\}] \wedge [\{E_c(q_j) \cap F = \{E_c(q_j) \vee \emptyset\}].$$

In words, the supposition asserts that the members of the connector subset $A_c(q_j)$ are all of the same connector category, either I or F, that the members of the connector subset $E_c(q_j)$ are likewise all the same connector category, either I or F, and that this is true for all q_j. The reader can empirically verify that the models described in [5]-[7] are compatible with this supposition once integrating functions in information channels [3, ch. 8] and information paths leading from rates to outputs (via auxiliaries) are removed and the stock-and-flow diagrams of these models are redrawn in their corresponding causal diagram formats.

The insertion of integrating functions in information channels can be easily performed after the stock-and-flow diagram model has been delineated if they are desired. The same is true for information paths leading from rates to outputs. However, it is always possible to use the same information used by a rate to reconstruct a rate at an output, thereby eliminating any requirement for an information path leading from a rate to an output.

Next the restrictions imposed upon choice of quantities in Q must be considered. To choose the quantities in Q properly, it is suggested that the significant subsystems that interact strongly with the problem of interest be itemized first [24]. Then the important quantities within each subsystem should be explicitly detailed, being careful to include quantities that would represent rates. Next, the important quantities used to transmit either information or substance between subsystems should be included in the list of quantities Q. Finally; care must be exercised in including separate variables for outputs.

Definitions, which are consonant with the consistency supposition, can now be related for each of the quantity and connector types and also for the structural element known as the feedback loop.

IV. Definitions

The first definition provides a method for determining parameters p_i and inputs u_i from the remaining quantity categories through inspection of the STM. Parameters and inputs are considered constant at lest for the duration of a single model run, and they are information inputs to rates (and occasionally outputs), either directly or by way of auxiliaries. Parameters are distinguished from inputs by virtue of an identified manager's capability to manipulate or change the latter.

D1. *Parameters and Inputs*: Any quantity q_j whose associated $A_c(q_j) = \varnothing$, the null set, is a parameter or an input; that is, $A_c(q_j) = \varnothing \Rightarrow q_j \in PU$.

This definition simply asserts that whenever a quantity is affected by nothing, it is an input or a parameter. Inspection of the STM shown in Fig. 2 reveals that q_9, q_{15}, q_{16}, and q_{17} are parameters or inputs because there are no entries in the columns associated with these quantities.

The next definition provides a technique for determining the outputs y_i just by inspection of the STM. Outputs are those quantities that are monitored or measured by the manager of the system.

D2. *Outputs*: Any quantity q_j whose associated $E_c(q_j) = \varnothing$, the null set, is an output; that is $E_c(q_j) = \varnothing \Rightarrow q_j \in Y$.

In words, this definition states that a quantity that affects nothing is an output. Inspection of the STM in Fig. 2 reveals that there is no quantity q_i that has an associated row consisting entirely of blanks, and consequently there are no outputs for the example. Note that when levels or auxiliaries are measured, there must be an information path leading from them to the associated output variables.

It is expected that there will be no quantities q_j for which $A_c(q_j) = \varnothing \wedge E_c(q_j) = \varnothing$. Such a quantity is unrelated to the system, may be removed from the set Q, and is said to be disjoint. Likewise, it is impossible to partition the set Q into subsets Q_1,Q_2, which are completely disjoint; that is, $R(Q_1,Q_2,) \cup R(Q_2,Q_1) = \varnothing$. If this were true, then it would be possible to study the sets Q_1 and Q_2 in complete isolation from each other. This notion is later stated as axiom A5.

Note that when $A_c(q_j)$ is empty, so is $A_q(q_j)$; likewise, when $E_c(q_j)$ is null, $E_q(q_j)$ is also. Thus the definitions provided above for parameters, inputs, and outputs could just as appropriately be defined in terms of A_q, E_q, as they were defined in terms of A_c,E_c. Moreover, the number of elements in the sets A_c, A_q and E_c, E_q is always the same. We refer to the number of elements in any set as the cardinality of the set and denoted it by $| \cdot |$. Thus $| A_c | = | A_q |$ and $| E_c | = | E_q |$.

In system dynamics [3], stocks can be recognized as accumulations or integrations of rates of flow. They integrate the results of action in a system. The following definition for stocks is intended to permit recognition of the same on the basis of the kind of connectors directed toward and directed away from the stock.

D3. *Stocks*: Any quantity q_j whose $A_c(q_j) \subseteq F$ and whose $E_c(q_j) \subseteq I$ is a stock; this we write as follows:

$$A_c(q_j) \subseteq F \wedge E_c(q_j) \subseteq I \Rightarrow q_j \in X.$$

In words this definition asserts that any quantity whose affector subset A_c is a subset of the set of flow connectors and whose effector subset E_c is a subset of the set of information connectors is a stock. It is apparent that the classification of stocks (or for that matter rates and auxiliaries) is contingent upon the previous classification of associated connectors. Suppose that all interactions along row 1 in Fig. 2 were identified as information links, whereas all interactions indicated in column 1 were known to be flows. Then q_1 would be classified as a stock. This definition is consistent with the notion that information generally proceeds from stocks to rates, whereas control the flows into and out of stocks (later stated as an axiom). The next definition is for rates.

D4. *Rates*. Any quantity q_j whose $A_c(q_j) \subseteq I$ and whose $E_c(q_j) \subseteq F$ is a rate; thus

$$A_c(q_j) \subseteq I \wedge E_c(q_j) \subseteq F \Rightarrow q_j \in R.$$

Likewise, this definition is consonant with the notion suggested above the asserts merely that any quantity whose inward-directed connectors A_c are information connectors and whose outward-directed connectors E_c are flow connectors is a rate. Referring to Fig. 2, if c_{61}, were an identified flow, whereas c_{76}, and c_{96} were known to be information links, then q_6 must by D4 be a rate.

Auxiliaries are those quantities placed within information paths that modify or transform the information as it is passed from stocks to rates. The following definition is given for auxiliaries.

