92

# USING THE LIAPUNOV PRINCIPLES IN THE ANALYSIS OF SYSTEM DYNAMICS MODELS

by

Dr. Qifan Wang
Shanghai Institute of Mechanical Engineering
People's Republic of China
and
Visiting Scholar
Sloan School of Management
Massachusetts Institute of Technology

July 1983
(revised December 1983)



USING THE LIAPUNOV PRINCIPLES IN THE ANALYSIS OF SYSTEM DYNAMICS MODELS

Dr. Qifan Wang

Shanghai Institute of Mechanical Engineering
and
Visiting Scholar
Sloan School of Management
Massachusetts Institute of Technology

## ABSTRACT

A new analytic method combining Piapunov's methods and eigenvalue analysis approach, a technique for identifying dominant loop analysis contributed by Nathan B. Forrester, is developed to search for the feasible and most policies in system dynamics models. The paper briefly introduces the Liapunov methods of stability analysis (the first and second methods of Liapunov). Liapunov's first method, under certain conditions, enables one to arrive at conclusions about a nonlinear system (original system) by studying the behavior of linearized systems. Liapunov's second method gives sufficient conditions for the stability of equilibrium states of nonlinear systems and gives necessary and sufficient conditions for the stability of equilibrium states of linear systems. Criteria of Liapunov's first method and Krasovskii's method, an extended method of Liapunov's second method, are both applied in the analytic method. The structure of the method and how the method is used are described. It is expected that the analytic method willbecome a new approach to testing models in order to seek the best feasible policies automatically in system dynamics.

## I. Preface

Challenges face the modeler who strives to understand and identify the structural origins of behavior in system dynamics. Traditional simulation methods for tesing models and decisionmaking are time-consuming and error-prone.

The purpose of this paper is to develop an analytic method based on Liapunov methods and the eigenvalue analysis approach, a technique for identifying dominant loop analysis contributed by Nathan B. Forrester for analyzing system dynamics models. The paper briefly introduces the Liapunov's methods of stability analysis, the first and the second methods of Liapunov. The first method of Liapunov, under certain conditions, enables one to arrive at conclusions about a nonlinear system (original system) by studying the behavior of linearized systems. The second of the two Liapunov methods and its extended method give sufficient conditions for the stability of equilibrium states of nonlinear systems and give necessary and sufficient conditions for the stability of equilibrium states of linear systems. Krasovskii's method, an extended method of Liapunov's second method, is recommened as a new criterion to test the stability of nonlinear systems. The structure of the method and how the method is used are stated. It is expected that the analytic method will become a new approach to testing models in order to seek the most feasible policies automatically in system dynamics.

The presentations here assume that the reader has read NBF's thesis. Therefore, his approach, mentioned above, is not described in this paper.

In order to explain the ideas presented in this paper and convince people of them, we discuss some basic concepts of Liapunov's methods and decscribe and prove some of them further in appendices. Appendix 1 introduces the basic concepts of stability, APPENDIX 2 DESCRIBES LIAPUNOV's second method and gives the proof of Krasovskii's method.



## II. The ideas in this paper

The basic idea of the method is using the Liapunov principles in the analysis of system dynamics models. The overview structure is shown in Figure 1. The main contribution of this paper is that Liapunov's first method, the second method, and its extended method, are incorporated with "eigenvalue analysis approach" to analyze the system model. Then a new approach for searching for feasible policies is formed. The new criterion is Krasovskii's method (that is "if $F^{t}(X)+F(X)<0$" ?), it gives sufficient conditions for nonlinear systems and necessary and sufficient conditions for linear systems. The criterion can pick out unreasonable policies causing system instability before going to the next procedure. Liapunov's first method, under certain conditions, enables one to arrive at conclusions about a nonlinear system (original system) by studying the behavior of linearized systems. Therefore, it can be used to decide whether the next procedure goes to "eigenvalue analysis approach" or to"policymaking". As a result, the new approach will speed up the whole process of searching for the best policy.

