# Tourism and Equilibrium Quantities: a dynamic perspective

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# Abstract

The systemic approach in studying tourism is firmly accepted in literature because of the complexity of the topic both from the supply side (the heterogeneity of goods and services making the tourism product) and the demand side (not every operator in the tourism network chain has a direct contact with tourists).

To face this complexity the Input-Output analysis, theorized by Wassily Leontief, is widely used in empirical studies of tourism. This methodology, however, gives just a snapshot, even very detailed in some cases, of the economic structure under study but gives very few insights from a dynamic point of view.

To overcome this limitation the Dynamic Input-Output (DI-O) model, implemented with System Dynamics methodology, is introduced in this paper.

Moreover, some considerations about the technical sustainability of the production process are made possible by the proposed model.

Keywords: Systemic approach; Tourism; Input-Output Model; Wassily Leontief; Dynamic Input-Output Model; System Dynamics.

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# 1. Introduction

In the economic literature a very well known model, which takes the complexity and interrelations of a system's activities into account, is the Input-Output model (also called the model of the structural interdependences).

Associated with Wassily Leontief's name (Leontief 1936), the Input-Output analysis has characterised, from a theoretical point of view, a current of thought describing production as a circular process.

On the applied and empirical side, the Input-Output model represents the foundations of economic policy and planning. In fact, the Input-Output model is a very powerful tool to describe and understand the sectoral interrelations in an economic system and to highlight the links between the intermediate and final demand. This kind of model allows the researcher to complete, in a disaggregated form, the analysis of the fundamental macroeconomic variables carried out in terms of aggregated supply and demand.

Moreover, the Input-Output methodology allows the analyst to follow the flows of goods between the different regions of an economy and to value the contribution of such regions to the national income.

An Input-Output model has a normative value as well. Indeed, by using the table of the sectoral interrelations, it is possible to assess the effects of final demand variations on the productive structure of a particular country and, as a consequence, on its whole economy. This ability makes this kind of model a very useful tool to control the efficiency of the sectoral policies.

Therefore, it is not by chance that the Input-Output matrices are the fundamental tool for economic planning and that their maximum diffusion has been observed in former Socialist countries with a planned economy.

Even in the most recent models such as Computational General Equilibrium (CGE) models, which assess the impact of a wide variety of economic policies and the effect of changes in the economic agents' expenditures, an Input-Output analysis represents the starting point for the research.

In the empirical studies about tourism the Input-Output methodology has spread and has become so popular for two main reasons: firstly because of the heterogeneity and variety of goods and services the tourism product is made up of (the supply side), involving a relatively great number of industrial sectors; and secondly for the systemic nature of tourism that can be only captured by a disaggregated and, at the same time systematic, study of the tourism economy.

Moreover, an Input-Output model can be applied to tourism in two ways not mutually exclusive: the model can be used as a base, together with a statistical sampling about the tourists' expenditure, to construct systems of accounting for tourism and/or it can be used for the economic regional analysis and to value the importance and the effects of tourism in a given destination.

Yet, since tourism is not a homogeneous and fully defined productive sector it is not possible to identify a row and a column headed to tourism in the matrix of the sectoral interdependences. To overcome this problem the tourism demand has to be considered a component of the total demand, and the production of the different sectors needed to satisfy that demand needs to be measured separately.

The integration of tourism activities among them and with other productive sectors and the tourism's capacity for generating income, occupation, imports and

exports, can therefore be assessed looking at the tourists' expenditures and including the direct and indirect production needed to satisfy the tourism demand.

Very good examples of the use of the Input-Output methodology for a regional economic planning are the articles of Campisi and Gastaldi (1996), McMenamin and Haring (1974), Pigozzi and Hinojosa (1985), Ralston et al (1986), Round (1983), Stevens et al (1983), Stevens and Rose (1985).

In Italy the so called "Scuola di Venezia" can surely be considered the first and main centre for tourism studies based on the Input-Output matrices, beginning with the article of Costa e Manente (1985) that has evolved into the annual Report of Italian Tourism at its twelfth edition in 2003.

Apart from this, studies dealing with the dimension of the regional tourism industry and its importance to the local economic system are more rare. In this research field the survey of Casini Benvenuti *et al.* (1983) for Tuscany, Shaffer (1985) for Hawaii, Manente and Minghetti (1996) for Venetian, Archer and Fletcher (1996) for Seychelles, Cao and Asay (2002) for Sardinia, and Lee and Choi (2004) for Gangwon and Jeju regions are a few examples.