D5. A*uxiliaries:*. Any quantity q_j whose $A_c(q_j) \subseteq I$ and whose $E_c(q_j) \subseteq I$ is an auxiliary; thus

$$A_c(q_j) \subseteq I \wedge E_c(q_j) \subseteq I \Rightarrow q_j \in V.$$

As an example, consider q_4 in Fig. 2. Its affector subset $A_c(q_4)$ consists of connectors $\{c_{54}, c_{16,4}\}$, whereas its effector subset $E_c(q_4)$ consists of connectors$\{c_{43}, c_{4,14}\}$, as can be discerned by inspecting the column and row associated with q_4. If the set $\{c_{54}, c_{16,4}, c_{43}, c_{4,14}\} \subseteq I$, then by D5, q_4 must be an auxiliary. This definition is consonant with the concept of an auxiliary as resident only within information channels.

Note that each quantity category above is defined in terms of the kinds of connectors directed either toward or away from the quantity. In a similar way, the definitions of the connector types below are given in terms of the kinds of quantities bounding them on either side.

Flow connectors generally indicate that substance is being moved from place to place within a system, such substance being controlled by rates. On the other hand, information connectors do not cause a transfer of substance within a system but just give information about the magnitude of the content. The following definitions are given for flow and information connectors.

D6. *Flow Connectors*: Any connector $c_{ij} = (q_i,q_j)$ whose q_i is a rate or whose q_j is a stock is a flow connector; mathematically, this is written.

$$q_i \in R \vee q_j \in X \Rightarrow c_{ij} \in F.$$

D7. *Information Connectors*: Any connector $c_{ij} = (q_i,q_j)$ whose q_i is not a rate and whose q_j is not a stock is an information connector; thus

$$q_i \in XPUV \wedge q_j \in VRY \Rightarrow c_{ij} \in I,$$

where

$$XPUV = X \cup P \cup U \cup V \qquad VRY = V \cup R \cup Y.$$

All definitions provided thus far are set-theoretic equivalents of similar definitions provided in [3]. To facilitate discussion in later sections, let us now define the concept of an endogenous quantity.

D8. *Endogenous Quantities*: Any quantity q_j whose $A_c(q_j) \neq \emptyset$ and whose $E_c(q_j) \neq \emptyset$ is an endogenous quantity.

Apparently, it would be possible to partition Q into endogenous quantities Q_e and nonendogenous (not necessarily exogenous) quantities Q_n; that is, $\Pi_Q = \{Q_e; Q_n\}$. Moreover, Q_e consists exclusively of rates, stocks, and auxiliaries, whereas Q_n consists exclusively of inputs, parameters, and outputs. Thus each of Q_e and Q_n can be partitioned as follows:

$$\Pi_{Qe} = \{R;X;V\} \qquad \Pi_{Qn} = \{U;P;Y\}.$$

In succeeding discussions, the abbreviation “end.” to mean *endogenous* shall be employed.

Next, a definition for a structural element that Forrester [4] refers to as the feedback loop is provided. As a minimum, the feedback loop consists of just two quantities q_j and q_k (and associated connectors). We represent such feedback loops by L_{min} and refer to them as *minor submodels*. The following definition is intended to formalize these notions.

D9. *Minor Submodels*: Any feedback loop consisting of just two quantities q_j and q_k is a minor submodel and is represented by L_{min}, where L_{min} is the following set:

$$L_{min} = \{q_j,q_k,A_c(q_j),E_c(q_j),A_c(q_k),E_c(q_k)\}.$$

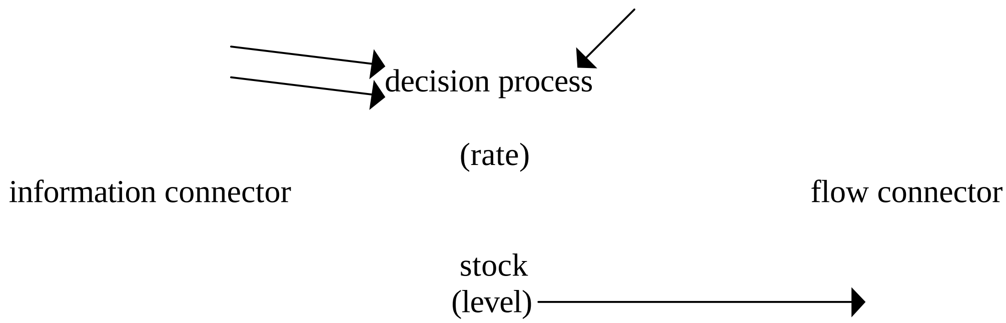


Fig. 3. Feedback loop.

An illustration of a minor submodel is provided in Fig. 3. Note that for such structures to exist both m_{jk} and m_{kj} must be nonzero in the STM.

V. Axioms

Using the assumptions and definitions previously alluded to, this section relates the axioms of system dynamics. The intent of the axioms is to capture the essence of the methodology reported in [3], [4] and applied in [5]-[7] and elsewhere. Since an axiom is a maxim or proposition that is widely accepted or regarded as self-evident, many of the axioms will appear tautologous to previous notions popularized by Forrester. The first axiom stocks the constituents used by Forrester to model a system.

A1. *Elements*: The basic components of a dynamic system S can be modeled by means of the following quantity categories – stocks X, rates R, inputs U, outputs Y, auxiliaries V, and parameters P, and by means of the following connector categories—flow F and information I. Symbolically, $S \sim D = \{Q, C\}$, where Q is the specified set of quantities, C is the specified set of connectors, and

$$Q = X \cup R \cup U \cup Y \cup V \cup P.$$

while $C = F \cup I$.

The next axiom is, perhaps the most fundamental to the Forrester methodology [3, Ch. 4].

A2. *Feedback*. Any feedback loop consists of a minimum of rate, flow connector, stock, information connector, and associated connector, as depicted in Fig. 3.

Mathematically, this is denoted as follows. Let LI represent the set of identified connectors and quantities that comprise a feedback loop. According to the axiom, the minimum set of such entities, represented by LI_{min}, is the following:

LI_{min}, = {r_i, x_j, $c_{ij} \in$ F $c_{ji} \in$ I, and adjacent connectors}.
(A2)

A comparison of A2 with D9 above yields the conclusion that one of q_j, q_k is a rate and one is a stock. As a result, statements can also be made as to the classification of connector subsets $A_c(q_j)$, $E_c(q_j)$, $A_c(q_k)$, and $E_c(q_k)$, as will be demonstrated later.

