IS DETERMINISTIC CHAOS ONLY A PROPERTY OF MODELS?

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ABSTRACT

This paper presents results of extended experimentation with selected models of social phenomena widely used by the system dynamists in their studies on deterministic chaos. The models selected include various versions of a simple model of migratory dynamics and a model of resource allocation in a firm, and a simple model of long-term economic fluctuations. Chaotic modes seem to appear in each of the experimented model, either due to non-robust or unrealistic rate formulation, or from unrealistic parameter or input specifications or both. Minor changes in the models experimented with, which improve their correspondence to reality, eliminate chaotic modes. The paper raises the issue of the relevance of the chaotic models to real-world phenomena and policy design for system improvement.

KEY WORDS: Deterministic Chaos, Systems, System Dynamics, Models, Computer Simulation

1. INTRODUCTION

Deterministic chaos, a behavior mode discovered in certain nonlinear system dynamics models has lately drawn much attention from scholars and practitioners. System Dynamics Review brought to its readers a fine distillation of the important writings on the subject in its Volume Four double issue published in 1988. Deterministic chaos appears in the models explored in this issue as a pattern of behavior each cycle of which is different from the preceding cycles while no two cycles are systematically related. A chaotic mode is usually exhibited only with certain parameter values and exogenous inputs lying within narrowly specified ranges. The relevance to real-world systems of the chaotic behavior appearing in these highly aggregate and simple models is, however, unclear; nor has experimentation with them to-date evolved any principles for system improvement [Andersen 1988; Mosekilde, Aracil and Allen 1988].

This paper attempts to examine the relevance of the chaotic modes appearing in the models to real-world phenomena and to the agenda of system improvement implicit in the use of system dynamics method. The paper assumes readers have a working knowledge of system dynamics and some familiarity with the models of chaos appearing in the system dynamics literature. Further experimentation with the models of chaos selected from those widely used by the system dynamists to illustrate the phenomenon, and interpretations of the experimental results serve as bases for analysis. Chaotic modes seem to appear in each model either due to non-robust or unrealistic rate formulation, or from unrealistic parameter or input specifications or both. Minor changes in the experimented models, that improve their correspondence to reality, eliminate chaotic modes.

Our experiments suggest that chaotic behavior may not be inherent in systems represented by the experimented models, but rather that it appears due to mis-specifications in the models. And if this observation can be generalized, the appearance of a chaotic mode in a model may only signal the existence of anomalies in the model, calling for a revision of its structure and parameter specifications as suggested by Forrester and Senge (1980).

Chaotic modes can also not easily be identified in the real world because of the long time- constants involved in the process, while their conditional appearance in models provides little help in interpreting system behavior from its decision structure. Thus, models of chaos, although interesting artifacts, might appear irrelevant to the agenda of unification of knowledge and policy design for system improvement, which are widely advocated as important foci for the system dynamics method [Forrester 1987].

2. EXPERIMENTAL DESIGN

Three areas in which models of chaos have been developed were selected for further analysis: migratory dynamics, production and operations management, and macro-economics. Five relatively simple models were selected for experimentation, including the Waycross and Weidlich models of migratory dynamics [Rasmussen and Mosekilde 1988, Mosekilde, et. al (1985), Reiner et. al. (1988), Richardson and Sterman 1988], two versions of a model of resource allocation in a firm used respectively by Mosekilde, et. al.(1988) and Andersen and Sturis (1988), and a simple model of the economic long wave developed by Sterman and discussed in Rasmussen, et. al. (1985). Limitations of time and resources did not allow an exhaustive examination of all models discussed in the literature.

The first task undertaken was to replicate the chaotic modes. This turned out to be a straight forward, although laborious process, thanks to the excellent documentation provided by the various authors whose work was used. To digress a little, we greatly appreciated the thoroughness and accuracy of the published materials accessed, since in our experience such clear documentation is quite uncommon. While replicating chaotic modes, we also carefully examined the structure of the models for robustness. We even constructed simple MACRO to keep track of the instances in which stocks assumed negative values (see Appendix). We also prepared extended phase plots covering all important stocks and flows in each model and carefully examined them for negative or otherwise absurd regions.

Because of their simple and highly aggregated structure, the models used in general made many simplifying assumptions. These assumptions were carefully debated by the authors for validity and accepted unless glaring errors or inconsistencies were found. In each case, a single most inconsistent assumption or equation was identified and minimally modified to eliminate the anomalies discovered. After this modification, the models were re-simulated with the same parameter sets as those giving rise to chaos in the original models. In all cases, the chaos disappeared. The exact nature of the modifications made was different in each case, but all enhanced realism. The details of the experiments conducted and an analysis of results appear in the following sections.

