Using Binomial Decision Trees and Real Options Theory to Evaluate System Dynamics Models of Risky Projects

Burcu Tan
University of Texas, McCombs Business School
1 University STA B6000,
Austin, TX 78712-0201, US
512-471-1671
Burcu.Tan@phd.mccombs.utexas.edu

Edward Anderson
University of Texas, McCombs Business School
1 University STA B6000,
Austin, TX 78712-0201, US
512-471-6394
Edward.Anderson@mccombs.utexas.edu

James Dyer
University of Texas, McCombs Business School
1 University STA B6000,
Austin, TX 78712-0201, US
512-471-5278
Jim.Dyer@mccombs.utexas.edu

Geoffrey Parker
Tulane University, Entergy Tulane Energy Institute
A. B. Freeman School of Business
9 McAlister Dr.,
New Orleans, LA 70118, US
504-865-5472
gparker@tulane.edu

Many important risky projects are characterized by stochastic processes embedded in non-linear, feedback structures with delays. System dynamics models may be used to estimate the cash flow resulting from these projects. If these projects include managerial flexibility (real options), a correct financial evaluation of these cash flow requires the use of real options methodology. We adapt prior work on real options valuation in the decision analysis literature to develop a methodology that avoids the need to estimate a risk-adjusted discount rate for the project with options. We illustrate this approach with a model drawn from the wind power industry, which is characterized by numerous uncertainties and high managerial flexibility. We conclude with a discussion comparing this methodology to the previous methods and describe under what conditions each one might be a more appropriate choice.

Keywords: System dynamics, real options, decision analysis

1. Introduction

It has been well recognized that the traditional discounted cash flow (DCF) methods of valuation fail to account for the value of managerial flexibility inherent in many types of projects. For instance, a simple net present value analysis does not capture the values of the options to delay, expand or abandon the project. The real options valuation approach is the state-of-the-art method to value capital investment projects that involve managerial flexibilities. It applies financial options theory to value options derived from managerial flexibility, which are called "real options" to reflect their association with real assets rather than with financial assets (Myers, 1987; Trigeorgis and Mason, 1987; Trigeorgis, 1988; Dixit and Pindyck 1994).

Traditional real options solution approaches typically rely on models that are highly stylized closed-form formulations based on the assumption that the value of the real asset over time can be modeled as a stochastic process (e.g. McDonald and Siegel, 1986; Paddock et al. 1988; Capozza and Li, 1994) or they are based on the use of a discrete dynamic programming approximation of a stochastic process (e.g. Trigeorgis, 1991; Trigeorgis, 1993; Kogut and Kulatilaka, 1994). Neither of these approaches can generally handle complex projects that include rework, learning curves or other stochastic processes embedded within nonlinear feedback structures characterized by delays (Forrester, 1961; Forrester, 1975).

In this paper, we adapt prior work on real options valuation in the decision analysis literature to support the use of Systems Dynamics (SD) methodology to evaluate real options. This work builds on a burgeoning stream of research extending the capabilities of SD methodology in modeling complex systems. Ford and Sobek (2005) built a product development project model that uses real options concepts to manage product design risk. Barghav and Ford (2006) examined the relationship between project management quality and the value of flexible strategies. Johnson et al. (2006) used an SD model to value flexibility in a large petrochemical project. These papers provide examples of how real options logic can be incorporated into SD models of projects; yet they analyze projects with a single option and they do not focus on formalizing an algorithm to estimate the market value of projects with complex option structures using SD models.

Closest to our paper, Tan et al. (2009) proposed a formal, decision-tree based algorithm to evaluate SD models of projects that account for managerial flexibility. That paper develops a methodology to transform the data generated by Monte Carlo simulations of the SD model into a decision tree representation. The decision tree is then evaluated using a risk-adjusted discount rate for the project. This is commonly called the "naïve approach" to modeling real options in the finance literature (Copeland and Antikarov, 2001) as it does not capture the changing risk characteristics of the expected future cash flows by adjusting the discount rate. Adding the options changes the expected future cash flows and thereby alters the risk characteristics of the project so that the risk-adjusted discount rate for the project without options may not be appropriate after the real options have been included in the model.

In this paper we propose an approach (which we shall refer to as *the diffusion approximation approach*) which modifies the traditional decision tree approach by using concepts from Copeland and Antikarov (2001) and Brandao et al. (2005a) to overcome this flaw. The method avoids the problem of selecting an appropriate risk-adjusted discount rate for the analysis by using a "risk neutral"[1] valuation and provides a more accurate estimate of the market value of the project. Similar to the approach in Tan et al. (2009), *the diffusion approximation approach* is a decision-tree based method that relies on system dynamics simulations; hence, it takes advantage of the complementary strengths of system dynamics and decision analysis in representing stochastic models and decision processes (Tan et al. 2009). However, unlike Tan et al. 2009, this paper espouses an approach that more accurately reflects the market value of the project.

The remainder of the paper is as follows. Section 2 provides an overview of the traditional decision tree approach and discusses its limitations. Section 3 introduces the diffusion approximation algorithm. Section 4 briefly presents the motivating example that was analyzed in Tan et al. (2009) and illustrates the steps of valuing this project using the diffusion approximation algorithm. A comparison of the two valuation approaches as well as a discussion of some limitations is provided in Section 5, followed by a short conclusion in Section 6.