## III. Basic concepts of stability according to Liapunov and Liapunov's first method.

According to the Liapunov principles, Liapunov stability is concerned with the trajectories of a system when the initial state is near an equilibrium point. From a practical point of view, this is very important because external disturbances (such as noise, component errors, etc.) are always present in a real system.

The basic concepts of Liapunov stabitity are stability, asymptotic stability, insabitity, autonomous and nonautonomous (see references).

Liapunov's first method including three theorems (see Appendix 1) are very useful, because they enable one to draw conclusions about the stability status of the equilibrium point O of a given nonlinear system by examining a linear system. The advantages of these results are self-evident. Roughly, Liapunov's first method gives the results as follows:

(1) If all the eigenvalues of the constant matrix A have nonzero real parts, then the stability of the equilibrium state of the original nonlinear system is the same as that after linearizing.

(2) Under certain conditions, if the linearized system is asymptotically stable, then the nonlinear system is also asymptotically stable.

(3) If the eigenvalues of A have negative real parts, then the equilibrium state is asymptotically stable. And if at least one of the eigenvalues of A has a positive real part, then the equilibrium state is unstable.

(4) If, however, at least one of the eigenvalues of A has a zero real part, and the remainder have negative real parts, linearization techniques are inconclusive. In other words, the local stability behavior of the equilibrium state of the system cannot be determined by a linearized equation, because this case falls outside the scope of both Theorems 1 and 3. We shall call this case the critical case. In the critical case the local stability behavior of the equilibrium state of the system depends on higher-order terms of the Taylor series expansion of $f(X)$. This can be realized as follows:In the case described above, one can say that the linearized system is on the verge of stability--or, if one is a pessimist, that it is on the verge of instability. That is, the stability status of the equilibrium point O is actually determined by the higher-order terms, which are being neglected in the linearization.





The conclusions by the stability analysis in the first approximation apply only in the neighborhood of the equilibrium state. Namely, the first method of Liapunov concerns only stability in the small region. Stability analysis in the large region is more difficult than stability analysis in the small region. Consequently, to study global asymptotic stability, it is still necessary to resort to Liapunov's direct method. The puzzle is solved by the second method of Liapunov.

IV. Liapunov's second method (also called Liapunov's direct method) and its extended methods [Kalman, Vidyasagar, pp. 148, and Ogata pp.[470-485]

Liapunov's second method and several extended methods based on Liapunov's second method are available to investigate nonlinear systems without linearization. The second method of Liapunov proves to be particularly powerful in the investigation of stability analysis of certain nonlinear systems where conventional criteria cannot give answers. The extended methods include Krasovskii's method for testing sufficient conditions for asymptotic stability of nonlinear systems, Schultz-Gibson's variable gradient method for generating Liapunov functions for nonlinear systems, Lur'e's method applicable to stability analysis of certain nonlinear control systems, Zubov's method for constructing domain of attraction and others. In this paper Liapunov's direct method itself and Krasovskii's method for constitute the new analytic method. The other extended methods of Liapunov are not adopted because their processing procedures are not convenient enough to use.

Liapunov's second method is based on a generalization of the idea that if the system has an asymptotically stable equilibrium state, then the stored energy of the system displaced within the domain of attraction decays with increasing time until it finally assumes its minimum value at the equilibrium state. This method consists of determining a fictitious "energy" function called a Liapunov function (candidate). The idea of the Liapunov function is more general than that of energy and is more widely applicable. Liapunov functions are functions of $x_1, x_2, \ldots, x_n$ and t. We denote Liapunov funtions as $V(x_1, x_2, \ldots, x_n)$ or simply as $V(X,t)$.

Liapunov's main stability theorem and Krasovskii's method are briefly presented as follows.