3. WAYCROSS AND WEIDLICH MODELS OF MIGRATION: *CHAOS FROM NON-ROBUST RATE EQUATION FORMULATION*

Both the Waycross and Weidlich models deal with two hypothetical ethnic groups (say Itrachians and Lomanians) and three neighborhoods (say Richmond, Jonesboro and Camden). While the two ethnic groups prefer to live with their own kind, living in an Itrachian neighborhood is of positive value to Lomanians while the Itrachians view living in a Lomanian neighborhood to be of negative value.

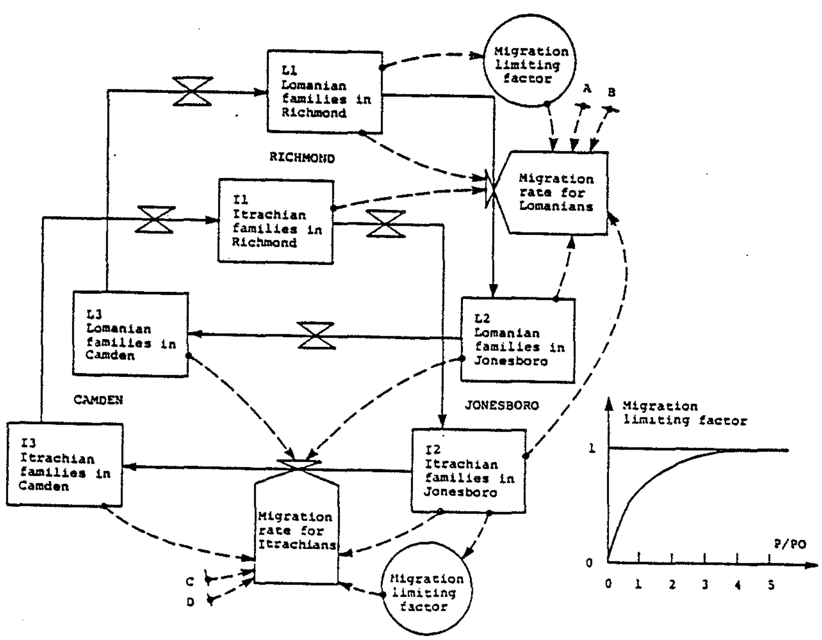
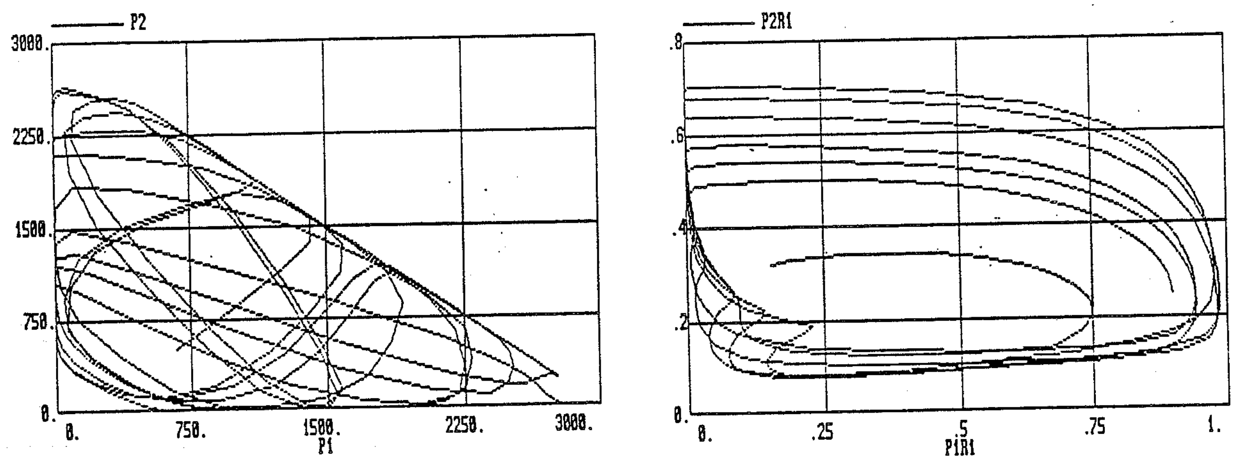


Figure 1: Flow diagram of Waycross/Weidlich models

Hence the population in the three neighborhoods is constantly on the move. The total number of families in each ethnic group is assumed to remain constant over time. A flow diagram of the Waycross model reproduced from Mosekilde et. al.(1988) is shown in Figure 1. The Weidlich model is similar in structure.

The Waycross model employs six net migratory flows while the Weidlich model uses twelve flows representing the in- and out- migration of each ethnic group in each district. Both models use complicated limitations to prevent out-migration exceeding source population, but unsuccessfully; according to our observations, chaos occurs in these models for selected parameter sets due to non-robust rate equations repeatedly causing flows to exceed the stocks feeding them. The chaotic modes for one of the parameter sets suggested by the respective authors are shown in Figures 2: a and b.



a) Waycross Model *b) Weidlich Model*

Figure 2:Phase plots of Waycross and Weidlich models with Chaotic parameter sets

Led by the evidence of negative stock values shown through our switching equations, we closely examined the migratory flows and found them to be non-robust in both models.

