INFLOW: from Influence to Flow Diagram in System Dynamics

Paolo V. Dolci, Maria A. Puddu

Università di Cagliari - Italy

ABSTRACT

In this paper we review an algorithm to identify the type of variables (levels, rates, auxiliaries) which appear in the influence diagram. This algorithm has been implemented on a personal computer at the Cagliari University.

THE INFLUENCE DIAGRAM

"The influence diagram records the way in which the system works" (Coyle 1977): right and stenographic. All the system's variables are in fact gathered in the influence diagram, connected by arrows with the variables at the tips influenced by those on the tail. A '+' near the arrow-head means that variations of both variables go in the same direction, a '-' the opposite, whereas no sign if the direction of the variation is incostant. The influence diagram is an oriented graph, with variables as nodes and lines of influence, the arrows, as arcs.

To have drawn the influence diagram is to have made an essential step towards the system's model, but the next step may be tiring, for we must identify each variable as a level, a rate or as an auxiliary, in order to be able to build up the flow diagram. Coyle (1977) suggests a trial-and-error method to find out the variables' type: Assouline (1981) proposed an algorithm to automize the procedure. That algorithm was implemented on a large computer; and we have (partially) repeated the experience using a personal, a IBM XT. The program is written in BASICA, but in the paper we present only the tehory, and the algorithm step-by-step (in fact we are still experimenting with the program, hunting for some forgotten bugs).

Assouline's algorithm is composed of three parts, but we present in this paper only the first, in which the influence diagram is scanned to recognize the type of present variables, adding if necessary some other to give internal consistency to flow diagram. The second part examines the outcome of the first one, with the aim to reduce, if possible, the system's order by 'degrading' some level, which has been singled out in the first stage, to an auxiliary. In the last part of the algorithm the antecedents of the variables are checked and the equations are formally written.

VARIABLES' SET

Let $X = \{x_1, x_2, \ldots, x_n\}$ be the set of the nodes of grapf - $x_i (i=1,2,\ldots,n)$ are the system's variables -, $C \subseteq X^2$ the set of arcs, and $C_{ij} \subseteq C$ the set of arcs connecting x_i to x_j. We shall indicate the influence diagram as $G=(X,C)$.

If G is connected, and $\forall (x_i, x_j) \in X^2 \Rightarrow \text{Card } C_{ij} \leqslant 1$, and $\forall x_i \in X \Rightarrow C_{ii} = \emptyset$, then the influence diagram can be represented as a boolean matrix M, the system's structure matrix (Lange 1981). M is square, with the number of lines equal to n, which is the number of the system's variables; its elements are $m_{ij} = \text{Card } C_{ij}$ (i.e. if $C_{ij} = \emptyset$, then $m_{ij} = 0$, else $m_{ij} = 1$).

Let $A_x(x_i)$ and $S_x(x_i)$ be the sets of nodes, respectively, preceding or following x_i in the influence diagram; and $A_c(x_i)$ and $S_c(x_i)$ the sets of arcs, respectively, terminating in, or starting from, x_i.

Let us define, for the moment, the following sets:

(1) $$U = \{x_i \in X : A_c(x_i) = \emptyset\}$$

there are no arcs terminating in the elements of U, so they do not depend on other variables: they are parameters or exogenous variables;

(2) $$Y = \{x_i \in X : S_c(x_i) = \emptyset\}$$

there are no arcs starting from the elements of Y, so they do not influence other variables: they can be either outputs of the system, or supplementary variables, the latter of which indicate the system's performance even though not being part of the system itself.

The graph which is the influence diagram is a model of the system, which in turn can be thought of as a graph, or more precisely a bounded network (Le Moigne 1977, Dolci 1985): so we can say that through the arcs of the influence diagram, as through the connections between the system's elements, run flows of matter, energy and information. Let ME be the set of matter or energy flows - ME type interactions, and I be the set of information flows - I type interactions. The two classes of interactions are equivalence classes, from which we can obtain a binary partition of influence diagram $G = (X, C)$ such that $ME \cup I = C$ and $ME \cap I = \emptyset$.