The model presented in this paper is part of a research project, which is just at the beginning, investigating the characteristics and macroeconomic effects of the tourism demand in a southern Italian region. After having pointed out the limits of the Input-Output analysis, this article shows how System Dynamics methodology can be used to overcome those limits.

# 2. Input-Output model's limits

In spite of the diffusion and importance of Input-Output models in tourism literature these models have several limits.

First of all, the production technologies described by the technical coefficients have a constant productivity and, thus, the model assumes that the demand of production inputs grows proportionally with the quantity produced. In other words, the methodology gives a snapshot, even very detailed in some cases, of the economic structure under study but gives very few insights from a dynamic point of view. Moreover, such a limiting assumption prevents a description of possible technological developments.

Secondly, the relationships are linear meaning any non-linearity in the economic system is excluded.

Thirdly, sectors are homogeneous inside. Of course, this problem can be solved with more disaggregated and, therefore, bigger matrices; but this solution increases the computational complexity and the difficulties in gathering the needed information.

Finally, using tables of sectoral interdependences expressed in currency units requires the assumption of constant prices to allow a reliable assessment of the production technology.

The model presented in this paper, even in its simplicity and generality, solves the first two problems listed in the limitations of the Input-Output analysis.

In fact, the Dynamic Input-Output (DI-O) model allows us to relax the assumption of technical coefficients' constant productivity and, therefore, gives much more insight into the dynamic behaviour of the economic system than the Input-Output model.

## 3. The Economic Model

In the following model a very simplified economy is considered. Two goods, wheat and iron, are produced and the production process uses the same goods as production units. The economy is closed meaning that neither imports nor exports take place with other economies.

Let  $q_1$  be the quantity of wheat and  $q_2$  the quantity of iron produced in a certain period of time. It is therefore possible to represent the production process as

$$X_{11} \oplus X_{21} \oplus L_1 \to q_1$$
$$X_{12} \oplus X_{22} \oplus L_2 \to q_2$$

where the symbol  $\oplus$  indicates the logic addition.

Let's consider, for instance, the wheat production (output 1 in the model). The first expression states that in the wheat industry in order to produce  $q_1$  units of wheat  $X_{11}$  units of wheat (as seeds),  $X_{21}$  units of iron (for ploughs, for instance) and  $L_1$  units of labour are used.

In the same way, the production of iron in the iron industry is expressed by the second formula.

The economic system normally produces a surplus equal to the difference between the total quantity of goods produced and the total consumption of production inputs:  $S_1 = q_1 - \sum_{i=1,2} X_{1i}$ ,  $S_2 = q_2 - \sum_{i=1,2} X_{2i}$ . This surplus is what the economy can

consume, invest or destroy without threatening the economic system in terms of inputs needed for it to work. Indeed, if  $S_1$ ,  $S_2 < 0$ , the system is not vital and it's doomed to extinction.

#### 3.1 The Input-Output analysis

The economic system described can be represented by a double-entry table in which a column labelled to the final demand of wheat and iron is added in order to consider that goods are used both as production inputs and as consumption goods.

	Sector 1	Sector 2	<b>Final demand</b>	<b>Total Demand</b>
Sector 1	$X_{11}$	$X_{12}$	<i>Y</i> 1	$q_1$
Sector 2	$X_{21}$	$X_{22}$	<i>Y</i> 2	$q_2$
Labour	$L_1$	$L_2$		
Production	$q_1$	$q_2$		

Table 3.1.1.: Table of intersectoral transactions

Table 3.1.1. is the Input-Output table, whereas, the table whose rows and columns contain the production units (it is a square matrix by construction) is the transactions matrix.

Provided the needed information, the two-sector example can be easily generalized to *n* production sectors with *n* being very big but finite.

The table, read in the direction of the rows, shows how the production of each sector  $(q_i)$  satisfies the demand for production units  $(X_{ij})$  and the final demand  $(y_i)$ . The same table, read in the direction of the columns, describe the demand of production units for each sector and the labour input.

The exchange of goods taking place among the different industries is described by the transactions matrix inside the table.

Despite the limitations due to the simplicity of the example, it's already plain that between the different economic sectors there exist tight interconnections: each industry sells its products to the other industries and from the same ones buys the production inputs. Each industry, moreover, sells to the end-consumers who buy the products with the salary they earned by working in the production process.