2. Overview of the Traditional Decision Tree Approach

The traditional decision tree approach developed by Tan et al (2009) for linking to an SD model is based on transforming data generated by Monte Carlo simulations of the SD model of the project into a decision tree. The key virtue of the method is taking advantage of the system dynamics methodology in modeling the underlying uncertainty. Real options valuations traditionally model uncertainty by assuming an analytically tractable stochastic process. Yet, the simple stochastic processes that are often used to value financial options may not capture the complex real-world behavior of uncertainties associated with real options. System dynamics, on the other hand, is a methodology developed to analyze and manage complex feedback systems. Consequently, it is powerful in handling nonlinearity and path-dependence. Modeling the structure that produces the complex dynamics associated with many risky projects using SD may improve the accuracy of the ultimate valuation. Yet, to incorporate the real options in the valuation of a project, one needs to model and optimize a sequential decision process, which is not an inherent capability of the SD simulation environment. To model the decision process, Tan et al. (2009) resort to decision tree analysis, which provides an intuitive approach in modeling managerial flexibilities and discrete approximations of the project uncertainty. Hence, the traditional decision tree approach aims to benefit from the

[1] Risk Neutral Measure is an important concept in the context of mathematical finance and risk neutral valuation is an important general principle in option pricing (Hull 2006). A risk-neutral measure is a probability measure in which today's arbitrage-free price of a derivative security is equal to the discounted expected value (under the measure) of the future payoff of the derivative. The measure is in general different than the "physical" measure of probability and is employed to determine the worth of derivative securities. Please refer to Dixit and Pyndick (1994) for further details.

complementary strengths of system dynamics and decision tree analysis. The steps of the algorithm are as follows:

I. *Identify the managerial flexibilities and decision sequences:* The first step is identifying the "real options" in the project. At each period, the manager needs to decide whether or not to exercise the available options; hence there is a sequence of decisions to be made throughout the horizon of the project. Each possible sequence of these decisions is called a *decision sequence*.

II. *Build the deterministic SD model that captures the project dynamics*

III. *Model the uncertainty by specifying the random variables and their distributions:* In general, there are multiple sources of uncertainty to be specified in a project.

IV. *Run Monte Carlo simulations of the SD model for each decision sequence*: A Monte Carlo run for a specific decision sequence gives a cash flow distribution for each period. Hence, Step 4 results in $M \times T$ cash flow distributions, where T is the time horizon of the project and M is the number of possible decision sequences.

V. *Obtain the discrete distribution approximations for the first period cash flow distribution for each decision sequence:* The *bracket median* approximation technique (Clemen 1997) is used to obtain a *k*-point discrete distribution approximation of the first period cash flow distributions.

VI. *Obtain the conditional discrete approximations for the remaining periods for each decision sequence:* The cash flow distributions for period t are conditional on the cash flow distributions of period $t-1$, as well as on the *decision sequence* that is chosen. These conditional continuous distributions are discretized applying the bracket median method recursively.

VII. *Solve the decision tree by backwards induction using the risk adjusted discount rate:* In practice, this step is handled easily by using decision analysis software such as DPL™.

The major limitation of the traditional decision tree approach just described is that the risk-adjusted discount rate for the project without options is used as the discount factor for the entire decision tree (Teisberg, 1995). Essentially, the risk-adjusted discount rate that a financial analyst should choose to value the project *with options* may be different from the one he should choose to value the project *without options* because of the alternatives' different risk levels.

The classical approach to incorporating market information is based upon identifying a replicating portfolio for the project under consideration and using the volatility information of this replicating portfolio to obtain an appropriate discount rate for the project. However, the replicating portfolio assumption is difficult to use in practice when evaluating individual corporate investments because it is hard to find a single replicating asset or even a portfolio of publicly traded assets with returns that are perfectly correlated with those from the project (Borison 2005), and the appropriate replicating portfolio may change if the risk of the project is changed by the addition of options.

This criticism can be overcome by using a decision tree based real options valuation method developed by Copeland and Antikarov (2002) and modified by Brandao, Dyer and Hahn (2005a). We will make some modifications to this method and call it the *diffusion approximation approach*. Copeland and Antikarov (2002) suggest the use of a Monte Carlo simulation of a pro forma spreadsheet model to obtain an estimate of the risk associated with the project without options, which is then used to construct the required decision tree. Instead we substitute simulation runs from an SD model and obtain a reliable, and theoretically correct (from the point of view of the finance literature) valuation of the investment projects.

3. The Diffusion Approximation Approach

In its simplest form, the diffusion approximation method assumes that the changes in the project's value over time approximately follows a geometric Brownian motion (GBM) diffusion process, which is a standard assumption in the finance literature (e.g. Copeland and Antikarov, 2001). The method relies on the *market asset disclaimer* (MAD) assumption, which assumes that the value of the project without options is the best unbiased estimator of the market value of the project. Hence, the expected NPV of the project without options is taken as the market price of the project as if it were traded (Copeland and Antikarov, 2001).[2] Then, the value of the project without options is assumed to change over time according to a GBM process, which is the same process used to model the changes in the price of a stock when the Black and Scholes (1973) option pricing model is used. The assumptions behind the GBM model may not hold for all projects, in which case other models of stochastic processes (e.g. mean reverting) may be used (Brandao et al 2005a, Hahn and Dyer 2008).