Depending on this partition, we can define the following sets:

(3) $$L = \{x_i \in X : A_c(x_i) \subseteq ME, \ S_c(x_i) \subseteq I\}$$

the elements of L are the levels, and they are the state variables of the system;

(4) $$T = \{x_i \in X : A_c(x_i) \subseteq I, \ S_c(x_i) \subseteq ME, \ \text{Card } S_c(x_i) \leqslant 2\}$$

the elements of T are the rates, the decision variables; the last set to be defined is the one of auxiliary variables:

(5) $$A = \{x_i \in X : A_c(x_i) \subseteq I,\ S_c(x_i) \subseteq I\}$$

It is thus evident that arcs preceding or following, a vertex are all of the same type (Fig. 1):

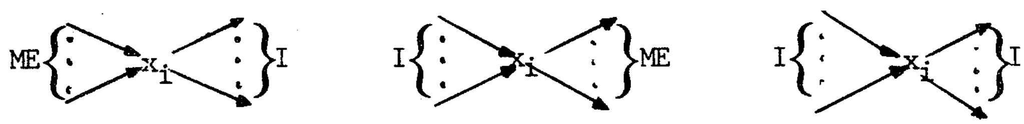

Fig. 1

Therefore, the only connections between elements of the sets L, T, A, are in the absence of delays, the ones of Fig. 2:

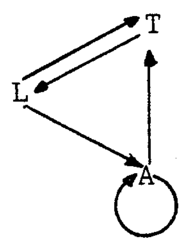

Fig. 2

The closed loops in such a graph, either go through a level or are composed only by auxiliary variables: beware not the dog, but simultaneous equations! We shall call these variables, solutions, if any, of simultaneous equations, loop auxiliaries.

Before illustrating the algorithm, let us label some sets, which will be used in the following pages:

L, the set of levels; we shall start with a set of 'imposed' levels, that is, variables which can be nothing but levels, not contiguous in the influence diagram; new levels will be singled out during the analysis, and they will join to L;

T : the set of rates;

TI : the set of intermediate rates generated by the algorithm to ensure the coherence of flow diagram;

A : the set of auxiliaries;

AL : the set of auxiliaries which represent an information on a level;

AT : the set of auxiliaries which represent an information on a rate;

ALT: the set of auxiliaries coming from the treatment of at least one piece of information on a level and one on a rate;

AC : the set of loop auxiliaries;

V : the set of variables not yet singled out.

THE ALGORITHM

1 - The starting point of the algorithm is to identify the set $A_x(L) = \{\cup A_x(x_i) : x_i \in L\}$, that is the set of antecedents of levels, 'imposed' or single out, applying the algorithm. This identification is done by scanning the columns corresponding to levels in the structure matrix M: a 1 in the k-th row means that x_k is an antecedent of the variable corresponding to the column. For instance, let us suppose that x_i is a level: if $m_{ki} = 1$ then $x_k \in A_x(L)$, that is x_k preceeds x_i in the influence diagram.

2 - Identify the set S_x of the successors of the elements of $A_x(L)$: if $m_{kj}=1$ then $x_j\ S_x(x_k)$, i.e. x_j is a successor of x_k in the influence diagram.

3 - Find the cardinality of $S_x(x_k)$, Card $S_x(x_k)$, by counting the 1s in the k-th row of M.

4 - If Card $S_x(x_k) = 1$, then $x_k \in T$, e.g.:

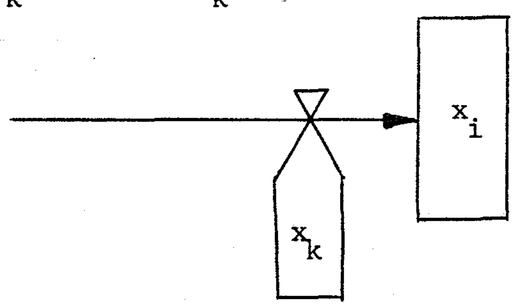

Fig. 3

else, if Card $S_x(x_k) = 2$, go to 5, else go to 6. It is impossible to have Card $S_x(x_k) = 0$, because at least $x_i \in X_x(x_k)$.