LIAPUNOV'S MAIN STABILITY THEOREM

Consider the vector differential equation

$$\dot{X}(t)=f[t,X(t)], \quad t\geq 0$$

Where $X(t) \in R^n$

the equilibrium point 0 at time $t_o$ of the function is stable if there exists a continuously differentiable, locally positive definite function V such that

$$\dot{V}(t,X)\leq 0, \quad \forall\ t\geq t_o, \quad \forall\ X \in B_r \text{ for some ball } B_r$$

(omitted proof, see Kalman and Vidyasagar, PP. 148)

KRASOVSKII's METHOD

Consider the vector differential equation

$$\dot{X}(t)=f[t,X(t)], \quad t\geq 0$$



Where $X(t) \in R^n$

A state $X_e \in R^n$ where

$f(t,X_e)=0$ for all t

is supposed to be an equilibrium state of the system.

The Jacobian matrix for the system is

$$F(X)=\frac{\partial f(X)}{\partial X}$$

$$=\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \quad \text{(Jacobian Matrix.)}$$

Define

$$\hat{F}(X)=F^{\tau}(X)+F(X)$$

where $F(X)$ is the Jacobian matrix and $F^{\tau}(X)$ is the conjugate transpose of $F(X)$. $\hat{F}(X)$ is clearly Hermiton.

Since $F^{\tau}(X)+F(X)$ is negative definite, it follows that $\dot{V}$ is negative definite. Hence $V(X)$ is a Liapunov function. Therefore, the origin is asymptotically stable. If $V(X)=f^{\tau}(X)f(X)$ tends to infinity as $|X|\rightarrow\infty$, then the equilibrium state is asymptotically stable in the large.

It should be pointed out that Krasovskii's theorem differs from usual linearization approaches; it is not limited to small departures from the equilibrium state. Furthermore, $V(X)$ and $\dot{V}(X)$ are represented in terms of $f(X)$ or $\dot{X}$ rather than in terms of X.

The distinguishing feature of all these theorems is that they enable one to draw conclusions about the stability status of an equilibrium point, without solving the system equations, by constructing a suitable Liapunov function.

In addition, notice that the theorem described above gives sufficient conditions for nonlinear systems and necessary and sufficient conditions for linear systems. Thus an equilibrium state of a nonlinear system may be stable even if the conditions specified in this theorem are not satisfied.

## V. Descriptions of the main parts in the analytic method and the main procedures of the method

The key idea of the method is applying Liapunov's principles in the analysis of system dynamics models. The overview structure of the method is as shown in Figure 1. The main parts of the method are

1). Krasovskii's method ("if $F^T(X)+F(X))<0$" ?), one of the extended second Liapunov methods. It is used to judge whether the system is stable in the large rigion or not.

2). Eigenvalue analysis approach, a technique for identifying dominant loops contributed by Nathan B. Forrester.

3). Linearizing model, based on Liapunov's first method. Liapunov's first method, under certain conditions, enables one to arrive at conclusions about a nonlinear system (original system) by studying the behavior of linearized systems. The original system is stable when the linearized model has been judged stable enough by the criteria.



4). Criteria of the first Liapunov method, which judge whether the linearized model is proper or not. As we know, any linearized model is just an approximation of its nonlinear origin system in the neighborhood of the operating point. Liapunov's first method gives powerful support in theory and the useful criteria to examine the behavior of a linearized model.

The main procedures of the method are as follows:

1). Given a nonlinear model and certain policies from "policymaking." (go directly to Step 4 for a linear system.)

2). "if $F^T(X)+F(X)<0$" ? Yes, go directly to "eigenvalue analysis"; If not, go to "linearizing model."

3). linearizing model. Applying criteria of Liapunov's first method to judge "if all eigenvalues are negative ?"
Yes, go to "eigenvalue analysis";
No (i.e., at least one of the eigenvalue of A has a positive real part or a zero real part), go to "policymaking" and change policy.

4). Eigenvalue analysis approach, identifying dominant loops.
Are all the tests done and the results good?
Yes, go to the "end."
No, go to "policymaking" then return to eigenvalue analysis approach or go back to the beginning and through step 1 to 3.