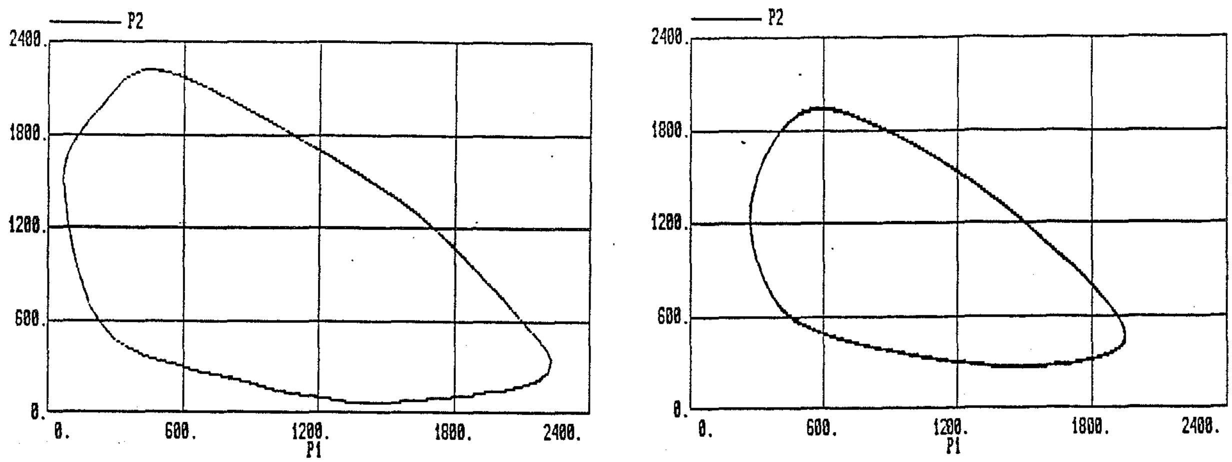
The Waycross model assumes that the inclination of a population (P_1 or P_2) to move from district i to district j (IMP_{1ij}, or IMP_{2ij}) is given by a linear combination of the differences of the two populations between the two districts.

For example, $$IMP_{1ij} = A*(P_{1j} - P_{1i}) + B*(P_{2j} - P_{2i})$$

where i and j represent respectively donor and recipient districts. Migration is then obtained by dividing the inclinations by a constant.

So far, the model is still linear and produces growing oscillations, which periodically leads to negative population in one or more of the districts after some time - a clearly meaningless result. Non-linear limiting factors have been introduced into the model to slow down the rate of migration out of a district as the number of remaining families in it approaches zero. Additionally, a number of shift functions are also applied, adjusting the limiting factors corresponding to the populations being reduced (Mosekilde et al. 1985). Unfortunately, the complicated limiting factors and shift functions cannot stop the populations from becoming negative during the simulation, which seems to create aperiodic behavior, evolving in a random fashion giving the impression of chaos.

Two solutions were applied separately to correct the stock negativity problem. First, a crude solution, the in- and out- migration rates were separated and a MIN function used to ensure that outflows do not exceed stocks. Second, a relatively more refined solution, the weighting functions were normalized by the total population making sure that a fractional flow out of a stock at no time exceeds the stock. Modified sets of equations for the two cases appear in the appendix and the corresponding simulations are shown in Figures 3: a and b for the same parameter set as for Figure 2. The system settles into a limit cycle instead of continuing in a chaotic mode. The modified models were also simulated for the other parameter sets, identified as chaotic in the original model and also for many others. There was no evidence of chaos, which is not surprising since the model contains only two conservative systems inter-connected through negative feedback, which is very likely to settle into a limit cycle unless something outrageous is happening to the stocks.



a)Limiting outflows not to exceed stocks *b) Normalizing fractional flow rates*

Figure 3: Limit cycles generated by the modified versions of the Waycross model

The Weidlich model separates in- and out- migratory flows and employs a set of exponential weighting functions to compute fractional flow rates. However, this does not alleviate the problem of flows assuming strange values. The fractional migratory flow of a population (P_1 or P_2) from district i to district j (FR_{1ij}, or FR_{2ij}) is given by a an exponential combination between the differences of the two populations of the two districts.