Note that the axiom does not allow or permit constructions in which a single quantity is involved in a loop with itself. However, feedback loops involving more than two quantities are entirely permissible. Other consequences of this axiom can be inferred: for example, no feedback loop may consist exclusively of auxiliaries; or for that matter, neither may a feedback loop consist entirely of rates, or entirely of stocks. The feedback loop must possess at least one stock and one rate. As the feedback loop embodies more than two quantities, it may also possess auxiliaries or additional stocks and rates.

The next two axioms are concerned with the equations associated with rates and stocks. The rate axiom is stated in [3, ch. 4, and ch. 9].

A3. *Rate Equations*: Rate equations are models of decision processes that control rates of flow. These decision processes may be explicit (conscious) or implicit (governed by nature). Moreover, decision processes or rates use information about the values of stocks, parameters, and inputs to control rates of flow into and out of stocks. This information may, at times, be transmitted by means of auxiliaries. (In rare instances, rates may also use "measured" information about other decision processes. This situation is modeled by means of a smoothing function, a construction that has been eliminated from the fabrication of the basic feedback structure by the consistency supposition. Such constructions can be added after the basic structure has been formulated.) In a more rigorous vein, the axiom could be worded so as to require that there exists a solution r_i to a decision process represented by $f_i\{A_q(r_i)\}$ [16]. The solution r_i controls rates of flow and is based upon information about levels, parameters, and inputs as represented by $A_q(r_i)$.

In an off-line batch simulation environment, the r_i are specified by dimensionally consistent equations. In an on-line interactive simulation environment, each of the r_i could be specified by the decision maker(s) directly responsible for decision process f_i; as could be accomplished in a gaming simulation situation. Alternatively, each decision maker would be allowed to specify the "form" of his decision process f_i; this form would be translated into a specific structural equation for r_i in terms of $A_q(r_i)$. The next axiom involves the stocks (levels) and their associated equations [3, chs. 5 and 7].

A4. *Stock Equations*: The level or stock equation is an accounting of the net change in the amount of the accumulation within the level or stock as the simulation proceeds from time-step to time-step.

Let $x_i(n+1)$ represent the solution of the stock equation S_i. Then the axiom could be worded in such a fashion as to require that there exists an $x_i(n+1)$ satisfying S_i. Clearly then, $x_i(n+1)$ could be expressed as a function of S_i, using $x_i(n+1) = S_i[x_i(n), \{A_q(x_i)\}]$. Then the axiom maintains that there exists a solution to the stock equation at time point n + 1 prescribed by $S_i[\ldots]$, and that this solution is dependent upon the value of the stock at time point n and the affector subset $A_q(x_i)$.

The next two axioms are concerned with the nonendogenous quantities Q_n, which includes inputs, parameters, and outputs. From the definition of endogenous quantities, it is known that

$$Q_n = \{q_j | A_c(q_i) = \varnothing \vee E_c(q_j) = \varnothing\}.$$

Even so, the modeler does not include within the set Q of quantities any that are completely isolated from the rest of the set. Such *disjoint* elements neither affect nor are affected by a particular system. In addition, it is impossible to partition the set Q into disjoint subsets Q_1 and Q_2 such that $\Re(Q_1,Q_2) \cup \Re(Q_2,Q_1) = \varnothing$. These concepts are stated in the following axiom, called the "axiom of connectedness."

A5. *Connnectedness*: The set of quantities Q includes no elements which are disjoint from the rest of the set, as specified by C; moreover, it is also impossible to partition the set Q into disjoint subsets Q_1 and Q_2.

Next, consideration must be given to the kinds of connectors directed from inputs and parameters or directed toward outputs. There are four possibilities as depicted in Fig. 4. The I intent is to include within the boundary of the model the dynamics of the problem [3, ch. 4], and generally inputs and parameters are not permitted to become time-varying. Thus the model must include within its boundary *submodels* of any subsystems that "drive" or otherwise influence a subsystem of interest. This is the essence of the sixth axiom.

A6. *The Closed Boundary*: There are no flows across the boundary of a model D, and parameters and inputs are not allowed to become time-varying, at least initially.

The result of this axiom is that constructions (b) and (d) in Fig. 4 are not permitted, as shall be argued in Theorem T4. Finally, the following axiom regarding conservation of flow is required. The essence of this axiom is that branching of flow lines is not permitted when the model structure does not explain how the flow would be divided.

A7. *Conservation of Flow*: As a substance flows from stock to stock, the quantity that is flowing is neither created nor destroyed, but nevertheless is controlled.

In terms of the flow diagram this axiom will not permit constructions shown in Fig. 5.

Having considered assumptions, definitions, and axioms, the implications of these primitives can be explored. It is appropriate to digress momentarily and comment on the class of problems to which this approach may be applicable. With the possible exception of the consistency supposition, each primitive is just a set theoretic treatment of analogous notions found in [3], [4]. Since the assumption of consistency can be relaxed after the basic structure has been delineated, it seems fair to assert that the problem class to which the approach is applicable is essentially the same as the class treated by system dynamics.

VI. Implications of the Definitions and Axioms

In the development of any theory involving a list of quantities Q and a set of connectors C, there are two approaches that can be taken. The theory can focus on the quantities, or the theory can focus upon the connectors (interactions) between the quantities. An explanation of the quantity categories by reference to the connectors is referred to as *field theory*, whereas an explanation of the relations (interaction or connector categories) by reference to the attributes of the quantities is *monadic theory*. In the succeeding development, use is made both of monadic and field concepts. The development is based upon the preceding definitions and axioms, and