## VI. Discussion

1. Generally speaking, the process of testing models and policies is usually iterative, time-comsuming, and costly. It needs a lot of tests to approach the policymaking goal. Traditional simulation methods

for tesing models and decisionmaking are not only time-consuming but also error-prone. It will be an especially heavy burden to test all the possible policies, feasible or purely wrong, while people lack experience at the beginning of policymaking. Clearly, the detour is not worthwhile. The analytic method presented in this paper combines Liapunov's methods and the eigenvalue analysis approach into a new integrated approach, which takes advantage of each criterion in different steps to pick out the bad choices. Thus, the process of searching for the mostfeasible policies in system dynamics models is simplified and shortened. And it is expected that the analytic method will become an integrated piece of software to test models automatically in system dynamics.

2. The Krasovkii methods gives a simple and very useful criterion (if $F^{\tau}(X)+F(X)<0$ ?) to the analytic method. The criterion can be used widely no matter what type of system it is (nonlinear or linear). Once the policy changes, it changes the parameters of the system model. As a first filter, the criterion only allows the policies that have corresponding "$F^{\tau}(X)+F(X)<0$" to go to "eigenvalue analysis approach" for identifying dominant loops. In other words, the function of Krasovkii's method is to sift out bad policies (they go to the second filter -- linearizing model, and criteria of Liapunov's first method). Then the remaining policies will be more feasible, and it would be easier for eigenvalue analysis to search for the important feedback loops. Consequently, the criterion will save people time and money in seeking the best policy. Another advantage of Krasovskii's criterion is also evident. Because of its applicability and simplicity to nonlinear systems, the criterion is easy to program and incorporate into the program of the analytic method.



3. Liapunov's first method gives a powerful support in theory and some useful criteria to examine the behavior of a linearized model. It is well known that linearization causes little impact on model behavior. Any linearized model is just an approximation of its nonlinear origin system in the neighborhood of the operating point. People may doubt whether the model strays far from the original linearization point, and what the relationship between the linearized model and the real system is. Liapunov's first method, stated in Appendix 1, includes three theorems. These theorems are helpful in reaching conclusions about a nonlinear system by examining a linear system. It convinces people that Liapunov's first method, under certain conditions, enables one to arrive at conclusions about a nonlinear system (original system) by studying the behavior of linearized systems. Theorem 1 states that if a linearized system is uniformly asymptotically stable, then the nonlinear origin system is no doubt uniformly asymptotically stable too in the same region. Theorems 2 and 3 convince people that if all eigenvalues of A have negative real parts the equilibrium point 0 of this system is asymptotically stable; and if at least one eigenvalue of $A_0$ has a positive value, the equilibrium point 0 of the system is unstable. Therfore, Liapunov's first method also supports the theoretical foundations of NBF's eigenvalue approach in theory. In addition, theorems 2 and 3 are very powerful criteria in the analytic method, as they can pick out and exclude the unreasonable numbers of A in the process of testing.

VII. Conclusion

1. Liapunov's first method strengthens the theoretical foundation of linearization and convinces people of the conclusion about a nonlinear system by examing linearized systems.

2. A new analytic method for seeking the feasible policies is presented. It is a combination of criteria of Liapunov's methods and eigenvalue analysis approach. The new method is more efficient and has more functions. Its advantage is clear:it saves time and cost in the searching process. Furthermore, Krasovskii's method and the criteria of Liapunov's first method are easy to program and incorporate with NBF's orginal eigenvalue approach. Therefore, it is expected that the analytic method will become an integrated software tool to test models automatically in system dynamics.



## REFERENCES

Forrester, Nathen B., A Dynamic Synthesis of Basic Macroeconomic Theory: Implications for Stabilization Policy Analysis. Ph.D. Thesis, System Dynamics Group, Sloan School of Management, M.I.T., Cambridge MA, August 1982.

Hahn, Wolfgang, Theory and Application of Liapunov's Direct Method, Englewoof Cliffs NJ: Prentice-Hall, Inc., 1963.

Kalman, R.E. and J.E. Beryram, "Control System Analysis and Design Via the Second Method of Liapunov:II. Discrete-time Systems," ASME J. Vol. 82, No.2 June, 1960, pp. 394-400.