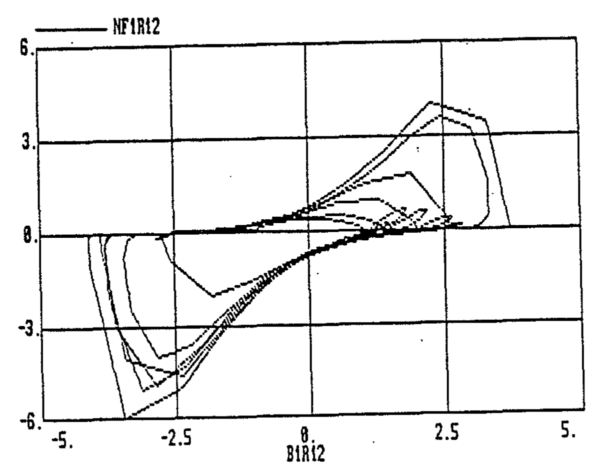


Figure 4: Net flow between two regions vs population imbalance in Weidlich model for the chaotic parameter set

For example, $FR_{1ij} = Exp[k_{1i}*(P_{1j} - P_{1i}) + k_{1j}* (P_{2j} - P_{2i})]$

It is evident from this function that a zero value of the argument of the exponential function will be returned for many values of population not necessarily representing a balance in size. Figure 4 shows the scatter plot between one of the population imbalances in one of the pairs of districts and the net flow between them. The net flow corresponding to an imbalance of zero is never zero, while it stays at zero for many high values of imbalance, both positive and negative. Such flow equations are evidently clumsy and should be replaced by one of the modified sets suggested for the Waycross Model, which would create a limit cycle rather than chaos.

4. MODELS OF RESOURCE ALLOCATION IN A FIRM: *CHAOS FROM UNREALISTIC INFORMATION BASIS FOR A DECISION*

Anderson and Sturis (1988) and Rasmussen and Mosekilde (1988) have used the same model of resource allocation in a firm to produce chaotic modes, although they refer to it differently and use slightly differing parameters and slopes of table functions. The model deals with resource allocation between production and sale activities in a firm whose total resources are fixed. Figure 5 is a flow diagram of the model reproduced from the paper by Andersen and Sturis (1988).

A comparison of the actual and desired inventory (DI) is the basis for the resource allocation decision; the desired inventory is fixed. Production is a linear function of workforce in the production activity, but involves a 30-day third-order production delay. Sales are linearly related to the number of customers who are recruited in proportion to the sales-force and normally exit the customer pool after an average stay of about eight years (actually, 3,000 days). The loyalty of the customers, however, also depends on the ratio of the inventory level to the fixed desired inventory, presumably a proxy for delivery delay.

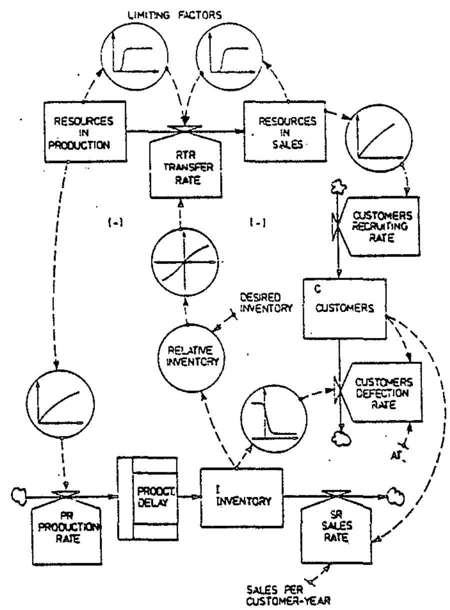


Figure 5: Flow diagram of the resource allocation model

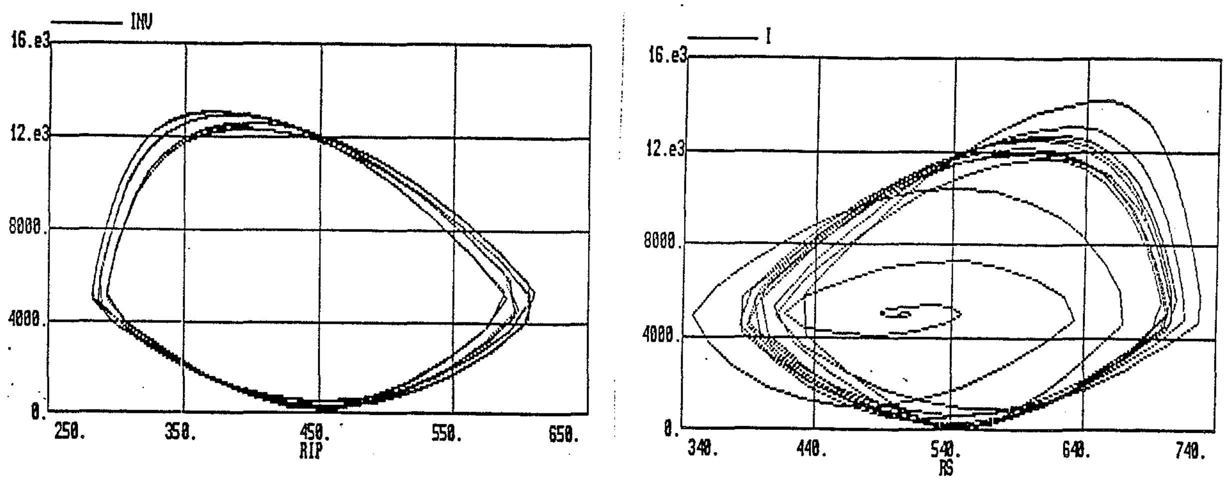
Both Andersen and Sturis (1988) and Rasmussen and Mosekilde (1988) found that the model's chaotic regime contains complicated sequences of multiple periodicities. Figures 6: a and b show chaotic modes produced by the two models over one of the suggested parameter spaces.