We use the GBM model because the use of a binomial lattice approximation to a GBM process is well established in the literature (Hull, 2003) and it is straightforward to build a corresponding decision tree to value the project once the parameters of the process are provided. In the decision tree representation, the project values are discounted with the risk-free rate since the risk-neutral probabilities are used, which eliminates the need to estimate different risk-adjusted discount rates as options are added to the project.

The first three steps of the diffusion approximation approach are exactly the same as the traditional decision tree approach: 1) Identify the decision variables and decision sequences 2) Build the deterministic SD model 3) Specify the distributions for the uncertain variables. The remaining steps that distinguish the diffusion approximation algorithm are as follows.

---

[2] The MAD assumption is used in order to create a complete market for an asset that is not traded in the market. It is a strong modeling assumption made to justify the use of risk-neutral valuation. Nevertheless, it eliminates the reliance on the existence of a replicating portfolio. Instead, it uses the project itself as the twin security and is claimed to "make assumptions no stronger than those used to estimate the project NPV in the first place" (Copeland and Antikarov, 2001, p. 67). For further discussion of the MAD assumption, see also Borison 2005and Smith 2005.

The fourth step requires calculating the expected NPV of the project without options at time *t=0* using a DCF analysis. To do that, we run Monte Carlo simulations of the project without options, obtain the NPV for each iteration using the WACC[3] and use the mean value of these iterations as an estimate of the expected NPV.

Step 5 of the diffusion approximation algorithm is to obtain the cash flow payout rate δ_t in each period, which is defined as the ratio of the expected cash flow in period *t* to the remaining value of the project in period *t*. Let $\tilde{V}_t$ and $\tilde{C}_t$ be random variables representing the uncertain project values and cash flows in period t, and $\overline{V}_t$ and $\overline{C}_t$ be their corresponding means. The project value at time *t* is simply the present value of the remaining project cash flows. Hence, the cash flow payout rate in period t is defined as $\delta_t = \overline{C}_t / \overline{V}_t$. The cash flow payout rate is used to calculate the cash flows that are paid out at the end of each time period as a function of the project value. This implies that the cash flows will vary over time reflecting the uncertainty in the project value, but that they will remain a constant fraction of the residual value of the project in each time period (Copeland and Antikarov 2001).

Step 6 is to estimate the volatility (σ) of the project returns. Smith (2005) suggests an approach to estimating this volatility that can be used in the SD simulation environment. First, we model the GBM approximation of the project cash flows using the present value computed in Step 4 and the cash flow payout rates computed in Step 5. Then, we search for a volatility that best mimics the uncertainty in the original SD model; i.e., that minimizes the difference between the cash flow distributions generated by the original SD simulation model and the cash flow distributions obtained by the GBM approximation.

Step 7 is calculating the parameters of the binomial approximation of this GBM diffusion process. A binomial lattice is a probability tree with binary chance branches that go up (u) or down (d) with the unique feature that the outcome resulting from moving up and then down is the same as the outcome from moving down and then up. In particular, the binomial lattice model assumes that with probability p the value of the project V will go up to Vu, and with probability $1-p$ it will go down to Vd at the end of one period. The parameter u is greater than 1 (reflecting a proportional increase), whereas $d = 1/u$ (reflecting a proportional decrease).

[3] See Brandao et al (2005) for a discussion on the choice of the discount rate for this step.

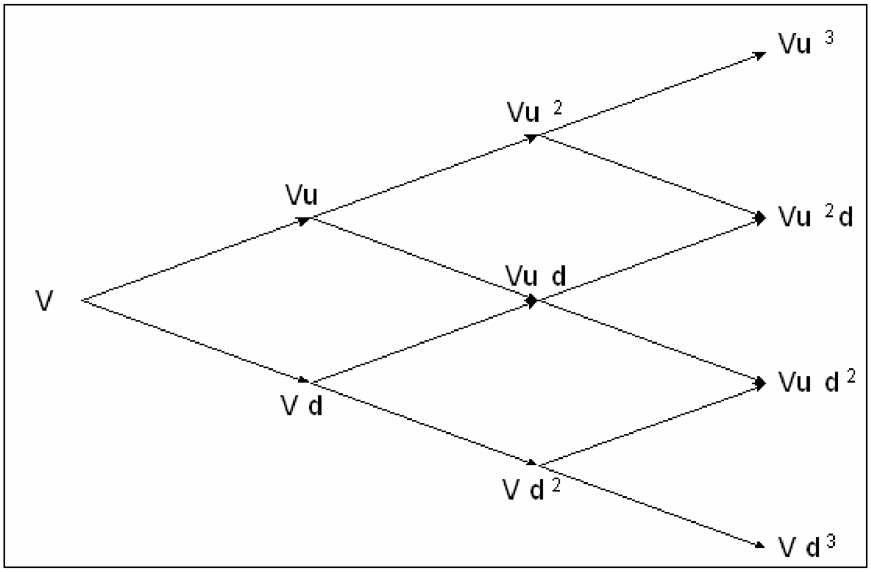

Figure 1: A Binomial Lattice

In order to calculate these three parameters *u*, *d* and *p*, it is sufficient to know the volatility σ of the GBM process, which is estimated in Step 6, and the risk-free discount rate *r* since $u = e^{\sigma\sqrt{\Delta t}}$, $d = 1/u$, and $p = \frac{1 + r\Delta t - d}{u - d}$ where Δt is the time period used in the binomial lattice. The probabilities *p* and $1 - p$ are the probabilities that a risk-neutral investor would assign to the two outcomes; therefore they are often called "risk-neutral" probabilities. Finally, we need the initial value *V* of the project to build the binomial lattice, which is approximated by the expected NPV of the project without options determined in Step 4.