Dividing each production unit in the transactions matrix by the quantity produced, one gets the following representation in terms of production coefficients:

$$a_{11} \oplus a_{21} \oplus l_1 \rightarrow 1$$
 unit of wheat  
 $a_{12} \oplus a_{22} \oplus l_2 \rightarrow 1$  unit of iron

where  $a_{ij} = X_{ij}/q_j$  is the quantity of the production input *i* used to get a unit of product *j* and  $l_j = L_j/q_j$  is the quantity of labour units employed to produce a unit of product *j*. In every Input-Output analysis of the economic system these ratios are considered constants. In other words, the Input-Output approach assumes that the production process is characterized by a constant yield of the production inputs and, therefore, the demand of production inputs (wheat and iron in this example) and labour inputs rises proportionally with the output produced.

Such an assumption makes the model static and excludes any non-linearity in the economic system.

#### 3.2 The Dynamic Input-Output (DI-O) model.

The DI-O model presented in this article relaxes the assumption that the technical coefficients of production are constant. Therefore, the coefficients of the Input-Output model  $a_{ij}$  are replaced with new coefficients  $b_{ij}$  that will vary according to the dimension of the production process. In the DI-O model, thus, the production coefficients of wheat and iron become a function of the quantity produced. In mathematical terms:

$$b_{ij} = f(q_j)$$

In this case, moreover, the functions describing the production processes can have a non-linear form.

The model is divided into a quantities system and a prices system where the equilibrium quantities,  $q_1^*$  and  $q_2^*$ , and the equilibrium prices,  $p_1^*$  and  $p_2^*$ , are respectively calculated.

The production coefficients are the parameters of the model whereas r (the profit rate),  $w_1$  (the salary in sector 1) and  $w_2$  (the salary in sector 2) are exogenous variables. Figure 3.2.1. schetches the DI-O model.



Figure 3.2.1.: The duality of the DI-O model

### 3.2.1 The quantities system

In the quantities system the equilibrium condition between supply and demand can be formalized by the following expressions:

$$b_{11}q_1 + b_{12}q_2 + y_1 = q_1$$
 [1.1a]

$$b_{21}q_1 + b_{22}q_2 + y_2 = q_2$$
 [1.1b]

The equations of the system [1.1] can be rearranged to take the unknown variables out of data. Then, the system becomes:

$$(1-b_{11})q_1 - b_{12}q_2 = y_1$$
 [1.2a]

$$-b_{21}q_1 + (1 - b_{22})q_2 = y_2$$
 [1.2b]

The system [1.2] can be written using vectors and matrices, with I being the identity matrix, in the following compact form:

$$(I-B)q = y \qquad [1.2]$$

with

$$(I - B) = \begin{pmatrix} 1 - b_{11} & -b_{12} \\ -b_{21} & 1 - b_{22} \end{pmatrix}$$
 the net output matrix;

 $q = [q_1, q_2]'$  the column vector of production levels;  $y = [y_1, y_2]'$  the column vector of final demand.

If the determinant  $\Delta$  of the matrix (I - B) is different from 0, in mathematical terms

$$\Delta = |\mathbf{I} - \mathbf{B}| = (1 - b_{11})(1 - b_{22}) - b_{12}b_{21} \neq 0$$

the inverse matrix  $(I - B)^{-1}$  of the net output matrix exists and, therefore, the solution vector of the system [1.2] can be found multiplying the two members of the system by  $(I - B)^{-1}$ :

$$(I-B)^{-1}(I-B)q = (I-B)^{-1}y$$

getting

$$q^* = (I - B)^{-1} y$$

where  $q^* = [q_1^*, q_2^*]$  is the vector of the equilibrium quantities.

Quantities  $q_1^* = f_1(y_1, y_2)$  and  $q_2^* = f_2(y_1, y_2)$  are the solutions of the system and they represent the sectoral productions that allow to satisfy the final demand and the demand of the production inputs. In other words, the solutions take into account both the final production and the sectoral interdependences.

Of course, the equilibrium solutions  $q_1^*$  and  $q_2^*$  must be positive. This means that the following conditions, called Hawkins-Simon conditions, must be satisfied:

$$l - b_{11} > 0$$
 [1.3a]

$$l - b_{22} > 0$$
 [1.3b]

$$\Delta = (1 - b_{11})(1 - b_{22}) - b_{12}b_{21} > 0$$
 [1.3c]

The  $1 - b_{ii} > 0$  (i = 1, 2) conditions state that each sector must be characterized by a net positive production of the output; the third condition states that the output of each production process must be bigger than the quantity demanded, directly and indirectly, in the economic system.