Led again by our switching functions which keep track of the incidence of negativity in the stocks, we examined carefully the logic of the related flow rates. The model in question treats customer reaction as a function of product availability. Product availability is then defined as the ratio of the actual inventory to the desired inventory, which is assumed to be fixed. This is anomalous since, treated this way, availability does not directly depend on demand. For example, an inventory level of 50 units will affect a demand of 100 units in the same way as a demand of 5 units, turning customers away in both cases.

In reality, customers would view delivery delay, or a proxy such as inventory coverage, as a basis for their orders, not a fixed fraction of the absolute amount of inventory. Within the frame work of the model, this can be easily accomplished by making desired inventory DI a function of order rate as follows:

DI.K=CUS.K*(ASPC/T)*NIC

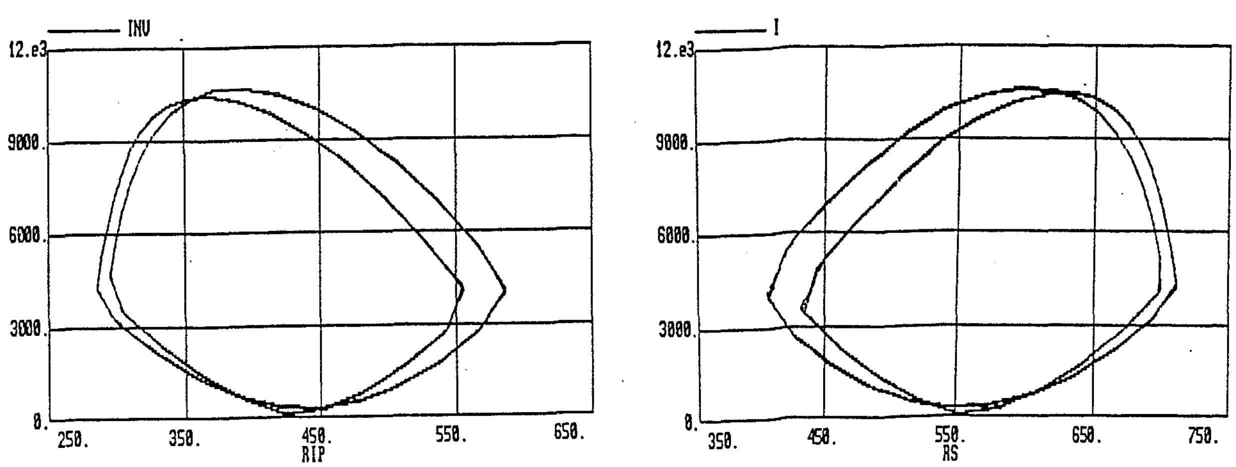
where CUS denotes Customers, ASPC/T is Average Sale Per Customer per unit Time and NIC is Normal Inventory Coverage. The ratio of inventory to desired inventory now represents normalized inventory coverage.



a) Andersen and Sturis *B) Rasmussen and Mosekilde*

Figure 6:Phase plots for two versions of the resource allocation model for chaotic parameter sets

Phase plots of simulations of the modified Anderson and Sturis and Rasmussen and Mosekilde versions of the model for the same parameter sets as in Figure 6 are placed at Figures 7:a and b. The behavior of neither model shows chaos, but sustained oscillations after the anomalies in the information basis for the customer response have been removed. Similar non-chaotic behavior was also obtained with other parameter sets identified as chaotic in the original model.



a) Andersen and Sturis *B) Rasmussen and Mosekilde*

Figure 7: Phase plots of two versions of the resource allocation model after modification

5. STERMAN'S MODEL OF LONG WAVE: *CHAOS FROM UNREALISTIC RESPONSE TO INFORMATION AND EXCESSIVE EXOGENOUS DISTURBANCE*

Sterman's model of long wave contains a simple and generally robust structure which produces chaotic behavior when subjected to an unrealistically high exogenous disturbance and by making one of its behavioral functions extremely steep. The model represents an aggregate production sector that orders capital from itself depending on the required production capacity. For normal parameter values, the model exhibits a characteristic limit cycle. The details of the structure of this model are discussed in Rasmussen et. al. (1985). A flow diagram reproduced from the original paper is given in Figure 8.