5 - (Card $X_x(x_k) = 2$) x_i is one of the successors of x_k; let x_j be the second. If we know what type of variable x_j is, go to 5.1, else, go to 5.3.

5.1 - If $x_j \in L$, then x_k T, e.g.:

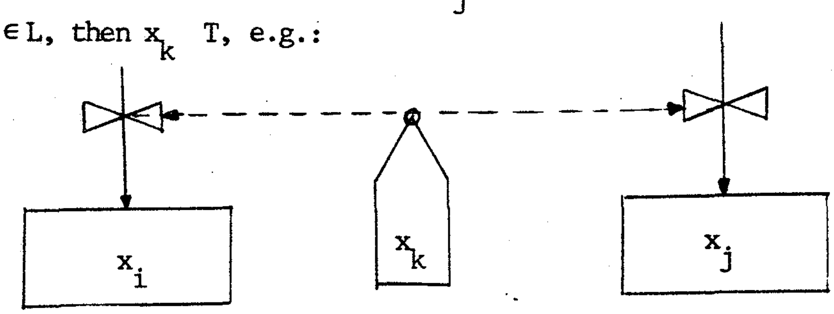

Fig. 4

else, go to 5.2.

5.2 - If $x_j \notin L$, then, $x_k \in AT$, and we generate an intermediate rate $\hat{x}_k$ between x_k and x_i: $\hat{x}_k \in TI$, e.g.:

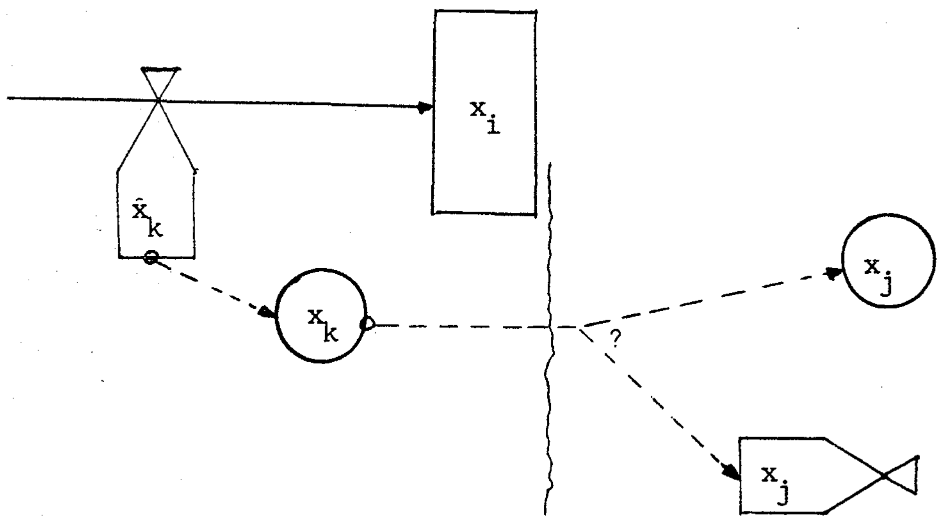

Fig. 5

5.3 - $x_j \in V$ (x_j's type unknown); there are still two more cases: there is at least one level among the successors of x_j, that is $S_x(x_j) \cap L \neq \emptyset$, or not. Specify the set $S_x(x_j)$: if $m_{jp} = 1$, then $x_p \in S_x(x_j)$. If $S_x(x_j) \cap L \neq \emptyset$, then $x_k \in AT$, and we generate an intermediate rate x_k between x_k and x_i: $x_k \in TI$, and as many intermediate rates as the levels following x_j; we will call them x_j^s, with s equal to the number of these levels. These last intermediate rates are to be interposed between x_j and the levels x_r, one or more, which follow it. So $x_j \in ALT$ and $\hat{x}_j^s \in TI$, e.g.: Fig. 6

5.4 - If there are not levels among the successors of x_j, that is $S_x(x_j) \cap L = \emptyset$, we have to examine the antecedents of x_j. To individualize the set $A_x(x_j)$ of the antecedents of x_j, scan the j-th column of M: if $m_{qj} = 1$ then $x_q \in A_x(x_j)$. Still two further possibilities: there is or there isn't at least one level in $A_x(x_j)$. In the first case continue, else go to 5.5. If $A_x(x_j) \cap L \neq \emptyset$ let us suppose that $x_r \in A_x(x_j)$ is the level, then $x_j \in AL$, $x_k \in AT$, and we generate an intermediate rate $\hat{x}_k$ between x_k and x_i: $\hat{x}_k \in TI$, e.g.: Fig. 7.