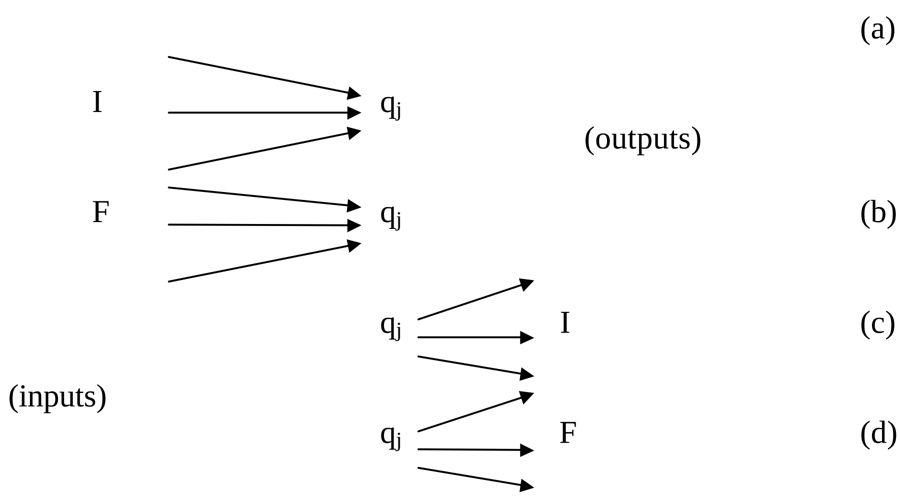


Fig. 4. Four Conceivable Non-endogenous Constructions

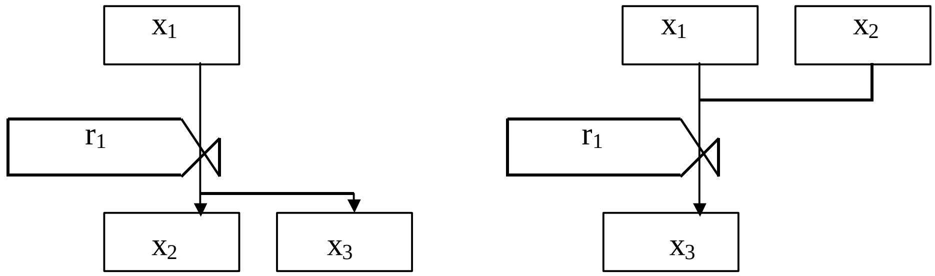


Fig. 5. Constructions involving flows that are not permitted.

and employs a theorem/proof format.

The Consequences of Consistency

The first theorem is a direct consequence of the consistency supposition S1.

T1: The following statements are true for any quantity q_j.

a) If $c_{ij} \in$ I, then $A_c(q_j) \subseteq$ I.
b) If $c_{ij} \in$ F, then $A_c(q_j) \subseteq$ F.
c) If $c_{ik} \in$ I, then $E_c(q_j) \subseteq$ I.
d) If $c_{jk} \in$ F, then $E_c(q_j) \subseteq$ F

P1: To prove the first two statements of the theorem, let CC represent a particular connector subset category: thus, CC is either F or I, and CC is the other. Now if $c_{ij} \in$ CC and $c_{mj} \in$ CC, both of which are by definition elements of $A_c(q_j)$, then not all elements of $A_c(q_j)$ are of the same generic category, and supposition S1 is violated. Thus, if $c_{ij} \in$ CC, $c_{mj} \in$ CC, and all of the elements within $A_c(q_j)$ are also elements of CC; then $A_c(q_j)$ $\subseteq$ CC. The last two statements of the theorem are proved by an identical rationale, except involving $E_c(q_j)$ rather than $A_c(q_j)$.

This theorem enables the identity of all connectors associated with a particular quantity q_j to be identified provided at least one connector directed toward and one connector directed away from the quantity is known. The corollary to T1 is the following.

COR1: Given a connector c_{ij} whose identity is known, then all connectors in the subsets $E_c(q_j)$ and $A_c(q_j)$ are also known.

P: Note that c_{ij} is common to both subsets $E_c(q_j)$ and $A_c(q_j)$. By T1, parts a) and b), the identify of A_c can be established; whereas by T1, parts c) and d), the identity of E_c is known.

As an example, consider the connector c_{12} in Fig. 2. If c_{12} is an information connector, then all connectors shown in row 1 must be information connectors, whereas all connectors shown in column 2 must also be information connectors. Accordingly in Fig. 2, $E_c(q_1) = \{c_{12}, c_{15}, c_{18}, c_{1,13}\}$, whereas $A_c(q_2)$, $= \{c_{12}, c_{32}\}$. Since c_{12} is common to both $E_c(q_1)$ and $A_c(q_2)$, its identity enables the identification of all connectors in both subsets.

Recognition of Endogenous Quantities

The next theorem is concerned with the recognition of endogenous quantities when the identity of a connector directed toward and a connector directed away from the quantity in question is known.

T2: The identity of an endogenous quantity can be determined provided at least one connector on either side of the quantity has been identified. In particular,

a) if $c_{ij} \in F \wedge c_{ik} \in I$ for some end. q_j, then $q_j \in X$;
b) if $c_{ij} \in I \wedge c_{ik} \in F$ for some end. q_j, then $q_j \in R$; and
c) if $c_{ij} \in I \wedge c_{ik} \in I$ for some end. q_j, then $q_j \in V$.

P2: The theorem can be established provided each of parts a), b), and c) are proven. From T1, $c_{ij} \in F \Rightarrow A_c(q_i) \subseteq F$ and $c_{ik} \in I \Rightarrow E_c(q_j) \subseteq I$. From D3, however, any quantity q_j whose $A_c(q_j) \subseteq F$ and whose $E_c(q_j) \subseteq I$ is a stock. This argument is sufficient to establish part a); the remaining parts b) and c) are established using an identical rationale except involving definitions D4 and D5, respectively.

As an example of the use of this theorem, suppose (in Fig.1) that $c_{13,10}$ is an identified flow connector, whereas $c_{10,12}$ has been identified as an information connector. Then by T2, q_{10} is a stock.

The Nonexistent Quantity Phenomenon

One is inclined to ask at this point whether it would be possible to classify a quantity when one and only one connector is known. The response is affirmative, but only when the connector is known to be a flow connector. Consider that with respect to the two connector categories previously defined, four permutted possibilities are conceivable for any particular endogenous quantity q_i, These are shown in Fig. 6. Of the four possibilities shown, only the first three are possible. There is no quantity type of which q_4 is characteristic – no quantity exists which will admit flow connectors directed both toward and away from it when the quantity and it connectors are depicted in the causal diagram. This is due to the interpretation that is applied to the connector c_{ij} as depicted in the causal structure. In general, a connector (q_i,q_j) indicates that q_i somehow directly affect, causes, influences, or has an impact upon q_j somehow directly affects, causes, influences, or has an impact upon q_j. By definition D6, a flow connector c_{ij} is any connector whose q_i is a rate or whose q_j is a stock. By axiom A3 flows into and out of stocks are controlled by rates. By axiom A4, a level is an accounting of the net change in the amount of the

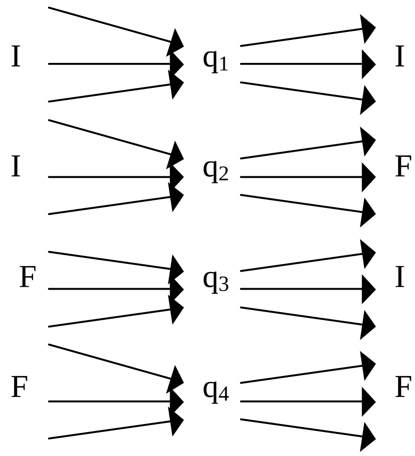


Fig. 6. Four conceivable endogenous quantity constructions.