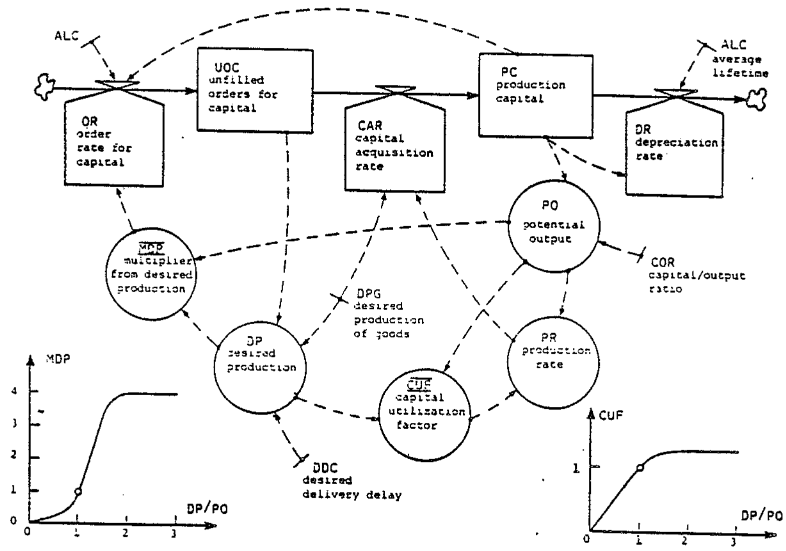


Figure 8:Flow diagram of Sterman's model of long wave

A critical relationship in the model is a table function used to obtain a multiplier from desired production (MDP), which depends on the ratio between desired production and potential output. A chaotic mode, reproduced in Figure 9, is obtained from the model by using a very steep table function for MDP together with a sinocidal disturbance of rather large amplitude (20%) in the desired production of goods (DPG), which is assumed to be constant. This chaotic mode, however, disappears when the amplitude of the exogenous disturbance is decreased to 5% or when the slope of MDP is reduced to a realistic value, as shown in Figure 10. The modes appearing after these changes show limit cycles represented in the phase plots of Figures 11: a and b.