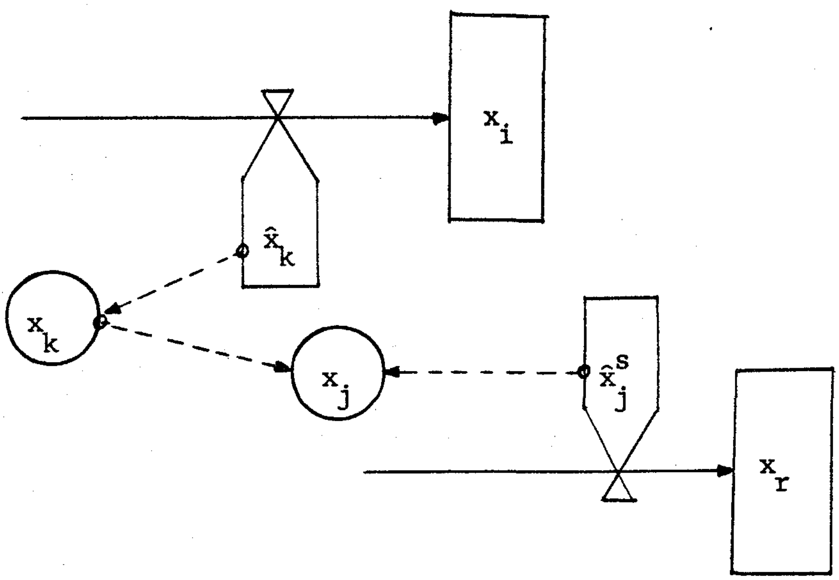

Fig. 6

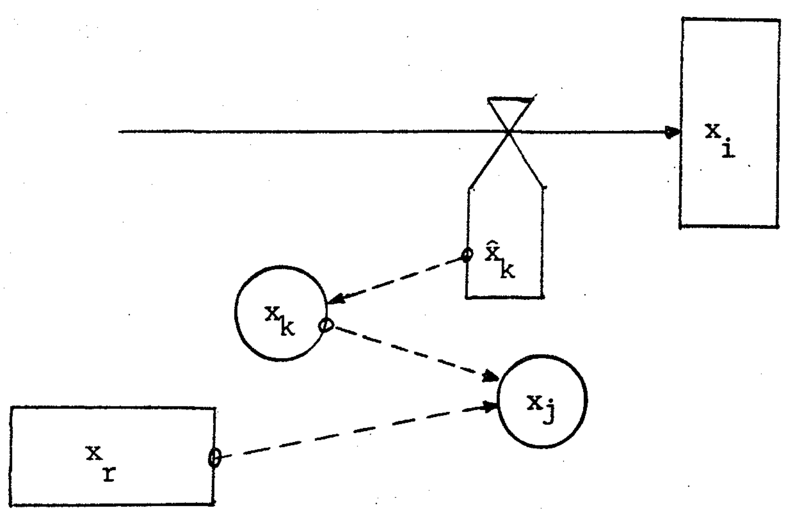

Fig. 7

5.5.- No levels among x_j's antecedents: $A_x(x_j) \cap L = \emptyset$. If at least one closed loop passes through x_j, then $x_j \in L$, $x_k \in T$, e.g.:

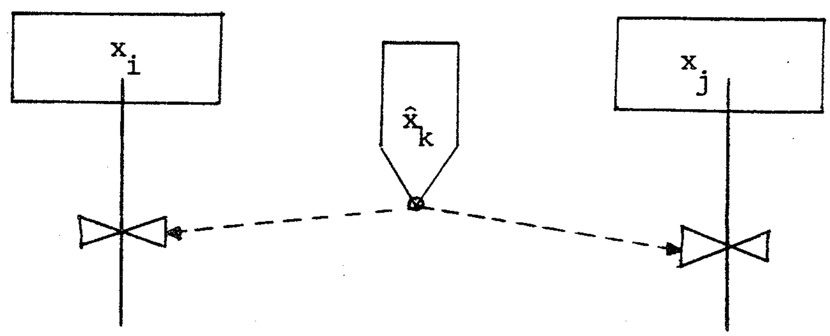

Fig. 8

else go to 5.6. Remember to update L, the set of levels, and A, the set of these levels' antecedents.

5.6 - No closed loops through x_j: as in 5.4, $x_j \in AL$, $x_k \in AT$, and we have to generate an intermediate rate $\hat{x}_k$ between x_k and x_i, $\hat{x}_k \in TI$, e.g.:

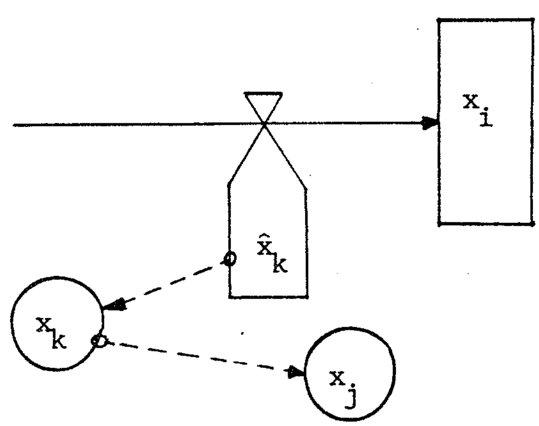

Fig. 9

6 - (Card $S_x(x_k) > 2$) $x_k \in AT$, and we have to generate an intermediate rate $\hat{x}_k$ between x_k and x_i, $\hat{x}_k \in TI$, e.g.:

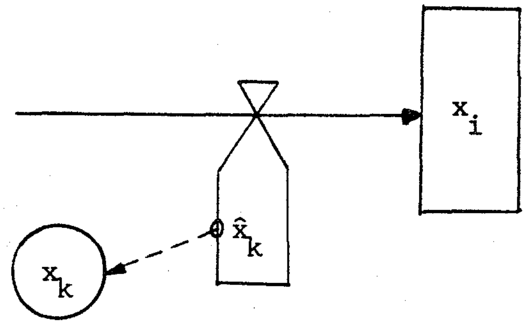

Fig. 10

7 - Once singled out, the antecedents of levels, including those identified during the analysis, we have to label the variables which are still unknown. Let AC = $\emptyset$; for each $x_s \in V$, let us count the closed looops through x_s and composed only by elements of V: let $nc(x_s, V)$ be this number.

8 - If, for each $x_s \in V$, $nc(x_s, V) = 0$, then all the unknown variables are auxiliaries, i.e.: $x_s \in A$ and $V = \emptyset$, else go to 9.

9 - For at least one $x_s \in V$, $nc(x_s, V) \neq 0$; so, if the closed loop through x_s passes across at least one element of $L \cup AC$, then $x_s \in A$, else go to 9.1.

9.1 - No elements of the closed loop through x_s belong to $L \cup AC$: then $x_s \in AC$. The elements of AC are solutions of simultaneous equations, which must be either eliminated, introducing delays, or solved apart, separately.

REFERENCES

Assouline G. "Aide à l'Analyse et à la Construction des Grands Modèles de la Dynamique des Systèmes", Thèse de Doctorat, Université Paris IX Dauphine, 1981.

Coyle R.G., "Management System Dynamics", Wiley, 1977.

Dolci P.V., "A System View of Transportation System", Proceeding 1985 Int. Conf. of the System Dynamics Soc., Keystone, Col., USA, 1985.

Le Moigne J. - L., "La Théorie du Système Général, PUF, 1977.

Lange O., "La parte e il tutto", Rosenberg & Sellier, 1981.