Using the equilibrium quantities,  $q_1^*$  and  $q_2^*$ , it is also possible to calculate the total occupation  $N^*$  and the vector of sectoral occupation related to the vector of final demand  $y = [y_1, y_2]$ ':

$$N^* = l_1 q_1^* + l_2 q_2^*$$
 [1.4]

where  $l_1q_1^*$  is the number of workers employed in the wheat industry and  $l_2q_2^*$  is the number of workers employed in the iron industry. The solution found gives the sectoral occupation:

$$N^* = [l_1 q_1^*, l_2 q_2^*]$$
[1.5]

or, in compact form,

$$N^* = L q^* = L(I-B)^{-1} y$$

where

 $L = [l_1, l_2]$  is the vector of labour units.

These solutions highlight how production and occupation levels depend of the structure and size of the final demand:

$$q_1^* = f_1(y_1, y_2); \ q_2^* = f_2(y_1, y_2); \ N_1^* = l_1 f_1(y_1, y_2); \ N_2^* = l_2 f_2(y_1, y_2)$$
 [1.6]

The mathematical equations described up to now are the equations of the system dynamic model that implements the quantities system of the DI-O model.

The name of the variables in the system dynamics model have been chosen in a way to remind us the name of the variables and parameters of the mathematical model.

In order to dynamically check the economic consistency of the results coming out of the quantities system according to the [1.3c] condition, the system dynamics model has been linked with a spreadsheet.

In particular, two connections between the dynamic model and a spreadsheet, called "diomSpreadsheet", have been set.

The first connection links the variable "net output matrix" of the quantities system with the diomSpreadsheet in the "quantities Dataset" sheet. By such a connection, the values of the net output matrix (I - B) are sent to the spreadsheet where the determinant of the said matrix is calculated. The value of the determinant is then returned to the model by the second connection between the "quantities Dataset" sheet and the variable "net output matrix determinant" in the "Model Conditions" constructor diagram where, according to the [1.3c], the condition is checked.

Running the simulation, the values of the transaction matrix's cells in the spreadsheet will be continuously uploaded according to the value calculated inside the model, and the determinant of the matrix will be recalculated for each set of values. If this determinant is less than or equal to zero a stop function will work to halt the simulation.

The same happens if, in the "Hawkins-Simon Conditions" variable of the System Dynamics model, one or both the [1.3a] and [1.3b] conditions (the wheat net output and the iron net output variables) are not fulfilled.

Figure 3.2.1.1. shows the connections between the model and the spreadsheet in the co-models section of the system dynamics model.

	Name	- A	Status	Transfer Direction
	Main     net output matrix     net output matrix determinant     net prices matrix     net prices matrix determinant     Connection of Spreadsheet Dataset	t 1	Connected To Connection of Spreadsheet Dataset 2.net output matrix determinant Connected To Connection of Spreadsheet Dataset 4.net prices matrix determinant	out in out in
	ret output matrix     ornection of Spreadsheet Dataset     ret output matrix determinant     ret output matrix determinant     ret output matrix determinant	t 2 t 3	Connected To net output matrix	in out
	Connection of Spreadsheet Dataset	t 4	Connected To net prices matrix	in out

Figure 3.2.1.1.: Connection between the DI-O model and the diomSpreadsheet.

# 3.2.2 The prices system

The system of equilibrium prices for the economy under study is given by:

$$p_1b_{11} + p_2b_{21} + (p_1b_{11} + p_2b_{21})r + w_1l_1 = p_1$$
[1.7a]

$$p_1b_{12} + p_2b_{22} + (p_1b_{12} + p_2b_{22})r + w_2l_2 = p_2$$
[1.7b]

where *r* is the profit rate and  $w_i$  is the salary of sector *i*.

Equation [1.7a] states that the wheat price is given by the cost of the inputs needed to get a unit of wheat  $(p_1b_{11} + p_2b_{21})$ , plus the profit calculated on the inputs' cost  $((p_1b_{11} + p_2b_{21})r)$ , plus the unit cost of labour  $(w_1l_1)$ .

Therefore, the quantity  $(p_1b_{11} + p_2b_{21})r + w_1l_1$  represents the value-added to each unit of wheat produced.

The same interpretation can be given to the equation [1.7b] regarding the production of iron.

System [1.7] can be rearranged to take the unknown variables, the prices of wheat and iron, out of data (prices system).