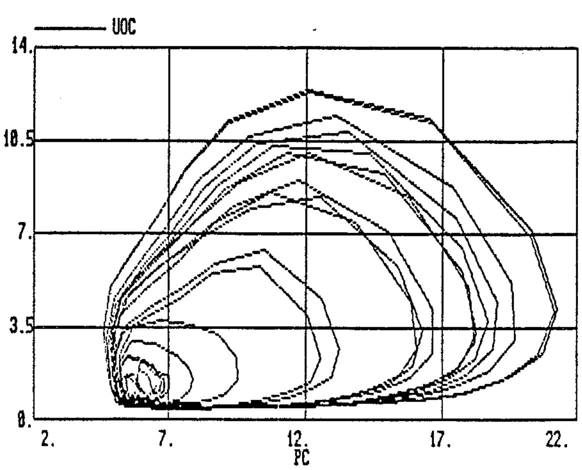


Figure 9: Phase plot showing chaotic mode

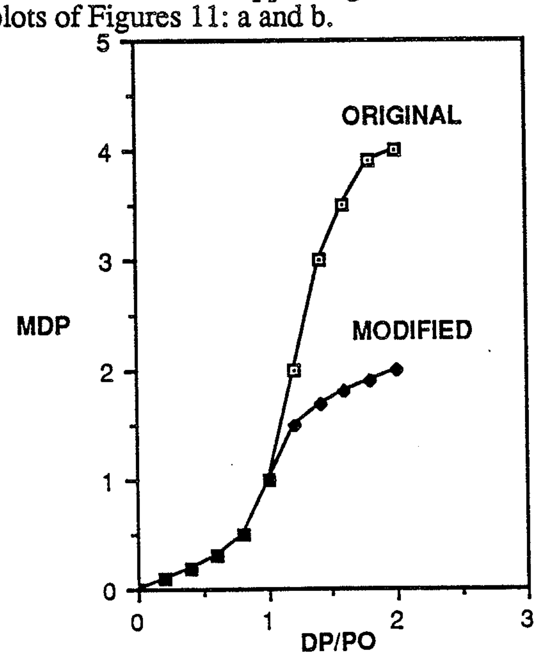
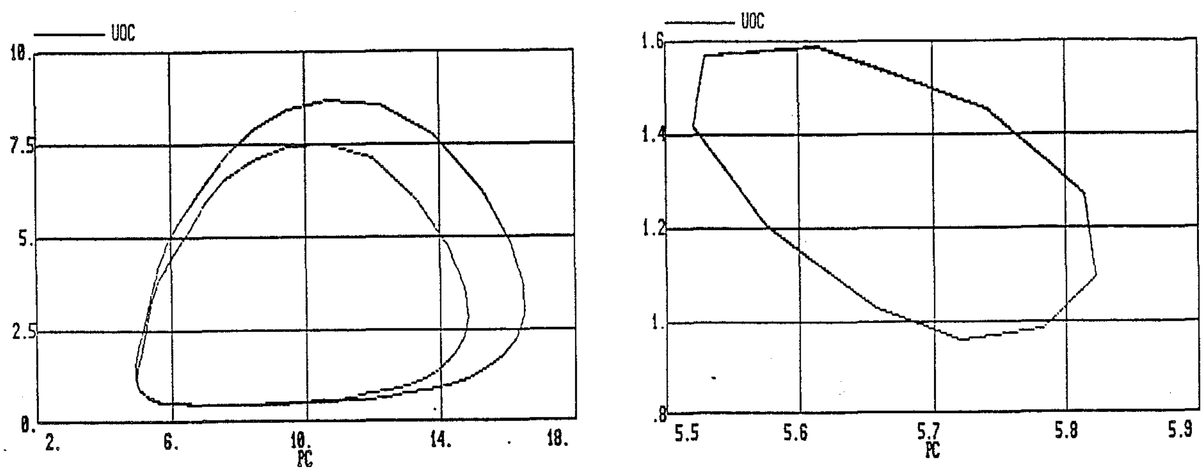


Figure 10: Table function creating chaotic mode



a) Reducing exogenous disturbance *b) Reducing slope of sensitive table function*

Figure 11: Limit cycles generated by Sterman's model after modifications

Most experienced system dynamists would agree that to preserve the integrity of a system and maintain the dominance of its internal trends, outside disturbances should be kept small so that they do not overpower the system forces. Also, while the behavior of a model may often not be sensitive to the slopes of its table functions, this parameter should be reasonable to make sure it corresponds to some extent with reality. Thus, while excessive exogenous disturbance introduces experimental error, an over-responsive table function raises doubts about the face validity of the model. A chaotic mode arising out of a combination of these two factors cannot be attributed to reality.

An abstract and highly aggregate model with a somewhat simple structure, such as Sterman's, would often lack the compensating feedback loops that render a model parameter insensitive [Saeed 1989]. The presence of a chaotic parameter region in such a model may only indicate that the parameters must be carefully and realistically estimated or model structure modified.

6. INTERACTIVE USE OF MODELS
CHAOS FROM MODULATION OF PARAMETER SENSITIVE MODELS WITH UNCOMMON PARAMETER SETS

Sterman has also reported the occurrence of chaos in models of human behavior using parameters related to a significant minority of the subjects participating in experiments using models interactively (20%). He also reports that the response of these subjects to decision-making information given to them was more aggressive than for the majority whose parameter set produced stable behavior (Sterman 1985).

Experiments with abstract models of systems having a limited feedback structure and a high sensitivity to parameters can indeed give varied results depending on the personal attitudes of the actors creating many of the parameter sets. More realistic models containing a compensating feedback structure similar to the real world would be parameter insensitive and would often display stable behavior over a wide variety of parameter sets. The question is whether the parameter-sensitive abstract models with unusual parameter sets have any real world counterparts. Our prior experience in systems modelling and the experimentation with the various models discussed in this paper suggests otherwise.

7. CONCLUSION

This paper has demonstrated with the help of experimentation with well-known models of chaos appearing in the system dynamics literature that the source of chaos might be an imperfection in the model structure and parameters rather than a manifestation of a real-world process.

		Sources of Chaos			
	Models	Non-Robust Rate Equation Formulation	Unrealistic Information Basis	Unrealistic Response to Information	Excessive Exogenous Disturbance
1.	Migratory Dynamics: Waycross/ Weidlich	*			
2.	Business Policy: Rassmussen/ Andersen		*		
3.	Macro-Economics: Sterman, Long Wave			*	*

Table 1: Sources of Chaos in the Experimented Models

Five models abstracting migratory, managerial, and macroeconomic processes were experimented with. The occurrence of chaos was traced to the four main sources summarized in Table 1. These are, non-robust rate equations, an unrealistic information basis for a decision, an unrealistic order of magnitude of response to information, and excessive exogenous disturbance. Additionally, chaos might appear in behavioral experiments using simple models not incorporating a compensating feedback structure when these are modulated by non-moderate decision parameters.