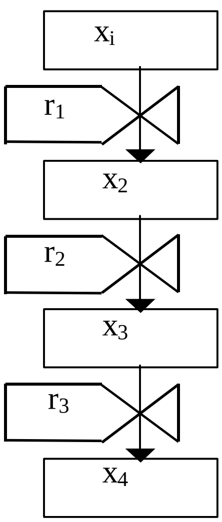


Fig. 7. Stock-and-flow and causal diagram models for a hypothetical conservative subsystem.

accumulation within the level due to flows of substance into and out of a level variable. These postulates all suggest that flow connectors represent the effect that rates ultimately have upon levels (stocks). Thus a flow connector may be directed from a rate or toward a stock. Any other construction(s) will produce a violation of D6, A3, or A4. Thus there is no quantity analogous to q_4 in Fig. 6 in the source methodology when considered in the causal diagram context.

An example is shown in Fig. 7. The direction of flow is downward as shown in the stock-and-flow diagram model on the left. Even so flow connectors shown in the causal diagram model proceed from rates to stocks because of the "causes" or "affects" interpretation that is applied to the connector. This argument is sufficient to establish the following theorem.

T3: In the casual diagram context there exists no such quantity q_j such that $A_c(q_j) \subseteq F \wedge E_c(q_j) \subseteq F$.

The implication of this theorem is the following corollary involving rates and stocks.

COR 3: Given $c_{ij} \in F$; then $q_i \in R$ and $q_j \in X$.

P: By COR1, $E_c(q_i) \subseteq F$ and $A_c(q_j) \subseteq F$. Now $A_c(q_i)$ and $E_c(q_i)$ are unspecified. However, if $A_c(q_i) \subseteq F$, T3 is violated; therefore $A_c(q_i) \subseteq I$ and q_i must be a rate by D4. Likewise, if $E_c(q_j) \subseteq F$, T3 is violated; hence $E_c(q_j) \subseteq I$. This rationale is sufficient to establish the following theorem. Likewise, if $E_c(q_j) \subseteq F$, T3 is violated; hence $E_c(q_j) \subseteq I$ makes q_j a stock by D3 and establishes the corollary.

As an example of the use of this result again consider Fig. 1. If c_{61} were known to be a flow connector then by COR3, q_6 must be a rate, whereas q_1 must be a stock. When a connector c_{ij} is known to be an information connector, it is not possible to uniquely identify the adjacent quantities because, as shown in Fig. 6, when $c_{jk} \in I$, there is no restriction imposed upon adjacent connectors c_{ij} and c_{kl}. These may be either information connectors of flow connectors. We refer to the quantity q_j such that $A_c(q_j) \subseteq F \wedge E_c(q_i) \subseteq F$ as the nonexistent quantity.

From Field Theory to Monadic Theory

Up to now, the development has been field-theoretic in the sense that the attempt has been to categorize quantities on the basis of the connector between them. We turn now to a discussion of the kinds of interaction (connector) that can be identified by reference to the attributes of the quantities. The following theorem is intended to lessen the restriction required to identify connectors, as specified by definitions D6 and D7.

T4: Given a quantity q_j whose identity is known, then its associated connector subsets $E_c(q_i)$ and $A_c(q_i)$ are also known. Specifically,

a) for any $q_j \in R$, $A_c(q_i) \subseteq I \wedge E_c(q_i) \subseteq F$;
b) for any $q_j \in X$, $A_c(q_i) \subseteq F \wedge E_c(q_i) \subseteq I$;
c) for any $q_j \in V$, $A_c(q_i) \subseteq I \wedge E_c(q_i) \subseteq I$;
d) for any $q_j \in PU$, $A_c(q_i) = \emptyset \wedge E_c(q_i) \subseteq I$;
e) for any $q_j \in Y$, $A_c(q_i) \subseteq I \wedge E_c(q_i) = \emptyset$.

P4: Since R, X, V, P, U, and Y represent all of the quantity types, the theorem can be proved provided each of parts a) -e) can be established. Now part a) is just the contrapositive of D4 and is consistent with axioms A3 and A4. Part b) is the contrapositive of D3 and likewise is consistent with A3 and A4. Part c) is the contrapositive of D5. Part d) is the contrapositive of D1 and is consistent with A5 and A6, whereas part e) is the contrapositive of D2 and is also consistent with A5 and A6. For part d), if $E_c(q_i) \subseteq F$, then either A5 or A6 is violated; whereas, for part e), if $A_c(q_i) \subseteq F$, then likewise either A5 or A6 has been violated. This completes the proof.

As an example of the utility of this theorem, suppose a particular quantity q_j shown
in Fig. 2 has been identified. Then all interactions shown in the row and column associated with q_j are also known. Specifically, if in Fig. 2, q_4 were identified as an auxiliary, then all connectors shown in row 4 and column 4 must be information connectors.

Compounding the Previous Results

With the previous theorems, it is now possible to compound the implications of an identified entity (connector or quantity) in the causal model. For example, if the connector c_{ij} between an endogenous pair of quantities (q_i,q_j) is known to be a flow connector, then by COR3, $q_j \in R$, $q_j \in X$, and by T4, part a), $A_c(q_j) \subseteq I \wedge E_c(q_i) \subseteq F$; whereas by T4, part b), $A_c(q_j) \subseteq F \wedge E_c(q_j) \subseteq I$. This rationale is sufficient to establish the following theorem.