The system becomes:

$$(p_1b_{11}+p_2b_{21})(1+r)+w_1l_1 = p_1$$
[1.8a]

$$(p_1b_{12}+p_2b_{22})(l+r)+w_2l_2 = p_2$$
[1.8b]

System [1.8] can be rewritten using vectors and matrices, with I being the identity matrix:

$$pB(1+r)+LW=p$$

that rearranged becomes

$$p[I - B(1 + r)] = LW$$

with

$$[I - B(1 + r)] = \begin{pmatrix} 1 - b_{11}(1 + r) & -b_{12}(1 + r) \\ -b_{21}(1 + r) & 1 - b_{22}(1 + r) \end{pmatrix}$$
 the net prices matrix;

 $p = [p_1, p_2]$  the row vector of prices; W the diagonal matrix of sectional salaries  $w_i$ L the row vector of labour coefficients.

If the determinant  $\Delta$  of the matrix [I - B(1 + r)] is greater than 0, (in mathematical terms  $\Delta = |\mathbf{I} - \mathbf{B}(1+r)| = (1 - b_{11}(1+r))(1 - b_{22}(1+r)) - b_{12}b_{21}(1+r)^2 > 0$ ) the inverse matrix  $[\mathbf{I} - \mathbf{B}(1+r)]^{-1}$  of the net prices matrix exists and, therefore, the solution vector of the system [1.8] can be found multiplying the two members of the system by  $[I - B(1 + r)]^{l}$ :

$$p [I - B(1 + r)] [I - B(1 + r)]^{-1} = LW [I - B(1 + r)]^{-1}$$

getting

$$p^* = LW[I - B(1 + r)]^{-1}$$

where  $p^* = [p_1^*, p_2^*]$  is the column vector of the equilibrium prices. The positiveness condition of  $\Delta$  identifies the maximum value of the profit rate allowed by the production technique in the system under study. The solutions of the prices system are:

$$p_1^* = g_1(w_1, w_2, r)$$
[1.10a]  

$$p_2^* = g_2(w_1, w_2, r)$$
[1.10b]

where the functions 
$$g(.)$$
 are given by the production techniques and highlight the sectoral interdependence in the price determination.

Indeed, if the salary in the wheat industry rises then the price per unit of wheat rises and, as a consequence, the production cost of iron rises, which will increase the iron price as well.

The same happens if the salary in the iron industry rises.

Using the equilibrium prices  $p_1^*$  and  $p_2^*$ , it is possible to determine the equilibrium cost of the tourist products in a tourist destination. Indeed, by the matrix of the tourism products, which summarize in the symbol  $\Pi$  the heterogeneity and plurality of tourism products:

$$\boldsymbol{\Pi} = [x_{zj}]$$

with

z = 1, 2,, n	number of tourism products in a destination
j = 1, 2,, m	goods and services in a unit of tourism product

it is possible to calculate the cost of the single unit (a tourist's presence) of each tourism product as:

$$v = \Pi p'$$

where  $v = [v_z]$ ' is the column vector of the cost of *n* tourism products in the system under study.

Of course, to get the final price of the tourism products the profit margins of the tourism operators, travel agencies and tour operators for instance, must be added.

The mathematical equations describing the prices system are the equations of the System Dynamic model that implement the prices system of the DI-O model.

The same technique used for the quantities system has been used for the name of the variables of the prices system and the way to introduce condition [1.9] in the System Dynamics model.

Therefore, for condition [1.9] the connection is still with the "diomSpreadsheet" but this time with the "prices Dataset" sheet. By such a connection, the values of the matrix [I - B(1 + r)] are sent to the spreadsheet where the determinant of the matrix is calculated. The determinant value is then returned to the model by the second connection between the "prices Dataset" sheet and the variable "net prices matrix determinant" in the "System Conditions" constructor diagram where condition [1.9] is checked. If in running the model the determinant of the net prices matrix becomes zero or negative the simulation stops.

Figure 3.2.2.1. shows the conditions of the DI-O model.

The model implemented with System Dynamics methodology looks like Figure 3.2.2.2. where the blue colour is used for the variables belonging to the quantities system, the red colour is used for the variables belonging to the prices system, the green colour for the production variables and the grey colour for the demand variables.

# 3.2.3 Simulations results

Two different simulations have been run in order to show how production coefficients, equilibrium quantities, equilibrium prices and the cost of the tourist products change whether the Input-Output approach or the DI-O approach is followed.

To run the simulations an hypothetical growing tourist demand (the variable called "test demand") has been included in the model by a graph function. The tourism demand is divided into a demand for farm holiday and a demand for cultural holiday.



Figure 3.2.2.1.: The conditions in the DI-O model.