The lattice may also be "unfolded" and represented as an equivalent binomial tree, which increases the number of endpoints in the model decreasing computational efficiency, but allows these problems to be solved using "off the shelf" decision tree software with an intuitively appealing visual representation. Brandao et al (2005a, b) and Smith (2005) discuss the pros and cons of this transformation of the problem.

Once the project without options is modeled with a binomial decision tree approximation (Step 8) of the GBM process, options can be added to the decision tree by using decision nodes. For example, an abandon option can be modeled by adding a decision node without any subsequent chance nodes (i.e. no further cash flows), whereas simple expansion and contraction options can be modeled as percentage changes in the cash flows (for details, see Brandao et al. 2005a).

These eight steps are summarized in Table 1.

Steps of the Algorithm	
Step 1	*Identify the managerial flexibilities and decision sequences*
Step 2	*Build the deterministic SD model that captures the project dynamics*
Step 3	*Model the uncertainty by specifying the random variables and their distributions*
Step 4	*Run Monte Carlo simulations of the SD model for the project without options and calculate the expected NPV*
Step 5	*Obtain the cash flow payout rates for each period*
Step 6	*Estimate the volatility of the project returns*
Step 7	*Calculate the parameters of the binomial approximation to the GBM process (i.e. u, d and p)*
Step 8	*Build the binomial lattice*
Step 9	*Add the options and solve the decision tree by using the risk free rate*

Table 1: Summary of the Algorithm

4. Illustration of the Solution Approach with a Motivating Example

We will use the same hypothetical wind power project described in Tan et al. (2009) to illustrate the diffusion approximation algorithm. For convenience, we directly quote their description of the project: A hypothetical firm needs to evaluate an investment opportunity to build a 40-MW wind farm with the option to add 50 MW within the first 4 years. The firm can also delay the beginning of the project by up to 2 years. The expansion option may be considered after the 40-MW wind farm comes online, which takes a year. The option can be distributed over the remaining 2 years, but due to economies of scale, the firm does not want to have less than 25 MW built at a time. So, the firm can either *expand high* (build all 50 MW at once) or *expand low* (build 25 MW one year) with the option to build another 25 MW in the successive year or *suspend* investment, i.e. continue operating the wind farm at its current capacity. Given these managerial flexibilities, the list of decision sequences can be constructed as follows:

Strategy ID	Decision in 2008	Decision in 2009	Decision in 2010	Decision in 2011
1	I	N/A	S	S
2	I	N/A	S	L
3	I	N/A	S	H
4	I	N/A	H	S
5	I	N/A	L	S
6	I	N/A	L	L
7	D	I	N/A	L
8	D	I	N/A	H
9	D	I	N/A	S
10	D	D	I	N/A

Table 2: Strategies for the Example Problem

The next step is building the SD model that captures the project dynamics. Figure 2 is a sector diagram of the SD model for evaluating the wind-power project. The most important sectors of the model are the natural gas price sector and the supplier sector in which the learning curve is modeled. A more detailed description of the model is provided in Tan et al. (2009).

Step 3 is adding the uncertainty by specifying the random variables and their distributions. Three major uncertainties are captured by the model: natural gas price, the learning curve and the expiration date of the production tax credit[4]. Each uncertainty has several components, making the project harder to evaluate with most traditional methods.