T5: If for any end. pair (q_i,q_j), $c_{ij} \in F$, then $q_i \in R \wedge q_j \in X \wedge A_c(q_i)$

$$\subseteq I \wedge E_c(q_i) \subseteq F \wedge A_c(q_j) \subseteq F \wedge E_c(q_i) \subseteq I.$$

As an application of this result consider connector c_{21}. If c_{21} is a flow connector, then q_2 is a rate, q_1 is a stock, the sets $A_c(q_2) = \{c_{12},c_{32}\}$, $E_c(q_1) = \{c_{1,13,}c_{18}, c_{12},c_{15}\}$are information subsets, whereas the sets $E_c(q_2) = \{c_{21}\}$, $A_c(q_1) = \{c_{21},c_{61}\}$are flow subsets. In terms of the STM, all connectors along the row of q_1 and column of q_2 are information connectors, whereas all connectors along the column of q_1 and row of q_2 are flow connectors. The sequel to this theorem is the following.

T6: If for any endogenous pair (q_i,q_j), $c_{ij} \in I$, then $E_c(q_i) \subseteq I \wedge A_c(q_j) \subseteq I \wedge q_i \in XV \wedge q_j \in VR$.

P6: By COR1 and T1, $c_{ij} \in I \Rightarrow E_c(q_i) \subseteq I \wedge A_c(q_j) \subseteq I$. By definitions D3 to D5 the only endogenous quantities q_i for which $E_c(q_i) \subseteq I$ are stocks or auxiliaries; thus $q_i \in X \cup V$. Again, by definitions D3-D5 the only endogenous quantities q_j for which $A_c(q_j) \subseteq I$ are auxiliaries or rates; thus $q_j \in V \cup R$.

The next theorem enables identification of adjacent quantities when a quantity q_j is known.

T7: If an endogenous quantity q_j is known, then such knowledge imposes specific limitations upon the subsets $A_c(q_j)$ and $E_c(q_j)$ as follows:

a) if $q_j \in X$, then $A_q(q_j) \subseteq R \wedge E_q(q_j) \subseteq VRY$;
b) if $q_j \in R$, then $A_q(q_j) \subseteq VXPU \wedge E_q(q_j) \subseteq X$;
c) if $q_j \in V$, then $A_q(q_j) \subseteq VXPU \wedge E_q(q_j) \subseteq VRY$.

P7: The theorem can be established provided each of its parts can be substantiated. Considering part a), if $q_j \in X$, they by T4, part b), $A_c(q_j) \subseteq F$. This each $c_{ij} \in A_c(q_j)$ is a flow connector to which its associated q_i must, by T5, be a rate; thus $A_c(q_j) \subseteq R$. Likewise, by T4, part b),$E_c(q_j) \subseteq I$. By definition D1-D5, the only quantities which may have information connectors directed toward them are auxiliaries V, rates R, or outputs Y; thus $E_q(q_j) \subseteq V \cup R \cup Y = VRY$. Parts b) and c) are established using an identical rationale involving definitions D1-D5 and Theorem T4.

In terms of the STM the effect of this theorem is that along any row whose associated $q_j \in R$, those q_k for which $m_{jk} \neq 0$ must be stocks. Likewise, the entries along the column associated with any $q_j \in X$ for which $m_{ij} \neq 0$ must be rates.

Feedback Loops and Minor Submodels

In the following, the concern is with minor submodels, as denoted by L_{min}. L_{min} consists of just two quantities, q_j and q_k, one of which is rate and one of which is a stock. Thus for any $q_j \in L_{min}$, $q_j \in X \vee q_j \in R$. The following theorem provides a useful technique for identifying rates in minor submodels.

T8: For any $r_i \in R$, $1 \leq | E_c(r_j)| \leq 2$.

This theorem claims that the cardinality of the effector subset associated with any rate must be either 1 or 2; in other words, every rate must possess at least 1 and at most 2 outward directed flow connectors.

P8: Since all rate quantities are endogenous, their affector and effector subsets $A_c(r_i)$, $E_c(r_i)$ must have cardinalities which are greater than zero; that is, they must possess at least 1 inward-directed connector and at least 1 outward-directed connector. Consequently, $| E_c(r_j)| \geq 1$. Now, consider the implication of an $| E_c(r_j)| > 2$. As shown in Fig. 5, when the cardinality of $E_c(r_j)$ is greater than 2, the corresponding flow diagram is incongruous with A7: every rate controls a flow which must be conserved. Consequently, $E_c(r_j)$ can never be greater than 2 and the theorem is proven.

The use of these theorems are illustrated in the next section. Other implications for the minor submodel could be stated in additional theorems, but because of their lack of utility in the example problem of the next section, were omitted. For example, it could be shown that whenever a component (a quantity or connector) in a minor submodel L_{min} has been identified, the identity of all remaining components in L_{min} can be inferred immediately.

VII. Application to the Example Depicted in Figs. 1 and 2

The implications of the previous section are applied to the causal diagram shown in Fig. 1 and its square ternary matrix shown in Fig. 2. By means of the definitions and theorems just provided, the sets Q and C can be partitioned into their subset categories. The partition enables the flow or stock-and-flow diagram to be delineated as a graphical representation of the quantity and connector categories and their interactions.

Consider first quantities q_9, q_{15}, q_{16}, and q_{17}. Since the columns associated with these quantities contain no entries, the quantities are, by D1, parameters or inputs and the connector directed away from them are, by T4, part d), information connectors. Now consider what happens as one proceeds from q_9 toward q_6. Quantity q_6 must be either a rate or an auxiliary (by D3-D5 the only endogenous q_j for which $A_c(q_j) \subseteq I$ are auxiliaries or rates). However, q_6 has a connector that is directed toward q_1 – a connector that is a flow if q_6 is a rate, or an information transfer if q_6 is an auxiliary (D4 and D5). If q_6 is a rate, then q_1 must be a stock or level (T7, part b)); if q_6 is an auxiliary, then q_1 must be either a rate or an auxiliary (D5 and T6).

What is q_1? By virtue of the minor submodel involving just q_1 and q_2, it is possible to identify q_1 as the stock. By A2, q_1 is either a rate or a stock. By T8, the rate cannot have more than two connectors directed away from it. Consequently, q_1 cannot be the rate, as $|E_c(q_1)| = 4$. Thus q_1 is a stock, and all connectors directed toward it from q_2 and q_6 are flow connectors (T4, part b)). Consequently, q_2 and q_6 or rates (COR 3). The connectors directed toward q_2 and q_6 or away from q_1 are information transfers (T4, parts a) and b)).