Figure 3.2.2.2.: The DI-O model.

According to the percentage of wheat and iron these two different holidays make use of, the demand of wheat and iron for tourism is respectively computed and added to the demand of wheat and iron for non-tourism use.

Simulations have been run on a time horizon of 20 years, which is a time interval normally used in tourism analysis.

Figure 3.2.3.1 shows the behaviour over time of the coefficients and the equilibrium variables in a traditional Input-Output analysis, whereas figure 3.2.3.2 shows the behaviour over time of the same variables in the DI-O model.

Since the model considers the interrelationships between the productive sectors, both in terms of quantity produced and price of the goods, when the production coefficients are constant over time, as they are in the Input-Output analysis, the equilibrium prices and the price of the tourist products stay constant. Indeed, since the coefficients of the matrix [I - B(1 + r)] in the system [1.8] don't vary when the production level changes the equilibrium prices stay at their initial level. Moreover, the production process of wheat and iron grows proportionally with the growing demand of these commodities for tourism. Indeed, the comparison between the shape of the demand functions of wheat and iron and the shape of the functions describing the quantity produced of said goods reveals this. Therefore, everything works as one would expect from a traditional Input-Output analysis (Figure 3.2.3.1).

The behaviour of the system changes completely when the simulations are run taking into account the variation of the production coefficients accordingly to the growing tourism demand (Figure 3.2.3.2).

First of all, the graphs of the production coefficients show that the production process moves along an efficiency curve.

Second, the demand curve of wheat and iron and the curves of the production processes don't have the same shape anymore. Something has happened in the production process. Some economies of scale have been achieved. Between period 5 and 11, when both the production processes experience the strongest economies of scale, the slope of the production curves is visibly lesser than the slope of the demand curves. As a consequence, the final demand of wheat and iron at the end of the time horizon (respectively 200 and 100 widgets per year) has been met with a lower production level (1630 instead of 1794 wdg/yr for wheat and 246 instead of 256 wdg/yr for iron).

The dynamics of the system are even more evident when looking at the behaviour of the prices. In fact, the equilibrium prices and the tourism product prices will show a non-constant behaviour because this time the coefficients of the net prices matrix vary with the level of production.

In particular, as long as the production processes experience some economies of scale production prices will decrease because the industries involved in tourism will be able to produce more efficiently. This efficiency will translate into lower prices for tourists' holidays or in more earnings for the tourism industry if the final price, namely the price for the tourist, is kept constant.



Figure 3.2.3.1: Behaviour of production, coefficients and equilibrium variables in the I-O model.



Figure 3.2.3.2: Behaviour of production, coefficients and equilibrium variables in the DI-O model.

The model also allows us to make some considerations about the technical sustainability of the production process. Indeed, the variable "matrix of production inputs" in the DI-O model shows how the needed quantity of production inputs (wheat, iron and labour units) in the wheat and iron industries vary over time because of the final demand's behaviour. Therefore, for the production process to be sustainable from a technical point of view the quantities of production units showed in the above-mentioned graph must always be available.

Moreover, comparing the simulations made in the two different analyses, it is possible to see that the DI-O model shows values for the production input over time lower than the ones showed in the normal Input-Output analysis. Looking at the behaviour of the economic system from the dynamic analysis point of view, therefore, the production process will look more easily sustainable than the same process studied with the static analysis.

# 4. Conclusions

This paper describes and compares two different models to analyze an economy with two sectors: the Input-Output model and the proposed Dynamic Input-Output (DI-O) model.

The models have in common the basic structure, but in the DI-O model the production coefficients change according to the size of the production process. In other words, the model presented in this paper allows the economy under study to realize economies of scale. Therefore, the DI-O model relaxes the very limiting Input-Output assumption that the production coefficients are constant.

Moreover and differently from every Input-Output analysis, the DI-O model allows non-linearity in the production process because the production coefficients of the model are a function of the quantity produced.

The DI-O model can be easily generalized to a number n of different sectors, and the system dynamics model that implements the DI-O model has been already built in a way to be easily extended to a bigger number of productive sectors when provided with the needed information.

Looking at the graphs, it is plain how the behaviour of the system changes considerably whether an Input-Output analysis or a DI-O analysis is carried out.

In spite of its very simple structure, the model presented also allows us to make some considerations about the technical sustainability of the production processes. Indeed, each production process, as influenced by the final and sectoral demand, must be compatible with the stocks of capital, labour and production inputs available.

Such stocks are calculated in the variable called "matrix of production inputs" in the model.

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