Ogata, K., State Space Analysis of Control Systems, Englewoof Cliffs NJ: Prentice-Hall, Inc., 1967.

Schiltz, D.G., and J.E. Gibson, "The Variable Gradient Method for Generating Liapunov Functions," AIEE Trans. Part II, 81(1962), pp. 203-209.

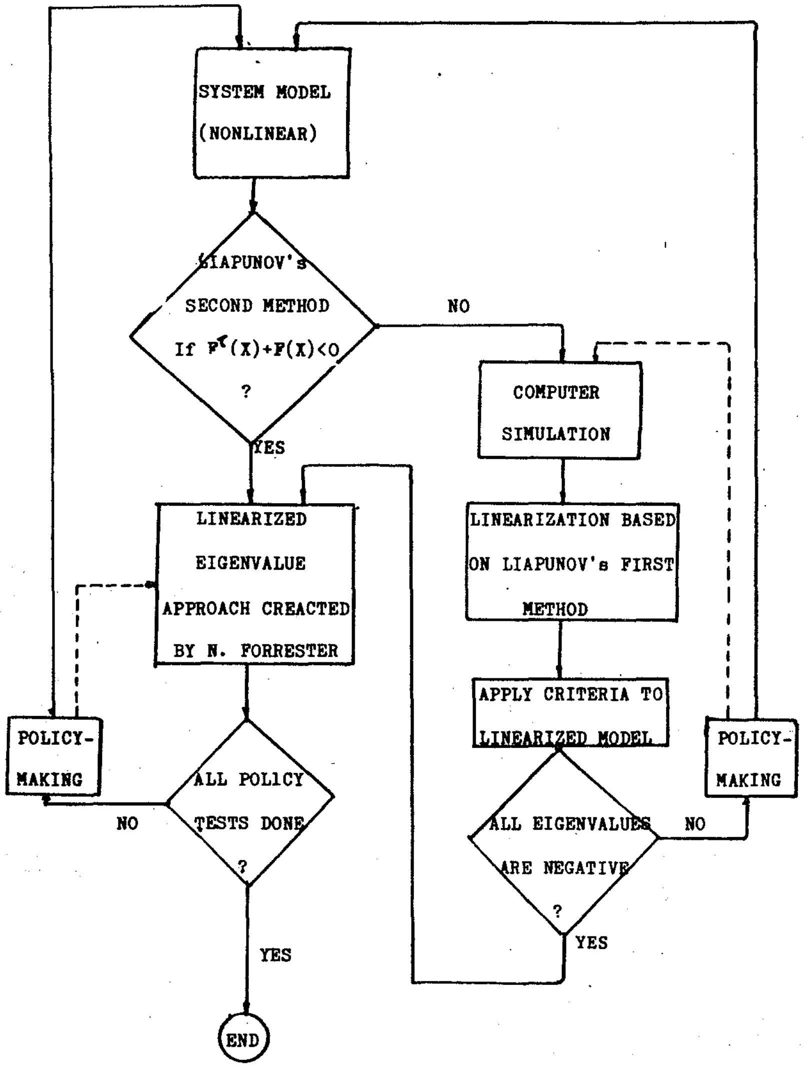


Figure 1. Flow chart of the method

99



## APPENDIX I Liapunov's first method

According to the Liapunov principle, Liapunov stability is concerned with the trajectories of a system when the initial state is near an equilibrium point. From a practical point of view, this is very important because external disturbances (such as noise, component errors, etc.) are always present in a real system.

We now offer precise definitions of the concept of Liapunov stability and instability. [Vidyasagaa, pp. 5, 132-138 and Ogata, pp. 438-442]

Consider the vector differential equation

$$\dot{X}(t)=f[t,X(t)], \quad t\geq 0 \quad (1)$$

Where $X(t) \in R^n$

We assume that the function has a unique solution over $[0,\infty)$ corresponding to each initial condition for $X(0)$, and that this solution depends continuously on $X(0)$, if f satisfies a global Lipschitz condition.