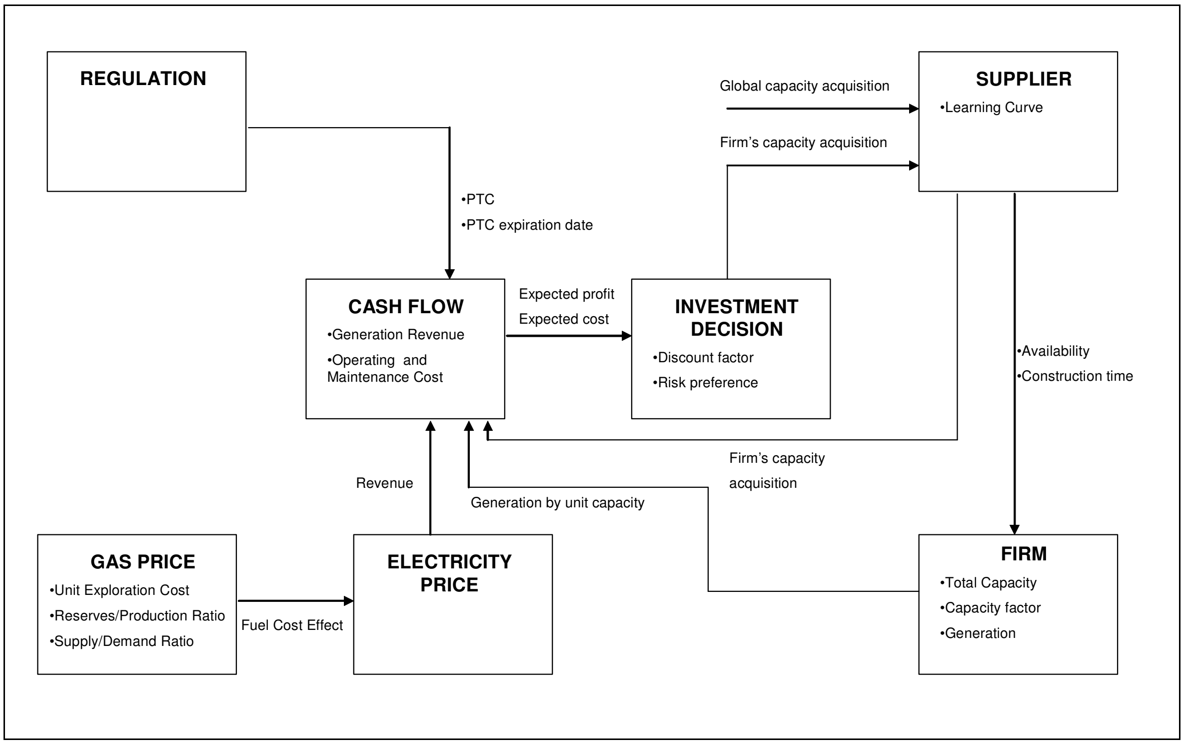

Figure 2: Sector Diagram of the SD Model

At Step 4, the expected NPV of the project without options at time $t=0$ is calculated using a DCF analysis. To do that, we use the Monte Carlo simulation of the strategy "invest-suspend-suspend", which reflects the project without options. Then, we calculate the NPV of each simulation iteration using the WACC[5] and estimate the mean value of these iterations to obtain an estimate of the expected NPV. In the example project, the expected NPV of the project without options is found to be \$7.79 million. Then, as the next step, cash flow payout rates are to be estimated for each time period. To do that, the present value of the remaining cash flows is calculated for each sample path generated by the Monte Carlo simulation, which yields the random project values $\tilde{V}_t$ at

[4] Renewable energy producers currently receive a 1.9 cent benefit for each kilowatt-hour of generation, known as the production tax credit (PTC). The uncertain expiration date of the PTC has been a major consideration in wind capacity investment decisions

[5] See Brandao et al (2005) for a discussion on the choice of the discount rate for this step.

period t. Then, the average cash flow at period t, $\bar{C}_t$, is divided by the of average project value $\bar{V}_t$ to obtain the following cash payout rates:

Period	δ_t	Period	δ_t
1	0.027	**11**	0.183
2	0.107	**12**	0.162
3	0.128	**13**	0.180
4	0.132	**14**	0.195
5	0.139	**15**	0.218
6	0.136	**16**	0.246
7	0.148	**17**	0.302
8	0.152	**18**	0.369
9	0.157	**19**	0.530
10	0.169	**20**	1.000

Table 3: Cash Flow Payout Rates

The PV and the cash flow payout rates are used to estimate the volatility of the GBM approximation. First, using this PV and the cash flow payout rates, and assigning an arbitrary value for volatility, a GBM approximation is modeled (Hull, 2003). Then, we search for the volatility that best mimics the uncertainty in the original SD model. One way of doing this is comparing the 10^{th}, 50^{th} and 90^{th} percentiles of the cash flow distributions obtained through the GBM approximation *for a given volatility* to the corresponding values given by the original SD model. Note that this results in $3\times T$ pairs to be compared, where T is the number of periods. We take the sum of squared errors between these pairs to obtain a measure of fit for the GBM approximation under the given volatility. We repeat this procedure for a predetermined set of candidate values for volatility. Then, we choose the value that minimizes the sum of squared errors. We find that when $\sigma = 0.06$, the GBM approximation mimics the cash flow distributions given by the original SD model quite closely (Figure 3). In this case, we limited the search to a predetermined set of candidate values for volatility; however, when more precision is required one can solve a stochastic optimization problem to determine the volatility that minimizes the difference between the original and the approximated cash flows.

As the next step (Step 7), we calculated the parameters of the binomial approximation of this GBM diffusion process. We set $\Delta t = 0.5$ years to increase the accuracy of the binomial approximation, and we obtained $u = 1.04$, $d = 0.96$ and $p = 0.78$. We also need the initial value V of the project to build the binomial lattice, which is approximated by the expected NPV of the project without options.

