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MODELS OF CYCLICAL GROWTH

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Regularities of big cycles of conjuncture and sustainable development belong to the most important ones for the theory and strategy of economic development. This monograph penetrates these regularities in depth on rich statistical data with a help of modern mathematics (the theory of non-linear differential equation, extended Kalman filter) and computer technique. By enhancing mathematical and instrumental methods of the economic theory and system dynamics, this monograph may help in elaborating a long-term strategy of Russia's development allowing for higher growth rates of the domestic economy than economic growth rates in the USA and other industrialised countries.

Investment and foreign trade planners, teachers of economic theory and mathematical modelling, researchers of capitalist economy, students and post-graduate students of economic theory and mathematics at higher schools will benefit from reading this advanced book.

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Alexander V. Ryzhenkov

Abstract

The Dutch artist Vincent van Gogh once expressed a fine thought: "I consider making studies like sowing and making pictures like reaping… It is easier to proceed from drawing to painting, than, conversely, to make pictures without drawing the necessary studies…" The same approach is typical for the system dynamics modelling. Rephrasing V. Van Gogh, we may say that a system of differential equations is the skeleton, statistics are the muscles of the sturdy system dynamics model.

This book defines a deterministic form of a *hypothetical Law of capital accumulation* (HL) as a system of non-linear ordinary differential equations based on real data. The state variables are the relative wage, employment ratio, unit gross rent, unit depletion and degradation of the natural capital, man-made capital-output ratio, natural capital-output ratio and indicated natural capital-output ratio. An operational policy within the scope of the existing and potentially feasible regulatory institutions for achieving sustainable development is also included. Complexity results from the high degree of interrelatedness of these structural elements, non-linearity of the functional forms and informational delays in this model. The fundamental equation of neo-classical growth is a special case of the more general dynamic regularity, presented as a direct consequence of the HL.

The present author makes use of four idealisations of an actual economic development of industrialised capitalist economy that could be considered as four subsequent steps for the derivation of the Law of capital accumulation. This Law subsumes all of them. Its rough-

ness corresponds to the real systems' property of relative structural stability.

A *first order* idealisation is a steady state growth that is locally or even globally stable. Accordingly, technical progress and growth of labour force tend to result in steady economic growth while long waves represent important fluctuations about this trend. The Kaldor's stylised facts are valid for steady state growth path still it is a too remote image of reality where all equilibrium states are locally unstable and cannot maintain themselves for any significant length of time.

A *second order* idealisation is mathematically a limit cycle that results from a locally unstable stationary state. Since no linear system can lead to a limit cycle, a dynamic economic model must be essentially non-linear. A limit cycle can be generated via Andronov – Hopf bifurcation if a steady state loses its stability when a control parameter passes a critical magnitude. This is not the only possible mechanism for manifestation of closed orbits. They may go around a neutral steady state like in the structurally unstable Goodwin model with the two phase variables (relative wage and employment ratio). Alternatively, they may exist in a system that cannot be linearised at a locally unstable stationary state and at points with the maximal or minimal employment ratios at all. This book argues that the latter case of the substantially non-linear dynamic system is the mostly realistic since it takes into account incompatibility of full employment with the institutional structure of capitalism.

The Kaldor's stylised facts are true on average for a limit cycle. This property contradicts the Marx Law of the tendency of the general profit rate to fall that has been supported by US official statistics for 1948–2000. The society accumulates the produced capital, however its activity brings about the excessive depletion and degradation of the natural environment that is detrimental for the unbiased general profit rate and net rent rate over the long term. Accumulation of produced assets and accumulation of developed natural assets are necessary for strongly sustainable development.

A *third order* idealisation is a transition motion to a limit cycle in a more structurally stable dynamic system. As proved, such a system cannot be linearised in neighbourhood of a locally unstable stationary state. The Law of the tendency of the general profit rate to fall

operates on the transient to a periodic attractor for centuries. A shortcoming of the third order idealisation is that a transient regime is too regular and repetitive. It leads to a periodic motion where the general profit rate is constant on the average.

It has been shown that the real US economy exhibits a long wave as a dominant quasi-cycle. A long wave is not a mere fluctuation superimposed on steady, balanced growth; rather, a long wave is a part of the very process of non-equilibrium growth in the capitalist economy. A *fourth order* idealisation is a long wave as a stochastic attractor and quasi-periodic trend with a quasi-period of about 29–33 years in the US economy after the Second World War.

The stochastic model that generates this long wave has been written in a canonical state space form. Setting a probabilistic form of the HL, it reflects additionally discrepancies (including effects of shorter quasi-cycles) in the dynamic equations and in measurement equations. An application of the extended Kalman filtering to the US macroeconomic data has identified unobservable components of this Law. The retrospective statistical analysis (1958–1991), univariate sensitivity analysis (1991–2034) and forecasting (1991–2107) have supported the analytical treatment. An exploratory and a normative scenarios have contrasted the strongly sustainable development to an unsustainable evolution.

The present downturn in the long wave is to be viewed not only as regularly recurrent phase of the long wave. Its additional pains are, likely, characteristic of childbirth of the natural capitalism. The 'old' industrial capitalism is thus experiencing the dialectical negation, or creative destruction. The system dynamics approach could be helpful for shortening and lessening disorder and distress of this major global transformation. The conscious element of the Law plays a decisive role in providing better governance of the ecological-economic reproduction on the increasing scale when ecology remains one of the major political issues.

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