And an $X_o \in R^n$ is said to be an equilibrium point of the system at time $t_o$ if

$$f(t,X)\equiv 0. \quad \forall\, t\geq t_o$$

The basic concepts of Liapunov stability are stability, asymptotic stability, insabitity, automous and nonautonomous.

Liapunov's first method (also known as Liapunov's indrect method) convinces one of the feasibility of linearizing a nonlinear system.

Liapunov's first method solves this problem. The main value of this method is that, under certain conditions, it enables one to arrive at conclusions about a nonlinear system by studying the behavior of a linear system. [Vidyasagaar, pp.186-189, and Ogata, pp. 450-452]

**THEOREM1** Consider the nonautonomous system*

$$\dot{X}(t)=f[t,X(t)]$$

Suppose that

$$f(t,0)=0$$

and that $f(t,\cdot)$ is continuously differentiable. Define

$$A(t)=\left[\frac{\partial f(t,X)}{\partial X}\right]_{X=0}$$

$$f_1(t,X)=f(t,X)-A(t)X$$

and assume that

$$\lim_{\|X\|\to 0} \sup_{t\geq 0} \frac{\|f_1(t,X)\|}{\|X\|}=0 \quad (**)$$

A(.) is bounded

Under these conditions, if the equilibrium point 0 of the system

$$\dot{Z}(t)=A(t)Z(t)$$

---

* Bacause there was nothing to be gained by assuming autonomy, we considered the result for a nonautonomous system. [Vidyasagar, pp. 190]

is uniformly asymptotically stable over [0, ), then the equilibrium point 0 of the system

$\dot{X}(t)=f[t,X(t)]$

is also uniformly asymptotically stable over $[0,\infty)$.
(proof omitted)

THEOREM2 Consider the autonomous system

$\dot{X}(t)=f[X(t)]$

Provided that $f(0)=0$ and that $f$ is continuously differentiable, define

$$A=\left[\frac{\partial f}{\partial X}\right]_{x=o}$$

Under these conditions, the equilibrium point 0 of this system is (uniformly) asymptotically stable if all eigenvalues of A have negative real parts.

THEOREM3 Consider the system

$\dot{X}(t)=f[X(t)]$

Provided that $f(0)=0$ and that $f$ is continuously differentiable, Suppose in addition that

$$A=\left[\frac{\partial f(t,X)}{\partial X}\right]_{x=o}=A_o \quad \text{(a constant matrix)}, \quad \forall\, t\geq 0$$

and (**) holds. Under these conditions, the equilibrium point 0 of this system is unstable if at least one eigenvalue of $A_o$ has a positive real part.



## APPENDIX 2 Liapunov's second method and its extended method

Liapunov's second method (also called Liapunov's direct method) and its extended method, [Kalman, Vidyasagar, pp. 148, and Ogata pp. 470-485]

In a free linear dynamic system if the equilibrium state is locally asymptotically stable, then it is asymptotically stable in the large region. In a nonlinear free dynamic system, however, an equilibrium state can be locally asymptotically stable without being asymptotically stable in the large. Therefore, the situations of asymptotic stablity of equilibrium states of these two systems are quite different. Obviously, stability analysis of linearized models of nonlinear systems is completely inadequate to examine asymptotic stability of equilibrium states of nonlinear systems. Liapunov's second method and several extended methods based on Liapunov's second method are available to investigate nonlinear systems without linearization. The second method of Liapunov proves to be particularly powerful in the investigation of stability analysis of certain nonlinear systems where conventional criteria cannot give answers. The extended methods include Krasovskii's method for testing sufficient conditions for asymptotic stability of nonlinear systems, Schultz-Gibson's variable gradient method for generating Liapunov functions for nonlinear systems, Lur'e's method applicable to stability analysis of certain nonlinear control systems, Zubov's method for constructing domain of attraction and others. In this paper Liapunov's direct method itself and Krasovskii's method are presented for constituting the new approach. The others are not adopted because their processing procedures are not convenient enough to use.