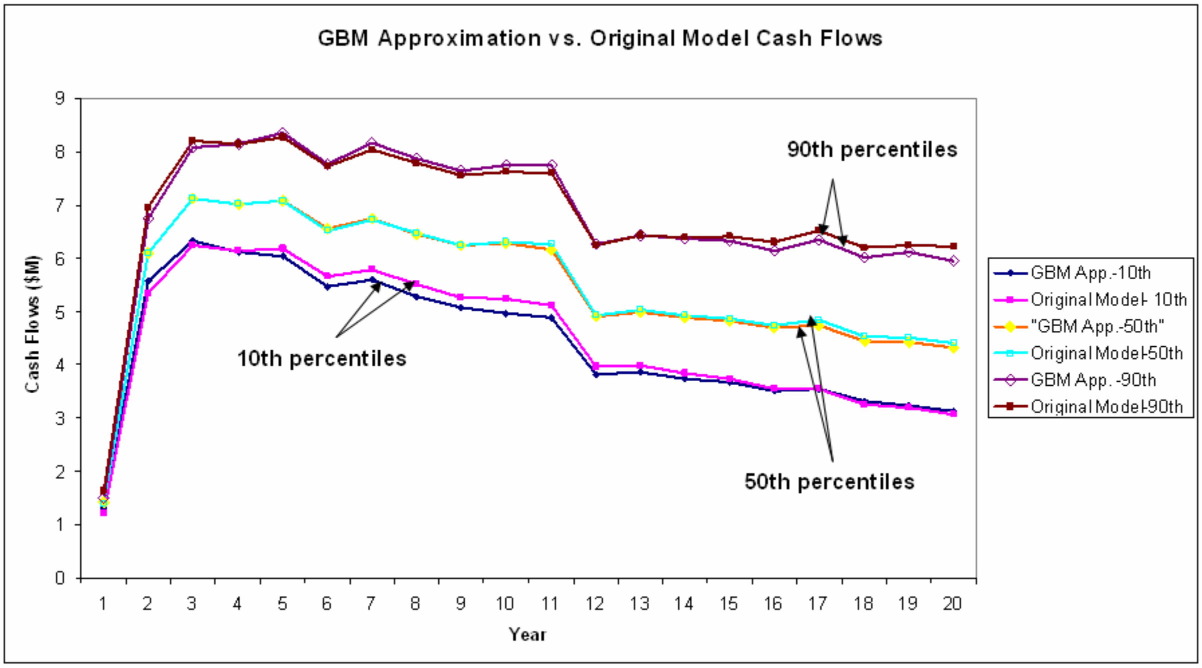

Figure 3: GBM Approximation vs. the Original Model Cash Flows

The final step is adding the options to the decision tree by using decision nodes (for details, see Brandao et al. 2005a). For the example project, the cash flows change proportionally with the changes in the capacity. For example, the option to expand by 25 MW ("Expand Low") increases the capacity by 62.5%. The revenues from electricity generation are proportional to the capacity; hence, the increase in revenue when the option is exercised can be modeled by simply increasing the cash flows by 62.5%. Yet, this scheme does not directly allow for incorporating the uncertainty in the cost of capacity (the learning curve uncertainty) because the cost of capacity is a one time payment at the exercise time of the option, which does not affect the cash flow stream afterwards. To handle this issue, we modeled the learning curve uncertainty as a *private risk*.

In many projects, there are project-specific risks that cannot be hedged by trading securities, such as technological risks. In view of that, real options studies make the distinction between public (market-priced) risks and private (project-specific) risks (Smith and Nau. 1995). When the investment under concern is dominated by private risks, dynamic programming based approaches (such as decision tree analysis) should be preferred rather than the traditional option pricing techniques that were developed mainly for market-priced risks (Dixit and Pindyck, 1994). Many projects have both kinds of risks, though; in which case the recommended strategy is to separate the public and private risks and use the appropriate risk adjustment for each one (Borison, 2005, Brandao et al. 2005a, Smith and Nau, 1995).

Fortunately, treating different types of risks separately is straightforward once a decision tree is created. Public and private risks may be represented with separate chance nodes. Risk-neutral probabilities are used for the former and subjective probabilities are used for the latter. For the example project, the cost of capacity is discretized so that at any period *t*, it is *high, nominal* or *low*. The values for each of these branches were obtained by discretizing the Monte Carlo simulation data for the cost of capacity. This is done the same way as the cash flow distributions are discretized: A three-point bracket median method is used while preserving the path-dependence of the learning curve uncertainty by carefully computing the conditional probabilities of the branches. For example, chance nodes HighC, NominalC and LowC in Figure 4 discretize the learning curve uncertainty for the first period. Note that the uncertainty in the cost of capacity does not affect the volatility of the subsequent cash flows.

The decision tree was built and solved using DPL™ (Figure 4). The expected PV of the project is estimated to be $57.48 million. The optimal policy suggests that the investment should be undertaken and it is optimal to expand high afterwards. Note that the traditional decision tree approach in Tan et al. (2009) results in an expected PV of $55.386 million and suggests investing immediately and expanding afterwards, the amount depending on the cash flow realization (Tan et al. 2009).