Next, consider quantities q_7, q_8, and q_{15}. Quantity q_8 must be an auxiliary or a rate by virtue of the information connectors directed toward it (T6). Quantity q_7 must be an auxiliary or a stock by virtue of the information connectors directed away from it (T6). In this instance, there is not enough information to identify either q_8 or q_7; if the identity of one of those could be determined, the identity of the other would be prescribed. At this point, additional information is required. If the procedure being described were algorithmatized, the algorithm would at this point interrogate the modeling consultant and ask for an identification of q_8. Assuming the information that is returned designates q_8 as an auxiliary, then the path directed from it and toward q_7 is an information transfer (T4; part c)), and q_7 must also be an auxiliary (T2, part c)).

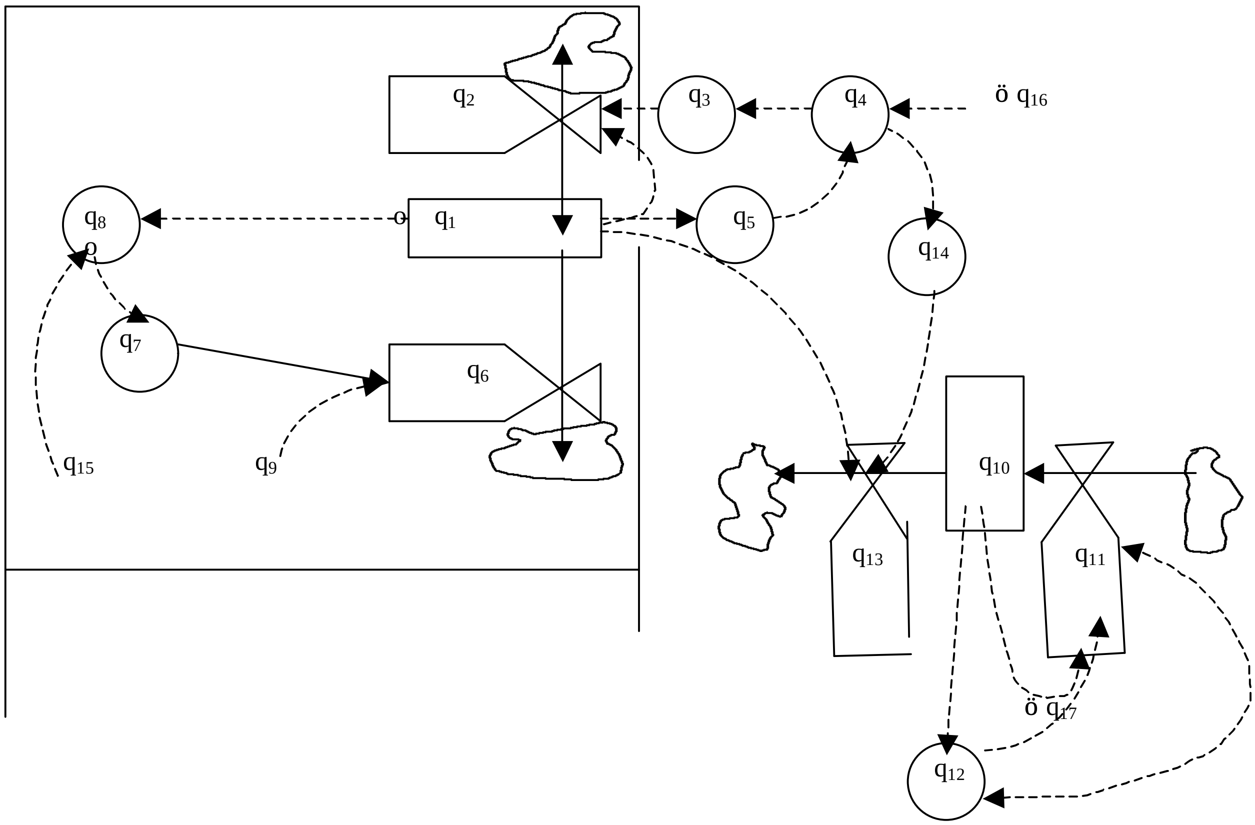


Fig. 8. Stock-and-flow diagram model corresponding to causal diagram model shown in Fig. 1.

TABLE II - Names of Quantities in Example

Quantity	Dimension	Name
1	AA	Deer Population
2	AA/DD	Deer Net Growth Rate
3	I/DD	Growth Rate Factor
4	dim/less	Food Ration
5	CC/AA	Food per Deer
6	AA/DD	Deer Predation Rate
7	AA/(BB·DD)	Deer Kill Rate
8	AA/ZZ	Deer Density
9	BB	Predator Population
10	CC	Food Supply
11	CC/DD	Food Generation Rate
12	DD	Food Regeneration Time
13	CC/DD	Food Consumption Rate
14	CC/(AA·DD)	Food Consumption per Deer
15	ZZ	Area
16	CC/AA	Food Needed per Deer
17	CC	Food Capacity

This analysis has been sufficient to identify all quantities and connectors inside the area of the stock-and-flow diagram model (SDM) shown in Fig. 8 enclosed by a dash/dot line. A similar rationale can be applied to the decomposition of the remaining quantities and connectors in the CDM to yield the SDM shown. In all, two interrogations are required to resolve ambiguities. The example employed is the "Kaibab Plateau" model [18, p. 377], which depicts the growth and decline of a deer population on the north rim of the Grand Canyon (the Kaibab Plateau). The exercise, if carried out in its entirely, would enable the correct identification of all connectors and quantities shown if Fig. 1, producing Fig. 8, which is comparable in form to [18, Fig. S15.2, p. 382]. The names of the quantities used in the example are indicated in Table II. The information depicted in Fig. 8 could also be represented in matrix format through modification of the STM, as shown in Fig. 9.

VIII. Development of Equations

The next step in methodology as discussed in Section I is the determination of the model equations from the stock-and-flow diagram model. Equations for stock variables can be formulated just by inspection of the SDM, whereas equations for rates, auxiliaries, and outputs require the use of dimensional consistency in addition to the information provided in the modified STM (Fig. 9), while all parameters and inputs require specification to a constant. The procedure described herein is one that could be algorithmatized.

It should be clear that the stock equation associated with q_1 is given by $x_1(t + \Delta t) = x_1(t) + \Delta t\{r_2 - r_6\}$. Similarly, the level equation for q_{10} is given by $x_{10}(t + \Delta t) = x_{10}(t) + \Delta t\{r_{11} - r_{13}\}$. Alternatively, these relationships could be formulated in differential equation format: $\dot{x}_1 = r_2 - r_6$, and $\dot{x}_{10} = r_{11} - r_{13}$.