We are of the view that experimentation with models alone without reference to realism and without specific a policy focus is more alchemy than life science. Since the traditional practice of system dynamics to date has emphasized combining science with real-world problem-solving, we are not sure how the study of chaos can be related to this objective. Albeit, we found chaos to be an interesting artifact and enjoyed experimenting with the models of chaos out of intellectual curiosity. More work is needed to find a real-world relevance for the phenomenon of chaos.

APPENDIX

1. MACRO for keeping track of incidence of flows exceeding stocks

```
MACRO COUNTER(ARG)
L COUNTER.K=COUNTER.J+(DT)($COUNT.JK)
N COUNTER=0
R $COUNT.KL=CLIP(0,1/DT,ARG.K,0)
MEND
```

where ARG is the stock variable being monitored

2. Modified equations for migratory flows in Waycross Model using MIN function to avoid outflows exceeding stocks

```
A IMP12.K=A*(P2.K-P1.K)+B*(I2.K-I1.K)
A EMP12.K=IMP12.K*(B12.K*DP1.K+(1-B12.K)*DP2.K)
R MRP12.KL=CLIP(P12.K,-P21.K,EMP12.K,0)
A P12.K=MIN(EMP12.K,P1.K)/AMD
A P21.K=MIN(-EMP12.K,P2.K)/AMD

A IMP23.K=A*(P3.K-P2.K)+B*(I3.K-I2.K)
A EMP23.K=IMP23.K*(B23.K*DP2.K+(1-B23.K)*DP3.K)
R MRP23.KL=CLIP(P23.K,-P32.K,EMP23.K,0)
A P23.K=MIN(EMP23.K,P2.K)/AMD
A P32.K=MIN(-EMP23.K,P3.K)/AMD

A IMP31.K=A*(P1.K-P3.K)+B*(I1.K-I3.K)
A EMP31.K=IMP31.K*(B31.K*DP3.K+(1-B31.K)*DP1.K)
R MRP31.KL=CLIP(P31.K,-P13.K,EMP31.K,0)
A P31.K=MIN(EMP31.K,P3.K)/AMD
A P13.K=MIN(-EMP31.K,P1.K)/AMD
```

3. Modified equations for migratory flows in Waycross Model using population weights to assure fractional out-migration rates are less than unity.

```
A IMP12.K=(A*(P2.K-P1.K)+B*(I2.K-I1.K))/(3000*(A+B))
* Inclination to Migrate for P between 1 & 2
R MRP12.KL=IMP12.K*clip(p1.k,p2.k,imp12.k,0)*MF
* Migration Rate for P from 1 to 2
C MF=.2                                Mobility Fraction

A IMP23.K=(A*(P3.K-P2.K)+B*(I3.K-I2.K))/(3000*(A+B))
R MRP23.KL=IMP23.K*clip(p2.k,p3.k,imp23.k,0)*MF

A IMP31.K=(A*(P1.K-P3.K)+B*(I1.K-I3.K))/(3000*(A+B))
R MRP31.KL=IMP31.K*clip(p3.k,p1.k,imp31.k,0)*MF
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REFERENCES

Andersen, D. F. 1988. Foreword: Chaos in System Dynamics Models. System Dynamics Review. 4(1-2):3-13.
Andersen, D. F. and J. Sturis. 1988. Chaotic Structures in Generic Management Models. System Dynamics Review . 4(1-2):218-245.
Forrester J. W. 1987. Lessons from System Dynamics Modelling. System Dynamics Review. 3(2):136-149
Forrester J. W. and P. M. Senge. 1980. Tests for Building Confidence in System Dynamics Models. In Legasto et. al. (eds). System Dynamics. Amsterdam: North Holland
Mosekilde, E., S. Rasmussen, H. Jorgensen, F. Jaller, and C. Jensen. 1985. Chaotic Behavior in a Simple Model of Urban Migration. Proceedings of the 1985 International System Dynamics Conference. Keystone, CO:System Dynamics Society.
Mosekilde, E., J. Aracil, and P. M. Allen. 1988. Instabilities and Chaos in Nonlinear Dynamic Systems. System Dynamics Review. 4(1-2):14-55.
Rassmusen, S. and E. Mosekilde. 1988. Bifurcation and Chaos in a Generic Management Model. European Journal of Operations Research. 1: 80-88.
Rasmussen, S., E. Mosekilde and J. D. Sterman. 1985. Bifurcation and Chaotic Behavior in a Simple Model of the Long Wave. System Dynamics Review. 1:92-110
Richardson G. P. and J. D. Sterman. 1988. A Note on Migratory Dynamics. System Dynamics Review. 4(1-2):200-207
Saeed, K. and S. Z. A. Gillani. 1989. Policy Design With A Parameter Sensitive Model. An Illustration with a Real Case. Bangkok: AIT
Sterman, J. D. 1988. Deterministic Chaos in Models of Human Behavior: Methodology Issues and Experimental Results. System Dynamics Review. 4(1-2):148-178.