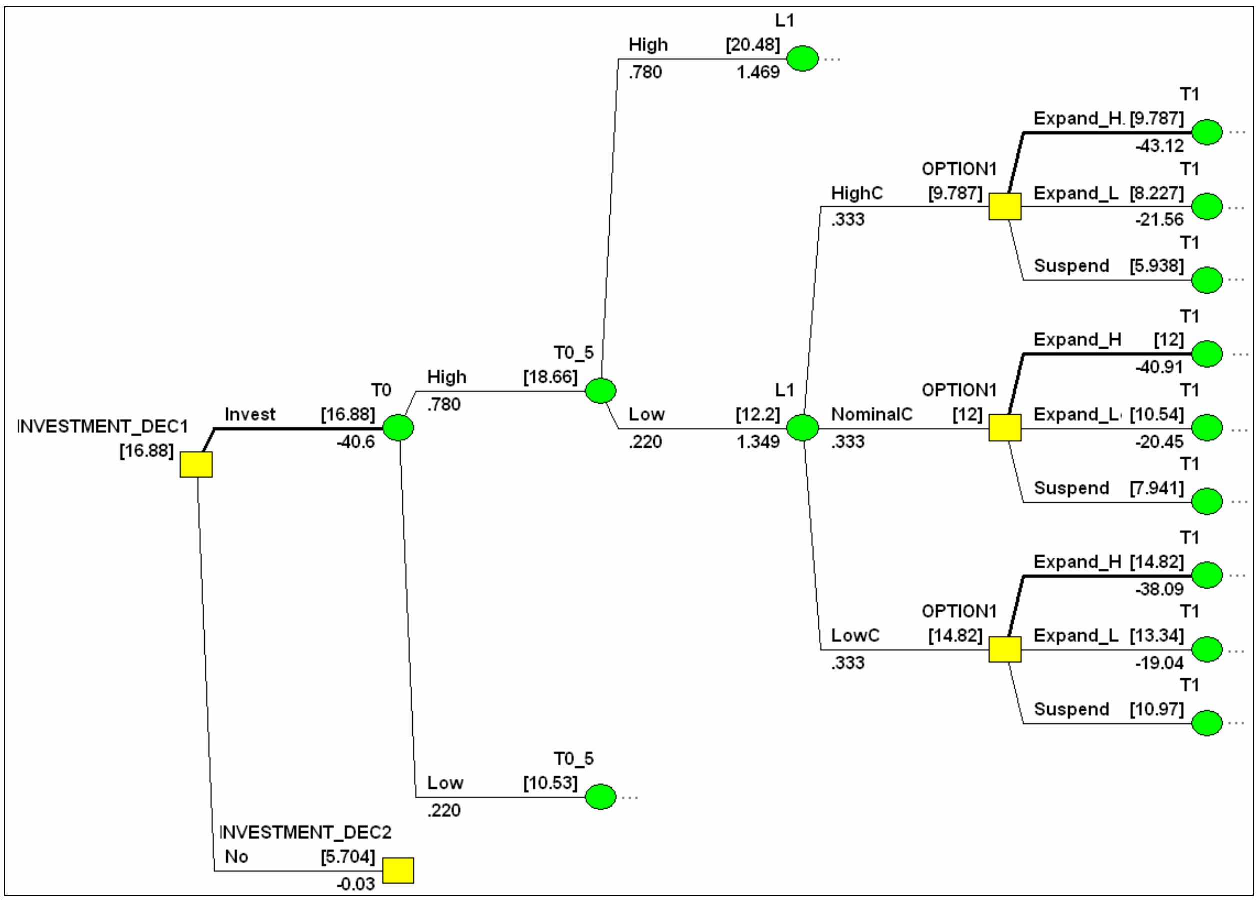

Figure 4: The Optimal Policy Obtained Using the BDH method

5. Benefits and Limitations

For the example project, the traditional decision tree approach and the diffusion approximation approach yielded similar results. However, this will not always be the case. The diffusion approximation algorithm overcomes the major flaw of the traditional decision tree approach (Tan et al. 2009), which is the use of the same risk-adjusted discount rate for the project with and without options regardless of the changing risk character. In general, for projects with long lives or for projects whose risk-profiles change significantly, the errors caused by using the wrong discount rate are magnified, and the differences between the two approaches become larger (Smith and McCardle 1998, Teisberg 1995).

Nonetheless, diffusion approximation algorithm has some limitations of its own. The algorithm makes two strong assumptions. One is that the cash flow of the project without options can be represented by a GBM (or other similarly tractable) stochastic process. This implies that the algorithm will only yield a reasonably accurate valuation if each period's cash flow from the project without options has an approximately lognormal distribution. Fortunately, because of the numerous influences upon the individual cash flows that one can represent in a system dynamics model, this is often the case.

A second restriction is that the diffusion approximation approach assumes the cash flows of the project with an option are proportionate to the cash flows of the project without the options. For example, if the firm exercises an expansion option to increase its capacity by x %, the revenues should change by a linear function of x. If this is not the case, then the diffusion approximation approach may not be an appropriate modeling approach. In such cases, the traditional decision tree approach might be a better alternative since it provides a greater fidelity to the details of the modeled project and is not subject to the assumptions associated with the use of the GBM diffusion approximation. Still, for a reliable valuation the analyst needs to make sure that the valuation results are not highly sensitive to the choice of discount rate.

6. Summary and Conclusion

Smith (1999) makes the observation that when evaluating risky projects, there has existed a fundamental trade-off between what he terms "detail complexity" and "dynamic complexity." He suggests that financial theory has tended to sacrifice detail complexity, the fidelity of a model at a detailed level, to better capture market information to appropriately discount risky cash flows. On the other hand, he suggests that decision analysis has often focused on detail complexity at the expense of keeping some model dynamics unrealistically simple, for example by using a single risk-adjusted discount rate to value future cash flows even if the risks of these cash flows change when different project options are selected. In this paper, we have attempted to show that by using a system dynamics model as an input to evaluating managerial flexibility it is possible to improve this trade-off between dynamic and detail complexity.

We proposed a method that relies on an SD model of the project to model the project uncertainty and a binomial tree approximation of the uncertainty to employ risk neutral valuation. The former brings high fidelity to model details due to the unique capabilities of SD in modeling complex feedback systems, whereas the latter avoids the need to estimate a risk-adjusted discount rate for the project with options ensuring better fidelity to dynamic complexity compared to traditional decision tree methods. Hence, our method potentially improves the trade-off between dynamic and detail complexity especially for evaluating projects whose viability is determined by the interaction of stochastic processes within a complex nonlinear feedback structure.