Equations for rates auxiliaries, and outputs are formulated as follows. By proceeding down the column (in the modified STM) associated with any v_i or r_i, the set $A_q(q_i)$, the affector subset, is determined. Clearly, $q_i = f[\{A_q(q_i)\}]$. For example, consider r_2 (r_2 is a function of x_1 and v_3). The possible functional combinations are

$$r_2 = \pm x_1 \pm v_3 \qquad r_2 = \pm x_1/v_3$$
$$r_2 = v_3/x_1 \qquad r_2 = \pm x_1 \cdot v_3.$$

(all blank positions are zeros)

	dimensions	type	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	AA	x_1		I			-I			I					I				
2	AA/DD	r_2	F																
3	I/DD	v_3		I															
4	dimless	v_4			I											I			
5	CC/AA	v_5				I													
6	AA/DD	r_6	-F																
7	AA/(BB.DD)	v_7						I											
8	AA/ZZ	v_8							I										
9	BB	p_9						I											
10	CC	x_{10}					I						-I	I					
11	CC\DD	r_{11}										F							
12	DD	v_{12}											-I						
13	CC/DD	r_{13}										-F							
14	CC/(AA.DD)	v_{14}													I				
15	ZZ	p_{15}								-I									
16	CC/AA	p_{16}				-I													
17	CC	p_{17}													I	-I			

Fig. 9. Modified square ternary matrix corresponding to stock-and-flow diagram model shown in Fig. 8.

Signs can be determined by the signs associated with the connectors. Only one of these equations is dimensionally consistent: $r_2 = \pm x_1 \cdot v_3$.

Now consider the next variable v_3. Evidently, v_3 is a function only of v_4. In this particular instance it is impossible to establish dimensional consistency; it would therefore be appropriate to surmise that v_3 is a tabular function of v_4 and that tubular data are required to define and describe the relationship. The relationship will be denoted here as simply $v_3 = f_3(v_4)$, where $f_3(\cdot)$ is to be determined by tabular information provided by those who are knowledgeable about the system concerned. The remaining relationships for each of the rates and auxiliaries can be established by a similar rationale:

$$v_4 = v_5/p_{16}$$
$$v_5 = x_{10}/x_1$$
$$r_6 = v_7 \cdot p_9$$
$$v_7 = f_7(v_8)$$
$$v_8 = x_1/p_{15}$$
$$r_{11} = (p_{17} - x_{10})/v_{12}$$
$$v_{12} = f_{12}(x_{10}/p_{17})$$
$$r_{13} = x_1 \cdot v_{14}$$
$$v_{14} = f_{14}(v_4).$$

These equations would be written in any 4GL language and ordered properly so that all variables appear on the left-hand side of the equal sign before being used on the right-hand side. The result, shown in Table III, is a subroutine capable of being called a routine that performs the numerical integration. Once parameter values, initial stocks, and tabular functions have been established by a main program, the simulation model shown in Table III could be executed for a given time horizon with the results printed and plotted at regular intervals.

TABLE III
RESULTANT DIFFERENTIAL EQUATION SUBPROGRAM

```
SUBROUTINE MODEL (Z, DZ, N, TIMME)
REAL Z(N), DZ(N )
COMMON /2/  F3(15), F7(15), F12(15), F14(15)
COMMON /3/  F9, P15, P16, P17
X1 = Z(1)
X10 = Z(2)
V5 = X10/X1
V4 = V5/P16
V3 = TABLE (F3,…., V4)
V8 = X1/P15
V12 = TABLE (F12,…., X10/P17)
V14 = TABLE (F14,…., V4)
V7  = TABLE (F7,…., V8)
R2 = X1·V3
R6 = V7·P9
R11 = (P17 - X10)/V12
R13 = X1·V14
DZ(1) = R2 - R6
DZ(2) = R11 - R13
RETURN
END
```

IX. Conclusion

In this paper a new approach to the problem of converting CLD's to SFD's is presented. The approach merges notions taken from digraph theory [17] and system dynamics [3]. The basic postulates of system dynamics were reformulated into set-theoretic definitions, axioms, and a supposition. The reformulation does provide refreshed insight and understanding regarding the hypotheses that underlie and support system dynamics. Whenever the underlying hypotheses of any methodology are explicitly exposed, it becomes easier to question the premises and suggest alternative hypotheses.

Also in this paper, the logical consequences of the axiom set A1-A7 made explicit in [3], [4] or implicit by its application [5]-[7] were explored in the causal diagram context. It was found that certain constructions in the signed digraph dictated the kind or type of quantities and connectors that must necessarily comprise the construction. It was also suggested that algorithms would be capable of detecting the identifying characteristics of these constructions and of performing the quantity and connector classifications in an automated fashion. Such classifications effectively induce a partition on the quantity and connector sets Q and C. Once such classifications are complete, algorithms might then perform the equation compositions. State equations follow directly from the information provided in the modified STM. Rate and auxiliary equations require the additional use of the notion of dimensional invariance.

The equation composing step would be followed by execution of the simulation model, also accomplishable by the use of algorithms. If the computer aids were designed to operate as interactive time-sharing software, then these algorithms would perform interrogations whenever additional information was required, to which a team of participating managers and planners could respond. As previously indicated, planner participation improves the credibility of the model.

As mentioned in the introduction, other strategies for formulating SFD's are possible. Such strategies may usefully employ many of the concepts introduced herein--consistency, the square ternary matrix, set-theoretic definitions and axioms, even the theorems. In the absence of empirical and experimental comparison of various strategies in controlled model formulation exercises, it is difficult to ascertain what strategy seems most appropriate for the broadest class of problems. Certainly each would have advantages and disadvantages. Therefore, the approach of the paper has been to illustrate how set and graph theoretic definitions, axioms, and theorems might be utilized within the context of one of the strategies while avoiding discussions about detailed algorithms employed by any one strategy.

In conclusion, a theory for computer-aided construction of simulation models is discussed in this paper. The computer aids, once developed, could be employed to 1) check the validity and integrity of manually constructed models, 2) assist the modeler to expeditiously construct models, and 3) encourage manager and planner participation in the model construction process.

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