Liapunov's second method is based on a generalization of the idea that if the system has an asymptotically stable equilibrium state, then the stored energy of the system displaced within the domain of attration decays with increasing time until it finally assumes its minimum value at the equilibrium state. This method consists of determination of a fictitious "energy" function called a Liapunov function (candidate). The idea of the Liapunov function is more general than that of energy and is more widely applicable. Liapunov functions are functions of $x_1, x_2, \ldots, x_n$ and t. We denote Liapunov funtions as $V(x_1, x_2, \ldots, x_n)$ or simply as $V(X,t)$.

Liapunov's main stability theorem and Krasovskii's method are presented as follows.

**LIAPUNOV's MAIN STABILITY THEOREM**

Consider the vector differential equation.

$\dot{X}(t)=f[t,X(t)], \quad t\geq 0$

Where $X(t) \in R^n$
the equilibrium point 0 at time $t_o$ of the function is stable if there exists a continuously differentiable, locally positive definite function V such that

$\dot{V}(t,X)\leq 0, \quad \forall\, t\geq t_o, \quad \forall\, X\in B_r$ for some ball $B_r$

(omitted proof, see Kalman and Vidyasagar, PP. 148)

**KRASOVSKII's METHOD**
Consider the vector differential equation

$\dot{X}(t)=f[t,X(t)], \quad t\geq 0$
Where $X(t) \in R^n$

A state $X_e \in R^n$ where

$f(t,X_e)=0$ for all t

is supposed to be an equilibrium state of the system.

101



The Jacobian matrix for the system is

$$F(X)=\frac{\partial f(X)}{\partial X}$$

$$=\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \quad \text{(Jacobian Matrix.)}$$

Define

$\hat{F}(X)=F^{\tau}(X)+F(X)$

where $F(X)$ is the Jacobian matrix and $F^{\tau}(X)$ is the conjugate transpose of $F(X)$. $\hat{F}(X)$ is clearly Hermiton.

Then try to fine a l.p.d.f. V and $\dot{V}(t,X)<0$

Let

$$V=\|\dot{X}\|=\dot{X}^{\tau}\dot{X}=f^{\tau}(x)f(X)$$

Then

$$\dot{V}(X)=\dot{f}^{\tau}(X)f(X)+f^{\tau}(X)\dot{f}(X)$$
$$=\left[\frac{\partial f(X)}{\partial X}\frac{dX}{dt}\right]^{\tau}f(X)+f^{\tau}(X)\left[\frac{\partial f(X)}{\partial X}\frac{dX}{dt}\right]$$

$$\dot{V}(X)=[F(X)f(X)]^{t}f(X)+f^{t}(X)[F(X)f(X)]$$
$$=f^{t}(X)F^{t}(X)f(X)+f^{t}(X)F(X)f(X)$$
$$=f^{t}(X)[F^{t}(X)f(X)+F(X)f(X)]$$
$$=f^{t}(X)[F^{t}(X)+F(X)]f(X)$$

If

$F^{t}(X)+F(X)<0$ (negative difinite)

and

$f(X)=0$ when $X=0$

$f(X)\neq 0$ when $X\neq 0$

Then $V(X)$ positive definite

$\dot{V}(X)$ negative definite.

Now we have proved that

$F(X)<0$

If

$F^{t}(X)+F(X)<0$ and when $X=0$

PROOF There is

$$0>X^{t}[F^{t}(X)+F(X)]X=X^{t}F^{t}(X)X+X^{t}F(X)X$$
$$=(X^{t}F(X)X)^{t}+X^{t}F(X)X$$
$$=2*X^{t}F(X)X$$

then $2*X^{t}F(X)X<0$

then $F(X)<0$.

then it can be proved further $f(X)=0$ when $X=0$

$f(X)\neq 0$ when $X\neq 0$.

Since $F^{t}(X)+F(X)$ is negative definite, it follows that $\dot{V}$ is negative definite. Hence $V(X)$ is a Liapunov function. Therefore, the origin is asymptotically stable. If $V(X)=f^{t}(X)f(X)$ tends to infinity as $\|X\|\to\infty$, then the equilibrium state is asymptotically stable in the large.