References

Black, F., Scholes, M. 1973. The Pricing of Options and Corporate Liabilities. *Journal of Political Economy,* 81 (3): 637-654.

Brandao, L. E., Dyer, J. S., Hahn, W. J. 2005a. Using Binomial Decision Trees to Solve Real-Option Valuation Problems. Decision Analysis, 2 (2): 69-88.

Brandao, L. E., Dyer, J. S., Hahn, W. J. 2005b. Response to: Alternative Approaches for Solving Real Options Problems: A Comment on Brandão, Dyer and Hahn (2005), 2 (2): 103-109.

Brandao, L. E., Dyer, J. S. 2005. Decision analysis and real options: A discrete time approach to real option valuation. *Annals of Operations Research*, 135(1): 21-39.

Brennan, M. J., Schwartz, E.S. 1985. Evaluating Natural Resource Investments. *The Journal of Business,* 58 (2): 135-157.

Borison, A. 2005. Real Options Analysis: Where Are the Emperor's Clothes?. *Journal of Applied Corporate Finance* 17 (2): 17-32.

Clemen, R.T. 1997. *Making Hard Decisions: An Introduction to Decision Analysis*. Duxbury Press, Belmont, CA.

Copeland, T., Antikarov, V. 2001. *Real Options*. Texere LLC, New York. .

Dixit, A.K., Pindyck, R.S. (1994). *Investment under Uncertainty.* Princeton University Press, Princeton, NJ.

Feinstein, S. P., Lander, D. M. 2002. A Better Understanding of Why NPV Undervalues Managerial Flexibility. *The Engineering Economist,* 47 (4): 418-435.

Ford , D., Sobek, S. 2005. Adapting Real Options to New Product Development by Modeling the Second Toyota Paradox. *IEEE Transactions on Engineering Management*, 52 (2): 175-185.

Ford, D.N., Bhargav S. 2006. Project management quality and the value of flexible strategies, *Engineering, Construction and Architectural Management,* 13(3): 275–289.

Forrester, J. W. 1961. *Industrial Dynamics.* M.I.T Press, Cambridge.

Hahn, W. J., Dyer, J. S. 2008. Discrete time modeling of mean-reverting stochastic processes for real option valuation. *European Journal of Operational Research,* 184 (2008): 534-548.

Hull, J. 2006. *Options, Futures and Other Derivatives*, Prentice Hall, New Jersey.

Johnson, S. T., Taylor, T., Ford, D. 2006. Using System Dynamics to Extend Real Options Use: Insights from the Oil and Gas Industry. *Proceedings of the System Dynamics Conference,* Nijmegen, Netherlands, 2006.

Longstaff, F. A., Schwartz, E. S. 2001. Valuing American Options by Simulation: A Simple Least-Squares Approach. *The Review of Financial Studies* 14 (1): 113-147.

McDonald, R., Siegel, D. 1986. The Value of Waiting to Invest. *Quarterly Journal of Economics,* 101: 707-727.

Nau, R., McCardle, K. 1991. Arbitrage, rationality and equilibrium. *Theory and Decision* 31 (2-3):199–240.

Paddock, J.L., Siegel, D. R., Smith, J. L. 1988. Option Valuation of Claims on Real Assets: The Case of Offshore Petroleum Leases. *Quarterly Journal of Economics*, 103: 479-508

Roberts, E. B. 1974. "A Simple Model of R&D Project Dynamics." In E. B. Roberts, (ed.), *Managerial Applications of System Dynamics*, 293-314. Cambridge, MA: Productivity Press.

Schwartz, E., Smith, J. 2000. "Short-Term Variations and Long Term Dynamics in Commodity Prices". *Management Science* 46 (7), 893-911.

Smith, J. 1999. Much Ado about Options? *Decision Analysis Newsletter* 18(2): 4-8.

Smith, J. 2005. Alternative Approaches for Solving Real-Options Problems. *Decision Analysis* 2 (2): 89-102.

Smith, J., McCardle, K. 1998. Valuing Oil Properties: Integrating Option Pricing and Decision Analysis Approaches, *Operations Research* 46 (2), 198-217.

Smith, J., McCardle, K. 1999. Options in the Real World: Lessons Learned in Evaluating Oil and Gas Investments. *Operations Research,* 47, 1-15.

Smith, J., Nau R. 1995. Valuing Risky Projects: Option Pricing Theory and Decision Analysis. *Management Science* 14(5): 795-816.

Sterman, J. D. 2000. *Business Dynamics: Systems Thinking and Modeling for a Complex World.* McGraw-Hill/Irwin: New York.

Tan, B., Anderson, E. G., Dyer, J., Parker, G. 2009. Evaluating System Dynamics Models of Risky Projects Using Decision Trees: Alternative Energy Projects as an Illustrative Example. Under Revision in *System Dynamics Review*.

Teisberg, E. O. 1995. Methods for Evaluating Capital Investment Decisions under Uncertainty. Real Options, in *Capital Investment: New Contributions* (ed. Lenos Trigeorgis). Praeger Publishing: Westport, CT.

Triantis, A. 2005. Realizing the Potential of Real Options: Does theory meet practice?. *Journal of Corporate Finance*. 17(2):8-16.